Existence of Solution to First-Order Multipoint Boundary Value Problems

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Contents

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Ybhg. Prof.,

LAPORAN AKHIR PENYELIDIKAN“EXISTENCE OF SOLUTIONS TO FIRST-ORDER MULTIPOINT BOUNDARY VALUE PROBLEMS,”

Merujuk kepada perkara di atas, bersama-sama ini disertakan 3 (tiga) naskah Laporan Akhir Penyelidikan bertajuk “Existence of Solutions to First-Order Multipoint Boundary Value Problems”.

Sekian, terima kasih.

Yang benar,

Diana Sirmayunie Bt Mohd Nasir

Ketua

Projek Penyelidikan

Abstract

In this manuscript we study initial value problems and boundary value problems for a first order ordinary differential equations. First, we will investigate the initial value problems of the form

$$y'(t) = f(t, y(t)), \quad y(t_0) = y_0.$$

We establish the existence of solutions by the Banach Contraction Mapping Principals. Next we present the numerical methods for the above initial value problems, where the numerical comparison between the Euler and Runge-Kutta methods are being investigated. Then we establish the existence of solutions to multipoint boundary value problems

$$y' = f(t, y) \text{ for each } t \in I = [0, 1],$$

$$ay(0) + by(\eta) + cy(1) = \alpha.$$

We prove the existence of solutions to the multipoint by Schaeffer fixed point theorem and uniqueness of solutions by the Contraction Mapping Principal. Finally we investigate the existence of solutions for nonlocal problems

$$y' = f(t, y(t)), \text{ for each } t \in J = [0, T],$$

$$y(0) + g(y) = y_0.$$

Chapter 1

Introduction

This project is structured as follows. In chapter 1, we introduce notations, definitions and results that will be used in the remainder chapters of this project.

In Chapter 2, we will prove that there is a unique solution to the initial value problems for a first-order ordinary differential equations. We consider

$$y'(t) = f(t, y(t)),\ y(t_0) = y_0$$

where f is a given function, and t_0 and y_0 are given real numbers. A solution to this problem is a function $y(t)$ satisfying the differential equation $y' = f(t, y)$ with initial condition $y(t_0) = y_0$. We use the Banach Fixed Point Theorem to establish the existence of a unique solution.

In Chapter 3, we investigate the numerical method of the initial value problem from Chapter 2. We show that the Runge-Kutta method gives better results than Euler's method.