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Computation of Temperature Distributions on Uniform and Non-uniform Lattice Sizes Using Mesoscopic Lattice Boltzmann Method

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ABSTRACT

This work is concerned with the mesoscopic lattice Boltzmann computation of heat conduction problems on uniform and non-uniform lattice sizes. It also focuses to solve heat conduction problems in one- and two-dimensional Cartesian geometries. It is known that, the lattice Boltzmann method is a relatively new method and application to heat conduction problems is scarce. In the present work, heat transfer formulations of lattice Boltzmann method to solve heat transfer problems are presented and implementation of non-uniform lattices is described. To show the accuracy and stability of the present lattice Boltzmann method, number of iterations and CPU time are reported. In order to study the effect of lattice structure, uniform and non-uniform lattice sizes are performed. To lend credibility to the lattice Boltzmann results they are further compared with those obtained from a finite difference method. It is concluded that the present study in heat conduction produces results that are in excellent contribution of lattice Boltzmann method in the area of computational fluid dynamics.

Keywords: *Lattice Boltzmann method, uniform lattice, non-uniform lattice, finite difference method*

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Introduction

The lattice Boltzmann method (LBM) receives more and more attention in recent years as an alternative numerical approach in Computational Fluid Dynamics (CFD) [1]. The LBM received a tremendous impetus with their spectacular use in viscous fluid flow and heat transfer problems [2]. The application of LBM to pure heat conduction problems received less attention than the field of fluid flow problems [3]. Nowadays heat transfer research using LBM is getting momentum. It is known that, numerical schemes for the heat conduction are based mostly on continuum-based methods such as finite difference method (FDM), finite element method (FEM) and finite volume method (FVM). It is known that, the LBM is second-order accurate in time and space, which is sufficient for most engineering applications [4]. The main concept of LBM is to bridge the gap between micro-scale and macro-scale approach by not considering each particle behaviour alone but behaviour of a collection of particles as a unit [5]. The property of the collection of particles is represented by a particle distribution function. The distribution function acts as a representative for collection of particles. This scale is known as mesoscopic approach.

The present LBM can handle a problem in micro- and macro-scales with reliable accuracy. It also enjoys advantages of both macroscopic and microscopic approaches, with manageable computer resources. The lattice size used in the LBM, commonly used Lattice-Bhatnagar-Gross-Krook (LBGK) model is restricted to orthogonal grid with equal lattice spacing. It is known that, uniform grid is desirable in many practical applications. Recently, many studies have been dedicated to the extension of the LBM on non-uniform lattice size [6, 7].

Boundary and initial conditions play important role in LBM simulations. The majority of the LBM works so far dealt with uniform lattice size only. Mishra et al. [8] presented the transient heat conduction problems on uniform and non-uniform lattice sizes using LBM. Ho et al. [9] extended the lattice Boltzmann scheme for hyperbolic heat conduction equation with a source term. Recently, Perumal and Dass [10] studied natural convection with heat transfer in a square cavity for a wide range of Rayleigh number using LBM.

The primary objective of this work is to get acquainted with various aspects of the LBM and develop efficient codes that can numerically compute heat transfer involving various levels of complexities. Another objective of this work is to develop a 'C' program to solve the 1-D and 2-D heat conduction problems using LBM in uniform and non-uniform lattice sizes. The LBM is a relatively novel technique of flow computation, there is some scope for speculation as to the accuracy of the present LBM computations. Therefore the existing LBM results always have been compared favorably with finite difference methods.

This paper is organized as follows. In Section 2 numerical methods including LBGK model and finite difference method is described. Section 3 deals with heat conduction problems in one- and two-dimensional Cartesian geometries and the results are presented and validated. Section 4 concludes this paper.

Numerical Methods

Lattice Boltzmann method

The governing lattice Boltzmann equation with BGK approximation can be written as [9]

$$f_i(\mathbf{x}+c_i\Delta t, t+\Delta t) = f_i(\mathbf{x},t) - \frac{\Delta t}{\tau}\left(f_i(\mathbf{x},t) - f_i^{(eq)}(\mathbf{x},t)\right) \quad i = 0...\mathrm{b} \qquad (1)$$

Where $f_i(\mathbf{x}, t)$ and $f_i^{(eq)}(\mathbf{x}, t)$ are the particle and equilibrium distribution functions at $(\mathbf{x}, t)$, c_i is the particle velocity along the i^{th} direction and τ is the relaxation time due to particle collision. In LBM, the solution domains need to be divided into lattice points. Figure 1 shows the *D1Q2* and *D1Q3* lattice models.

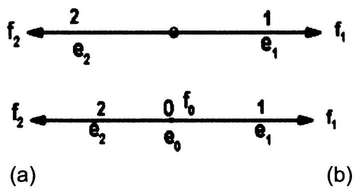


Figure 1: One-dimensional lattice models (a) *D1Q2* and (b) *D1Q3*

In Figure 1(a), Position 1 and 2 indicates the nearest neighbor lattice sites in the respective directions. In Figure 1(b), Position 0 indicates the stationary particle. The particle velocities measured in lattice units and their weights for different lattice models are shown in Table 1.

Table 1: Lattice particle velocities and weights of different lattice models

Model	Particle velocity (c_i)	Weights (w_i)
D1Q2	$c_1 = C, c_2 = -C$	$w_1 = 1/2, w_2 = 1/2$
D1Q3	$c_0 = 0, c_1 = C, c_2 = -C$	$w_0 = 1/2, w_1 = 1/4, w_2 = 1/4$
D1Q5	$c_0 = 0, c_{1,2} = \pm C, c_{3,4} = \pm 2C$	$w_0 = 6/12, w_{1,2} = 2/12, w_{3,4} = 1/12$
D2Q9	$c_0 = (0,0), c_{1,2} = (\pm 1,0).C,$ $c_{3,4} = (0, \pm 1).C,$ $c_{5\text{-}8} = (\pm 1, \pm 1).C$	$w_0 = 4/9, w_{1,2} = 1/9,$ $w_{3,4} = 1/36$

The equilibrium particle temperature distribution function can be written as [9]

$$f_i^{(eq)}(\mathbf{x}, t) = w_i T(\mathbf{x}, t) \tag{2}$$

The relaxation time τ for the one-dimensional models can be written as [8]

$$\tau = \frac{\alpha}{c_i^2} + \frac{\Delta t}{2} \tag{3}$$

The relaxation time τ for two-dimensional lattice model can be written as [8]

$$\tau = \frac{3\alpha}{c_i^2} + \frac{\Delta t}{2} \tag{4}$$

It is known that weights satisfy the relation $\sum_{i=1}^{N} w_i = 1$. The macroscopic temperature is obtained as

$$T = \sum_{i=0}^{N} f_i(\mathbf{x},t) = \sum_{i=0}^{N} f_i^{eq}(\mathbf{x},t) \tag{5}$$

It is known that, non-uniform grids are the lattice points which are unequally placed. Here we are considering non-uniform lattice points and the main advantage in this is that we can find the concentration of temperature in a particular area. Figure 2 shows the one-dimensional (1-D) non-uniform lattice points. The relaxation time is calculated based on smallest grid spacing.

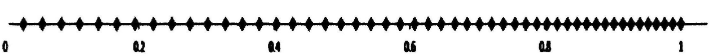


Figure 2: One-dimensional non-uniform lattice points for heat conduction

The 2-D non-uniform lattices have a similar formulation which was applied for 1-D non-uniform lattices. Figure 3 shows the two-dimensional (2-D) non-uniform lattice points. In the present work, it is necessary to update the particle distribution functions using interpolations for fine lattice points which are adjacent to the coarse lattice points. The interpolation formula for non-uniform lattices can be written as [7]:

$$f(x) = f(x_0)\frac{(x\ x_1)(x\ x_2)}{(x_0\ x_1)(x_0\ x_2)} + f(x_1)\frac{(x\ x_0)(x\ x_2)}{(x_1\ x_0)(x_1\ x_2)} + f(x_2)\frac{(x\ x_0)(x\ x_1)}{(x_0\ x_2)(x_2\ x_1)} \tag{6}$$

Implementation of boundary conditions in LBM requires careful effort, as the boundary conditions available at the macroscopic level need to be expressed in terms of particle distribution function [11]. For a node near a boundary, some of its neighboring nodes lie outside the flow domain. The unknown particle distribution function is determined from the known particle distribution function. Some boundary condition details of the heat transfer simulations by LBM can be found in Mishra et al. [8]. The solution procedure of the LBM at each time step comprise the streaming and collision steps, application of boundary conditions, calculation of particle distribution function followed by calculation of macroscopic variables.

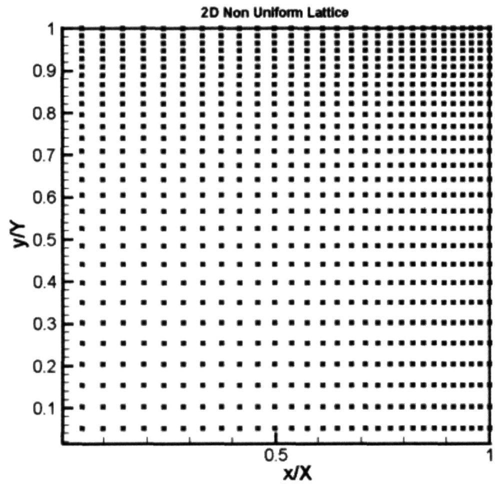


Figure 3: Two-dimensional non-uniform lattice points for heat conduction problems

Finite difference method

Finite difference method (FDM) for diffusion problem is discussed in this section. It is known that, this method is a conventional method for solving fluid flow and heat transfer problems [12]. Extension to two- and three-dimensional problems in FDM is a straightforward procedure. The main objectives of the approximation of the diffusion equation using the finite differences are two folds; first to show the difference and similarity between FDM and LBM; also, to compare the results of the two methods. An explicit finite difference approach can be used forward in time and central differences in space. Approximating the diffusion equation at a node i, yields,

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \alpha \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x^2} \tag{7}$$

The above Equation (7) can be formulated as

$$T_i^{n+1} = T_i^n(1-\omega) + (0.5T_{i+1}^n + 0.5T_{i-1}^n) \tag{8}$$

where $\omega = {2\alpha\Delta t}/{\Delta x^2}$. For stability condition, the coefficients of the right-hand side terms must be positive. Hence, the term (1 – ω) must be greater than or equal to zero, which implies that $\Delta t \le \frac{\Delta x^2}{2\alpha}$. The last term in Equation (2) represents the equilibrium value of T_i. It is appropriate to write the term $(0.5T_{i+1}^n + 0.5T_{i-1}^n)$ as $T^{(eq)}$. To resemble LBM, Equation (8) can be rewritten as

$$T_i^{n+1} = T_i^n(1-\omega) + \omega T_i^{(eq)} \tag{9}$$

Results and Discussion

Test Problem 1: One-dimensional Lattice Boltzmann models including the two-velocity model (*D1Q2*), the three-velocity model (*D1Q3*) and the five-velocity model (*D1Q5*) are used in the present work to compute the flow of heat conduction in a rod. In the one-dimensional heat conduction problem, the left and right side temperatures are known. The left and right sides are maintained at 1.0 and 0. Figure 4 shows the temperature distributions by the LBM for 1-D geometry. It is seen that *D1Q3* and *D1Q5* models give highly accurate results that exactly matches each other. The result given by D1Q2 model, however, is seen to be deficient. To plot temperature distributions, the *D1Q3* model is used in the present section as it consumes less memory and computational time compared with *D1Q5*.

Test Problem 2: A 1-D planar geometry with uniform Lattice points having west boundary at specified temperature and east boundary at known temperature are solved using Lattice Boltzmann method. Boundary conditions: $T_{initial} = 0$, $T_W = 1.0$, $T_E = 0.0$. Figure 5 shows the temperature distributions by uniform and non-uniform lattices for 1-D geometry. It is seen that, in the case of 1-D steady state heat conduction problems, the results obtained using the LBM uniform and non-uniform lattices are almost the same.

The CPU timing taken for processing as well as the number of iterations taken is compared for LBM uniform, non-uniform lattices as well as finite difference method and the results for 1-D problem are tabulated in Table 2.

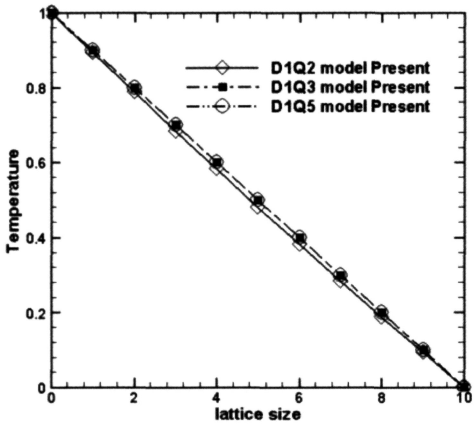


Figure 4: One-dimensional temperature distributions obtained by the LBM models

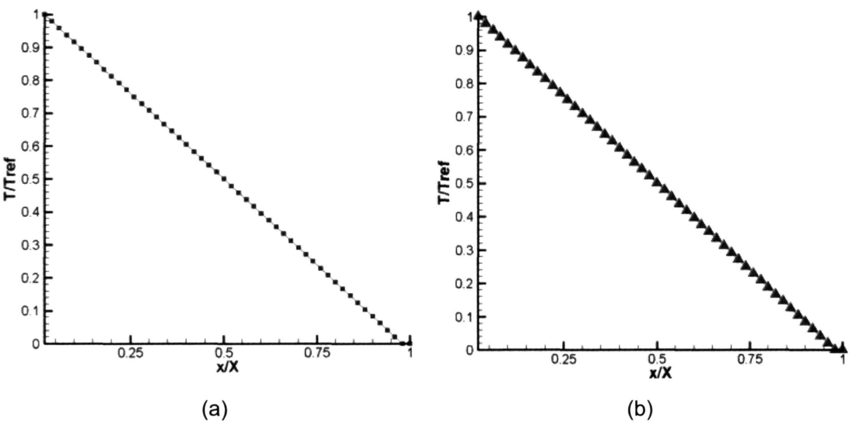


Figure 5: 1-D Temperature distribution obtained by LBM (a) uniform lattices and (b) non-uniform lattices

The present LBM is an alternative technique of flow computation, there is some scope for speculation as to the accuracy of the uniform and non-uniform computations. Thus, favorable comparison of the present LBM results with these accurate FDM results will grant legitimacy to them. It is also observed from the table 2 that, the number of iterations taken as well as the CPU timing is almost same for 1-D LBM for uniform lattice sizes as well as for FDM method. But the 1-D non-uniform lattice sizes using LBM gives accurate results, it takes more number of iterations and less CPU time.

Table 2: Comparison of number of iterations and CPU times of the 1-D LBM and FDM

Lattice points	LBM (uniform)		LBM (non-uniform)		FDM	
	Iterations	CPU time (sec)	Iterations	CPU time (sec)	Iterations	CPU time (sec)
50	2616	0.103	2843	0.124	3696	0.086
75	5150	0.156	5529	0.134	5450	0.125
100	8135	0.218	8759	0.172	8643	0.156

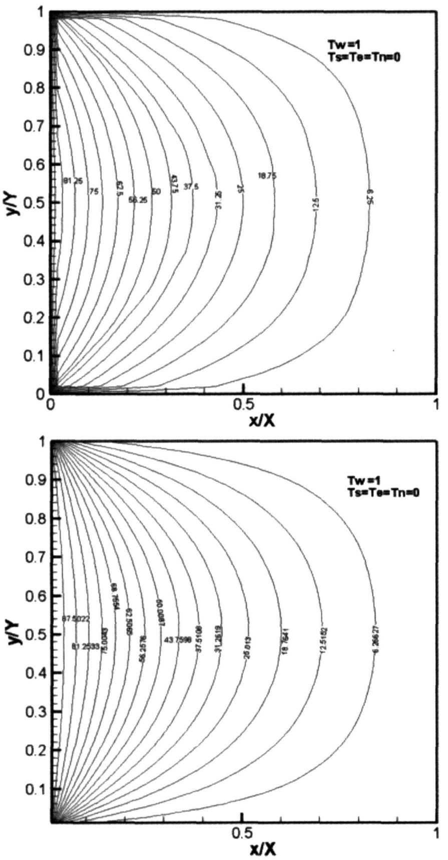
Test Problem 3: A 2-D square geometry have the west wall at specified temperature and rest of the walls at known temperature. Boundary conditions are as follows: $T_{initial} = 0$, $T_W = 1.0$, $T_E = T_N = T_S = 0.0$. Figure 6 shows the temperature distributions by uniform and non-uniform lattices for 2-D square geometry. The 2-D heat conduction problem is more complicated than 1-D problem. The number of parameters to be considered has increased. But the real advantage of LBM on comparison with uniform and non-uniform lattices can be seen in the case of 2-D steady state heat conduction problem.

The CPU timing taken for processing as well as the number of iterations taken is compared for LBM uniform, non-uniform lattices as well as finite difference method and the results for 2-D problem are tabulated in Table 3. It is observed that, the result obtained through lattice Boltzmann method is more accurate and values are converging quickly when compared with the same set of conditions for Finite Difference method. In the present work, we considered the different lattice sizes 30×30, 50×50 and 80×80 with the west boundary temperature given as 1.0. But, the LBM is much faster and accurate when compared with conventional FDM.

To understand the effect of LBM non-uniform lattice size in the present work 2-D lattice of 25×25 is considered. Figure 7(a) and (b) shows the temperature distributions by LBM uniform and non-uniform lattices for 2-D square geometry.

From Figure 7(a) and (b) it is seen that, near the west walls the non-uniform LBM results is better than uniform lattice size. The compressibility effect of LBM is clearly visible in Figure 7. In the present work, 2-D problems of different combinations of wall temperatures are also tested.

Test Problem 4: A 2-D square geometry having the south wall at specified temperature and rest of the walls at known temperature. Boundary conditions are as follows: $T_{reference} = 1.0$, $T_S = 1.0$, $T_E = T_N = T_W = 0.2 \times T_{reference}$. The flow pattern for the LBM non-uniform lattice size simulation and conventional FDM of heat

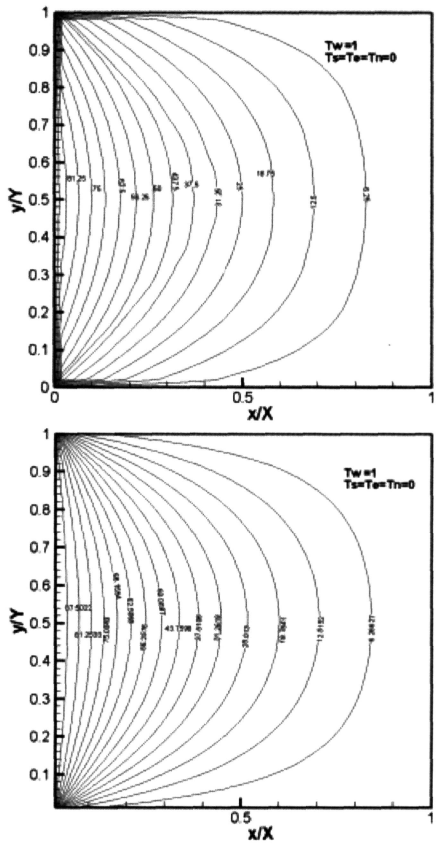


(a) (b)

Figure 6: 2-D temperature distribution obtained by uniform and non-uniform lattice sizes of 50 × 50

Table 3: Comparison of number of iterations and CPU times of the 2-D LBM and FDM

Lattice sizes	LBM (uniform)		LBM (non-uniform)		FDM	
	Iterations	CPU time (sec)	Iterations	CPU time (sec)	Iterations	CPU time (sec)
30 × 30	1173	4.514	1483	7.382	1315	7.566
50 × 50	1813	12.075	3585	14.81	3370	14.537
80 × 80	5890	21.60	7716	24.074	7537	26.08



(a) (b)

Figure 7: 2-D temperature distribution obtained by LBM on uniform and non-uniform lattice sizes of 25 × 25

conduction problem is illustrated in Figure 8. It is seen that the present isotherms LBM non-uniform results agrees excellence with conventional FDM.

Table 4 shows the comparison of CPU processing time and the number of iterations for LBM non-uniform lattices as well as FDM. It is also observed from the Table 4, the present LBM non-uniform lattice results are excellent agreement with conventional FDM. As the lattice size increases the CPU time and number of iterations also increases. The present LBM non-uniform lattice results are substantiate with conventional FDM results. It is seen that, the present LBM is second order accurate in space and time it converges faster than conventional FDM. The present non-uniform lattice gives accurate results near the walls on heat conduction problems.

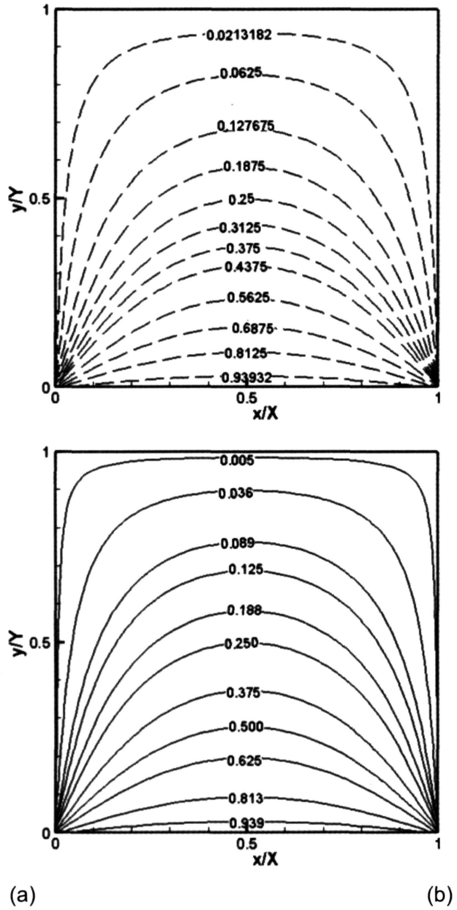


Figure 8: 2-D temperature distribution plots (a) the LBM non-uniform lattices of 50 × 50 (b) FDM grid size of 50 × 50

Table 4: Comparison of number of iterations and CPU times of LBM non-uniform lattice sizes and FDM

Lattice sizes	LBM (non-uniform)		FDM	
	Iterations	CPU time (sec)	Iterations	CPU time (sec)
30 × 30	5013	5.282	4815	7.566
50 × 50	6615	13.11	6370	13.537
80 × 80	7516	42.074	7137	42.08

Conclusion

In the present work, the LBM has been successfully implemented in solving heat conduction problems in 1-D and 2-D uniform as well as non-uniform lattice sizes. The results obtained through mesoscopic LBM is validated by solving the same problems using conventional FDM. Results of the present LBM on uniform and non-uniform lattices were found very well. To check the accuracy of the LBM non-uniform lattices, problems were solved using the FDM. In all the cases, the non-uniform LBM was found to provide accurate results. When solving heat conduction problems in 1-D and 2-D lattice sizes, the LBM was found to take much lesser iterations and CPU time on comparison with FDM for converged solutions. The present work successfully demonstrates that non-uniform LBM has come of age and is now an important alternative solution procedure in computational fluid dynamics.

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