T-Norm of Yager Class of Subsethood Defuzzification: Improving Enrolment Forecast in Fuzzy Time Series

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ABSTRACT

Fuzzy time series has been used to model observations that contain multiple values. This paper proposes the t-norm of Yager class of subsethood defuzzification to forecast university enrolments based on fuzzy time series and the data of historical enrolments which are adopted from Song and Chissom (1994). The proposed method applied seven and ten interval with equal length and the max-product and max-min as the composition operator in the fuzzy relations $F(t) = F(t-1) \circ R(t,t-1)$. *The result shows that the t-norm of Yager class of subsethood defuzzification models with (10, max-product) is the best forecasting method in terms of accuracy. The proposed method has also improved the forecasting results by previous researchers.*

Keywords: *forecasting enrolments, t-norm of Yager Class, subsethood defuzzification, max-min composition, max-product composition*

Introduction

The fuzzy set theory was originally developed to handle problems involving human linguistic terms, for example, *hot*, *warm*, *cool* and *cold* for temperature and *short, medium* and *tall* for height. In economic forecasting, the classical time series method cannot deal with forecasting problems in which the values of time series are linguistic terms represented by fuzzy sets. In view to this, Song and Chissom (1993) proposed a special dynamic process called fuzzy time series to overcome the drawback of the classical time series methods. In this application of fuzzy time series, Song and Chissom (1993; 1994) used two models to forecast students' enrolments in the University of Alabama by fuzzifying the historical data. Since then, several other studies have been carried out. However, there are still many critical issues pertaining to Song and Chissom's methods. Undoubtedly, the determination of defuzzification principles is one of these issues.

In the early days, defuzzification problem never had the chance to be formally defined or analysed (Oliveira 1995). Nowadays, many different studies on it have been developed. Defuzzification is the process to select an appropriate crisp value based on a fuzzy set in such a way that the selected crisp value may represent the fuzzy set. This step is necessary in most applications of fuzzy systems. For example, the output of fuzzy controllers must be in the defuzzified form since mechanical, electrical, pneumatic and other actuators can only accept and use these deterministic signals.

Song and Chissom (1993) combined two defuzzification principles: the mean of maxima (MOM) and the centre of area (COA) in their research. In 1994, Song and Chissom applied a three-layer back propagation neural network to convert the output of the variant model. Inspired by Song and Chissom's approach, Song and Leland (1996) proposed the concept of the optimal defuzzification mapping and used Song and Chissom's time variant fuzzy time series model to forecast the university enrolments. An adaptive learning of the optimal defuzzification mapping is developed to find the optimal parameters. Besides, research findings of Nazirah and Abu Osman (2000) also indicated that the number of intervals and the defuzzification techniques could influence the forecasting results. In their study, the universe discourse is divided into seven and ten intervals with equal length. Fuzzy intervals with defuzzification requirement (reasonable but not necessary (RNN)) is applied to obtain the result occurred at the maximum value of the membership function.

In other related study, Tsaur et al. (2005) proposed the concept of entropy to measure the degree of fuzziness and modified Song and Chissom's time invariant fuzzy time series model to forecast the university enrolments. The fuzzy output is defuziffied by using combined principle as in Song and Chissom's (1993). Nazirah and Abu Osman (2006) proposed the concept of subsethood defuzzification with algebraic product t-norm $\mu(x_i).x_i$ to forecast the university enrolments. They applied max-min and max-product composition operator in the fuzzy relation $F(t) = F(t-1) \circ R(t, t-1)$.

This paper attempts to approach the forecasting issue by proposing the t-norm of Yager class,

$$t_w(\mu(x_i), x_i) = 1 - \min\left[1, \left((1-\mu(x_i))^w + (1-x_i)^w\right)^{\frac{1}{w}}\right] \quad w \in (0, \infty)$$

in the subsethood defuzzification indicated in the study by Nazirah and Abu Osman (2006). The proposed method applied seven and ten interval with equal length and the max-product and max-min as the composition operator in the fuzzy relations, $F(t) = F(t-1) \circ R(t, t-1)$

The analysis shows that the t-norm of Yager class of subsethood defuzzification with (10, max-product) is the best forecasting method in terms of accuracy. The proposed method will also improve the forecasting results by other previous researchers.

Preliminaries

In 1965, Lotfi Zadeh proposed the idea of fuzzy set for dealing with the vagueness type of uncertainty (Wang 1997). A fuzzy set A defined on the universe X is characterised by a membership function such that $\mu_A : X \to [0, 1]$. The nearer the value of $\mu(x)$ is unity, the higher the grade of membership of x in A.

Intersection of Fuzzy Subsets: T-Norms (Triangular Norms)

Triangular norms (briefly t-norms) are an indispensable tool for the interpretation of the conjunction in fuzzy logics and subsequently, for the intersection of fuzzy sets. A t-norm is a binary operation t on the unit interval [0, 1] which is commutative, associative, monotone and has 1 as the neutral element, that is, it is a function $t : [0, 1] \times [0, 1] \to [0, 1]$ such that for all $a, b, c \in [0, 1]$:

Axiom t1: $t(a, b) = t(b, a)$,

Axiom t2: $t[t(a, b), c] = t[a, t(b, c)]$,

Axiom t3: $t(a, b) \leq t(a, c)$ for $b \leq c$,

Axiom t4: $t(a, 1) = a$.

Subsethood Defuzzification

The subsethood defuzzification (SD) is based on sigma-count measurement and mean value theorem (Oliveira 1995). The SD coincides with the unifying structure of fuzzy set theory, that is, parallel with accordance to the Kosko's subsethood theorem (Kosko 1992).

Without the loss of generality, the universe discourse is normalised to $X = [0,1]$. For a given fuzzy set A, the averaging defuzzification in terms of set measures is given by $\hat{x} = \dfrac{M_p(A \cap \mathrm{M}_A)}{M_p(A)}$ where the sigma-count measure of a set denoted as:

$M_p(X) = \sqrt[p]{\mu^p{}_x(x_1) + \mathrm{K} + \mu^p{}_n(x_n)}$, the mirror set of support A is expressed as $\mathrm{M}_A = \sum_{i=1}^{n} \mu_\mathrm{M}(x_i)/x_i$ and $\mu_\mathrm{M}(x_i) \cong x_i$. The intersection of fuzzy sets A and M_A denoted as $A \cap \mathrm{M}_A$, is defined as $\mu_{A \cap \mathrm{M}_A}(x) = \mu_A(x)\, t\, x_i$ where t is a triangular norm. From the averaging defuzzification, the SD method can be written as:

$$x_{SD} = \frac{\sqrt[p]{\sum_{i=1}^{n} (\mu_A(x_i)\, t\, x_i)^p}}{\sqrt[p]{\sum_{i=1}^{n} \mu^p{}_A(x_i)}}$$

where for $i = 1, 2, \ldots n$, $\mu_A(x_i)$ is the degree of the membership of the i-th element in the support of A.

Enrolment Forecasting

Song and Chissom (1994) developed a time-variant fuzzy time series model to forecast students' enrolment in the University of Alabama. A three-layer neural network was applied to defuzzify the output of the fuzzy time series model. In 2006, Nazirah and Abu Osman proposed the subsethood defuzzification with t-norm of algebraic product to approach issues on students' enrolment forecasting at the University of Alabama.

The following section will show that the subsethood defuzzification with t-norm of Yager class yields better than the forecast result by other researchers mentioned before. The enrolment forecasting using the fuzzy time series (Song and Chissom 1994) with the t-norm of Yager class subsethood defuzzification is described step by step as follows.

Seven and ten equal intervals are used along with the max-min and max-product composition operator.

Step 1: Define the universe of discourse U within the historical data. Let U = [13000, 2000] and for seven and ten equal intervals, the length is 1000 and 700 respectively.

Step 2: Ten or seven (based on the number of equal interval) linguistic value must be determined to define fuzzy sets on the universe U. The A_i (i = 1, 2, 3, ...) are the possible linguistic values of *enrolment*. For ten intervals, each A_i is defined by the intervals $u_1, u_2, u_3, \ldots, u_{10}$ as follows:

$$A_1 = 1/u_1 + 0.5/u_2 + 0/u_3 + 0/u_4 + \ldots + 0/u_9 + 0/u_{10}$$
$$A_2 = 0.5/u_1 + 1/u_2 + 0.5/u_3 + 0/u_4 + \ldots + 0/u_9 + 0/u_{10}$$
$$A_3 = 0/u_1 + 0.5/u_2 + 1/u_3 + 0.5/u_4 + \ldots + 0/u_9 + 0/u_{10}$$
$$\ldots$$
$$A_9 = 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + \ldots + 0/u_7 + 0.5/u_8 + 1/u_9 + 0.5/u_{10}$$
$$A_{10} = 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + \ldots + 0/u_7 + 0/u_8 + 0.5/u_9 + 1/u_{10}$$

Table 1: The Fuzzified Historical Enrolments (Ten Intervals)

Year	Actual enrolment	Fuzzified enrolment
1971	13055	A_1
1972	13563	A_1
1973	13867	A_2
1974	14696	A_3
1975	15460	A_4
1976	15311	A_4
1977	15603	A_4
1978	15861	A_5
1979	16807	A_6
1980	16919	A_6
1981	16388	A_5
1982	15433	A_4
1983	15497	A_4
1984	15145	A_4
1985	15163	A_4
1986	15984	A_5
1987	16859	A_6
1988	18150	A_8
1989	18970	A_9
1990	19328	A_{10}
1991	19337	A_{10}
1992	18876	A_9

Step 3: Choose a model basis $w = 4$ and at a given time t, calculate the fuzzy relation

$$R^{w}(t,t-1) = f^{T}(t-2)\times f(t-1)\cup f^{T}(t-3)\times f(t-2)\cup \text{K}\ f^{T}(t-w)\times f(t-w+1)$$

and fuzzy forecasted $F(t) = F(t-1)\circ R(t,t-1)$ where $\circ$ is the max-min and max-product composition.

Step 4 : Interpret the forecast outputs. By using the SD with t-norm of Yager class, $t_w(\mu(x_i), x_i) = 1 - \min\left[1, \left((1-\mu(x_i))^{w} + (1-x_i)^{w}\right)^{\frac{1}{w}}\right]$,

the forecast enrolment can be written as

$$x_{SD} = \frac{\sqrt[p]{\sum_{i=1}^{n}\left[1-\min\left(1, \left((1-\mu(x_i))^{w} + (1-x_i)^{w}\right)^{\frac{1}{w}}\right)\right]^{p}}}{\sqrt[p]{\sum_{i=1}^{n}\mu^{p}(x_i)}}$$

Applying the above principles, the predicted enrolments are tabulated in Table 2 and shown in Figures 1 and 2.

Results

The following table shows the forecasting results of four different pairs of interval and composition operator.

Table 2: Forecast Enrolments Using T-Norm of Yager Class of Subsethood Defuzzification

Year	(10, max-min)	(10, max-product)	(7, max-min)	(7, max-product)	Actual enrolments
1975	15151	15169	15168	15168	15460
1976	16002	16065	15762	15459	15311
1977	15452	15450	15524	15262	15603
1978	15452	15450	15936	15936	15861
1979	16807	16805	15936	15936	16807
1980	17275	17702	16924	16924	16919
1981	16852	16850	16630	16630	16388
1982	16852	16850	16988	16988	15433

1983	17275	16065	16924	16924	15497
1984	15457	15450	16454	16454	15145
1985	15452	15450	15936	15936	15163
1986	15452	15450	15936	15936	15984
1987	16807	16805	15936	15936	16859
1988	17275	17702	16924	16924	18150
1989	19091	18576	19242	18778	18970
1990	19552	19550	18850	18850	19328
1991	19552	19701	19242	19242	19337
1992	19649	19650	19212	19212	18876
1993	19649	19650	19212	19212	
(w,p)	(1.41, 45)	(1.52, 10)	(1.55, 1)	(1.55, 1)	

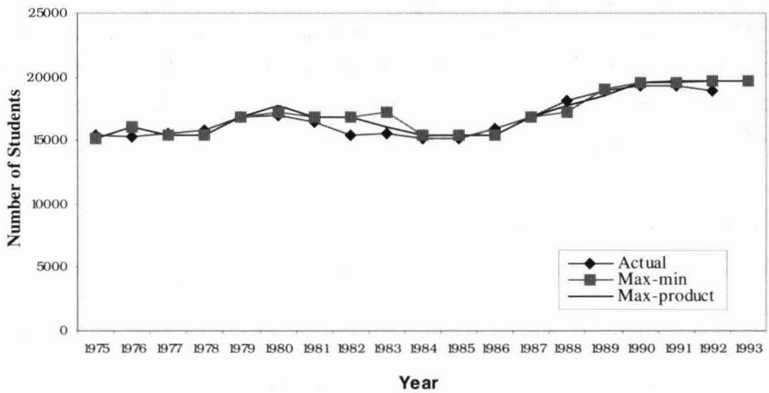

Figure 1: Forecast Enrolments and Actual Enrolments (Ten Intervals)

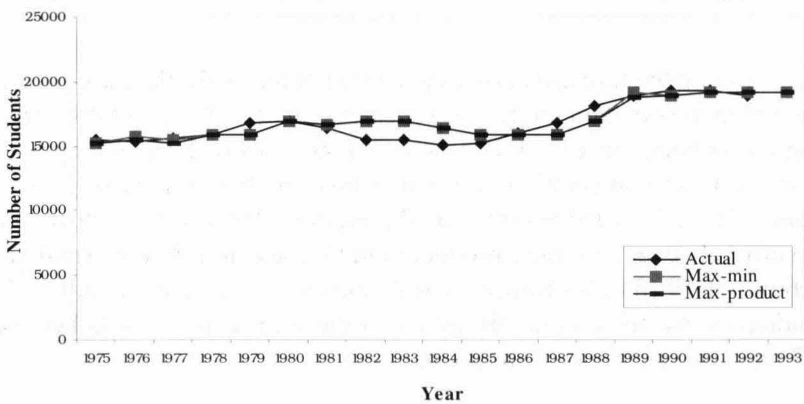

Figure 2: Forecast Enrolments and Actual Enrolments (Seven Intervals)

Discussions

In this study, the t-norm of Yager class of subsethood defuzzification is presented to forecast university students' enrolments based on fuzzy time series. The historical data are adopted from Song and Chissom (1994). The forecast enrolment in Step 4 is calculated for $w \in (0, \infty)$ and $1 \leq p \leq 50$ using Mathcad software. It is quite difficult to find w with the best forecasting model since there is no obvious pattern between the forecasting error and p. However, after numbers of effort in trial and error, finally the value of w with the best forecasting enrolments (Table 2) is found. The best forecast enrolments (with the smallest forecasting errors) for (10, max-product) and (10, max-min) occurred at ($w = 1.51$, $p = 45$) and (1.52, 10) respectively. However, for (7, max-product) and (7, max-min), the best forecasting occurred at the same point (1.55, 1).

The accuracy of this forecasting method as proposed in the model is measured by using four criteria namely mean absolute deviation (MAD), mean square error (MSE), root mean square error (RMSE) and mean absolute percentage error (MAPE).

Table 3: Comparison of Performance Indicator for the Proposed Methods

	MAD	MSE	RMSE	MAPE
(10, max-product)	456.83	311268.83	557.91	2.77
(10, max-min)	498.33	456706.11	675.80	3.06
(7, max-product)	574.22	586458.11	765.81	3.52
(7, max-min)	580.94	592490.17	769.73	3.56

Among the forecast enrolments with ten intervals, the max-product composition operator has the best accuracy since its MSE, RMSE, MAD and MAPE are the lowest. For the seven intervals with equal length, the max-product composition operator is also the best forecasting model since all the forecasting errors are the smallest. However, the difference in errors between (7, max-product) and (7, max-min) is very small and 16 out of 19 (84.2%) forecast enrolments show the same results. This indicates that there is no difference in the forecasting results between (7, max-product) and (7, max-min).

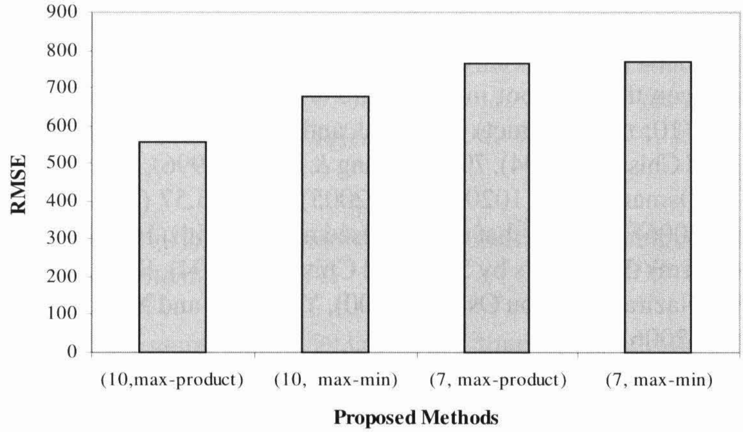

Figure 3: Comparison of RMSEs for the Proposed Methods

Based on the results in Table 3 and Figure 3, it is obvious that the t-norm of Yager class of the subsethood defuzzification with (10, max-product) is ranked first followed by (10, max-min), (7, max-product) and (7, max-min).

Table 4: Comparison of RMSEs with Various Models

	Song & Chissom (1994)	Song Leland (1996)	Nazirah & Abu Osman (2000)	Yu (2005)	Nazirah & Abu Osman (2006)	Proposed method (10, max-product)
RMSE	880.73	796.36	883.25	1020.38	765.57	557.91

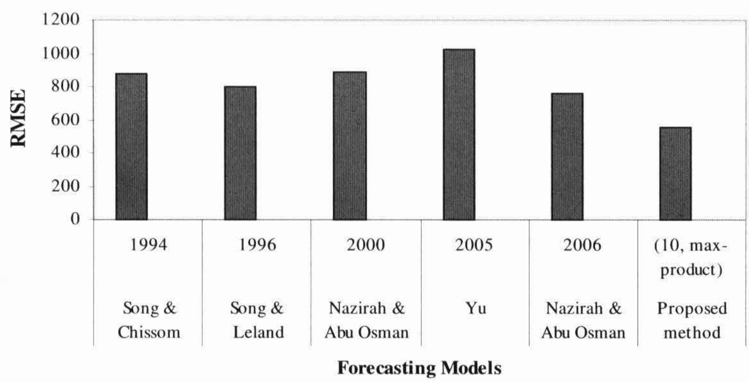

Figure 4: Comparison of RMSEs with Various Models

Table 4 compares the RMSE of different forecasting models with the proposed method (10, max-product). From Table 4 and Figure 4, it can be seen that the root mean square error (RMSE) for the proposed method (10, max-product) is 557.91 and this is better than 880.73 in Song and Chissom (1994), 796.36 (Song & Leland 1996), 883.25 (Nazirah & Abu Osman 2000), 1020.38 (Yu 2005) and 765.57 (Nazirah & Abu Osman 2006). It shows that the proposed method with (10, max-product) outperforms the models by Song and Chissom (1994), Song and Leland (1996), Nazirah and Abu Osman (2000), Yu (2005) and Nazirah and Abu Osman (2006).

Conclusion

This study proposes the t-norm of Yager class of subsethood defuzzification to forecast university students' enrolments based on fuzzy time series and the data of historical enrolments were adopted from Song and Chissom (1994). The model with (10, max-product) is the best forecasting method compared to (10, max-min), (7, max-product) and (7, max-min). The (10, max-product) models has also improved the forecasting results by Song and Chissom (1994), Song and Leland (1996), Nazirah and Abu Osman (2000), Yu (2005) and Nazirah and Abu Osman (2006).

The difficulty of finding the relationship between p and the average forecasting error is one of the problems in this study. In order to overcome this shortcoming, it is suggested genetic algorithms to be applied in the future studies (Day 1995, as cited in Hwang 1998).

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