Natural Convection In Polar Enclosure

R. A. Rasih
Faculty of Mechanical Engineering,
UniversitiTeknologi MARA Johor
KampusPasirGudang, Bandar Seri Alam,
81750 Masai, Johor
radzirasih@gmail.com

Azman Ahmad Bakir
Faculty of Mechanical Engineering,
UniversitiTeknologi MARA Pulau Pinang,
13500 PermatangPauh, Pulau Pinang
azmanbakir@ppinang.uitm.edu.my

Ahmad Faiz Zubair
Faculty of Mechanical Engineering,
UniversitiTeknologi MARA Pulau Pinang,
13500 PermatangPauh, Pulau Pinang

Mohd Noor Halmy Abdul Latif
Faculty of Mechanical Engineering,
UniversitiTeknologi MARA Johor
KampusPasirGudang, Bandar Seri Alam,
81750 Masai, Johor

ABSTRACT

Natural convection in fluid-filled, 2D cylindrical sector enclosure heated from below and cooled from above is studied numerically for a wide variety of Rayleigh number. A numerical model by using finite difference method with first order upwind was chosen for the solution of the mass, momentum and energy transfer governing equations. Stream-vorticity function is applied in order to reduce the pressure term in momentum equation. Simulations are performed for several values of both the length in radial direction with angle direction ratio of the enclosure in the range between (19/16)π to (19/4)π, and the Rayleigh number based on the angle of enclosure in the range between 102 to 106. These parameters influence upon the flow patterns, the temperature distributions and the heat transfer rates. Rayleigh number afftected the natural convection process by increase the heat transfer rate from hot temperature to cold temperature.

Key Words: Finite difference method, cylindrical sector enclosure, natural convection

INTRODUCTION

Natural convection in enclosures has been extensively studied both numerically and experimentally due to its important application in many engineering and science applications such as collection of solar energy, operation and safety of nuclear reactors, energy efficient design of buildings and rooms and heat exchangers. These are always complex interactions between the finite fluid content inside the enclosure with the enclosure walls. The study of natural convection is very complex because the velocity equations are coupled due to the buoyancy force. Many geometries and boundary conditions are present in practice but little study has been carried out for more complex shapes of the enclosure. Furthermore, rectangular cavities always chosen as an interesting area for the study of natural convection even though the study consists of many obstacles inside it. In contrast, the natural convection in cylindrical coordinate system is less interest due to its complicity in solving Navier-Stokes equation compared to Cartesian coordinate.

Ostrach [1] pointed out that thermal gradients can be varied by change its position in horizontal or vertical axis. A numerical and experimental analysis on natural convection heat transfer from a horizontal cylinder enclosed in a rectangular cavity was carried out by Cesinit et al. [2]. Holographic interferometer was used to measure the temperature distribution in the air and the heat transfer coefficients and then compared with numerical predictions obtained by finite element based on stream-vorticity formulation of the momentum equations. The study was emphasized on Rayleigh number and geometry of the cavity.

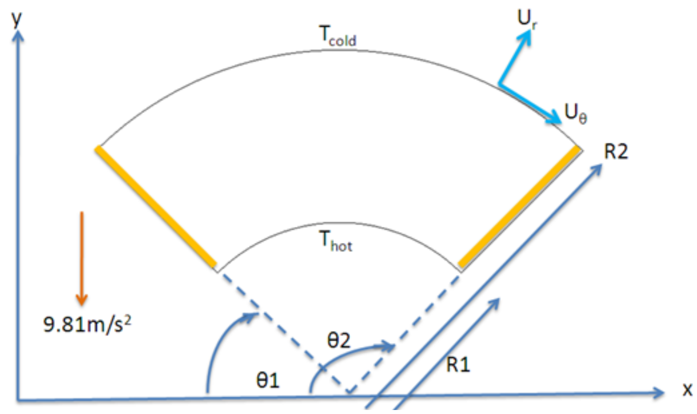
The influence of aspect ratio in rectangular cavities has been discussed by Aydinet. al [3] and Corcione [4]. They found that the heat transfer rate from the heated or cooled sidewalls is dependent only on Rayleigh number if the cavity aspect ratios larger than a specific value. Aspect ratio also discussed by Ho and Chang [5]. They conducted a study by numerically and experimentally about the effect of natural convection heat transfer in a vertical rectangular enclosure with two-dimensional discrete heating. They found that the effect of an enclosure aspect ratio of the average Nusselt number of the discrete heaters tends to decrease with the increase of the modified Rayleigh number. Frederick [6] conducted a study of natural convection in rectangular cavities and found that the overall Nusselt number decrease rapidly with increasing aspect ratio. The circulation rate always increases with the Rayleigh number and aspect ratio. Tong [7] also studied the effect of aspect ratio on natural convection in water near its density maximum. He reports that the flow patterns and temperature distributions in rectangular enclosures is impacted by the aspect ratio of enclosure.

Chandrasekhar [8] summarized these studies presented recalculations for the solutions for the critical Rayleigh number and the cellular patterns, as well as the associated wave numbers and wave length. He showed that the critical Rayleigh number is 1707.8 for the case in which both top and bottom surfaces are fixed,

and 1100.7 if the top surface is free while the bottom surface is fixed.
This study will focus on the natural convection in a polar coordinate system with different aspect ratio and Rayleigh number in order to investigate the flow and heat transfer characteristics. The cavity will be in a cylindrical sector form with adiabatic boundary condition at the sidewalls while heated from bottom and cooled from above.

MATHEMATICAL FORMULATION

Figure 1 Schematic Diagram of A Polar Enclosure Heated From Bottom and Cooled From Top



A fluid-filled, polar enclosure of height R2-R1 with an angle of θ2-θ1 is heated from the bottom, kept at temperature Thot and cooled from the top, maintained at temperature Tcold, as sketched in Figure 1. As the sidewalls are adiabatic, Table 1 shows the configuration of the angle and length. The buoyancy-driven flow is considered to be two-dimensional, steady and laminar. The fluid is assumed to be incompressible, with constant physical properties and negligible viscous dissipation. The buoyancy effects upon momentum transfer are taken into account through the Boussinesq approximation. Once the above assumptions are employed into the conservation equations of mass, momentum and energy, and the following governing equations are derived from Cartesian coordinate and then transform into stream-vorticity function. After that, the stream-vorticity transport and energy equation is transform into polar coordinate including the buoyancy term. The following governing equation is obtained:

Vorticity equation:

$$-\left[\frac{\partial^2\Psi}{\partial r^2}+\frac{1}{r}\frac{\partial\Psi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Psi}{\partial\theta^2}\right] = \omega \qquad (1)$$

Vorticity transport equation:

$$(\boldsymbol{V}.\nabla)\omega = \nu(\nabla^2\omega)+\beta g\left[cos\theta\frac{\partial T}{\partial r}-sin\theta\frac{\partial T}{r\partial\theta}\right] \qquad (2)$$

Energy Equation:

$$(\boldsymbol{V}.\nabla)T = \alpha\nabla^2 T \qquad (3)$$

where

$$\boldsymbol{V}.\nabla = -u_r\frac{\partial}{\partial r}+\frac{u_\theta}{r}\frac{\partial}{\partial\theta}\ ;\ \nabla^2 = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} \qquad (4)$$

$$u_r = \frac{1}{r}\frac{\partial\Psi}{\partial\theta}\ ;\ u_\theta = -\frac{\partial\Psi}{\partial r} \qquad (5)$$

with boundary condition:

$\theta = \theta_1$, u_r, u_θ, $\omega = 0$, $dT/d\theta = 0$; insulated left wall
$\theta = \theta_2$, u_r, u_θ, $\omega = 0$, $dT/d\theta = 0$; insulated right wall
$r = R_1$, u_r, u_θ, $\omega = -2\Psi/dr^2$, $T = T_H$; bottom hot wall
$r = R_2$, u_r, u_θ, $\omega = -2\Psi/dr^2$, $T = T_C$; top cold wall

NUMERICAL SIMULATIONS

Finite difference formulation is used to discretize the considered partial differential equations. The resulting algebraic equations for vorticity and temperature were solved by using upwind scheme with first order accuracy. The conditional statement in the convection terms was applied in order to maintain the stability of the convergence criteria. The iteration method using Guess Seidel was applied for solving the discretization equation of the stream function and temperature. A home computer program using Matlab language was constructed to handle the considered problem. In order to ensure that the flow and heat transfer characteristics are not affected by the mesh, different grids were used, (20x20), (50x50), (100x100) and (150x150) respectively as shown in Figure 2. The grid density (100x100) was used in this work because the temperature different at angle θ = π/2 is small between 100 and 150 grids. Below shows the procedure of the iteration process:

(i) Guess initial value of Ψ_{ij}, ω_{ij} and T_{ij}.
(ii) Define boundary condition
(iii) Calculate new value for ω_{ij}.
  - Use conditional statement by using backward difference if u_r or u_θ is positive value while forward difference is applied when u_r or u_θ is negative value.
(iv) Calculate new value of Ψ_{ij}.
(v) Calculate new value of T_{ij} by using the same method like ω_{ij}.
(vi) Repeat procedure (iii) to (v) until satisfies convergence criteria = 10^{-5}.

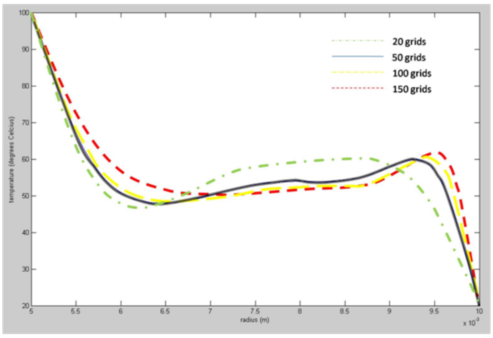


Figure 2. Grid independence test for temperature plot at angle θ = π/2 at Ra = 2x105.

The important dimensionless parameter to investigate the heat transfer characteristics are Rayleigh number (Ra), Reynolds number (Re) and Prandtl number (Pr). Dimensionless numbers are defined as below:

$$Re = \frac{U_R L}{\nu} \; ; Pr = \frac{\nu}{\alpha} \tag{6}$$

$$U_R = \frac{\beta g (T_1 - T_2) L^2}{\alpha} \tag{7}$$

$$Ra = Re.Pr = \frac{\beta g (T_1 - T_2) L^3}{\nu \alpha} \tag{8}$$

Where β = volumetric thermal expansion coefficient
ν = kinematicviscosity
α = thermaldiffusivity
g = gravityspeed

The configuration of aspect ratio is differed by angle and inner radius while outer radius is fixed. The Rayleigh is varied from 2x102 until 2x106 passes the critical Rayleigh number which is 1708. Rayleigh-Benard convection was studied with the two-dimensional axis symmetric code, simplified from the full three-dimensional code. While maintaining the Prandtl number of 4 for Refrigerant-134a, Rayleigh number is differed by change the value of β. T1 is the inner hot temperature which set to be 100oC while T2 is the outer low temperature which set to be 20oC to investigate the effect of aspect ratio. The critical Rayleigh number was chosen in order to study possibility of AR at certain value to predict the value of critical Rayleigh number.

$$Aspect\ Ratio, AR = \frac{\frac{1}{2}(R_2 + R_1)(\theta_2 - \theta_1)}{R_2 - R_1} \tag{9}$$

Table 1 Configuration of Angle with Ra (critical) = 1708

Angle, θ_2- θ_1 (radian)	R_1 (m)	R_2 (m)	R_2-R_1 (m)	Aspect Ratio, AR
π/8	0.01	0.009	0.001	$\frac{19}{16}\pi = 3.731$
π/4				$\frac{19}{8}\pi = 7.461$
π/2				$\frac{19}{4}\pi = 14.923$

Table 2Configuration of Angle with Ra (critical) = 1708

Angle, θ_2- θ_1 (radian)	R_1 (m)	R_2 (m)	R_2-R_1 (m)	Aspect Ratio, AR
π/8	0.01	0.009	0.001	$\frac{19}{16}\pi = 3.731$
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Effect of Rayleigh Number

Figure 3 shows the streamline and isotherm contour of natural convection in Cartesian coordinate [4] with polar coordinate. The presence of hot bottom and cold top, fluid near the bottom surface will be heated in the result of conduction and its density will be decreased and so move up. On the contrary, the fluid near the upper surface which has higher density, and is cooler will move down. This will cause to produce columns of cold and warm fluid that move up and down. Moving of these columns will cause of vorticity appearance in the enclosure. Thus both Cartesian coordinate and polar coordinate shows the same pattern of natural convection in an enclosure.

Figure 4 shows the streamline and isotherm contour for different Rayleigh number at AR=1.178. The flow field consists respectively of two counter rotating cells, symmetric about the vertical mid-plane of the cavity. The isotherm contour at Ra=2x103 shows that the temperature is not linear anymore. This is due to the convection mode take part the heat transfer process from conduction mode after pass the critical Rayleigh number which is 1708. All of the cells are stable until Ra=2x106, the cells become asymmetric and unstable. At this high Ra, the steady state assumption is not valid and the residual cannot converge. Thus at high Ra, unsteady state problems must be use because the fluid flow is time dependent.

Figure 3 Streamline and Isotherm Contour For Cartesian Coordinate [4] With Polar Coordinate

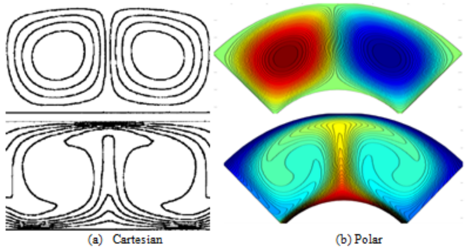


Figure 4 Streamline and Isotherm Contour for Ra = 2x102, 2x103, 2x105, 2x106 .

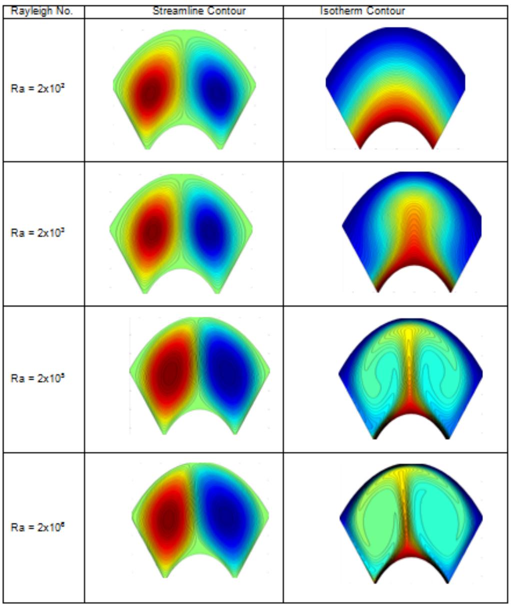
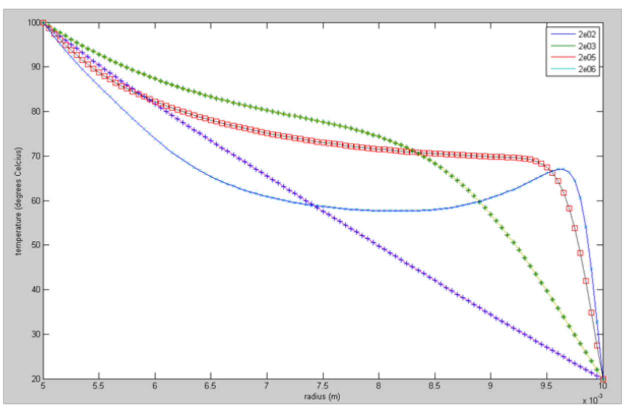


Figure 5. Temperature at Symmetrical Line For Ra = 2x102(+) , 2x103(*), 2x105(□),2x106 (x).



As mentioned above, and is obvious from Figure 5, in the lower range of Ra numbers (e.g. Ra=200), they are very close to the typical temperature distribution that corresponds to the limit of pure conduction. It is because of low buoyancy-driven circulation inside the cavity relative to viscosity force. Gradually, with increasing of Ra number, convection contribution will be increased.

Effect of Aspect Ratio

Figure 6 shows the streamline and isotherm contour of different aspect ratio. It can be seen that with the increasing aspect ratio, number of vorticity also will be increased which shows a good agreement with M. Corcione [4]. For instance, when the aspect ratio is 3.731, four vorticities appeared which are circulating in the middle of radial direction. When aspect ratio is increase, there is higher possibility of appearance of more columns of cold and hot air which cause more vorticity. These effects could be in the result of braking of regular layering in the upper and bottom of the enclosure. All isotherm contours also show that the temperature is not linear anymore because the fluid flow is simulated at critical Rayleigh number. To make clear, Figure 6 shows the temperature plot along symmetrical line which $\theta = \pi/2$. This means that the convection process take place at critical Ra even the aspect ratio is different.

Figure 6 Streamline and Isotherm Contour For AR = 3.731, 7.461, 14.923 at Ra = 1708

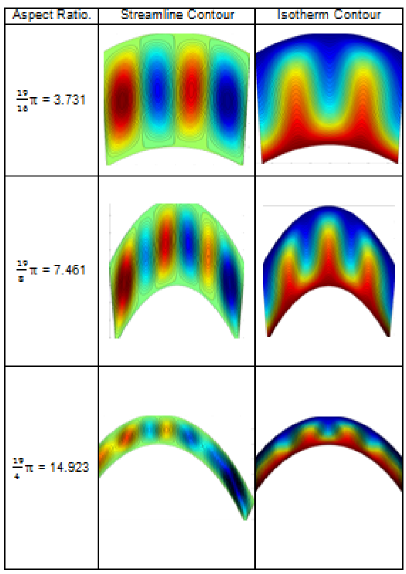
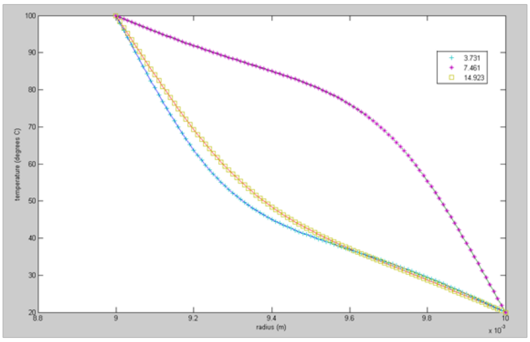


Figure 7 Temperature Plot at Symmetrical Line For AR = 3.731, 7.461, 14.923 at Ra = 1708



CONCLUSIONS

Natural convection heat and momentum transfer in fluid filled, polar enclosures heated from the bottom and cooled from the top has been studied numerically for different aspect ratios of the enclosure in the range between (19/16)π to (19/4)π, and for values of the Rayleigh number based on the volumetric thermal coefficient in the range between 102 to 106. Rayleigh number affected the natural convection process by increase the heat transfer rate from hot temperature to cold temperature. The existences of buoyancy forces will enforce the convection to take part the heat transfer process from conduction. It is investigated that with correct selected parameters, such as configurations and dimensions of the enclosure, we can control the heat transfer and obtains the optimal heat transfer in it.

ACKNOWLEDGEMENTS

The author would like to thank for RAGS grants (600-RMI/RAGS 5/3 (43/2012)) and UniversitiTeknologi MARA (UiTM) for their support and funding.

REFERENCES

S. Ostrach, S., 1988.Natural convection in enclosures, Int. J. Heat Transfer, 110, 1175 1190.

Cesini, G., Paroncini, M., Cortella, G., Manzan, M., 1999. “ Natural convection from a horizontal cylinder in rectangular cavity”, Int. J. Heat and Mass Transfer, 42, 1801-1811

Aydin, O., Unal, A., Ayhan, T., 1999. Natural convection in rectangular enclosures heated from one side and cooled from the ceiling, Int. J. Heat Mass Transfer, 42, 2345–2355.

Corcione, M., 2003. Effects of the thermal boundary conditions at the sidewalls upon naturalconvection in rectangular enclosures heated from below and cooled from above. Int. J. of Thermal Sciences, 42, 199–208

Ho, C.J., Chang, J.Y., 1994. A study of natural convection heat transfer in a vertical rectangular enclosure with two-dimensional discrete heating: effect of aspect ratio, Int. J. Heat Mass Transfer, 37 (6), 917–925.

Frederick, R.L., 1999. On the aspect ratio for which the heat transfer in differentially heated cavities is maximum, Int. Comm. Heat Mass Transfer, 26 (4), 549–558.

Tong, W., 1999. Aspect ratio effect on natural convection in water near its density maximum temperature, Int. J. Heat Fluid Flow, 20 (6), 624–633.

Chandrasekhar, S., 1961. Hydrodynamic and HydromagneticStability.Oxford at the Clarkndon Press.