UNIVERSITI TEKNOLOGI MARA

AN INTEGRATED Q–RUNG ORTHOPAIR FUZZY DOMBI GENERALIZED HERONIAN MEAN–AURA FRAMEWORK FOR MULTI-CRITERIA DECISION-MAKING UNDER UNCERTAINTY

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ABSTRACT

Decision-making in complex and uncertain environments presents ongoing challenges in Multi-Criteria Decision-Making (MCDM), particularly when criteria are heterogeneous, conflicting, and evaluated through subjective expert judgments. Classical distance-based methods such as TOPSIS and VIKOR are primarily designed for benefit-type and cost-type criteria and may not adequately represent target-type criteria, for which the preferred value lies within the observed range. In addition, conventional aggregation operators often assume criterion independence, while many decision support tools provide limited transparency and flexibility for uncertain decision information. Therefore, there is a need for a framework that can accommodate benefit, cost, and target-type criteria, represent expert uncertainty and inter-criteria interaction, and support transparent decision-making. This study develops the Adaptive Utility Ranking Algorithm (AURA), a distance-based MCDM method designed to improve flexibility and interpretability in ranking alternatives. AURA incorporates adaptive normalization for benefit, cost, and target-type criteria, evaluates alternatives against positive, negative, and average benchmark solutions, and applies a correction factor to improve discrimination between closely ranked alternatives. To aggregate uncertain expert evaluations, the q-Rung Orthopair Fuzzy Dombi Generalized Heronian Mean (qROFDGHM) operator is proposed. The operator combines q-rung orthopair fuzzy sets, Dombi operations, and the Heronian mean to represent expert hesitancy, nonlinear aggregation behaviour, and interactions among criteria. Its mathematical properties, including idempotency, monotonicity, boundedness, and closure, are established. The proposed qROFDGHM–AURA framework is applied to the evaluation of six educational technology alternatives for English language learning. The alternatives are assessed by ten experts across six main criteria and eighteen sub-criteria. Sensitivity analyses are conducted to examine the effects of the AURA parameters α and p, as well as the qROFDGHM parameters q, (a, b), and λ. The results show that the main ranking pattern remains stable under the tested parameter settings. Comparative analysis against six benchmark MCDM methods, namely COBRA, WASPAS, MOORA, SAW, VIKOR, and TOPSIS, shows that Gamification and Quizzing Platforms (A_3) and Mobile Learning Applications (A_4) consistently occupy the leading positions, whereas ASR Tools (A_6) are ranked last. Correlation analysis produces Spearman coefficients ranging from 0.943 to 1.000 and Kendall coefficients ranging from 0.867 to 1.000. The overall Kendall coefficient of concordance is approximately $W = 0.92$, indicating strong agreement among the ranking methods. In addition, the AURA method is implemented in an open-source, Python-based decision support system that supports data upload, criterion-type selection, step-by-step computation, and automated Excel reporting. The AURA software processes a synthetic decision matrix containing 1000 alternatives and 20 criteria in 1.10 seconds, demonstrating the scalability of the ranking component. Overall, the study contributes a qROFDGHM–AURA framework that integrates fuzzy aggregation with adaptive ranking for multi-expert decision-making under uncertainty. The framework provides a transparent and systematic approach for evaluating complex alternatives and may be extended to other MCDM domains.

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LIST OF SYMBOLS

Symbols

X	Universe of Discourse
x	Element of the Set X
A_i	i-th Alternative
C_j	j-th Evaluation Criterion
m	Number of Alternatives
n	Number of Criteria
i, j	Indices of Alternatives and Criteria
$\mu_E(x)$	Membership Degree of Element x in Set E
$\nu_E(x)$	Non-Membership Degree of Element x in Set E
$\pi_E(x)$	Hesitancy Degree of Element x in Set E
q	q-Rung Parameter ($q \geq 1$)
$\chi = (\mu, \nu)$	q-Rung Orthopair Fuzzy Number (qROFN)
$S(\chi)$	Score Function of qROFN
$A(\chi)$	Accuracy Function of qROFN
a, b	Parameters of the Generalized Heronian Mean ($a, b \geq 0$)
$GHM^{a,b}$	Generalized Heronian Mean Operator
$GWHM^{a,b}$	Generalized Weighted Heronian Mean Operator
w_j	Weight of Criterion C_j
$\mathbf{w}$	Criteria Weight Vector
λ	Dombi Operational Parameter ($\lambda > 0$)
$T_{D,\lambda}(x, y)$	Dombi t-Norm
$S_{D,\lambda}(x, y)$	Dombi t-Conorm
$f(\cdot)$	Additive Generator Function (Increasing)
$g(\cdot)$	Additive Generator Function (Decreasing)
$f^{-1}(\cdot)$	Inverse Additive Generator
$g^{-1}(\cdot)$	Inverse Additive Generator
$\oplus$	Dombi-Based Addition Operator

$\otimes$	Dombi-Based Multiplication Operator
φ	Scalar Multiplication Parameter ($\varphi > 0$)
τ	Power Parameter ($\tau > 0$)
$X = [x_{ij}]_{m \times n}$	Decision Matrix
x_{ij}	Performance Value of Alternative A_i under Criterion C_j
r_{ij}	Normalized Performance Value
$R = [r_{ij}]_{m \times n}$	Normalized Decision Matrix
v_{ij}	Weighted Normalized Value
$V = [v_{ij}]_{m \times n}$	Weighted Normalized Decision Matrix
$x_j^{\max}$	Maximum Value of Criterion C_j
$x_j^{\min}$	Minimum Value of Criterion C_j
r_j	Target Value of Criterion C_j
v_j^+	Positive Ideal Solution (PIS)
v_j^-	Negative Ideal Solution (NIS)
$\bar{v}_j$	Average Solution of Criterion C_j
d_i^{ref}	Distance of Alternative A_i from Reference Point
d_i^{corr}	Corrected Distance
σ	Distance Correction Factor
p	Minkowski Distance Order ($p \geq 1$)
α	Adaptive Trade-Off Parameter ($\alpha \in [0, 1]$)
S_i	Final AURA Score of Alternative A_i

LIST OF ABBREVIATIONS

Abbreviations	
AHP	Analytic Hierarchy Process
ANP	Analytic Network Process
AURA	Adaptive Utility Ranking Algorithm
AVG	Average Solution
COBRA	Comprehensive Distance-Based Ranking
CORPAS	Complex Proportional Assessment
DGHM	Dombi Generalized Heronian Mean
DM	Decision Matrix
DSS	Decision Support System
ELECTRE	Élimination et Choix Traduisant la Réalité
FUCOM	Full Consistency Method
GHM	Generalized Heronian Mean
HM	Heronian Mean
ITARA	Indifference Threshold-based Attribute Ratio Analysis
MACBETH	Measuring Attractiveness by a Categorical-Based Evaluation Technique
MAUT	Multi-Attribute Utility Theory
MABAC	Multi-Attributive Border Approximation Area Comparison
MAVT	Multi-Attribute Value Theory
MCDM	Multi-Criteria Decision-Making
MOORA	Multi-Objective Optimization by Ratio Analysis
PIS	Positive Ideal Solution
NIS	Negative Ideal Solution
QFD	Quality Function Deployment
q-ROFS	q-Rung Orthopair Fuzzy Set
qROFDGHM	q-Rung Orthopair Fuzzy Dombi Generalized Heronian Mean
qROFDWGHM	Weighted q-Rung Orthopair Fuzzy Dombi Generalized Heronian Mean
SAW	Simple Additive Weighting
TOPSIS	Technique for Order Preference by Similarity to Ideal Solution

VIKOR	Višekriterijumska Optimizacijal Kompromisno Rešenje
WASPAS	Weighted Aggregated Sum Product Assessment
WSM	Weighted Sum Model
WPM	Weighted Product Model

CHAPTER 1
INTRODUCTION

1.1 Research Background

Decision-making in modern contexts has become increasingly complex as decision-makers are often required to evaluate multiple, interrelated and sometimes conflicting criteria simultaneously. Thus, Multi-Criteria Decision-Making (MCDM) serves as an essential framework to address this challenge by providing systematic approaches for ranking, selecting, and prioritizing alternatives (Khodamipour et al., 2022; Mardani et al., 2016). Classical techniques such as the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) and the VIseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) have been successfully applied across domains like healthcare, finance, supply chain management, and education (Esangbedo & Tang, 2023; Ziemba, 2022). However, these methods often rely on rigid normalization schemes and distance measures that fail to adapt to the dynamic nature of modern decision environments, especially when uncertainty and target-type criteria are involved (Al-Quran et al., 2024).

To address these limitations, this study introduces the Adaptive Utility Ranking Algorithm (AURA), a new distance-based ranking method designed to provide greater flexibility and adaptability in MCDM. The AURA method unifies benefit, cost, and target-type criteria under a single ranking structure through an adaptive normalization mechanism. Unlike conventional methods that only assess performance based on extreme values (maximum or minimum), AURA is capable of evaluating alternatives when the desired value lies somewhere in the middle of the range. This feature allows decision-makers to model realistic scenarios where the best performance may not necessarily correspond to the highest or lowest value. Moreover, AURA includes a correction factor that enhances ranking discrimination and reduces ambiguity when alternatives exhibit similar performance, making it a more stable and interpretable decision-making tool.

While developing a robust algorithm is important, its adoption also depends

on accessibility and practical usability. Many decision support systems (DSS) remain limited by their closed-source architecture and lack of integration with fuzzy or uncertain models (Clarke et al., 2021; Zhu et al., 2021). To bridge this gap, the AURA method is implemented within an open-source, Python-based Streamlit platform. This system provides a user-friendly interface that allows users to upload datasets, define criterion types (benefit, cost, or target), assign weights, and automatically generate visualized rankings. The integration of algorithm and system enhances transparency, reproducibility and scalability, ensuring that AURA can be effectively applied across research and industry settings.

Beyond ranking, MCDM frameworks also depend on effective aggregation of expert judgments, particularly under uncertain or hesitant conditions. Fuzzy set theory, originating from Zadeh's classical formulation (Zadeh, 1965), has evolved through various generalizations such as intuitionistic fuzzy sets (Atanassov, 1986), Pythagorean fuzzy sets (Yager, 2014), and q-rung orthopair fuzzy sets (q-ROFS) (Yager, 2017). These extensions allow a more expressive representation of membership and non-membership, improving the handling of uncertainty and hesitancy in expert evaluation. In this study, a novel aggregation operator, termed the q-Rung Orthopair Fuzzy Dombi Generalized Heronian Mean (qROFDGHM), is proposed to enhance the modeling of interdependencies among criteria and to capture the inherent fuzziness of expert assessments. This development is grounded in existing advances in q-rung orthopair fuzzy sets, Dombi operators, and Heronian mean aggregation frameworks, which have been widely applied to model uncertainty and interdependent criteria in complex decision-making environments (Wei et al., 2018; Zhong et al., 2019).

The integration of qROFDGHM with AURA forms a comprehensive fuzzy–adaptive framework that combines advanced fuzzy aggregation and adaptive ranking. In this framework, linguistic assessments from multiple experts are first expressed as q-rung orthopair fuzzy numbers and aggregated using the qROFDGHM operator to generate a collective fuzzy decision matrix. The aggregated values are then defuzzified into crisp scores and analyzed using the AURA method to produce interpretable and reliable rankings. This combined qROFDGHM–AURA model is validated using a real-world case study in educational technology selection and benchmarked against established MCDM methods, including TOPSIS, VIKOR, and

SAW, to demonstrate its consistency, robustness, and practical applicability. Such comparative benchmarking against classical MCDM techniques is a well-established validation strategy in decision science research (Esangbedo & Tang, 2023; Ziemba, 2022).

Overall, this research bridges theoretical innovation and practical implementation by developing an adaptive decision-making algorithm (AURA), implementing it through an open-source decision support system, proposing a new fuzzy aggregation operator (qROFDGHM), and integrating both into a unified fuzzy–adaptive MCDM framework. This holistic approach contributes to advancing both the mathematical foundation and real-world utility of MCDM, particularly in environments characterized by uncertainty, interdependence, and complex evaluation structures.

1.2 Problem Statement

Despite the significant progress achieved in the field of MCDM, several methodological and practical challenges continue to hinder the effectiveness of existing approaches. These challenges concern the treatment of different criterion types, the accessibility of decision support tools, the aggregation of uncertain expert evaluations, and the integration of fuzzy aggregation with adaptive ranking methods.

The first problem concerns the rigid and type-specific normalization procedures used in classical distance-based MCDM methods. Classical methods such as TOPSIS and VIKOR have been widely adopted across various application domains due to their computational simplicity and interpretability (Esangbedo & Tang, 2023; Ziemba, 2022). However, these methods rely on static normalization and distance formulations that are mainly designed for benefit-type or cost-type criteria. Mathematically, TOPSIS normalizes each column using the vector norm $r_{ij} = x_{ij}/\sqrt{\sum_i x_{ij}^2}$ and ranks alternatives according to their closeness to the fixed extreme values $\max_i v_{ij}$ and $\min_i v_{ij}$ (Hwang & Yoon, 1981). Similarly, VIKOR determines the best and worst values based on $f_j^* = \max_i x_{ij}$ and $f_j^- = \min_i x_{ij}$ (Opricovic & Tzeng, 2004). These formulations assume that the preferred value must be located at one of the extremes of the observed data range. Consequently, they are not appropriate when a criterion has an intermediate target value as its desired level of performance. In such situations, alternatives that

are close to the actual target may be penalized, whereas alternatives with extreme values may be favoured. Therefore, a unified normalization mechanism is needed to evaluate benefit, cost, and target-type criteria within a single decision-making structure (Al-Quran et al., 2024).

The second problem is the limited availability of accessible and transparent decision support systems for modern MCDM applications. Although many modified ranking methods have been proposed, a considerable number remain at the theoretical level without being translated into practical computational tools. Decision support systems that are closed-source or restricted to a particular decision structure limit the usability of MCDM techniques, especially for practitioners who do not possess advanced programming or mathematical expertise (Clarke et al., 2021; Zhu et al., 2021). Existing reviews of MCDM software indicate that many available packages are proprietary, provide limited control over parameters, and offer insufficient support for fuzzy or target-type inputs (Ayan et al., 2023; Borges Souza et al., 2021; Gül & Güneri, 2021). This weakens the transparency, reproducibility, and auditability of the decision-making process. Hence, there is a need for an open, transparent, and reproducible decision support platform that implements a modern MCDM algorithm in a user-friendly environment.

The third problem relates to the aggregation of expert judgments in the presence of uncertainty, hesitation, and interdependence among criteria. In complex decision-making problems, criteria may influence one another, while experts may express uncertainty or hesitation in their assessments. Although fuzzy extensions such as intuitionistic fuzzy sets, Pythagorean fuzzy sets, and q-rung orthopair fuzzy sets (q-ROFS) have improved the representation of uncertainty, there remains a lack of generalized aggregation operators that can simultaneously model inter-criterion interaction and nonlinear dependence (Al-Quran et al., 2024; Liu & Wang, 2018; Yager, 2017). Conventional aggregation operators, such as the weighted arithmetic mean and weighted geometric mean, generally assume that criteria are independent, which may not reflect real decision-making conditions (Wei et al., 2018; Zhong et al., 2019). As a result, the aggregated values may not accurately represent collective expert preferences. Therefore, an aggregation operator is needed that can represent expert hesitancy through q-ROFS, incorporate nonlinear flexibility through Dombi operations,

and capture pairwise relationships among criteria through the Heronian mean.

The fourth problem is the absence of a unified framework that combines advanced fuzzy aggregation with adaptive ranking. Although various fuzzy MCDM frameworks have been developed, advanced fuzzy aggregation operators are commonly integrated with conventional ranking methods such as TOPSIS or VIKOR (Sarkar et al., 2023; Wang et al., 2024). At the same time, adaptive ranking developments are rarely combined with an expressive fuzzy aggregation stage (Esangbedo & Tang, 2023; Krstić et al., 2022). This separation limits the ability of existing frameworks to process linguistic uncertainty, expert hesitation, and mixed-type criteria within a single analytical procedure. Consequently, decision outcomes may not fully reflect realistic evaluation conditions and may be less interpretable to decision-makers. Therefore, a unified framework is required to transform uncertain linguistic assessments into an interpretable and target-aware ranking outcome through the integration of fuzzy aggregation and adaptive ranking.

In summary, this study addresses four interrelated issues in MCDM: rigid normalization procedures for different criterion types, limited availability of transparent decision support systems, insufficient aggregation mechanisms for uncertain and interdependent evaluations, and the lack of an integrated fuzzy–adaptive decision-making framework. These issues provide the basis for the research objectives presented in the following section.

1.3 Research Questions

This study addresses the following research questions:

a) How can a unified, adaptive, distance-based ranking algorithm be developed to evaluate benefit-, cost-, and target-type criteria within a single structure?

b) How can uncertainty and interdependency among criteria be modelled through a q-rung orthopair fuzzy Dombi generalized Heronian mean (qROFDGHM) aggregation operator?

c) How does the integrated qROFDGHM–AURA framework perform when applied to and validated in educational technology evaluation, relative to established

MCDM methods?

1.4 Research Objectives

The objectives of this study are:

a) To develop the AURA method and implement it within a Python-based Streamlit decision support system for MCDM.

b) To propose a novel qROFDGHM aggregation operator by integrating the Heronian mean with Dombi operations under the q-rung orthopair fuzzy environment.

c) To apply and validate the integrated qROFDGHM–AURA framework in educational technology evaluation through sensitivity, comparative, and correlation analyses.

1.5 Justification of the Study

This section justifies the development of the AURA and its integration with the proposed qROFDGHM operator.

The development of AURA is motivated by a limitation of TOPSIS in handling criteria with an intermediate preferred value. In TOPSIS, each criterion is evaluated relative to fixed extreme values, namely the maximum value for a benefit criterion and the minimum value for a cost criterion. The relative closeness coefficient, rewards alternatives that are closer to the positive ideal solution and farther from the negative ideal solution (Hwang & Yoon, 1981; Opricovic & Tzeng, 2004). However, some decision-making situations require a target value rather than the highest or lowest possible value. Examples include a preferred response time, an appropriate class size, or a balanced cost-to-feature ratio. In such cases, the extreme values may not represent the actual optimum. AURA addresses this limitation by allowing the preferred value of a target-type criterion to be located within the observed criterion range. It also incorporates a correction factor to improve the separation between alternatives with closely related distance values. Therefore, AURA extends the conventional

distance-based approach by accommodating benefit, cost, and target-type criteria within a single procedure.

AURA belongs to the distance-based MCDM family. Among existing methods, the Comprehensive Distance-Based Ranking (COBRA) method is the closest in terms of its evaluation structure because it assesses alternatives using multiple reference points, including the positive, negative, and average solutions (Krstić et al., 2022). However, AURA differs from COBRA in two main aspects. First, AURA accommodates target-type criteria through its adaptive normalization procedure. Second, it incorporates a dispersion-based correction factor to improve discrimination between closely ranked alternatives. The empirical comparison reported in Chapters 3 and 5 shows strong agreement between AURA and COBRA, with Spearman correlation coefficients reaching 1.000. This indicates that both methods share similar ranking principles while AURA provides additional flexibility for mixed criterion types.

The proposed framework is relevant to several groups of users. Decision-makers and analysts in areas such as education, supply chain management, healthcare, and public policy may use the framework to evaluate alternatives involving benefit, cost, and target-type criteria under uncertain multi-expert assessments. Researchers in MCDM and fuzzy decision-making may benefit from the proposed qROFDGHM operator and the methodological integration of fuzzy aggregation with adaptive ranking. The AURA software also provides an open-source and reproducible implementation of the ranking procedure that can support further methodological development. In addition, educators and students may use the step-by-step AURA software interface to understand the main stages of distance-based MCDM without requiring advanced programming knowledge. Within the present application, the framework is particularly relevant to institutions that need to evaluate educational technologies for English language learning.

1.6 Significance of the Study

This research contributes both theoretically and practically to the advancement of MCDM by integrating methodological innovation, fuzzy aggregation, and open-source implementation into a single coherent framework.

In terms of theoretical part, this study advances the field of decision making by introducing the AURA, a new distance-based ranking method capable of

evaluating benefit, cost and target-type criteria within a unified normalization structure. The target-type formulation enables AURA to handle mid-range optimal solutions, addressing a key limitation of existing MCDM models like TOPSIS and VIKOR. Furthermore, the proposed qROFDGHM operator expands the theoretical foundation of fuzzy aggregation by combining Dombi operations with the Heronian mean, enabling more accurate representation of uncertainty, hesitation and interdependencies among criteria. Thus, both of these developments contribute to the growing body of knowledge on fuzzy and adaptive decision-making methods.

Meanwhile, as for the methodological part, the study integrates the qROFDGHM operator with the AURA method to establish a comprehensive fuzzy–adaptive framework. This integration demonstrates how uncertainty and adaptive ranking can coexist within a single analytical structure, allowing for the systematic transformation of linguistic fuzzy data into crisp decision outcomes. The framework not only enhances interpretability but also promotes consistency and robustness in ranking, providing an improved alternative to existing distance-based or fuzzy MCDM methods.

As for the practical aspect, the study bridges theory and application by developing an open-source, Python-based Streamlit decision support system that implements the AURA method. This system provides a transparent, user-friendly, and scalable environment that allows practitioners and researchers to upload datasets, configure decision parameters, and automatically obtain results with detailed normalization and ranking outputs. Such a tool democratizes access to advanced MCDM techniques, promotes reproducibility, and encourages adoption across various domains such as education, engineering, and management.

Overall, the findings and outcomes of this study are expected to contribute to both academic research and real-world decision-making by offering a more flexible, accurate, and accessible framework for handling complex multi-criteria problems characterized by uncertainty and interdependence.

1.7 Scope and Limitations of the Study

The scope of this study comprises three main parts in the field of MCDM. This includes the methodological part, computational implementation and practical application within the proposed AURA and qROFDGHM area.

In terms of methodological scope, the research concentrates on distance-based MCDM techniques that employ adaptive normalization and fuzzy aggregation. The AURA method is specifically designed to handle three types of criteria which are benefit, cost, and target-type within a unified computational structure. Meanwhile, the qROFDGHM operator focuses on the aggregation of expert judgments represented in the form of q-ROFS, where uncertainty, hesitation, and interdependencies among criteria are explicitly modeled. The integration of these two components forms the foundation of the proposed fuzzy–adaptive MCDM framework.

In the computational implementation, this study employs the Python programming language as the primary platform for algorithmic development and system implementation. The Streamlit library is used to design an interactive and open-source decision support system that allows users to upload data, configure decision parameters, and generate ranking results in real time. The implementation relies specifically on the open-source Python libraries `NumPy` and `Pandas` (matrix construction and data handling), `Matplotlib` (charting), `XlsxWriter` (automated Excel reporting), and `Streamlit` for the browser-based interface, running on `Python` ≥ 3.11. This system is optimized to handle large-scale datasets efficiently through Python's computational libraries and vectorized processing, ensuring scalability for extensive MCDM applications. Computational experiments and validations are performed on datasets derived from real-world decision-making problems, particularly within the context of educational technology selection.

As for the limitation, the application is only focused on the domain of decision-making problems that involve multiple evaluation criteria, uncertain judgments, and linguistic assessments from multiple experts. Here, our proposed framework is validated using a specific case study in educational technology selection, which serves as a representative example of complex decision scenarios that include benefit, cost, and target-type criteria.

Despite its contributions, this study acknowledge certain limitations. First, the accuracy and effectiveness of the AURA method depend on the appropriate determination of parameter values, such as correction and weighting factors, which may vary across decision contexts. Second, the qROFDGHM operator assumes consistent expert evaluation behavior within the q-ROFS framework, which may not always

hold true in heterogeneous decision groups. Third, while the Streamlit-based DSS is capable of handling large datasets, its current implementation is designed for single-user operation; integrating multi-user or distributed real-time computation would require further system development.

Overall, the study maintains a balance between methodological rigor and practical usability. The defined scope ensures focused contributions within the areas of adaptive ranking, fuzzy aggregation, and decision support implementation, while the stated limitations provide direction for potential future research and system enhancements.

1.8 Thesis Outline

This thesis consists of six chapters, each contributing to the overall development and validation of the proposed fuzzy–-adaptive decision-making framework based on the integration of the AURA and the qROFDGHM operator. The structure of the thesis is as Figure 1.1.

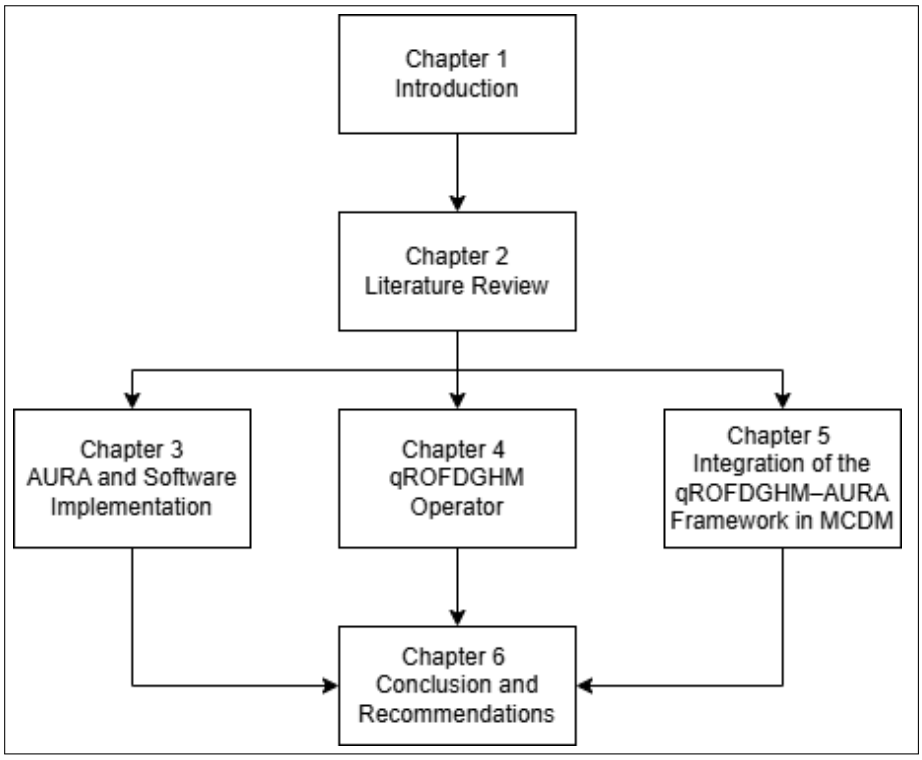


Figure 1.1 Structure of the Thesis

Figure 1.1 should be read as a foundation-then-development structure rather

than a single linear chain. Chapter 2 (Literature Review) establishes the shared theoretical foundation: the limitations of classical MCDM, the q-rung orthopair fuzzy preliminaries, the Dombi operations, and the Heronian mean. Because all three later chapters build on this common foundation, the flow from Chapter 2 branches into Chapters 3, 4, and 5 rather than proceeding strictly in sequence. Chapter 3 develops the AURA ranking method (answering RO1) and draws on the distance-based MCDM material of Chapter 2. Chapter 4 develops the qROFDGHM operator (answering RO2) and draws on the fuzzy-set, Dombi, and Heronian material of Chapter 2. Chapter 5 then integrates the two independent contributions of Chapters 3 and 4 into the unified qROFDGHM–AURA framework and validates it (answering RO3). Chapters 3 and 4 are thus parallel methodological developments that converge in Chapter 5, which is why the diagram shows Chapter 2 feeding all three.

Chapter 1: Introduction

This chapter introduces the research background, problem statement, objectives, research questions, significance, scope, and limitations of the study. It outlines the motivation behind developing an adaptive MCDM framework and concludes by presenting the organization of the thesis.

Chapter 2: Literature Review

This chapter reviews existing works in MCDM, focusing on distance-based and fuzzy-based approaches. It covers related studies on q-ROFS, Dombi operations, Heronian mean, and recent developments such as the AURA and COBRA methods. The chapter concludes with a gap analysis that justifies the need for the proposed qROFDGHM–AURA framework.

Chapter 3: The AURA and Software Implementation

This chapter details the mathematical formulation, normalization process, and computational framework of the AURA method. It also presents the design and implementation of the AURA decision support system using the Python–Streamlit environment, highlighting its workflow, interface, and computational capabilities.

Chapter 4: The q-Rung Orthopair Fuzzy Dombi Generalized Heronian Mean (qROFDGHM) Operator

This chapter introduces the theoretical foundation of the proposed aggregation operator. It provides definitions, properties, and proofs for the qROFDGHM, integrating

Dombi operations with the Heronian mean under q-ROFS. Illustrative numerical examples are provided to demonstrate the operator's behavior and validity.

Chapter 5: Integration of the qROFDGHM–AURA Framework in MCDM

This chapter presents the integration process between qROFDGHM and AURA, forming the complete fuzzy–adaptive framework. It describes the step-by-step procedure involving linguistic evaluation, fuzzy aggregation, defuzzification, and adaptive ranking. A real-world case study on educational technology selection is conducted, followed by comparative analysis with established MCDM methods.

Chapter 6: Conclusion and Recommendations

This final chapter summarizes the key findings and contributions of the study. It discusses the theoretical, methodological, and practical implications, revisits the research objectives, and provides recommendations for future work and possible extensions of the proposed framework.

CHAPTER 2
LITERATURE REVIEW

2.1 Introduction

This chapter provides an overview of the theoretical background and related studies on multi-criteria decision-making (MCDM) and its extensions in uncertain environments. It outlines the fundamental principles of MCDM, the role of decision support systems in facilitating structured evaluation, and the significance of q-Rung Orthopair Fuzzy Sets (q-ROFS) in modelling expert uncertainty. The discussion also highlights the development of aggregation operators, including Heronian and Dombi-based formulations, which form the mathematical basis for the proposed framework. The chapter concludes by identifying the existing research gaps that justify the integration of the q-Rung Orthopair Fuzzy Dombi Generalized Heronian Mean (qROFDGHM) operator with the AURA method in this study.

2.2 Multi-Criteria Decision-Making (MCDM)

MCDM is a crucial part of decision science, addressing the challenge of making decisions given multiple conflicting criteria. In today's complex society, decision-making encompasses various fields, including business, engineering, healthcare, environmental management, and public policy (Taherdoost & Madanchian, 2023). When faced with real-world problems, individuals often need to evaluate many potential solutions against various criteria. To help people make rational, unambiguous, and defensible judgments, a structured approach based on MCDM techniques has been developed. The standard procedure for using the MCDM method is to: (i) identify the alternatives and criteria relevant to the problem; (ii) assign weights to each criterion; (iii) evaluate the performance of each alternative against the criteria; and (iv) rank the alternatives based on their overall performance across all criteria.

Over time, these MCDM methods have evolved significantly by integrating both traditional and modern approaches to enhance decision-making processes across a wide range of fields (Zhang & Duan, 2024). Traditional MCDM methods can be classified

into several categories based on their underlying characteristics, such as scoring, distance-based, pairwise comparison, outranking, and utility-based or value-based approaches (Penadés-Plà et al., 2016). Table 2.1 presents a numerical grouping of MCDM methods, highlighting some of the representative methods within each category.

Table 2.1
MCDM Methods

MCDM Group	**MCDM Method**	**Acronym**	**References**
Scoring	Weighted Sum Model	WSM	Fishburn (1967)
	Simple Additive Weighting	SAW	Podvezko (2011)
	Complex Proportional Assessment	CORPAS	Podvezko (2011)
	Weighted Product Model	WPM	Zavadskas et al. (2012)
Distance-Based	Technique for Order of Preference by Similarity to Ideal Solution	TOPSIS	Hwang and Yoon (1981)
	Multi-Objective Optimization by Ratio Analysis plus the full Multiplicative form	MOORA	Brauers et al. (2010)
	Weighted Aggregated Sum Product Assessment	WASPAS	Zavadskas et al. (2012)
	Vlsecriterijumska Optimizacija Kompromisno Resenje	VIKOR	Mardani et al. (2016)
	Comprehensive Distance-Based Ranking	COBRA	Krstić et al. (2022)
Pairwise Comparison	Analytic Hierarchy Process	AHP	Saaty (1987)
	Measuring Attractiveness through a Categorical-Based Evaluation Technique	MACBETH	Costa and Vansnick (1994)
	Analytic Network Process	ANP	Saaty and Vargas (2013)

MCDM Group	MCDM Method	Acronym	References
	Full Consistency Method	FUCOM	Pamučar et al. (2018)
Outranking	Elimination et Choix Traduisant la Réalité	ELECTRE	Roy (1968)
	Preference Ranking Organization Method for Enrichment Evaluations	PROMET-HE	Mareschal (2005)
	Multi-Attributive Border Approximation Area Comparison	MABAC	Pamučar and Ćirović (2015)
	Indifference Threshold-based Attribute Ratio Analysis	ITARA	Hatefi (2019)
Utility/Valuate	Multi-Attribute Utility Theory	MAUT	Sarabando and Dias (2010)
	Multi-Attribute Value Theory	MAVT	
Others	Quality Function Deployment	QFD	Chan and Wu (2002)

Penadés-Plà et al. (2016) mentioned that the most basic MCDM techniques are the scoring techniques. Their foundation is evaluating the options using basic mathematical concepts. The basic concept behind the distance-based method is to calculate the distance between every alternative and a specific point. On the other hand, the pairwise comparison methods help obtain the weight of the different criteria and compare alternatives with respect to a subjective criterion. As for the outranking methods, a preference relation is established on a number of alternatives that shows the level of dominance among them. Lastly, the utility methods define expressions that determine the degree of satisfaction of the criteria.

Therefore, the method that is created in this study belongs to the distance-based MCDM methods family similar as TOPSIS, VIKOR, MOORA, WASPAS, and COBRA, with special attention given to these methods among the ones mentioned above. All of these approaches use the same ranking logic, which assesses alternatives according to different kinds of distances from a reference alternative. Since Simple Additive

Weighting (SAW), a scoring method, is still a widely used traditional MCDM techniques, proving its ongoing relevance and applicability, it is also included in the comparative study.

Majority of the MCDM studies employs SAW method. It is applied in various disciplines to evaluate alternatives based on several criteria in order to present an easy way of making decisions. In the SAW approach each criterion is associated with a weight where the scores of the alternatives are normalized, and the weighted sum is calculated to determine the preferred alternative. Its simplicity and effectiveness in solving decision-making problems in various uncertain environments make this method particularly appreciated. The SAW method has been successfully implemented in ranking educational institutes using fuzzy logic (Hiremani et al., 2024), integrated with recurrent neural networks (RNN) for resource allocation in cloud computing (Purohit et al., 2023), and revised for manufacturing environments (Biswas & Chaki, 2022). However, despite its strengths, SAW assumes a linear nature of criteria weighting, limiting its ability to capture complex relationships. Extensions such as fuzzy SAW and hybrid approaches have been proposed to address these issues (Wang, 2020).

Meanwhile, the TOPSIS method ranks alternatives according to their geometric closeness degree to the positive–ideal and negative–ideal solutions. It is considered simple, logically consistent, and popular due to its straightforward mathematical formulation. Nevertheless, TOPSIS has some flaws, particularly its reliance on additive distance aggregation and its assumption that moving closer to the positive ideal and farther from the negative ideal always leads to the best choice (Opricovic & Tzeng, 2004). Applications include green supplier selection (Calık, 2021), waste–to–energy technology selection (Alao & Popoola, 2021), vehicle routing (Micale et al., 2019), and cost–effectiveness evaluation of green infrastructure (Zeng et al., 2021).

Similar to TOPSIS, the VIKOR method considers utility, regret, and distance from ideal and worst solutions to compare options. Unlike TOPSIS, VIKOR emphasizes compromise solutions, ensuring that the top-ranked alternative is closest to the ideal answer. VIKOR has been applied in diverse domains such as web-based expert chatbots (Purwanto & Susilawati, 2024), medical diagnostics (Nivedita et al., 2024), and sequential classification ranking (Xu & Li, 2024).

On the other hand, the MOORA method ranks alternatives by evaluating their

relative distances from a reference point. It is appreciated for reducing computational complexity, providing consistent results, and requiring fewer parameters than VIKOR. MOORA has been applied in renewable energy project selection (Petrov, 2021), academic staff recruitment (Sitorus et al., 2024), and pilot area determination (Simamora et al., 2024).

In contrast, the WASPAS method combines the weighted sum model and the weighted product model, providing reliable results across various applications. It has been applied in education program selection (Ramadani et al., 2024), librarian recruitment (Suendri & Wardani, 2024), and waste reduction system design (Wayahdi & Ruziq, 2024).

As for distance-based MCDM, the COBRA method has gained attention for combining the strengths of multiple approaches. By integrating distance calculations, COBRA avoids the ambiguity of choosing which distance metric to use. It has been applied in e-commerce strategy selection (Popović et al., 2022) and IT supplier evaluation using neutrosophic sets (Razak et al., 2025).

Beyond describing these methods, a critical appraisal is required to expose the specific weaknesses that motivate the present study. Table 2.2 summarizes the strengths, weaknesses, and residual gaps of the distance-based and scoring methods most relevant to AURA. The recurring weaknesses are mathematical rather than incidental.

Table 2.2
Critical Comparison of Representative MCDM Methods

Method	**Strengths**	**Weaknesses (mathematical)**	**Residual gap**
SAW	Simple, transparent, additive	Assumes full compensation and linear value; no interaction among criteria	Cannot model target criteria or interaction
TOPSIS	Intuitive closeness to ideal; widely validated	Fixed extreme ideals; rank reversal on alternative addition/removal; vector normalization distorts target criteria	No target-type criterion; sensitive to reference set
VIKOR	Compromise solution; group utility/regret balance	Requires the v trade-off parameter; extreme-anchored f_j^*, f_j^-; ties need accuracy rule	No target-type criterion; parameter dependence
MOORA	Few parameters; stable	Ratio normalization sensitive to scale; benefit/cost only	No target criterion; no interaction
WASPAS	Combines WSM and WPM; tunable λ	Still benefit/cost only; product term unstable near zero	No target criterion
COBRA	Multiple reference points (PIS, NIS, average)	Benefit/cost only; no dispersion correction for near-ties	Closest to AURA but lacks target handling and correction factor

Three mathematical limitations cut across these methods and directly motivate the proposed AURA–qROFDGHM framework. The first limitation concerns normalization. TOPSIS, VIKOR, MOORA, WASPAS, and COBRA normalize criteria toward an extreme reference. Therefore, a criterion with an optimum at an interior target value cannot be appropriately represented. AURA addresses this gap through its target-type normalization and correction factor. The second limitation concerns the assumption of criterion independence. The scoring and distance-based methods discussed above aggregate criteria additively or multiplicatively under an implicit independence assumption, thereby ignoring possible interactions among criteria. This limitation is addressed through the Heronian mean. The third limitation concerns uncertainty representation. These crisp methods cannot natively represent expert

hesitancy, which is addressed through the q-rung orthopair fuzzy environment. The proposed framework is designed to address these three limitations simultaneously, which no single existing method does.

Despite these advancements, a common gap persists: existing methods rely on fixed reference points and traditional normalization schemes, which limit their adaptability in real-world decision-making contexts. The Adaptive Utility Ranking Algorithm (AURA) is proposed to addresses this gap by introducing a flexible normalization scheme and a unified distance measure. Unlike conventional methods that restrict ideal solutions to extremes, AURA allows the optimal point to be located anywhere within the data range, enhancing flexibility, computational efficiency, and applicability in diverse decision-making scenarios.

2.3 Decision Support Systems and Software in MCDM

Decision Support Systems (DSS) are interactive computer-based tools that assist decision-makers in analyzing data, evaluating alternatives, and choosing optimal solutions in complex environments. In the context of MCDM, DSS are particularly valuable as they integrate mathematical models with user-friendly software to enhance both interpretability and usability (Borges Souza et al., 2021; Gül & Güneri, 2021). Traditional MCDM techniques such as TOPSIS, VIKOR, and SAW have been widely implemented in DSS across diverse domains including healthcare, engineering, and environmental management due to their structured nature and ease of interpretation (Esangbedo & Tang, 2023; Khodamipour et al., 2022).

However, conventional DSS implementations often face limitations such as rigid normalization mechanisms, lack of flexibility in handling benefit, cost, and target-type criteria, and difficulties in scaling to large, high-dimensional datasets (Ayan et al., 2023; Ziemba, 2022). Moreover, most classical DSS software lacks interactivity, automated reporting, or the capability to accommodate user-defined benchmarks, making them less adaptable for modern data-driven decision-making (Clarke et al., 2021; Zhu et al., 2021).

To overcome these challenges, recent advances have emphasized integrating fuzzy logic, neutrosophic sets, and hybrid approaches into DSS software, thereby enabling the handling of uncertainty, imprecision, and conflicting information in practical decision scenarios (Al-Quran et al., 2024; Hammad et al., 2025; Yang et al.,

2025). This evolution has led to a new generation of decision support software capable of balancing methodological rigor with practical usability.

Therefore, the AURA represents a recent innovation in this area. AURA unifies benefit, cost, and target-type criteria within a single scoring function, supported by a Python-based Streamlit interface that provides scalability, transparency, and user accessibility. Its open-source implementation bridges the gap between theoretical models and applied DSS, supporting applications across finance, healthcare, supply chain, and education. Furthermore, empirical validations demonstrate that AURA achieves ranking consistency with established methods like TOPSIS and VIKOR while maintaining computational efficiency on large-scale datasets.

In summary, the integration of advanced MCDM methodologies into decision support software has transformed DSS from rigid computational tools into adaptive, scalable, and user-friendly systems. With tools like AURA, DSS are increasingly positioned as essential enablers of data-driven strategic decision-making in complex and uncertain environments.

2.4 q-Rung Orthopair Fuzzy Sets

The concept of fuzzy sets (FS) was first introduced by Zadeh (1965), allowing partial membership values between 0 and 1 to represent uncertainty. However, FS could not adequately capture hesitation due to its binary complementarity between membership and non-membership degrees. To address this, Atanassov (1986) proposed the intuitionistic fuzzy set (IFS), which incorporates both membership (μ) and non-membership (ν) degrees, with the condition $\mu + \nu \leq 1$. The inclusion of a hesitancy degree enhanced flexibility, but the restricted range imposed limitations in highly uncertain decision-making environments (Voskoglou, 2023).

Building upon IFS, Yager (2014) introduced the Pythagorean fuzzy set (PFS), where the squared sum of membership and non-membership degrees satisfies $\mu^2 + \nu^2 \leq 1$. This extension increased expressive power, as decision-makers could assign values that IFS could not accommodate. PFS was successfully applied in areas such as risk analysis, goal programming, and supplier selection (Calık, 2021; Peng & Yang, 2015). Despite its enhanced representational capacity, PFS remained restrictive when handling extreme uncertainty and conflicting information.

To further generalize these frameworks, Yager (2017) proposed the q-Rung Orthopair Fuzzy Set (q-ROFS). In q-ROFS, the condition is extended to $\mu^q + \nu^q \leq 1$, where $q \geq 1$. This generalization means that IFS and PFS are special cases of q-ROFS when $q = 1$ and $q = 2$, respectively. As q increases, the permissible domain of membership and non-membership pairs expands, offering significantly greater flexibility for representing vagueness, hesitation, and contradiction in decision-making contexts. For example, values such as $(\mu = 0.6, \nu = 0.9)$, which are invalid in IFS and PFS, can be valid under q-ROFS with sufficiently large q.

The adoption of the q-rung orthopair fuzzy set (q-ROFS) environment is justified by its wider admissible membership space, flexibility in representing uncertainty, and generality. The admissible region is governed by the constraint $\mu^q + \nu^q \leq 1$, where the feasible area of (μ, ν) expands as the value of q increases. IFS, defined by $\mu + \nu \leq 1$, and PFS, defined by $\mu^2 + \nu^2 \leq 1$, are obtained as special cases of q-ROFS when $q = 1$ and $q = 2$, respectively. Hence, q-ROFS generalizes both models and can accommodate assessment pairs such as $(0.6, 0.9)$ that are not permissible under either IFS or PFS. This enables experts to express relatively high membership and non-membership degrees simultaneously while satisfying the q-rung constraint. In comparison with spherical and picture fuzzy sets, which require an additional neutral or abstinence degree and satisfy the condition $\mu^2 + \eta^2 + \nu^2 \leq 1$, q-ROFS retains a two-dimensional representation. This reduces computational complexity while maintaining flexibility through the adjustable parameter q. It also avoids requiring the educational-technology experts in this study to provide a third assessment degree. In contrast to Fermatean fuzzy sets, which correspond to the fixed case $q = 3$, q-ROFS allows the rung parameter to be selected according to the decision context and data characteristics. Furthermore, q-ROFS preserves the orthopair structure required by the score function $S = \mu^q - \nu^q$ used throughout this thesis. This provides a consistent basis for converting fuzzy assessments into interpretable ranking values. Single-valued neutrosophic sets were not adopted because they involve a separate indeterminacy component, whereas the present study relies on the membership and non-membership structure of q-ROFS. Hesitant q-ROFS were also not considered because each expert provides only one Likert-scale judgment for each criterion, making multiple possible membership values unnecessary. Overall, q-ROFS provides an appropriate balance between expressive capability, model generality, and

computational simplicity for the present application.

Since its introduction, q-ROFS has attracted wide research interest. Several studies focused on developing score and accuracy functions to facilitate ranking of q-rung orthopair fuzzy numbers (q-ROFNs) (Peng & Dai, 2019; Peng et al., 2018). In parallel, distance measures for q-ROFS were proposed to enhance MCDM applications (Wang et al., 2024). Further advancements include the exploration of correlation measures for q-ROFS (Ch et al., 2025), as well as the extension to hesitant q-ROFS to capture scenarios where experts provide multiple possible membership values (Ghoushchi et al., 2024).

A substantial body of literature has focused on aggregation operators for q-ROFS, which play a crucial role in synthesizing information across criteria. Early works by Liu and Wang (2018) introduced q-ROFS aggregation operators such as weighted averaging and geometric operators. Subsequent research extended these to include Bonferroni mean (Wei et al., 2018), Heronian mean (Zhong et al., 2019), as well as Archimedean-based operators (Alimohammadi et al., 2024). These operators not only capture interdependencies between attributes but also provide flexibility in modeling nonlinear interactions, making them highly applicable in complex MCDM environments.

The applicability of q-ROFS has been demonstrated across diverse fields. In healthcare, it has been used for medical diagnosis and treatment evaluation under uncertain information (Nivedita et al., 2024). In supply chain and logistics, q-ROFS frameworks have improved supplier selection and reverse logistics assessment (Calık, 2021; Ulutaş, 2021). In sustainable urban transport, Sarkar et al. (2023) applied dual hesitant q-ROFS with advanced aggregation operators to identify optimal solutions. Other applications include education, policy evaluation, and financial decision-making, highlighting q-ROFS as a versatile and robust framework for modern decision support systems.

The reviewed applications of q-rung orthopair fuzzy sets can be grouped into four main areas. The first area involves medical and diagnostic decision-making under uncertain conditions (Nivedita et al., 2024). The second includes supplier selection, reverse logistics, and supply-chain evaluation (Calık, 2021; Ulutaş, 2021). The third covers sustainable transport and environmental planning, particularly through hesitant

and dual q-rung orthopair fuzzy extensions (Sarkar et al., 2023). The fourth area involves applications in education, policy evaluation, and financial assessment. Across these application areas, q-ROFS is commonly combined with an aggregation operator and a ranking method. This structure is also adopted in the present study. However, previous studies rarely integrate an interaction-aware aggregation operator, such as a Dombi–Heronian mean, with an adaptive ranking method that can accommodate target-type criteria. This limitation indicates an integration gap that the present study aims to address.

In summary, q-ROFS represents a significant advancement in fuzzy set theory, providing a unified and flexible structure that generalizes IFS and PFS. Its ability to accommodate wider ranges of membership and non-membership values enables decision-makers to effectively capture uncertainty, hesitation, and conflicting information. With continuous development in score functions, distance measures, correlation analysis, and aggregation operators, q-ROFS has become an indispensable tool in MCDM, offering more reliable and adaptable solutions in complex real-world decision-making scenarios.

2.5 Aggregation Operators

Aggregation operators (AOs) are fundamental tools in MCDM as they provide a systematic way of combining multiple attributes or evaluations into a single representative value (Beliakov et al., 2016). Classical AOs such as the arithmetic mean, geometric mean, and weighted average assume that all criteria are mutually independent. However, this independence assumption often fails in real-world decision-making where criteria may be interrelated or dependent. To address this limitation, more advanced operators such as the Bonferroni mean, Choquet integral, and Heronian mean (HM) were developed to capture interdependencies among criteria (Liu & Li, 2017; Liu et al., 2018).

The Heronian mean operator is particularly useful as it incorporates relationships among input arguments without requiring repeated computations. Unlike other operators that ignore interrelationships, HM captures interactions between attributes, ensuring a more comprehensive evaluation in MCDM problems (Liu et al., 2018). Its adaptability has motivated researchers to integrate HM with fuzzy and neutrosophic frameworks,

enhancing its capability to manage uncertainty and imprecision.

Recent studies have focused on extending HM to more complex fuzzy environments. For instance, the Archimedean Heronian mean has been applied to complex intuitionistic fuzzy sets to enhance decision-making accuracy (Alimohammadi et al., 2024), while the linguistic intuitionistic fuzzy Heronian mean has been employed in group decision-making (Liu et al., 2018). Similarly, hybrid models have emerged, such as the fuzzy robust PCA combined with HM, improving classification accuracy in high-dimensional datasets (Kurama, 2024).

Within the q-ROFS framework, AOs have been widely developed to aggregate uncertain information. Liu and Wang (2018) first introduced q-ROFS aggregation operators including weighted averaging and geometric operators. Wei et al. (2018) extended this by developing Bonferroni mean operators, while Zhong et al. (2019) proposed Dombi power Heronian mean operators under q-ROFS.

The integration of Dombi operations with HM has recently gained attention, particularly in complex fuzzy and neutrosophic environments. Dombi-based Heronian mean operators are capable of capturing non-linear dependencies while maintaining interpretability, which is critical in MCDM (Yaacob et al., 2024). Moreover, their adaptability makes them suitable for diverse application domains including healthcare, transportation, environmental management, and policy evaluation.

The Heronian mean is selected because it can account for the interaction among evaluation criteria while remaining mathematically interpretable. In real decision-making problems, criteria are often interrelated rather than fully independent. For example, in educational technology evaluation, engagement and interactivity may reinforce each other. A purely additive aggregation operator treats these criteria separately and may not adequately reflect their combined contribution to the overall assessment. The Heronian mean aggregates input values through pairwise combinations of criteria. This pairwise structure enables the operator to account for possible relationships among criteria rather than treating every criterion as fully independent. Compared with the Bonferroni mean, the Heronian mean represents pairwise interactions through a more compact aggregation structure. Compared with the Choquet integral, it does not require the specification of a fuzzy measure for all 2^n subsets of criteria. In addition, the parameters a and b provide flexibility in adjusting

the relative contribution and interaction pattern of the aggregated arguments. Therefore, the Heronian mean offers a suitable balance between the ability to represent criterion interaction and the need to retain a practical and interpretable aggregation process. For this reason, it is selected as the interaction mechanism in the proposed framework.

Dombi operations are selected because they provide flexibility in modelling the aggregation behaviour of fuzzy information while retaining a mathematically tractable structure. The Dombi t-norm and t-conorm are governed by a single positive parameter, λ, which controls the behaviour of the aggregation process. This parameter allows the operator to be adjusted according to the level of interaction or strictness required in a particular decision-making context. In contrast, algebraic and Einstein operations follow fixed functional forms and do not provide an adjustable parameter for modifying the aggregation behaviour. Although Hamacher and Yager operations also include parameterised forms, the Dombi family is particularly suitable for the present study because its additive generator and inverse generator allow the aggregated membership and non-membership degrees to be derived in closed form. This analytical property is important for developing the proposed q-rung orthopair fuzzy aggregation operator in Chapter 4. Therefore, Dombi operations offer a suitable combination of parameter flexibility, interpretability, and analytical convenience. These features support their integration with the Heronian mean under the q-rung orthopair fuzzy environment and justify their selection over the algebraic, Einstein, Hamacher, and Yager alternatives in the proposed framework.

In conclusion, AOs have evolved significantly from simple averaging techniques to sophisticated models that capture interdependencies and handle uncertainty. Among these, the Heronian mean and its variants—especially when combined with operators such as Dombi, Archimedean, and Bonferroni—offer robust frameworks for aggregating information in fuzzy and q-ROFS contexts. Thus, this evolution demonstrates the growing importance of AOs in enhancing decision-making under uncertainty and complexity.

2.6 Research Gap

Indeed, the review of literature across aggregation operators, fuzzy set extensions, and decision support software highlights several persistent limitations that

motivate this research. Although MCDM has matured through classical methods such as TOPSIS, VIKOR, and SAW (Khodamipour et al., 2022; Ziemba, 2022), these models still rely on rigid normalization and separate formulations for benefit, cost, and target criteria (Esangbedo & Tang, 2023). While fuzzy and neutrosophic extensions (e.g., intuitionistic, Pythagorean, and q-rung orthopair fuzzy sets) have improved the ability to capture uncertainty (Al-Quran et al., 2024; Liu & Wang, 2018; Yager, 2017), most existing decision frameworks have yet to integrate these sets seamlessly into adaptive and scalable computational models.

Similarly, aggregation operators such as Bonferroni, Heronian, and Dombi-based means provide powerful ways to synthesize fuzzy information (Wei et al., 2018; Zhong et al., 2019), but their deployment in practical software tools remains fragmented. On the other hand, decision support systems (DSS) implemented in Python, R, or MATLAB have increased accessibility, yet they often lack integration with modern fuzzy models, scalability to big data, and transparency for practitioners (Clarke et al., 2021; Zhu et al., 2021).

Recent advances such as the AURA address several of these concerns by unifying benefit, cost, and target criteria within a single scoring function and implementing a scalable Streamlit-based interface. However, important gaps remain in the existing MCDM literature. As summarized in Table 2.3, classical MCDM methods continue to rely on rigid and type-specific normalization procedures, with no unified mechanism for handling benefit, cost, and target criteria simultaneously. In addition, the integration of q-rung orthopair fuzzy sets with adaptive and scalable ranking algorithms remains limited. The practical implementation of interaction-aware aggregation operators is also fragmented, particularly within accessible decision support software. Existing fuzzy decision support systems may further lack sufficient transparency, reproducibility, and scalability. Moreover, many newly proposed methods are validated through only a limited set of analyses, without jointly considering sensitivity, comparative, and correlation-based evaluations. Therefore, the proposed qROFDGHM–AURA framework is developed to address these interrelated gaps through a unified fuzzy aggregation, adaptive ranking, and transparent decision support structure.

Table 2.3
Summary of Research Gaps in MCDM Literature

Theme	Existing Focus	Identified Gap
Classical MCDM	TOPSIS, VIKOR, SAW widely applied (Khodamipour et al., 2022; Ziemba, 2022)	Rigid normalization; separate formulations for benefit, cost, target criteria (Esangbedo & Tang, 2023)
Fuzzy Set Extensions	IFS, PFS, q-ROFS enrich uncertainty modeling (Al-Quran et al., 2024; Liu & Wang, 2018; Yager, 2017)	Limited integration of q-ROFS with adaptive, scalable algorithms
Aggregation Operators	Bonferroni, Heronian, Dombi means used in fuzzy MCDM (Wei et al., 2018; Zhong et al., 2019)	Deployment in practical DSS remains fragmented
Decision Support Systems	DSS software in Python/R/Matlab, user-friendly, visual (Clarke et al., 2021; Zhu et al., 2021)	Lack of integration with fuzzy models; weak scalability and transparency
Recent Advances (AURA)	Adaptive scoring unifies criteria; Python/Streamlit DSS	No incorporation of q-ROFS; need open-source extensibility
Target-type criteria	Benefit/cost dichotomy dominant in TOPSIS, VIKOR, MOORA, WASPAS, COBRA (Hwang & Yoon, 1981; Krstić et al., 2022)	No unified normalization for mid-range (target) optima; correction for near-ties absent
Operator–ranking integration	q-ROFS operators usually paired with a single classical ranker (Sarkar et al., 2023; Wang et al., 2024)	Interaction-aware Dombi–Heronian aggregation rarely coupled with adaptive ranking
Validation rigour	Many studies report a single ranking without robustness checks (Esangbedo & Tang, 2023)	Limited joint use of sensitivity, comparative, and correlation/concordance validation

In summary, although substantial progress has been achieved in classical MCDM methods, fuzzy set extensions, aggregation operators, and decision support software, these developments remain largely disjointed. Existing studies typically emphasize either advanced fuzzy modeling without adaptive ranking mechanisms or decision support implementations without integrating expressive fuzzy aggregation operators. To date, no unified framework integrates the q-rung orthopair fuzzy Dombi generalized Heronian mean operator within an adaptive scoring algorithm and an

open-source, scalable decision support system. Therefore, this research bridges this gap by developing an integrated qROFDGHM–AURA framework implemented in Python, ensuring methodological coherence, computational scalability, and practical transparency.

The fundamental concepts, preliminaries, and terminologies for q-rung orthopair fuzzy sets, Dombi operations, and the Heronian mean are presented in Chapter 4, particularly in Section 4.2. This arrangement allows the mathematical foundations to be introduced immediately before the development of the proposed qROFDGHM operator.

CHAPTER 3
THE ADAPTIVE UTILITY RANKING ALGORITHM (AURA) AND SOFTWARE IMPLEMENTATION

3.1 Introduction

This chapter focuses on the development of the Adaptive Utility Ranking Algorithm (AURA) and its corresponding software implementation. It explains the overall structure, computational procedure, and integration process of the AURA method as a Multi–Criteria Decision–Making (MCDM) approach.

The chapter is organized into two main parts. The first part describes the theoretical framework and mathematical formulation of AURA, including its normalization, weighting, and ranking mechanisms. The second part presents the software implementation developed using Python and Streamlit, which enables users to perform the AURA procedure interactively through a graphical interface. Numerical examples and case studies are also provided at the end of this chapter to demonstrate the practicality, efficiency, and consistency of the proposed method and software.

3.2 The Adaptive Utility Ranking Algorithm (AURA)

AURA is a newly proposed MCDM method designed to enhance flexibility and computational efficiency in evaluating alternatives under different types of criteria. Unlike traditional methods that rely on fixed normalization schemes, AURA introduces an adaptive framework capable of handling benefit, cost, and target–type criteria within a single unified procedure.

AURA is developed based on the distance-based principles of TOPSIS (Hwang & Yoon, 1981) and VIKOR (Opricovic & Tzeng, 2004), as well as the multiple-reference concept of COBRA (Krstić et al., 2022). However, AURA extends these approaches through three main features. First, it introduces an adaptive normalization procedure for benefit, cost, and target-type criteria. Unlike TOPSIS and VIKOR, which determine the ideal value of a criterion from the extreme values in the observed range, AURA allows the reference value for a target-type criterion, denoted by r_j, to be specified within

the interval $[x_j^{\min}, x_j^{\max}]$. The target-type normalization in Equations (3.2)–(3.4) gives a normalized value of 1 when $x_{ij} = r_j$ and decreases linearly as the deviation between x_{ij} and r_j increases.

Second, AURA evaluates each alternative with respect to three benchmark solutions, namely the positive ideal solution (PIS), negative ideal solution (NIS), and average solution (AS). This enables an alternative to be assessed not only against the best and worst performance levels but also against the central tendency of the alternatives. Third, AURA incorporates a dispersion-based correction factor, σ, as defined in Equation (3.10). This factor increases the separation between alternatives with similar raw distance values and helps reduce ties or rank ambiguity. The corrected distances are subsequently combined in the final AURA score given in Equation (3.12), where the parameter α controls the balance between the ideal–anti-ideal contrast and the distance from the average solution. These features provide AURA with a target-aware normalization process, a tri-benchmark evaluation structure, and a correction mechanism within a single distance-based ranking procedure.

The following describes the AURA procedure in six systematic steps as shown in Figure 3.1

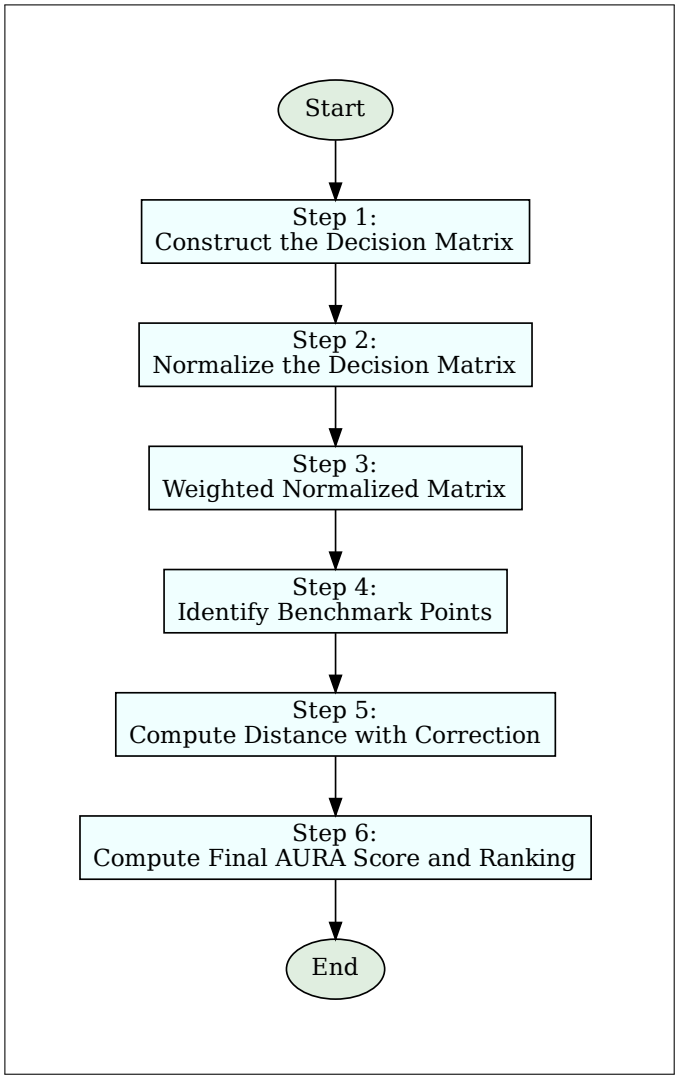


Figure 3.1 Flowchart of the AURA Method

Step 1: Construct the Decision Matrix

Let $X = [x_{ij}]_{m\times n}$ be the decision matrix that represents the performance of m alternatives A_i $(i = 1, 2, \ldots, m)$ under n evaluation criteria C_j $(j = 1, 2, \ldots, n)$. Each element x_{ij} denotes the performance value of alternative A_i with respect to criterion C_j. Thus, the decision matrix is expressed as in Equation (3.1)

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}. \tag{3.1}$$

Prior to applying the AURA method, the collected data should be validated and, if

required, scaled to ensure consistency across different measurement units. This matrix serves as the input for the subsequent normalization and weighting procedures.

Step 2: Normalize the Decision Matrix

Normalization converts heterogeneous criteria into a common, dimensionless scale so that they can contribute comparably to the overall evaluation. In the AURA method, the normalization scheme depends on the type of criterion, thereby preserving the intended direction of preference. Let x_{ij} denote the original performance value of alternative A_i with respect to criterion C_j, where $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. After applying the normalization process, the transformed value is denoted by r_{ij}, which represents the normalized and dimensionless performance score of A_i under C_j.

- **Max–type (Benefit) criterion** — higher values are preferred:

$$r_{ij} = \frac{x_{ij}}{x_j^{\max}}, \tag{3.2}$$

- **Min–type (Cost) criterion** — lower values are preferred:

$$r_{ij} = \frac{x_j^{\min}}{x_{ij}}, \tag{3.3}$$

- **Target–type criterion** — an intermediate or specific target value r_j is preferred:

$$r_{ij} = 1 - \frac{|x_{ij} - r_j|}{x_j^{\max} - x_j^{\min}}, \tag{3.4}$$

These normalization schemes ensure that all criteria are transformed onto a comparable scale while maintaining their preference directions. The benefit and cost normalization procedures in Equations (3.2) and (3.3) follow the conventional max/min ratio normalization commonly used in distance-based MCDM methods (Hwang & Yoon, 1981; Krstić et al., 2022). In contrast, the target-type normalization in Equation (3.4) is introduced in this study to accommodate criteria for which the preferred performance level lies within the observed range. It is formulated such that the normalized value is highest when the observed value equals the specified target and decreases linearly as the deviation from that target increases.

The reference value r_j in Equation (3.4) represents the decision-maker's preferred target level for criterion C_j. It is specified as an input through the software interface described in Section 3.4. When no explicit target value is provided, the software uses the mean value of the corresponding criterion as the default reference value. No reference value r_j is required for benefit-type or cost-type criteria. The resulting normalized decision matrix $R = [r_{ij}]_{m\times n}$ is bounded within $[0, 1]$, where a larger value of r_{ij} indicates better performance of alternative A_i with respect to criterion C_j.

Step 3: Weighted Normalized Decision Matrix

After normalization, the relative importance of each criterion is incorporated through a weight vector $\mathbf{w} = (w_1, w_2, \ldots, w_n)$, where $w_j \geq 0$ and $\sum_{j=1}^{n} w_j = 1$. Each normalized element r_{ij} is multiplied by its corresponding criterion weight to form the weighted normalized decision matrix $V = [v_{ij}]_{m\times n}$, defined as

$$v_{ij} = w_j \cdot r_{ij}. \tag{3.5}$$

This weighting process ensures that criteria with higher importance contribute more significantly to the overall evaluation. The weights can be determined either subjectively based on expert judgment or objectively using statistical techniques such as entropy, CRITIC, or MEREC. The resulting matrix V represents the weighted performance of each alternative under all evaluation criteria. The weighted normalization in Equation (3.5) follows the standard weighted decision-matrix construction common to TOPSIS and related distance-based methods (Hwang & Yoon, 1981; Krstić et al., 2022); it is adopted here rather than newly derived.

Step 4: Identify Benchmark Points

In the AURA method, three benchmark or tri-benchmark points are employed to evaluate the overall performance of alternatives across all criteria. These benchmark points are defined based on the weighted normalized decision matrix $V = [v_{ij}]_{m\times n}$ as follows:

- **PIS:**

$$v_j^+ = \max_i \left(v_{ij}\right), \tag{3.6}$$

which represents the most desirable performance for criterion C_j.

- **NIS:**

$$v_j^- = \min_i \left(v_{ij}\right), \tag{3.7}$$

which represents the least desirable performance for criterion C_j.

- **AS:**

$$\bar{v}_j = \frac{1}{m}\sum_{i=1}^{m} v_{ij}, \tag{3.8}$$

which represents the mean performance value of all alternatives under criterion C_j.

This tri-benchmark strategy allows the performance of each alternative to be assessed not only relative to the best and worst cases but also with respect to the average scenario. Consequently, it enhances the discriminatory capability of the AURA method by providing a more balanced and informative evaluation framework. The PIS and NIS in Equations (3.6) and (3.7) are the standard PIS/NIS of TOPSIS (Hwang & Yoon, 1981), while the inclusion of the average solution in Equation (3.8) as a third reference follows the multiple-reference principle of COBRA (Krstić et al., 2022).

Step 5: Compute Distance with Correction

The distance of each alternative from the benchmark points identified in Step 4, namely the PIS, NIS, and AS, is computed using the Euclidean or generalized Minkowski distance measure of order p. For a given reference point, the distance of alternative A_i is defined as

$$d_i^{\text{ref}} = \left(\sum_{j=1}^{n} \left|v_{ij} - v_j^{\text{ref}}\right|^p\right)^{\frac{1}{p}}, \quad \text{ref} \in \{+, -, \text{AVG}\}, \tag{3.9}$$

where $p \geq 1$ denotes the order of the Minkowski distance.

The computed distances reflect how far each alternative is from the corresponding benchmark point. To further enhance ranking discrimination, particularly when distance values are very close to one another, the AURA method introduces a correction factor σ. This factor is defined as the range of the distance values for the

selected reference point:

$$\sigma = \max_i \left(d_i^{\text{ref}}\right) - \min_i \left(d_i^{\text{ref}}\right). \tag{3.10}$$

Using the correction factor in Equation (3.10), the corrected distance of alternative A_i is calculated as

$$d_i^{\text{corr}} = d_i^{\text{ref}} + \sigma \left(d_i^{\text{ref}}\right)^2. \tag{3.11}$$

This correction slightly increases the separation between alternatives with similar distance values while preserving their original ranking order. As a result, the final ranking outcomes can be more clearly distinguished.

Equation (3.9) adopts the generalized Minkowski distance of order p, which is commonly used in distance-based MCDM (Hwang & Yoon, 1981; Opricovic & Tzeng, 2004). When $p = 2$, it reduces to the Euclidean distance. In contrast, Equations (3.10) and (3.11) are introduced in AURA to improve the discrimination between alternatives with closely related distance values.

The correction factor σ in Equation (3.10) measures the range of distances from a selected reference solution. It is equal to zero when all alternatives have the same distance and increases as the distance values become more dispersed. The corrected distance in Equation (3.11) combines the original distance with a term based on σ and the squared distance value. This transformation preserves the original order of the distances because the function $x + \sigma x^2$ is strictly increasing for $x \geq 0$ and $\sigma \geq 0$. Hence, if $d_i^{\text{ref}} < d_k^{\text{ref}}$, then $d_i^{\text{corr}} < d_k^{\text{corr}}$.

The squared component produces a relatively small adjustment for alternatives close to a reference solution and a larger adjustment for alternatives with greater distances. This strengthens the separation among alternatives without altering their original distance order. An illustrative example of the correction effect is presented in Table 3.1.

Table 3.1
Illustrative Example Showing the Effect of the Correction Factor σ.

Alternative	**Distance (d_i)**	**Corrected Distance (d_i^{corr})**
A_1	0.10	0.1010
A_2	0.12	0.1214
A_3	0.20	0.2040

As shown in Table 3.1, the corrected distances produce a slightly larger gap between closely ranked alternatives, such as A_1 and A_2, thereby improving ranking stability and reducing the likelihood of ties. This example is provided for illustrative purposes only.

Step 6: Compute Final AURA Score and Ranking

Finally, the overall performance of each alternative is evaluated by combining its distances from the tri-benchmark points using a tunable parameter $\alpha \in [0, 1]$, which controls the trade-off between ideality and average performance. Based on the corrected distances obtained in Step 5, the AURA score of alternative A_i is defined as

$$S_i = \frac{1}{2}\left[\alpha\left(d_i^+ - d_i^-\right) + (1-\alpha)d_i^{\text{AVG}}\right], \tag{3.12}$$

where d_i^+, d_i^-, and d_i^{AVG} denote the corrected distances of alternative A_i from the PIS, NIS, and AS, respectively.

The AURA score in Equation (3.12) is developed by combining two distance-based evaluation perspectives. The first perspective considers the contrast between the PIS and the negative ideal solution. A preferred alternative should be closer to the PIS and farther from the NIS. This condition is represented by the term $\left(d_i^+ - d_i^-\right)$, where a smaller, or more negative, value indicates a more preferred alternative. The second perspective considers the distance from the average solution, represented by d_i^{AVG}. This component evaluates the position of an alternative relative to the overall central tendency of the alternatives. The parameter $\alpha \in [0, 1]$ combines these two components and allows the decision-maker to determine their relative importance. When $\alpha = 1$, the score is based entirely on the contrast between the PIS and NIS. When $\alpha = 0$, the score is determined only by the distance from the average solution. Values between zero and one provide a balance between these two perspectives.

The factor $\frac{1}{2}$ rescales the final score without affecting the ranking order. Since preferred alternatives have smaller values for both components, the final AURA scores are ranked in ascending order, where a smaller value of S_i indicates a better-performing alternative.

3.3 Implementation (Method-Focused)

This section presents a method-focused evaluation of the AURA procedure. It includes a numerical example to demonstrate the computational steps, sensitivity analyses to examine the effects of parameter and weight variations, comparative analysis with established MCDM methods, and simulation-based analysis to assess ranking behaviour under different decision-matrix sizes.

3.3.1 Numerical Example

On the source and rationale of the data. The decision matrix used in this section is an illustrative dataset constructed for the sole purpose of demonstrating the AURA computational procedure step by step; it is not the empirical study of the thesis. The empirical validation, with its data-collection process, expert panel, and educational-technology context, is presented in Chapter 5. This numerical example evaluates blockchain-based models for enhancing social capital within Malaysian microfinance systems. Six beneficial criteria are considered to assess performance. The criteria and their functional roles are summarized in Table 3.2.

Table 3.2
Evaluation Criteria and Functional Description

Code	Criterion	Function / Benefit
C_1	Transparency	Enhances openness and accountability in financial transactions, promoting stakeholder trust.
C_2	Security	Ensures protection against fraud, cyber threats, and data breaches.
C_3	Operational Efficiency	Reduces transaction time and cost, enabling rapid and cost-effective services.
C_4	Financial Inclusion	Expands access to financial services for underserved and minority groups.
C_5	Scalability	Enables expansion across institutions and geographic regions.
C_6	Regulatory Compliance	Ensures alignment with Malaysian financial laws and regulations.

Eight blockchain-based microfinance alternatives are then evaluated, as presented in Table 3.3.

Table 3.3
Blockchain-Based Microfinance Alternatives

Code	Alternative	Primary Function
A_1	Smart Contracts for Loan Disbursement	Automates and enforces loan agreements without intermediaries.
A_2	Decentralized Identity Verification	Enables secure, user-controlled identity management.
A_3	Blockchain-Based Credit Scoring	Provides transparent and immutable credit records.
A_4	Cryptocurrency for Microfinance Payments	Facilitates low-cost and borderless transactions.
A_5	Peer-to-Peer Lending on Blockchain	Enables direct lending without traditional banks.
A_6	Tokenized Assets for Collateral	Allows digital asset representation for loan security.
A_7	Hyperledger-Based Consortium Model	Supports permissioned collaborative microfinance networks.
A_8	Public Blockchain for Financial Transparency	Provides open access to transaction histories for accountability.

To illustrate the application of the AURA method, a simple decision matrix is used in this subsection. All criteria in this numerical example are quantitative and of benefit type. Nevertheless, the proposed method can also accommodate cost-type and target-type criteria by applying the corresponding normalization procedures given in Equations (3.2)–(3.4). For a target-type criterion, the reference value r_j, defined in Equation (3.4), represents the preferred target level for criterion C_j. The criterion weights are denoted by w_j throughout this chapter. The symbol ω_j, which appears in selected tables and in Chapter 4, refers to the same criterion weight. The weights used in this example were determined through stakeholder consultation, and Table 3.4 summarizes the criteria and their corresponding weights.

Table 3.4
The Weights of Each Criterion for Blockchain-based Microfinance

Criteria	**Weight (ω_j)**
C_1	0.1200
C_2	0.2000
C_3	0.1600
C_4	0.3200
C_5	0.1500
C_6	0.0500

Step 1: Construct the Decision Matrix

Table 3.5 presents the 8×6 decision matrix where there are 8 alternatives and 6 criteria.

Table 3.5
The Decision Matrix of the Numerical Example

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	4	8	8	7	9	8
A_2	6	7	7	8	9	6
A_3	7	6	5	8	7	4
A_4	6	6	4	6	5	4
A_5	9	9	4	6	6	7
A_6	7	9	8	8	7	8
A_7	8	8	9	8	6	9
A_8	9	4	7	5	8	6

Step 2: Normalize the Decision Matrix

Normalization converts heterogeneous criteria into a common, dimensionless scale so that all criteria contribute comparably to the overall evaluation. Since all criteria in this numerical example are of the max-type (benefit), the AURA method applies the following normalization rule using Equation (3.2) and the result is as shown in Table 3.6:

Table 3.6
The Normalized Decision Matrix of the Numerical Example

Alt.	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.4444	0.8889	0.8889	0.8750	1.0000	0.8889
A_2	0.6667	0.7778	0.7778	1.0000	1.0000	0.6667
A_3	0.7778	0.6667	0.5556	1.0000	0.7778	0.4444
A_4	0.6667	0.6667	0.4444	0.7500	0.5556	0.4444
A_5	1.0000	1.0000	0.4444	0.7500	0.6667	0.7778
A_6	0.7778	1.0000	0.8889	1.0000	0.7778	0.8889
A_7	0.8889	0.8889	1.0000	1.0000	0.6667	1.0000
A_8	1.0000	0.4444	0.7778	0.6250	0.8889	0.6667

Example of normalization calculation:

For C_1 where $x_1^{\max} = 9$:

$$r_{11} = \frac{4}{9} = 0.4444, \qquad r_{21} = \frac{6}{9} = 0.6667. \tag{3.13}$$

For C_4 where $x_4^{\max} = 8$:

$$r_{14} = \frac{7}{8} = 0.8750, \qquad r_{84} = \frac{5}{8} = 0.6250. \tag{3.14}$$

Step 3: Weighted Normalized Decision Matrix

The weighted normalized decision matrix is obtained using Equation (3.5) by multiplying each normalized value r_{ij} with its corresponding criterion weight w_j. The resulting matrix is presented in Table 3.7.

Table 3.7
The Weighted Normalized Decision Matrix of the Numerical Example

Alt.	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.0533	0.1778	0.1422	0.2800	0.1500	0.0444
A_2	0.0800	0.1556	0.1244	0.3200	0.1500	0.0333
A_3	0.0933	0.1333	0.0889	0.3200	0.1167	0.0222
A_4	0.0800	0.1333	0.0711	0.2400	0.0833	0.0222
A_5	0.1200	0.2000	0.0711	0.2400	0.1000	0.0389
A_6	0.0933	0.2000	0.1422	0.3200	0.1167	0.0444
A_7	0.1067	0.1778	0.1600	0.3200	0.1000	0.0500
A_8	0.1200	0.0889	0.1244	0.2000	0.1333	0.0333

Example of weighted normalization calculation:

For C_1 where $w_1 = 0.12$:

$$v_{11} = w_1 \times r_{11} = 0.12 \times 0.4444 = 0.0533 \tag{3.15}$$

For C_4 where $w_4 = 0.32$:

$$v_{14} = w_4 \times r_{14} = 0.32 \times 0.8750 = 0.2800 \tag{3.16}$$

Step 4: Determine the Positive Ideal (PIS), Negative Ideal (NIS) and Average Solution (AS).

Since all criteria are benefit-type, the PIS, NIS, and AS for each criterion are obtained from the weighted normalized decision matrix $V = [v_{ij}]$ by using Equations (3.6), (3.7), and (3.8).

Table 3.8
The PIS, NIS, and AS for Each Criterion

	C_1	C_2	C_3	C_4	C_5	C_6
S_j^+	0.1200	0.2000	0.1600	0.3200	0.1500	0.0500
S_j^-	0.0533	0.0889	0.0711	0.2000	0.0833	0.0222
S_j^a	0.0933	0.1583	0.1155	0.2800	0.1188	0.0361

For example, for C_1, the PIS is $S_1^+ = 0.1200$ (maximum value in the C_1 column), the NIS is $S_1^- = 0.0533$ (minimum value), and the AS is $S_1^a = 0.0933$ (average of all C_1 values). The same procedure is applied to the remaining criteria.

Step 5: Compute the Distances from the Benchmark Solutions.

Using $p = 2$, the distances of each alternative from the benchmark solutions are computed by using Equations (3.9), (3.10), and (3.11). The order $p = 2$ (the Euclidean distance) is adopted here because it is the most widely used and balanced Minkowski order in distance-based MCDM, treating all criterion deviations symmetrically; the sensitivity analysis in Section 3.2 (Table 3.14) confirms that the ranking is stable for $p \in \{1, 1.5, 2, 2.5, 3\}$, so this choice does not bias the outcome.

Table 3.9
The Distances $d_i(S^+)$, $d_i(S^-)$, and $d_i(S^a)$ for Each Alternative

Alt.	$d_i(S^+)$	$d_i(S^-)$	$d_i(S^a)$
A_1	0.0830	0.1559	0.0611
A_2	0.0715	0.1642	0.0533
A_3	0.1100	0.1393	0.0560
A_4	0.1599	0.0655	0.0764
A_5	0.1301	0.1377	0.0800
A_6	0.0466	0.1871	0.0642
A_7	0.0563	0.1847	0.0684
A_8	0.1690	0.0996	0.1106

Example distance calculation for A_1 (with $p = 2$):

For $d_1(S^+)$*:*

$$d_1(S^+) = \left(\sum_{j=1}^{n} |v_{1j} - S_j^+|^2 \right)^{\frac{1}{2}} \tag{3.17}$$

$$= \Big(|0.0533 - 0.1200|^2 + |0.1778 - 0.2000|^2 + |0.1422 - 0.1600|^2 + |0.2800 - 0.3200|^2 + |0.1500 - 0.1500|^2 + |0.0444 - 0.0500|^2 \Big)^{\frac{1}{2}} \tag{3.18}$$

$$= 0.0830 \tag{3.19}$$

For $d_1(S^-)$*:*

$$d_1(S^-) = \left(\sum_{j=1}^{n} |v_{1j} - S_j^-|^2 \right)^{\frac{1}{2}} \tag{3.20}$$

$$= \Big(|0.0533 - 0.0533|^2 + |0.1778 - 0.0889|^2 + |0.1422 - 0.0711|^2 + |0.2800 - 0.2000|^2 + |0.1500 - 0.0833|^2 + |0.0444 - 0.0222|^2 \Big)^{\frac{1}{2}} \tag{3.21}$$

$$= 0.1559 \tag{3.22}$$

For $d_1(S^a)$*:*

$$d_1(S^a) = \left(\sum_{j=1}^{n} |v_{1j} - S_j^a|^2 \right)^{\frac{1}{2}} \tag{3.23}$$

$$= \Big(|0.0533 - 0.0933|^2 + |0.1778 - 0.1583|^2 + |0.1422 - 0.1156|^2 + |0.2800 - 0.2800|^2 + |0.1500 - 0.1188|^2 + |0.0444 - 0.0361|^2 \Big)^{\frac{1}{2}} \tag{3.24}$$

$$= 0.0611 \tag{3.25}$$

The same procedure is applied to the remaining alternatives to compute $d_i(S^+)$, $d_i(S^-)$, and $d_i(S^a)$, and the complete results are summarized in Table 3.9. To further enhance the discrimination ability among alternatives, a correction factor σ is introduced. This correction factor adjusts the distance values by considering the dispersion of the distance measures.

Table 3.10
The Correction Factor Based on the $d_i(S)$

	$d_i(S^+)$	$d_i(S^-)$	$d_i(S^a)$
σ	0.1224	0.1216	0.0572

The correction factor σ is obtained as the difference between the maximum and minimum distance values for each benchmark solution. The result are reported in Table 3.10.

Table 3.11
The Distances $D_i(S^+)$, $D_i(S^-)$, $D_i(S^a)$ for Each Alternative

Alt.	$D_i(S^+)$	$D_i(S^-)$	$D_i(S^a)$
A_1	0.0838	0.1588	0.0613
A_2	0.0722	0.1675	0.0535
A_3	0.1115	0.1417	0.0562
A_4	0.1630	0.0660	0.0767
A_5	0.1322	0.1400	0.0803
A_6	0.0468	0.1914	0.0644
A_7	0.0567	0.1888	0.0687
A_8	0.1725	0.1008	0.1113

Example of corrected distance calculation for A_1:

For $D_1(S^+)$:

$$D_1(S^+) = d_1(S^+) + \sigma \cdot d_1(S^+)^2 \tag{3.26}$$
$$= 0.0830 + 0.1224 \times (0.0830)^2 \tag{3.27}$$
$$= 0.0838 \tag{3.28}$$

For $D_1(S^-)$:

$$D_1(S^-) = d_1(S^-) + \sigma \cdot d_1(S^-)^2 \tag{3.29}$$
$$= 0.1559 + 0.1216 \times (0.1559)^2 \tag{3.30}$$
$$= 0.1588 \tag{3.31}$$

For $D_1(S^a)$:

$$D_1(S^a) = d_1(S^a) + \sigma \cdot d_1(S^a)^2 \tag{3.32}$$
$$= 0.0611 + 0.0572 \times (0.0611)^2 \tag{3.33}$$
$$= 0.0613 \tag{3.34}$$

The same correction process is applied to all alternatives, and the final corrected distances $D_i(S^+)$, $D_i(S^-)$, and $D_i(S^a)$ was presented in Table 3.11.

Step 6: Rank the Alternatives.

In the final step, the overall AURA score for each alternative is computed by combining its corrected distances from the PIS $D_i(S^+)$, negative ideal solution $D_i(S^-)$,

and average solution $D_i(S^a)$. A trade-off parameter $\alpha \in [0, 1]$ is introduced to balance the contrast between the ideal and anti-ideal solutions and the closeness to the average solution. The AURA score S_i is calculated using Equation (3.12). In this numerical example, the parameter is set to $\alpha = 0.5$.

Example calculation for A_1:

$$S_1 = \frac{1}{2}\left[0.5 \cdot \left(D_1(S^+) - D_1(S^-)\right) + (1 - 0.5) \cdot D_1(S^a)\right] \tag{3.35}$$

$$= \frac{1}{2}\left[0.5 \cdot (0.0838 - 0.1588) + 0.5 \cdot 0.0613\right] \tag{3.36}$$

$$= -0.0034. \tag{3.37}$$

The same procedure is applied to all remaining alternatives to obtain their corresponding AURA scores. The alternatives are then ranked in ascending order of S_i, where a lower score indicates a better-performing alternative.

To make the Step 6 contribution fully traceable, Table 3.12 presents the complete computation of the AURA score for every alternative from its corrected benchmark distances $D_i(S^+)$, $D_i(S^-)$, and $D_i(S^a)$ using Equation (3.12) with $\alpha = 0.5$.

Table 3.12
Complete Step 6 Computation of the AURA Score ($\alpha = 0.5$, $p = 2$)

Alt.	$D_i(S^+)$	$D_i(S^-)$	$D_i(S^a)$	$S_i = \frac{1}{2}[0.5(D_i^+ - D_i^-) + 0.5\, D_i^a]$	**Rank**
A_1	0.0838	0.1588	0.0613	$\frac{1}{2}[0.5(-0.0750) + 0.5(0.0613)] = -0.0034$	4
A_2	0.0722	0.1675	0.0535	$\frac{1}{2}[0.5(-0.0953) + 0.5(0.0535)] = -0.0105$	3
A_3	0.1115	0.1417	0.0562	$\frac{1}{2}[0.5(-0.0302) + 0.5(0.0562)] = \ \ 0.0065$	5
A_4	0.1630	0.0660	0.0767	$\frac{1}{2}[0.5(\ \ 0.0970) + 0.5(0.0767)] = \ \ 0.0434$	7
A_5	0.1322	0.1400	0.0803	$\frac{1}{2}[0.5(-0.0078) + 0.5(0.0803)] = \ \ 0.0181$	6
A_6	0.0468	0.1914	0.0644	$\frac{1}{2}[0.5(-0.1446) + 0.5(0.0644)] = -0.0200$	1
A_7	0.0567	0.1888	0.0687	$\frac{1}{2}[0.5(-0.1321) + 0.5(0.0687)] = -0.0159$	2
A_8	0.1725	0.1008	0.1113	$\frac{1}{2}[0.5(\ \ 0.0717) + 0.5(0.1113)] = \ \ 0.0458$	8

The final ranking result is shown is Table 3.13.

Table 3.13
Final Ranking using AURA Method

Alt.	Ranking Score	Rank
A_1	-0.0034	4
A_2	-0.0105	3
A_3	0.0065	5
A_4	0.0434	7
A_5	0.0181	6
A_6	-0.0200	1
A_7	-0.0159	2
A_8	0.0458	8

Based on the computed AURA scores, it is important to note that the distance index derived in the AURA framework represents a relative utility-based separation rather than a pure metric distance. As a result, the computed AURA scores may assume either negative or positive values. Nevertheless, the decision rule remains consistent: smaller AURA scores indicate more preferable alternatives and the final ranking is $A_6 \succ A_7 \succ A_2 \succ A_1 \succ A_3 \succ A_5 \succ A_4 \succ A_8$, which is also illustrated in Figure 3.2.

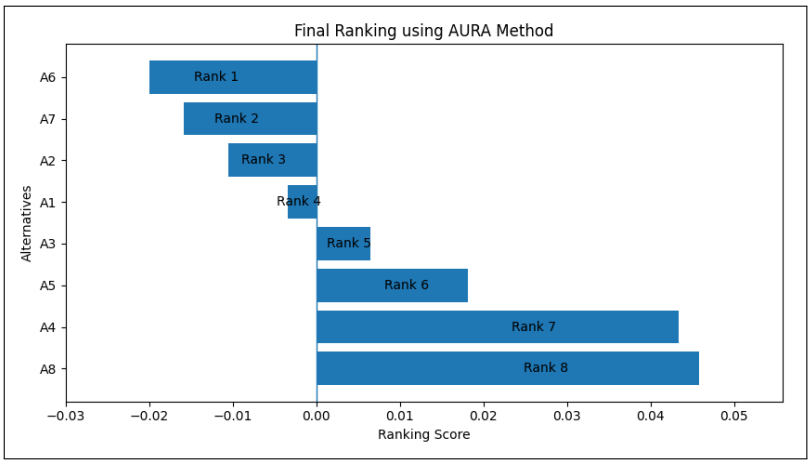


Figure 3.2 The Final Ranking of Alternatives using the AURA Method

The horizontal bar chart in Figure 3.2 illustrates the final ranking of eight alternatives obtained using the AURA method. In this figure, the AURA ranking scores are plotted along the x-axis, while the alternatives are displayed along the y-axis, with rank annotations provided for clarity. The results indicate that A_6 achieves the best performance with the lowest AURA score of -0.0200, followed by A_7 and A_2 with scores of -0.0159 and -0.0105, respectively. These alternatives demonstrate superior overall performance as they are closest to the ideal solution while remaining distant from the anti-ideal solution.

Alternatives A_1 and A_3 occupy intermediate positions with moderately low AURA scores, indicating acceptable but not outstanding performance. In contrast, A_5, A_4, and A_8 are ranked lower, with A_8 exhibiting the highest AURA score of 0.0458, suggesting the weakest performance among all alternatives. The distribution of both negative and positive AURA scores reflects a clear performance separation between high-performing and low-performing alternatives. Overall, this numerical example demonstrates the robustness and effectiveness of the AURA method in discriminating among alternatives and supporting decision-making in complex multi-criteria evaluation problems.

3.3.2 Sensitivity Analysis

This section investigates the sensitivity of the proposed AURA method to variations in key model parameters and decision-maker preferences. Sensitivity analysis is conducted to evaluate the robustness, stability, and reliability of the ranking results when critical parameters are altered. In particular, this analysis examines how changes in control parameters and criteria weights influence the relative ranking of alternatives.

The sensitivity assessment is carried out through three main perspectives: (i) variation of the balance parameter α, which regulates the trade-off between the ideal–anti-ideal contrast and the average-based evaluation; (ii) variation of the distance parameter p, which controls the aggregation behavior in the Minkowski distance measure; and (iii) variation of criteria weights, which reflects different decision-maker preference scenarios. The computations for the sensitivity analyses were performed using Excel, while the graphical illustrations were generated using Python to ensure consistent and clear visualization.

3.3.2.1 Sensitivity Analysis with Respect to α

In the AURA framework, the parameter $\alpha \in [0, 1]$ controls the relative importance of the contrast term $\big(D_i(S^+) - D_i(S^-)\big)$ and the average-based term $D_i(S^a)$. When $\alpha = 0$, the ranking is solely determined by the distance from the average solution, reflecting a more conservative and balanced evaluation. Conversely, when $\alpha = 1$, the ranking is dominated by the contrast between the ideal and anti-ideal solutions, emphasizing extremal performance. Intermediate values of α, particularly

$\alpha = 0.5$, provide a balanced assessment by simultaneously accounting for both perspectives. In this analysis, the parameter p is fixed at 2, while α is varied over the set $\{0, 0.25, 0.5, 0.75, 1\}$.

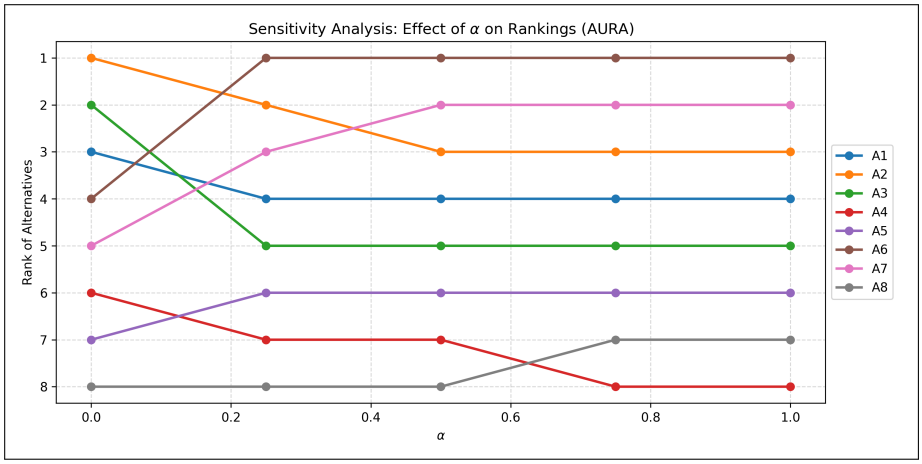


Figure 3.3 Effect of α on the Final Ranking using the AURA Method

Based on Figure 3.3, the ranking results exhibit a high degree of stability across different values of α. In particular, alternative A_6 consistently achieves the top rank for all considered values of $\alpha > 0$, indicating strong robustness and superior overall performance. Alternatives A_7 and A_2 also demonstrate stable behavior and remain among the top-ranked options across most values of α, suggesting that their performance is relatively insensitive to changes in decision-maker preferences.

In contrast, moderate rank variations are observed for alternatives A_1, A_3, and A_5, which occupy intermediate ranking positions. These alternatives are more sensitive to changes in α, as their relative performance lies near the boundary between ideal-based and average-based evaluations. Minor rank reversals are mainly observed among lower-ranked alternatives, particularly between A_4 and A_8 when $\alpha \geq 0.75$, highlighting sensitivity at the lower end of the ranking spectrum.

Overall, this sensitivity analysis confirms that the proposed AURA method produces robust and reliable ranking results, with only limited rank fluctuations under varying α values. The choice of $\alpha = 0.5$ is shown to be appropriate, as it preserves ranking stability while ensuring a balanced consideration of both ideal–anti-ideal contrast and average performance. These findings demonstrate the flexibility of the AURA method and its suitability for practical multi-criteria decision-making scenarios where decision-maker preferences may vary.

3.3.2.2 Sensitivity Analysis with Respect to p

Another sensitivity analysis is conducted to examine the impact of the distance parameter p on the ranking outcomes of the AURA method. The parameter p controls how deviations between alternatives and the benchmark solutions are aggregated in the Minkowski distance measure. In this analysis, the balance parameter α is fixed at 0.5, while p is varied over the set $\{1, 1.5, 2, 2.5, 3\}$ to evaluate the robustness of the ranking results under different distance aggregation behaviors.

Table 3.14 summarizes the ranking positions of all alternatives for the selected values of p. The table provides a concise comparison of ranking stability and highlights potential rank reversals caused by changes in the distance parameter.

Table 3.14
Sensitivity Analysis of Rankings with Respect to Parameter p ($\alpha = 0.5$)

Alt.	$p = 1$	$p = 1.5$	$p = 2$	$p = 2.5$	$p = 3$
A_1	4	4	4	4	4
A_2	3	3	3	3	3
A_3	5	5	5	5	5
A_4	8	8	7	7	7
A_5	6	6	6	6	6
A_6	1	1	1	1	1
A_7	2	2	2	2	2
A_8	7	7	8	8	8

From Table 3.14, it can be observed that the overall ranking structure remains largely stable across all tested values of p. In particular, alternative A_6 consistently achieves the first rank for all values of p, indicating strong robustness to variations in the distance parameter. Similarly, alternatives A_7 and A_2 maintain stable positions among the top-ranked alternatives, while A_1, A_3, and A_5 preserve their intermediate ranking positions throughout the analysis.

To further illustrate the ranking behavior and visualize potential rank reversals, Figure 3.4 presents the sensitivity plot of rankings with respect to varying values of p.

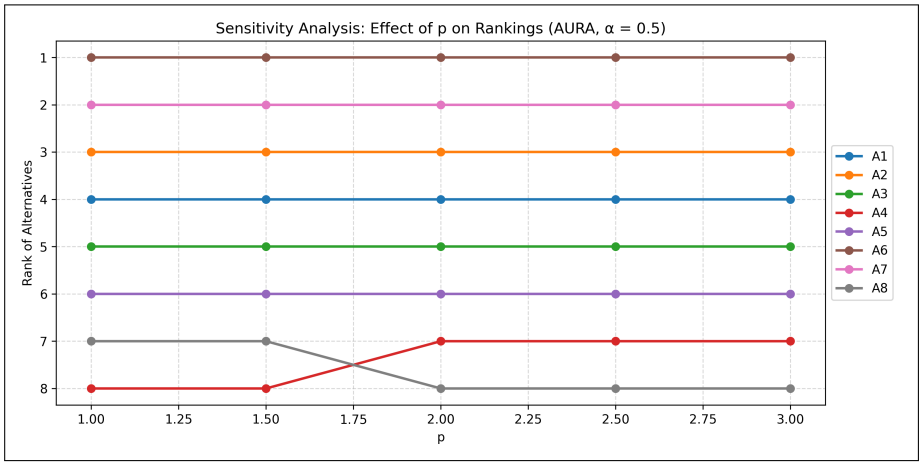


Figure 3.4 Effect of p on the Final Ranking using the AURA Method

As shown in Figure 3.4, the ranking trajectories of most alternatives are represented by nearly horizontal lines, indicating minimal sensitivity to changes in p. Minor rank reversals are observed only among the lowest-ranked alternatives, specifically between A_4 and A_8. When $p = 1$ and $p = 1.5$, A_8 is ranked higher than A_4; however, for $p \geq 2$, their positions are reversed. This behavior suggests that weaker alternatives are more sensitive to whether the distance measure emphasizes overall deviations (smaller p) or larger deviations (larger p).

Overall, this sensitivity analysis confirms that the proposed AURA method is not highly sensitive to the choice of the distance parameter p, as the ranking results remain stable with only limited variations occurring at the lower end of the ranking spectrum. The commonly adopted value $p = 2$ provides a balanced distance measure while preserving ranking robustness and consistency.

3.3.2.3 Sensitivity Analysis: Weight Variation

In addition to parameter-based sensitivity, a further sensitivity analysis is performed to examine the effect of weight variation on the relative ranking of alternatives obtained using the AURA method. Since the criteria weights represent decision-maker preferences, modifying the weight distribution may lead to changes in the overall evaluation and ranking outcomes. Therefore, this analysis is important to assess the robustness of the AURA rankings under different preference settings. In this experiment, the parameters are fixed at $\alpha = 0.5$ and $p = 2$, while the criteria weights are varied according to five weighting scenarios.

Specifically, the following scenarios are considered: *Baseline*, *Equal*, *Emphasize* C_1 *&* C_2, *Emphasize* C_4, and *Random*. The Baseline scenario adopts the original weight vector used in the numerical example, whereas the Equal scenario assigns equal importance to all criteria. The Emphasize scenarios are constructed to reflect preference shifts toward selected criteria, and the Random scenario represents a general perturbation to test ranking stability. Figure 3.5 illustrates the rank sensitivity of the eight alternatives across these five weight scenarios.

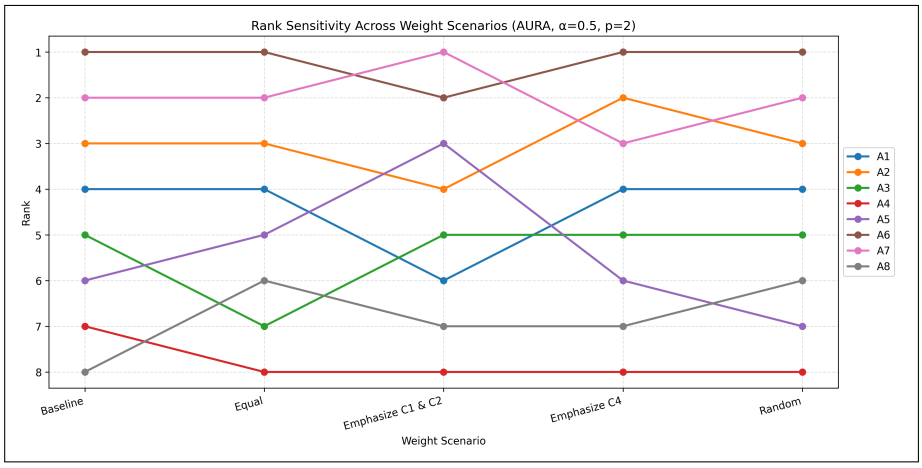


Figure 3.5 Rank Sensitivity Across Weight Scenarios

As shown in Figure 3.5, the AURA ranking results demonstrate a clear pattern of stability for strong and weak alternatives, while revealing moderate sensitivity for middle-ranked alternatives. In particular, alternative A_6 consistently remains within the top ranking positions across all scenarios, indicating strong robustness to weight variation. Similarly, A_7 also maintains a stable high ranking and becomes the best alternative under the *Emphasize* C_1 *&* C_2 scenario, suggesting that it performs particularly well when greater emphasis is placed on early-stage criteria. These outcomes imply that the superior performance of A_6 and A_7 is not dependent on a single weighting scheme, supporting their reliability as preferred alternatives.

On the other hand, A_4 consistently appears among the lowest-ranked alternatives across all weighting scenarios, reflecting persistently weak performance irrespective of how the criteria weights are adjusted. This behavior strengthens the discriminative power of the AURA method in identifying inferior alternatives in a stable manner.

Moderate rank changes are observed for alternatives A_1, A_2, A_5, and A_8, indicating that these alternatives are relatively more sensitive to preference changes

encoded through the weight vectors. For instance, A_5 improves noticeably under the *Emphasize* C_1 *&* C_2 scenario, whereas A_8 tends to remain in the lower tier but may shift slightly when the weight configuration changes (e.g., Equal and Random scenarios). Such fluctuations highlight that the relative positioning of middle-tier alternatives is more dependent on decision-maker priorities and weighting choices.

Overall, this weight sensitivity analysis confirms that the AURA method provides robust ranking outcomes for consistently strong alternatives (A_6 and A_7) and consistently weak alternatives (A_4), while also maintaining flexibility to reflect changes in decision-maker preferences for intermediate alternatives. This balance between robustness and adaptiveness enhances the practical applicability of the AURA method in real-world multi-criteria decision-making settings where weights may vary across stakeholders or scenarios.

3.3.3 Comparative Analysis

This section evaluates the effectiveness and reliability of the proposed AURA method by comparing its ranking results with several well-established multi-criteria decision-making (MCDM) methods, namely COBRA, TOPSIS, WASPAS, MOORA, VIKOR, and SAW. To ensure a fair comparison, all methods are applied to the same decision problem using identical criteria and alternatives. The comparative ranking results are summarized in Table 3.15 and visually illustrated in Figure 3.6.

Table 3.15
Comparison of AURA with Other MCDM Methods

Alt.	AURA	COBRA	TOPSIS	WASPAS	MOORA	VIKOR	SAW
A_1	4	4	4	4	4	4	4
A_2	3	3	3	3	3	2	3
A_3	5	5	5	5	6	5	5
A_4	7	8	8	8	8	7	8
A_5	6	6	6	6	5	6	6
A_6	1	1	1	1	1	1	1
A_7	2	2	2	2	2	2	2
A_8	8	7	7	7	7	8	7

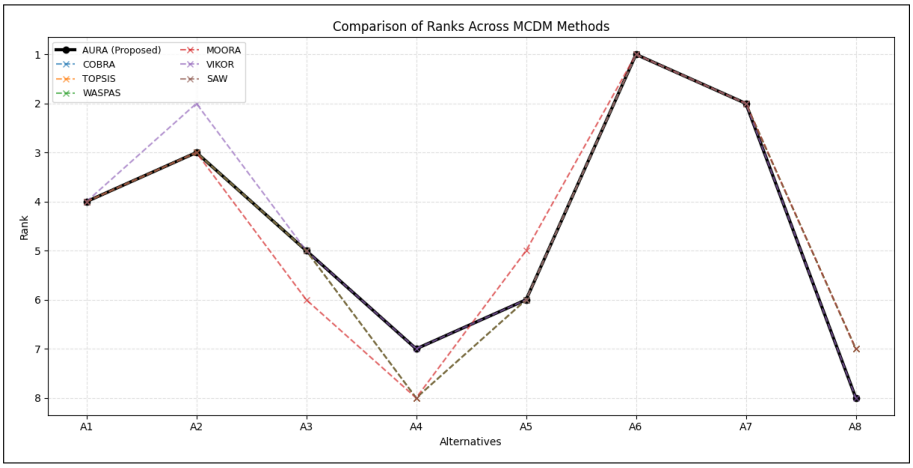


Figure 3.6 Comparison of AURA with Other MCDM Methods

From Table 3.15 and Figure 3.6, it can be observed that the ranking patterns produced by AURA are highly consistent with those obtained from the benchmark MCDM methods. In particular, alternative A_6 is unanimously identified as the best-performing option (rank 1) across all seven methods, indicating strong agreement regarding its overall superiority. Similarly, alternative A_7 consistently secures the second position, further confirming the robustness of the top-ranked alternatives.

At the lower end of the ranking, alternatives A_4 and A_8 are generally positioned among the weakest options. Although minor variations exist between methods, for instance, COBRA assigns A_4 to rank 8 while AURA places it at rank 7, and VIKOR assigns A_8 to rank 8 whereas other methods assign rank 7. The overall assessment of these alternatives as low-performing remains consistent. This indicates that the disagreement is limited to adjacent ranks and does not alter the overall decision outcome.

Moderate differences are observed for middle-ranked alternatives such as A_3 and A_5. For example, MOORA assigns A_3 to rank 6 while most other methods, including AURA, assign it to rank 5. Such variations are expected due to differences in aggregation mechanisms, normalization procedures, and distance formulations employed by the respective methods. Nevertheless, these discrepancies are relatively small and do not affect the identification of the best and worst alternatives.

Overall, the comparative results demonstrate that the proposed AURA method produces rankings that are highly compatible with established MCDM approaches while offering additional flexibility through its adaptive distance-based formulation. The strong agreement with COBRA, SAW, and TOPSIS, together with only minor rank shifts

for mid-tier alternatives, confirms the validity, stability, and practical reliability of the AURA method for complex decision-making problems.

To further quantify the similarity and consistency of the ranking results, Spearman's rank correlation coefficient was employed. The resulting correlation matrix is illustrated in Figure 3.7, which provides a pairwise comparison of ranking agreement among the AURA method and other established MCDM approaches.

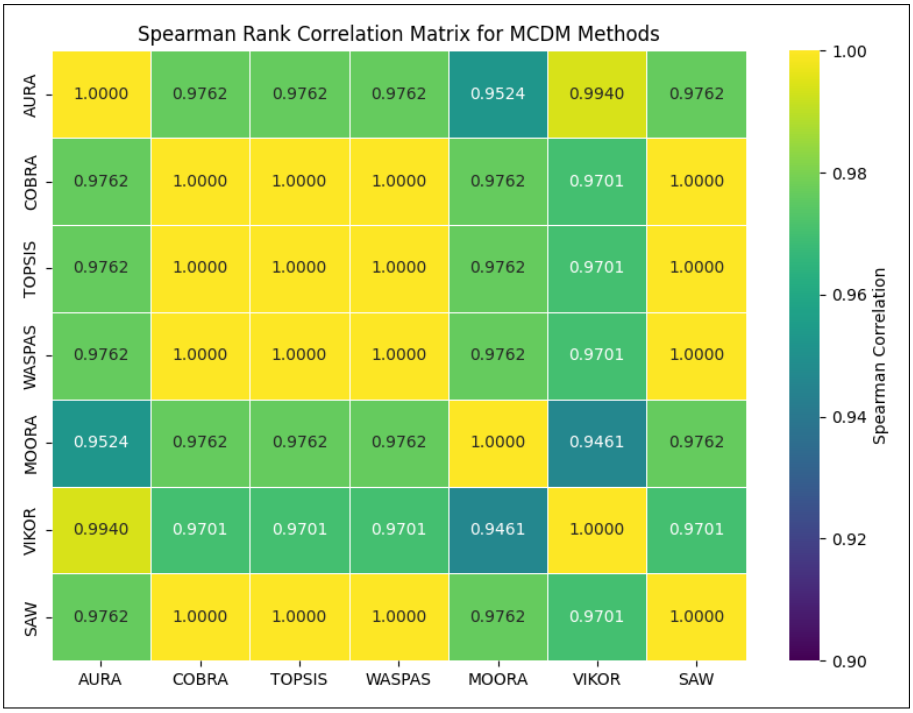


Figure 3.7 Spearman Rank Correlation Matrix for AURA and Other MCDM Methods

As shown in Figure 3.7, a generally high level of agreement is observed across all methods, indicating strong consistency in the decision outcomes. In particular, the proposed AURA method exhibits very strong positive correlations with VIKOR (0.9940), COBRA (0.9762), TOPSIS (0.9762), WASPAS (0.9762), and SAW (0.9762). These results confirm that the rankings generated by AURA are highly aligned with those produced by well-established MCDM techniques, thereby validating the reliability and credibility of the proposed approach.

The correlation between AURA and MOORA is slightly lower (0.9524), yet still remains within a strong agreement range. This difference can be attributed to variations in aggregation mechanisms and normalization procedures inherent to MOORA, rather than contradictory decision behavior. Such minor deviations highlight

the methodological diversity among MCDM approaches while preserving overall ranking consistency.

The average Spearman correlation coefficient for AURA methods is 0.9759, indicating an excellent level of concordance. This high average correlation demonstrates that AURA produces stable and dependable ranking outcomes while maintaining sufficient flexibility to capture alternative preference structures. Consequently, the proposed AURA method can be regarded as both competitive with and complementary to existing MCDM techniques.

Overall, the Spearman correlation analysis indicates a strong level of consistency between the proposed AURA method and the benchmark MCDM approaches, supporting its suitability as a reliable decision-support tool for complex multi-criteria decision-making problems. However, since Spearman's correlation primarily reflects monotonic relationships based on rank differences, an additional agreement-based measure is considered to provide further validation.

Accordingly, Kendall's tau correlation analysis is conducted to assess the degree of ranking agreement based on concordant and discordant pairs. As a more conservative measure for ordinal data, Kendall's tau (τ) offers complementary insights into ranking consistency. The resulting correlation matrix is presented in Figure 3.8.

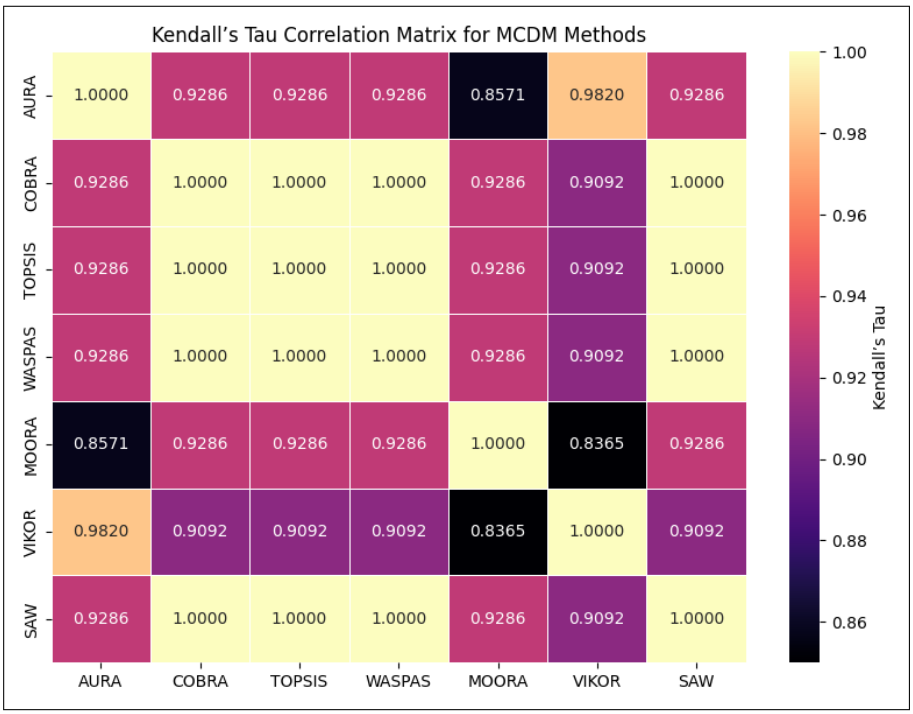


Figure 3.8 Kendall's Tau Correlation Matrix for AURA and Other MCDM Methods

As shown in Figure 3.8, a high level of agreement is observed among most of the MCDM methods. In particular, the proposed AURA method exhibits a very strong positive correlation with VIKOR ($\tau = 0.9820$), indicating near-identical ranking behavior between the two approaches. Strong correlations are also observed between AURA and COBRA, TOPSIS, WASPAS, and SAW (all $\tau = 0.9286$), demonstrating that the rankings generated by AURA are highly consistent with those produced by widely adopted MCDM techniques.

Meanwhile, the correlation between AURA and MOORA is slightly lower ($\tau = 0.8571$), though it still indicates a strong level of agreement. This deviation can be attributed to methodological differences in normalization and aggregation procedures rather than conflicting decision outcomes. Similar patterns are also evident among the benchmark methods, where COBRA, TOPSIS, WASPAS, and SAW exhibit perfect agreement ($\tau = 1.0000$), reflecting their closely related ranking structures.

Overall, the Kendall's tau analysis confirms the robustness and reliability of the proposed AURA method. When considered together with the Spearman correlation results, the findings provide strong statistical evidence that AURA produces stable and consistent ranking outcomes while maintaining sufficient flexibility to capture alternative preference structures in multi-criteria decision-making problems.

3.3.4 Simulation-Based Analysis

This subsection investigates the robustness and stability of the proposed AURA method through a simulation-based analysis. Simulation is employed to examine the behavior of ranking outcomes under stochastic variations in decision data and increasing problem complexity. Such an analysis is crucial for assessing the reliability of decision-making methods in real-world applications, where uncertainty and data variability are unavoidable.

To this end, three types of randomly generated decision matrices were constructed using PYTHON, representing small (5×5), medium (10×10), and large (20×20) decision problems. These matrix sizes correspond to increasing numbers of alternatives and criteria, allowing the scalability and ranking stability of the AURA method to be examined as the dimensionality of the decision problem increases. For each matrix size, multiple simulation trials were conducted, and the AURA method was

applied to derive ranking results for all alternatives.

To quantify ranking stability, two commonly used statistical indicators were employed, namely the mean standard deviation of ranks and ranking variance. Let $r_i^{(t)}$ denote the rank of alternative i obtained in trial t, where $i = 1, 2, \ldots, m$ and $t = 1, 2, \ldots, T$. The average rank of alternative i across all simulation trials is defined as

$$\bar{r}_i = \frac{1}{T} \sum_{t=1}^{T} r_i^{(t)}. \tag{3.38}$$

Subsequently, the dispersion of rankings across simulation trials is measured using the standard deviation and variance of ranks, following conventional statistical definitions. To provide an overall stability indicator, these quantities are averaged across all alternatives.

The numerical results of the simulation experiments are summarized in Table 3.16 and Table 3.17, which reports the mean standard deviation of ranks and ranking variance for each matrix size, respectively.

Table 3.16
Mean Standard Deviation of Ranks Across Simulation Trials

Matrix size	**Mean Standard Deviation**
5×5	1.4286
10×10	2.8868
20×20	5.7808

Table 3.17
Ranking Variance Across Simulation Trials

Matrix size	**Variance**
5×5	2.0408
10×10	8.3333
20×20	33.417

From Table 3.16, it can be observed that the mean standard deviation of ranking positions is relatively low for the small decision matrix, indicating strong ranking stability across simulation runs. As the matrix size increases to the medium case, a moderate increase in ranking dispersion is observed, reflecting the higher interaction among alternatives and criteria. For the large matrix, the higher mean standard

deviation is attributable to the increased complexity of high-dimensional decision spaces, where small perturbations in input data naturally lead to greater variability in ranking outcomes. This behavior is commonly observed in simulation-based evaluations of MCDM methods and does not indicate instability of the proposed approach.

A similar trend is evident in Table 3.17, where the ranking variance increases with matrix size. This increase reflects the amplification of uncertainty and interaction effects in larger decision matrices, rather than a structural limitation of the AURA method itself.

To complement the statistical measures, Figure 3.9 illustrates the distribution of ranking outcomes using boxplots. The boxplot for the small matrix exhibits a narrow interquartile range and short whiskers, confirming strong ranking stability. The medium matrix shows moderate dispersion, while the large matrix displays a wider spread, indicating higher sensitivity to input variations. Importantly, despite the wider spread observed for large matrices, the overall ranking patterns remain interpretable, indicating that AURA maintains coherent decision behavior even under highly complex scenarios.

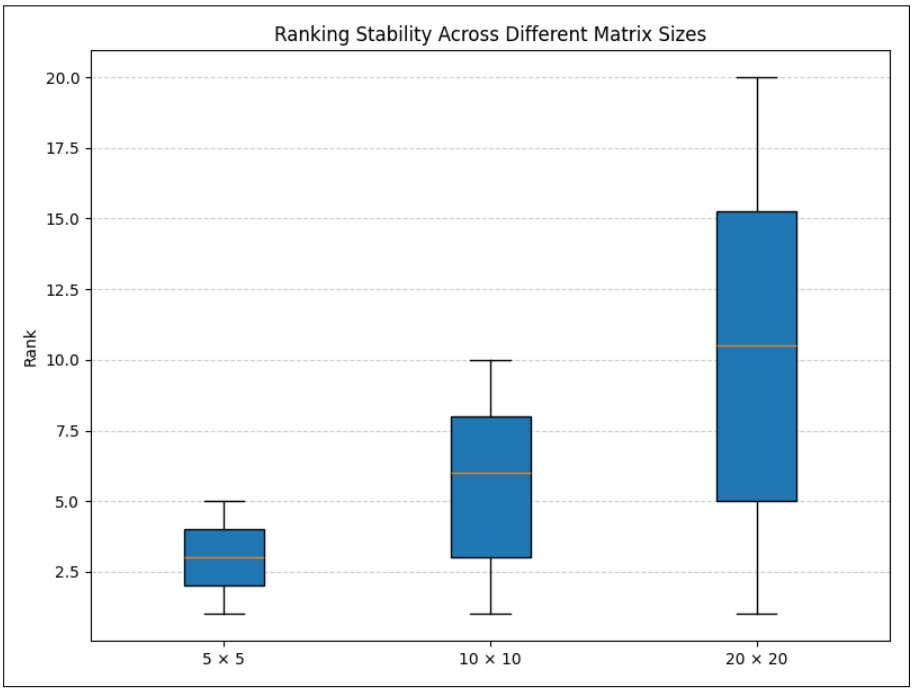


Figure 3.9 Ranking Stability Across Different Matrix Sizes

Overall, the simulation-based analysis demonstrates that the proposed AURA method provides stable and reliable rankings for small- and medium-scale decision problems, while exhibiting predictable and explainable sensitivity as problem

dimensionality increases. This behavior is consistent with the findings from the sensitivity and comparative analyses, further confirming the robustness and practical applicability of AURA in complex multi-criteria decision-making environments.

3.4 Software Implementation of AURA

This section documents the design and implementation of the AURA software, developed as a browser-accessible decision support tool. The implementation follows a modular architecture to ensure transparency, reproducibility, and scalability for large decision matrices.

3.4.1 System Architecture

The AURA software is implemented in Python and deployed through a lightweight web interface using Streamlit. Core numerical and I/O tasks are handled with established scientific libraries. The AURA application is deployed as a browser-accessible web app on the Streamlit Community Cloud, which serves the Python application directly from its source repository so that no local installation is required by the end user; the same source can alternatively be run locally with the command `streamlit run app.py`. The architecture is a single-tier, modular pipeline in which the Streamlit front end (input upload, parameter sliders, and result panels) calls a stateless computation core (normalization $\rightarrow$ weighting $\rightarrow$ benchmarks $\rightarrow$ distances $\rightarrow$ correction $\rightarrow$ scoring) and a reporting module that writes the multi-sheet Excel output. This separation keeps the numerical logic independent of the interface and is what allows the operator to be extended in future to fuzzy inputs. The overall stack is as follows:

- **Language:** Python (≥ 3.11).
- **Framework:** Streamlit for web-based GUI.
- **Computation:** NumPy and Pandas for matrix operations and data handling.
- **Visualization:** Matplotlib for charts and tabular views.
- **Reporting:** XlsxWriter for generating multi-sheet Excel summaries.
- **Packaging/License:** Open-source distribution under BSD-3-Clause.

Inputs are provided as structured Excel files. The first row contains the criterion labels, the second row stores normalized weights that sum to one, and the subsequent rows store the alternatives' data. The software parses the file, validates the schema, and constructs internal matrices for the AURA pipeline (normalization $\rightarrow$ weighting $\rightarrow$ benchmarks $\rightarrow$ distances $\rightarrow$ scoring).

3.4.2 Description of the Software

The AURA software is a lightweight, browser-based implementation of AURA designed to support transparent and scalable decision-making. Developed using Python and deployed through the Streamlit framework, the tool facilitates the evaluation and ranking of alternatives across multiple criteria in MCDM problems. It specifically supports benefit-type, cost-type, and target-type criteria within a unified computational framework, addressing the limitations of conventional methods such as TOPSIS and VIKOR.

Figure 3.10 shows the landing page of the application, where users are prompted to upload a structured Excel file containing a decision matrix. The required format includes criteria headers in the first row and their corresponding weights in the second row. In addition, the interface provides a slider for setting the balance parameter (α), which controls the trade-off between ideality and the average solution, and a dropdown menu to select the distance metric (p). By default, the parameters are set to $\alpha = 0.5$ and $p = 2$, which represent a balanced configuration commonly used in AURA experiments. This interface simplifies the initial setup and ensures that input files are processed correctly, making the software suitable even for decision makers with limited technical expertise.

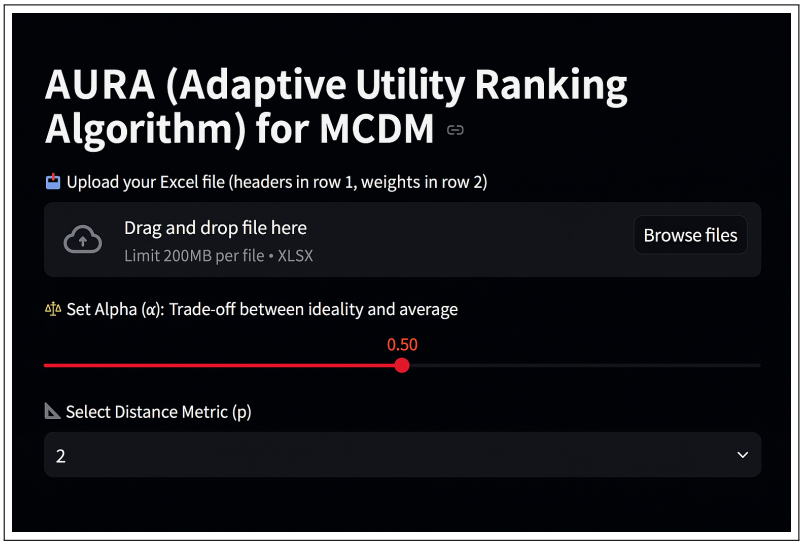


Figure 3.10 AURA Web Interface Prompting Excel-based Decision Matrix Upload.

Once uploaded, the input data is immediately displayed, including the alternative labels, criteria labels, and assigned weights. As illustrated in Figure 3.11, this preview enables users to validate the integrity of the uploaded matrix prior to executing the analysis. The software accepts any number of alternatives and criteria, as long as the formatting structure is preserved.

Alternative	Criteria 1	Criteria 2	Criteria 3	Criteria 4
Weight	0.3	0.2	0.3	0.2
Alt1	80	50	75	90
Alt2	85	60	70	85
Alt3	78	40	60	88
Alt4	90	55	72	92

Figure 3.11 Display of Raw Uploaded Decision Data with Criteria Weights.

Following validation, users are shown the raw uploaded data table and the initial decision matrix before normalization. As shown in Figure 3.12, the table includes all alternatives, criteria, and their associated weights, providing transparency and enabling verification of data accuracy before further processing.

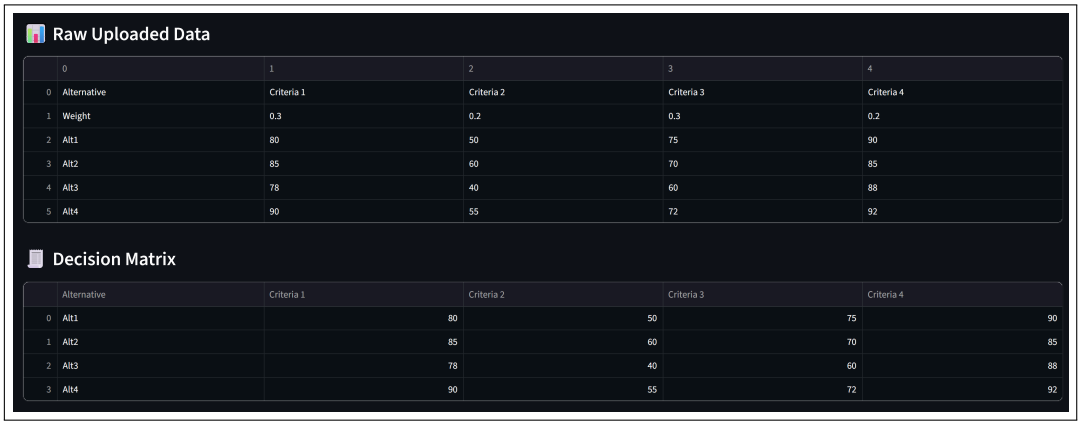


Figure 3.12 Raw Uploaded Data and Initial Decision Matrix Containing Values and Weights.

Before executing the algorithm by selecting the ”Run AURA” button, users define the type of each criterion-benefit, cost, or target-through an interactive selection panel. As illustrated in Figure 3.13, the system also allows users to assign custom target values for target-type criteria. This flexibility enhances the accuracy of the model in representing real-world preferences. Each alternative is evaluated across the normalized criteria space using AURA's unified scoring formula.

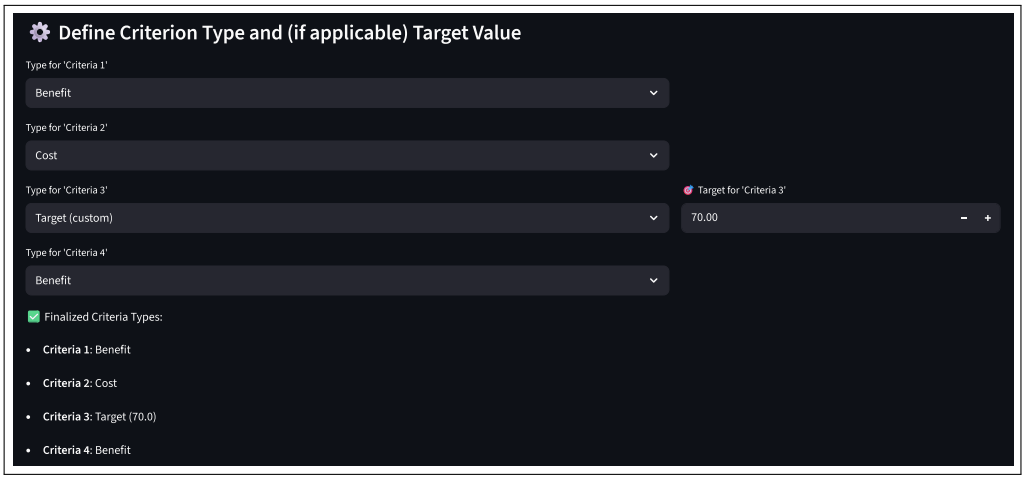


Figure 3.13 Interactive Each Criterion Type Selection with Support for Custom Target Values.

To ensure clarity and traceability, the AURA interface provides step-by-step panels, each corresponding to a major component of the algorithm: normalization, weighted matrix generation, benchmark identification, and distance computation. As shown in Figure 3.14, users can expand each section interactively to inspect the

intermediate outputs in real time. This modular approach facilitates both educational use and research transparency.

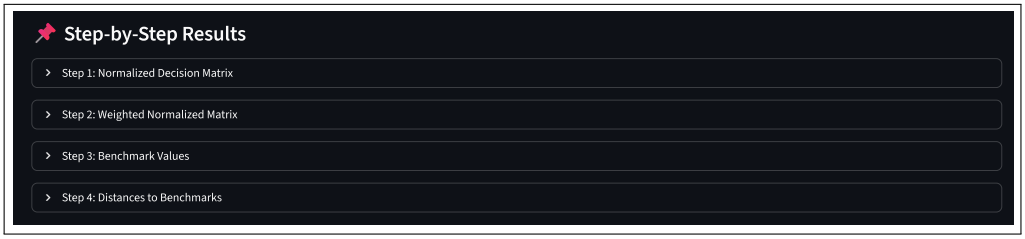


Figure 3.14 Interactive Step-by-step Result Interface for AURA Computations.

Final rankings are generated by computing the corrected utility distances from each alternative to three benchmark vectors: the PIS, the NIS, and the AS. Figure 3.15 displays the resulting ranked list of alternatives, ordered by their respective AURA scores. Lower scores indicate higher preference in this formulation.



Figure 3.15 Final AURA Scores and Ranking of Alternatives.

To support interpretability and communication, a bar chart visualization is automatically rendered. As shown in Figure 3.16, it provides a comparative view of score magnitudes across alternatives, facilitating decision-making under uncertainty and large-scale evaluation.

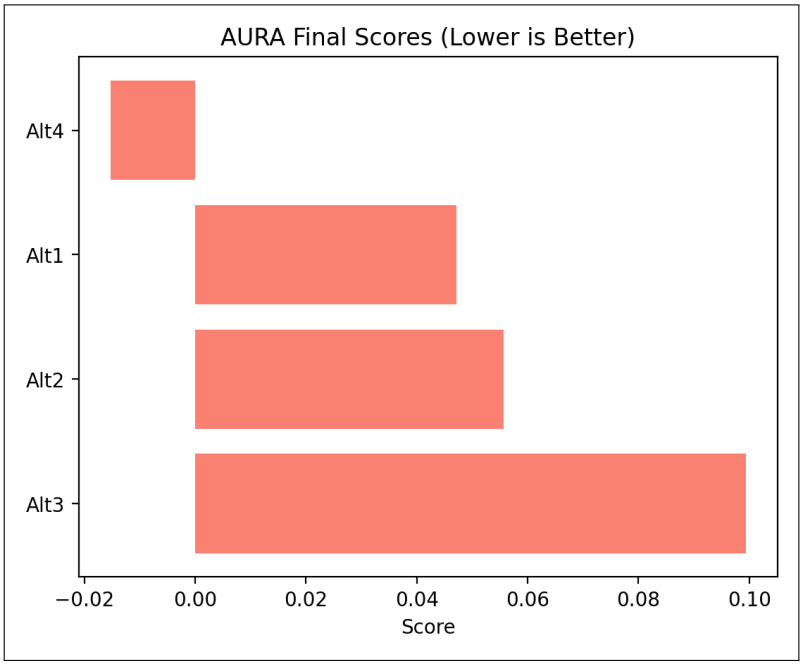
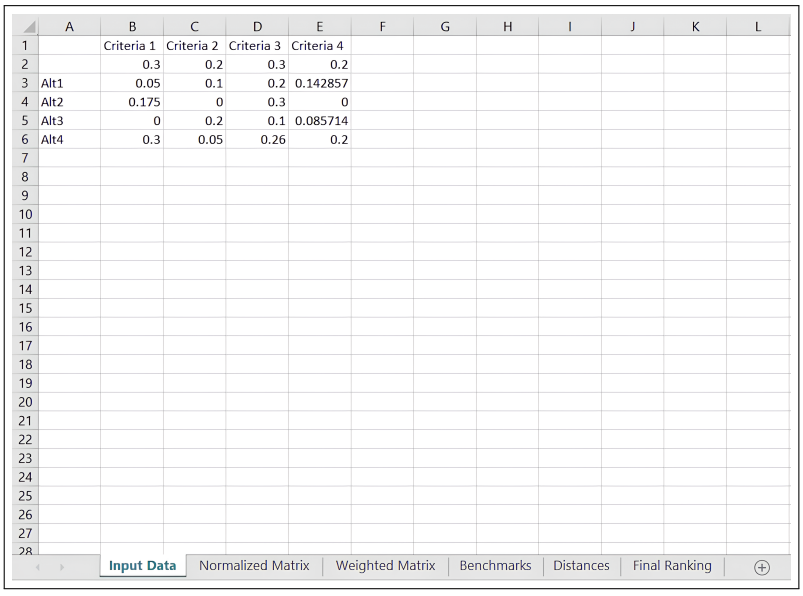


Figure 3.16 Bar Chart Visualization of Final AURA Scores.

In addition to on-screen visualization, the software automatically generates an Excel report containing all computational outputs. This includes the normalized matrix, weighted matrix, benchmark values, distance metrics, and final rankings in separate sheets. As shown in Figure 3.17, the report serves as a complete documentation artifact, suitable for inclusion in technical appendices or for auditing decisions in organizational contexts.



	A	B	C	D	E	F	G	H	I	J	K	L
1		Criteria 1	Criteria 2	Criteria 3	Criteria 4							
2		0.3	0.2	0.3	0.2							
3	Alt1	0.05	0.1	0.2	0.142857							
4	Alt2	0.175	0	0.3	0							
5	Alt3	0	0.2	0.1	0.085714							
6	Alt4	0.3	0.05	0.26	0.2							

Input Data | Normalized Matrix | Weighted Matrix | Benchmarks | Distances | Final Ranking

Figure 3.17 Generated Excel Report with Full Matrix Traceability Across Computation Steps.

Overall, the AURA software delivers a reproducible, modular, and user-centric tool that bridges the gap between theoretical MCDM models and practical applications. Its design accommodates both academic users seeking algorithmic transparency and decision-makers who require efficient, real-time analysis in high-stakes environments. Furthermore, the modular structure of the Python and Streamlit codebase ensures architectural flexibility for future development. This allows AURA to be readily extended to support more complex and uncertain data types-such as interval values, fuzzy numbers, or neutrosophic information-by integrating additional normalization and aggregation modules. As such, AURA is well-positioned for ongoing evolution to address the challenges of modern, data-rich decision-making contexts.

3.4.3 Scalability and Large-Scale Performance Analysis

To further assess the scalability and real-world applicability of the AURA method, a performance test was conducted using a large-scale synthetic decision matrix in Excel format, comprising 1,000 alternatives and 20 criteria. The matrix was structured to match the standard AURA input template, with the first row specifying criteria labels, the second row containing normalized weights that sum to one, and the subsequent rows providing the generated alternative data.

Random values between 50 and 100 were assigned to each alternative for each criterion to reflect realistic data variability. All computations were carried out using the AURA software on a standard laptop. The AURA algorithm successfully completed the ranking process for the 1000×20 matrix in 1.1000 seconds, with no observed computation delays, memory issues, or ranking instabilities as shown in Table 3.18.

These results confirm that AURA is highly scalable and suitable for large-scale, data-rich MCDM scenarios. The approach enables practical application in modern industrial, institutional, or research settings requiring efficient analysis of big decision matrices.

Table 3.18
Performance of AURA on a Large-scale Synthetic Decision Matrix (1000×20).

Scenario	**Computation Time (s)**	**Matrix Size**
Large-scale synthetic dataset	1.10	1000×20

3.5 Computational Analysis

This section evaluates the Adaptive Utility Ranking Algorithm (AURA) from two complementary angles. First, an application-driven analysis based on the implemented software demonstrates end-to-end usage, empirical validation, and a practical case study, concluding with impact observations. Second, a method-focused analysis reports a concise numerical example and sensitivity experiments conducted without reference to the software interface.

3.5.1 Application-Driven Analysis (via Software Implementation)

This subsection demonstrates the practical use of the AURA software through an illustrative example, empirical comparison with established MCDM methods, and a supplier-selection case study. It also discusses the practical impact of the software as a transparent and accessible decision-support tool.

3.5.1.1 Illustrative Example

To demonstrate the practical application of the AURA software, an illustrative case study is presented using an example dataset that consists of four alternatives and four evaluation criteria. The aim of the analysis is to evaluate and rank the alternatives based on Criteria 1, Criteria 2, Criteria 3, and Criteria 4.

As shown in Figure 3.11, Figure 3.12, and Figure 3.13 (referenced in the previous section), the user begins by uploading a structured Excel file containing the decision matrix. The weights for each criterion are defined manually based on expert consensus. Before running the analysis, users are required to define the type of each criterion—whether it is a benefit, cost, or target. Upon upload, the software automatically processes the matrix and presents a step-by-step breakdown of the computational workflow, including normalization, weighted aggregation, and benchmark identification, as illustrated in Figure 3.14.

Once all intermediate calculations are completed, the final utility scores are computed based on the corrected distances to the PIS, NIS, and AS. The resulting scores are then used to rank the four alternatives. The complete ranked list is presented in Figure 3.15, while a visual summary of score distributions is shown in

Figure 3.16. In this example, Alt_4 achieved the top rank, followed closely by Alt_1 and Alt_2, demonstrating AURA's capability to discriminate between closely performing alternatives.

A downloadable Excel report containing the normalized matrix, weighted matrix, benchmark values, distances, and final rankings is also generated for each execution. This output, referenced earlier in Figure 3.17, supports reproducibility and allows decision-makers to document and justify their decisions in institutional settings.

Overall, this example validates AURA's functionality in handling different types of criteria, managing multi-dimensional data, and providing interpretable outputs for real-world decision-making scenarios.

3.5.1.2 Empirical Validation and Comparative Analysis

To validate the effectiveness and reliability of the proposed AURA method, a comparative analysis was conducted against two well-established MCDM techniques: TOPSIS and VIKOR. Table 3.19 presents the final scores and rankings obtained for four alternatives (A_1–A_4) using all three methods. For AURA and VIKOR, lower scores indicate better alternatives, whereas for TOPSIS, higher scores indicate better alternatives.

Table 3.19
Comparison of AURA, TOPSIS, and VIKOR on a Sample Decision Matrix.

	AURA Method		**TOPSIS Method**		**VIKOR Method**	
Alternative	**Score**	**Rank**	**Score**	**Rank**	**Score**	**Rank**
A_1	0.0471	2	0.5911	3	0.5806	3
A_2	0.0557	3	0.7406	2	0.4875	2
A_3	0.0993	4	0.0579	4	1.0000	4
A_4	-0.0153	1	0.7952	1	0.0000	1

The results demonstrate that AURA's top ranking is fully consistent with both TOPSIS and VIKOR, with all three methods identifying A_4 as the top alternative. To quantitatively assess the agreement, the Spearman's rank correlation coefficient (ρ) and Kendall's (τ) were computed between AURA's rankings and those of TOPSIS and VIKOR. For both comparisons, Spearman's $\rho = 0.8000$ and Kendall's $\tau = 0.6700$, indicating a strong positive association between the rankings produced by the different methods.

This consistency confirms the reliability of AURA in replicating established ranking patterns, while its flexible scoring mechanism enables robust handling of mixed-type criteria. Computation times for all methods were negligible for this dataset, underscoring AURA's efficiency alongside its ranking validity.

3.5.1.3 Case Study Example: Supplier Selection Application

To further demonstrate the robustness and versatility of the AURA method, a real-world application was conducted using a supplier selection scenario, which is a classic benchmark problem in MCDM. In this case, five potential suppliers (S_1–S_5) were evaluated across six criteria, including cost, quality, delivery, flexibility, reputation, and service, with a mix of benefit-type and cost-type criteria and weights assigned based on expert judgment.

The decision matrix and weights were processed using the AURA software. The resulting scores and rankings are shown in Table 3.20. The results indicate that Supplier S_3 achieved the top rank, followed by S_5 and S_4.

Table 3.20
Supplier Selection Example: AURA Scores and Ranks.

Supplier	AURA Score	AURA Rank
S_3	-0.0935	1
S_5	-0.0460	2
S_4	-0.0216	3
S_1	0.2068	4
S_2	0.2406	5

This additional case study confirms that AURA can be effectively applied to real-world supplier selection problems, reinforcing the method's practical relevance and robustness across diverse decision-making scenarios.

3.5.1.4 Impact

The AURA software yields a significant contribution to the MCDM domain by offering an interactive, integrated, and customizable tool that addresses both theoretical and practical issues found among a range of approaches. Its ability to accommodate benefit, cost, and target-type criteria within a single scoring model distinguishes it from

classical MCDM tools, which typically require separate formulations and extensive pre-processing. This positions AURA as a unique solution capable of handling highly dimensional, real-world decision problems with hybrid evaluation objectives. Comparative benchmarking against established MCDM methods, such as TOPSIS and VIKOR, demonstrates high ranking consistency, reinforcing the method's validity.

Among the most notable contributions of AURA is the balance between the level of sophistication of its algorithms and its practical ease of use. The software can be deployed as a browser-accessible application without requiring local setup or programming skills. This reduces the cost of adoption for non-technical users in fields such as finance, public policy, supply chain management, healthcare, and education. Furthermore, the automated generation of Excel reports and visual analyses provides transparency and traceability, which are critical in institutional decision-making and audit requirements.

From a research standpoint, AURA fills a methodological gap by incorporating a correction-supported utility score that integrates distances to three benchmark solutions-the PIS, the NIS, and the AS-within a single scoring function. This innovation not only enhances the quality of ranking discrimination but also makes it possible to continuously analyze information across multiple datasets. Large-scale performance testing on datasets with up to 1000×20 confirms its computational efficiency and scalability for data-intensive environments. The algorithm is open-source and has a modular Python implementation, which makes it easy for researchers to adopt, adapt, or reproduce.

In education, the AURA interface serves as an intuitive teaching tool for instructors who wish to demonstrate MCDM principles without requiring students to write code. The step-by-step output interface is particularly helpful for learners to visualize how normalized data is transformed into rankings through benchmark comparisons and distance corrections.

Overall, AURA is a theory-to-practice tool that integrates modern decision-making theory with a readable, transparent software program. It can influence scholarly activities as well as practice to a considerable degree and may also be used as a pedagogical tool in teaching and training on MCDM-related topics.

CHAPTER 4
AGGREGATION OPERATOR

4.1 Introduction

This chapter formally proposes the q-Rung Orthopair Fuzzy Dombi Generalized Heronian Mean (qROFDGHM) operator and establishes its theoretical soundness. After presenting definitions and assumptions, it proves core properties (e.g., idempotency, monotonicity, boundedness) and shows how the operator integrates within an multi-criteria decision-making (MCDM) procedure. The chapter closes with a worked example and supporting analyses (numerical, sensitivity, comparative) to evidence the operator's effectiveness in uncertain, mixed-criteria settings.

4.2 Preliminaries

This section presents the fundamental concepts, definitions, and preliminaries related to q-rung orthopair fuzzy sets, Dombi operations, and Heronian mean operators, which form the basis of the proposed operator.

4.2.1 q-Rung Orthopair Fuzzy Sets

Definition 4.2.1. (Yager, 2017)

A q-rung orthopair fuzzy set (qROFS) E defined on a finite universe of discourse X is given by

$$E = \{\langle x, \mu_E(x), \nu_E(x)\rangle \mid x \in X\}, \tag{4.1}$$

where $\mu_E : X \to [0,1]$ denotes the degree of membership of the element $x \in X$ to the set E and $\nu_E : X \to [0,1]$ denotes the degree of non-membership of the element $x \in X$ to the set E, satisfying the condition

$$0 \leq \big(\mu_E(x)\big)^q + \big(\nu_E(x)\big)^q \leq 1, \qquad q \geq 1. \tag{4.2}$$

The degree of hesitancy (indeterminacy) of the element $x \in X$ to the set E is defined as

$$\pi_E(x) = \left(1 - \left(\mu_E(x)\right)^q - \left(\nu_E(x)\right)^q\right)^{\frac{1}{q}}. \tag{4.3}$$

For convenience, the ordered pair $(\mu_E(x), \nu_E(x))$ is referred to as a q-rung orthopair fuzzy number (qROFN) and is denoted by $\chi = (\mu, \nu)$, where μ and ν represent the membership and non-membership degrees, respectively. To compare two qROFNs, their score and accuracy functions are employed. The corresponding definitions are presented in the following.

Definition 4.2.2. (Liu & Wang, 2018)

Let $\chi_1 = (\mu_1, \nu_1)$ and $\chi_2 = (\mu_2, \nu_2)$ be two q-rung orthopair fuzzy numbers (qROFNs). Then, the following operations are defined:

1. $\overline{\chi_1} = \langle \nu_1, \mu_1 \rangle$
2. $\chi_1 \vee \chi_2 = \langle \max\{\mu_1, \mu_2\}, \min\{\nu_1, \nu_2\} \rangle$
3. $\chi_1 \wedge \chi_2 = \langle \min\{\mu_1, \mu_2\}, \max\{\nu_1, \nu_2\} \rangle$
4. $\chi_1 \oplus \chi_2 = \left\langle (\mu_1^q + \mu_2^q - \mu_1^q \mu_2^q)^{\frac{1}{q}}, \nu_1 \nu_2 \right\rangle$
5. $\chi_1 \otimes \chi_2 = \left\langle \mu_1 \mu_2, (\nu_1^q + \nu_2^q - \nu_1^q \nu_2^q)^{\frac{1}{q}} \right\rangle$
6. $\kappa\chi_1 = \left\langle (1 - (1 - \mu_1^q)^\kappa)^{\frac{1}{q}}, (\nu_1)^\kappa \right\rangle, \quad \kappa > 0$
7. $\chi_1^\kappa = \left\langle (\mu_1)^\kappa, (1 - (1 - (\nu_1)^q)^\kappa)^{\frac{1}{q}} \right\rangle, \quad \kappa > 0$

Definition 4.2.3. (Yager, 2017)

Let $\chi = (\mu, \nu)$ be a q-rung orthopair fuzzy number. The score function of χ is defined as

$$S(\chi) = \mu^q - \nu^q, \tag{4.4}$$

where $-1 \leq S(\chi) \leq 1$.

Definition 4.2.4. (Yager, 2017)

Let $\chi = (\mu, \nu)$ be a q-rung orthopair fuzzy number. The accuracy function of χ is defined as

$$A(\chi) = \mu^q + \nu^q, \tag{4.5}$$

where $0 \leq A(\chi) \leq 1$.

Definition 4.2.5. (Yager, 2017)

Let $\chi_1 = (\mu_1, \nu_1)$ and $\chi_2 = (\mu_2, \nu_2)$ be two arbitrary q-rung orthopair fuzzy numbers. Let $S(\chi_1)$ and $S(\chi_2)$ denote the score values of χ_1 and χ_2, respectively, and let $A(\chi_1)$ and $A(\chi_2)$ denote their corresponding accuracy values. Then, the ranking rules between χ_1 and χ_2 are defined as follows, where $\succ$ and $\sim$ denote strict preference and indifference between qROFNs, respectively:

1. If $S(\chi_1) > S(\chi_2)$, then $\chi_1 \succ \chi_2$;
2. If $S(\chi_1) = S(\chi_2)$, then
   a) If $A(\chi_1) > A(\chi_2)$, then $\chi_1 \succ \chi_2$;
   b) If $A(\chi_1) = A(\chi_2)$, then $\chi_1 \sim \chi_2$.

4.2.2 Heronian Mean (HM) Operators

This section then introduces the generalized Heronian mean and generalized weighted HM operators.

Definition 4.2.6. (Zhong et al., 2019) Generalized Heronian Mean (GHM)

Let x_i $(i = 1, 2, \ldots, n)$ be a collection of non-negative real numbers, where $n \in \mathbb{N}$ denotes the number of arguments, and let $a, b \geq 0$ with $a + b > 0$. Then, the generalized Heronian mean is defined as

$$GHM^{a,b}(x_1, x_2, \ldots, x_n) = \left[\frac{2}{n(n+1)} \left(\sum_{i=1}^{n} \sum_{j=i}^{n} x_i^a x_j^b \right) \right]^{\frac{1}{a+b}}. \tag{4.6}$$

Definition 4.2.7. (Zhong et al., 2019)

Generalized Weighted Heronian Mean (GWHM)

Let x_i $(i = 1, 2, \ldots, n)$ be a collection of non-negative real numbers and $w = (w_1, w_2, \ldots, w_n)$ be the associated weight vector satisfying $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$,

with parameters $a, b \geq 0$ and $a + b > 0$. Then, the generalized weighted Heronian mean is defined as

$$GWHM^{a,b}(x_1, x_2, \ldots, x_n) = \left[\frac{2}{n(n+1)} \left(\sum_{i=1}^{n} \sum_{j=i}^{n} (w_i x_i)^a (w_j x_j)^b \right) \right]^{\frac{1}{a+b}}. \quad (4.7)$$

4.2.3 Dombi t-norm and t-conorm

The Dombi operations, which comprise Dombi t-norms and t-conorms, were introduced as flexible operators for aggregating fuzzy information. These operations are effective in modeling intricate interactions between membership and non-membership values, making them suitable for situations where conventional operators, such as algebraic and Einstein t-norms, may be inadequate.

The Dombi t-norm and t-conorm based on additive generator functions are recalled in the following, as they form the basis of the proposed aggregation operators. (1) If $f(x)$ is a monotonically increasing function such that

$$f(x) : (0, 1] \to \mathbb{R}^+, \quad f^{-1}(x) : \mathbb{R}^+ \to (0, 1], \quad \lim_{t \to \infty} f^{-1}(t) = 0, \quad f^{-1}(0) = 1, \quad (4.8)$$

then the t-norm T is defined as

$$T(x, y) = f^{-1}(f(x) + f(y)). \quad (4.9)$$

(2) If $g(x)$ is a monotonically decreasing function such that

$$g(x) : (0, 1] \to \mathbb{R}^+, \quad g^{-1}(x) : \mathbb{R}^+ \to (0, 1], \quad \lim_{t \to \infty} g^{-1}(t) = 1, \quad g^{-1}(0) = 0, \quad (4.10)$$

then the t-conorm S is defined as

$$S(x, y) = g^{-1}(g(x) + g(y)). \quad (4.11)$$

According to Beliakov et al. (2016), the generator functions $f(x)$ and $g(x)$ satisfy the relationship $f(x) = g(1 - x)$.

Definition 4.2.8. (Jana et al., 2019)

Let $\lambda > 0$ be a real parameter and let $x, y \in (0,1)$. The Dombi t-norm (DTT), the Dombi t-conorm, and their corresponding additive generators are defined as follows:

$$T_{D,\lambda}(x,y) = f^{-1}(f(x)+f(y)) = \left(\frac{1}{1+\left(\left(\frac{1-x^q}{x^q}\right)^\lambda + \left(\frac{1-y^q}{y^q}\right)^\lambda \right)^{\frac{1}{\lambda}}} \right)^{\frac{1}{q}}. \tag{4.12}$$

$$S_{D,\lambda}(x,y) = g^{-1}(g(x)+g(y)) = \left(1 - \frac{1}{1+\left(\left(\frac{x^q}{1-x^q}\right)^\lambda + \left(\frac{y^q}{1-y^q}\right)^\lambda \right)^{\frac{1}{\lambda}}} \right)^{\frac{1}{q}}. \tag{4.13}$$

$$f(t) = \left(\frac{1-t^q}{t^q}\right)^\lambda, \qquad g(t) = \left(\frac{t^q}{1-t^q}\right)^\lambda, \tag{4.14}$$

$$f^{-1}(t) = \left(\frac{1}{1+t^{1/\lambda}}\right)^{\frac{1}{q}}, \qquad g^{-1}(t) = \left(\frac{t^{1/\lambda}}{1+t^{1/\lambda}}\right)^{\frac{1}{q}}. \tag{4.15}$$

Based on DTT, a set of operational rules of qROFNs can be established as definition 4.2.9:

Definition 4.2.9. (Zhong et al., 2019)

Let $\chi = (\mu, \nu)$, $\chi_1 = (\mu_1, \nu_1)$, and $\chi_2 = (\mu_2, \nu_2)$ be three arbitrary q-rung orthopair fuzzy numbers with $\mu, \nu, \mu_1, \nu_1, \mu_2, \nu_2 \in (0,1)$, and let $\varphi > 0$ and $\tau > 0$ be two real parameters. Then, the sum, product, scalar multiplication, and power operations between qROFNs, based on the Dombi t-norm $T_{D,\lambda}(x,y) = f^{-1}(f(x)+f(y))$ and the Dombi t-conorm $S_{D,\lambda}(x,y) = g^{-1}(g(x)+g(y))$, are defined as follows, respectively:

$$\begin{aligned}\chi_1 \oplus \chi_2 &= \left(g^{-1}(g(\mu_1)+g(\mu_2)),\, f^{-1}(f(\nu_1)+f(\nu_2)) \right) \\ &= \left(\left(1 - \frac{1}{1+\left(\left(\frac{\mu_1^q}{1-\mu_1^q}\right)^\lambda + \left(\frac{\mu_2^q}{1-\mu_2^q}\right)^\lambda \right)^{1/\lambda}} \right)^{1/q}, \left(\frac{1}{1+\left(\left(\frac{(1-\nu_1)^q}{\nu_1^q}\right)^\lambda + \left(\frac{(1-\nu_2)^q}{\nu_2^q}\right)^\lambda \right)^{1/\lambda}} \right)^{1/q} \right)\end{aligned} \tag{4.16}$$

$$\chi_1 \otimes \chi_2 = \left(f^{-1}(f(\mu_1)+f(\mu_2)),\, g^{-1}(g(\nu_1)+g(\nu_2))\right)$$
$$= \left(\left(\frac{1}{1+\left(\left(\frac{1-\mu_1^q}{\mu_1^q}\right)^\lambda+\left(\frac{1-\mu_2^q}{\mu_2^q}\right)^\lambda\right)^{1/\lambda}}\right)^{1/q}, \left(1-\frac{1}{1+\left(\left(\frac{\nu_1^q}{1-\nu_1^q}\right)^\lambda+\left(\frac{\nu_2^q}{1-\nu_2^q}\right)^\lambda\right)^{1/\lambda}}\right)^{1/q}\right) \tag{4.17}$$

$$\varphi\chi = \left(g^{-1}(\varphi g(\mu)),\, f^{-1}(\varphi f(\nu))\right)$$
$$= \left(\left(1-\frac{1}{1+\left(\varphi\left(\frac{\mu^q}{1-\mu^q}\right)^\lambda\right)^{1/\lambda}}\right)^{1/q}, \left(\frac{1}{1+\left(\varphi\left(\frac{1-\nu^q}{\nu^q}\right)^\lambda\right)^{1/\lambda}}\right)^{1/q}\right) \tag{4.18}$$

$$\chi^\tau = \left(f^{-1}(\tau f(\mu)),\, g^{-1}(\tau g(\nu))\right)$$
$$= \left(\left(\frac{1}{1+\left(\tau\left(\frac{1-\mu^q}{\mu^q}\right)^\lambda\right)^{1/\lambda}}\right)^{1/q}, \left(1-\frac{1}{1+\left(\tau\left(\frac{\nu^q}{1-\nu^q}\right)^\lambda\right)^{1/\lambda}}\right)^{1/q}\right) \tag{4.19}$$

4.3 Proposed q-Rung Orthopair Fuzzy Dombi Generalized Heronian Mean

This section extends the classical Generalized Heronian Mean (GHM) into the q-rung orthopair fuzzy environment. Based on Dombi operational laws, a qROFDGHM operator and its weighted form, namely the q-rung orthopair fuzzy Dombi weighted generalized Heronian mean (qROFDWGHM) operator, are introduced. Their fundamental properties are also investigated.

4.3.1 qROFDGHM Operator

Definition 4.3.1.

Let $\{\chi_1, \chi_2, \ldots, \chi_n\}$ be a collection of q-rung orthopair fuzzy numbers (qROFNs), where $\chi_i = (\mu_i, \nu_i)$ for $i = 1, 2, \ldots, n$, satisfying

$$\mu_i^q + \nu_i^q \leq 1, \quad q \geq 1 \tag{4.20}$$

Let a and b be two non-negative real numbers such that $a, b \geq 0$ and a and b are

not simultaneously zero. Then, the q-rung orthopair fuzzy Dombi generalized Heronian mean (qROFDGHM) operator is defined as

$$qROFDGHM^{a,b}(\chi_1, \chi_2, \ldots, \chi_n) = \left[\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \left((\chi_i)^a \otimes (\chi_j)^b \right) \right]^{\frac{1}{a+b}}. \tag{4.21}$$

Theorem 4.1.

Let $\{\chi_1, \chi_2, \ldots, \chi_n\}$ be a collection of q-rung orthopair fuzzy numbers (qROFNs), where $\chi_i = (\mu_i, \nu_i)$ for $i = 1, 2, \ldots, n$, satisfying $\mu_i^q + \nu_i^q \leq 1$ with $q \geq 1$. Let $a, b \geq 0$ and not simultaneously zero, and let $\lambda > 0$ be the Dombi parameter. Then

$$qROFDGHM^{a,b}(\chi_1, \chi_2, \ldots, \chi_n) = \chi^* = (\mu^*, \nu^*), \tag{4.22}$$

where

$$\mu^* = \left(1 - \frac{1}{1 + \left(\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{af(\mu_i) + bf(\mu_j)} \right)^{\frac{1}{\lambda}}} \right)^{\frac{1}{q}}, \tag{4.23}$$

$$\nu^* = \left(\frac{1}{1 + \left(\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{ag(\nu_i) + bg(\nu_j)} \right)^{\frac{1}{\lambda}}} \right)^{\frac{1}{q}}, \tag{4.24}$$

and

$$f(\mu_i) = \left(\frac{1 - \mu_i^q}{\mu_i^q} \right)^{\lambda}, \quad f(\mu_j) = \left(\frac{1 - \mu_j^q}{\mu_j^q} \right)^{\lambda}, \tag{4.25}$$

$$g(\nu_i) = \left(\frac{\nu_i^q}{1 - \nu_i^q} \right)^{\lambda}, \quad g(\nu_j) = \left(\frac{\nu_j^q}{1 - \nu_j^q} \right)^{\lambda}. \tag{4.26}$$

Proof. According to Definition 4.3.1, we have

$$qROFDGHM^{a,b}(\chi_1, \chi_2, \ldots, \chi_n) = \left[\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \left((\chi_i)^a \otimes (\chi_j)^b \right) \right]^{\frac{1}{a+b}}. \tag{4.27}$$

For $\chi_i = (\mu_i, \nu_i)$, by the Dombi operational laws follow Definition 4.2.8, the

Dombi scalar power yields

$$(\chi_i)^a = \left(\left(\frac{1}{1 + \left(a\, f(\mu_i)\right)^{\frac{1}{\lambda}}} \right)^{\frac{1}{q}}, \left(1 - \frac{1}{1 + \left(a\, g(\nu_i)\right)^{\frac{1}{\lambda}}} \right)^{\frac{1}{q}} \right), \tag{4.28}$$

$$(\chi_j)^b = \left(\left(\frac{1}{1 + \left(b\, f(\mu_j)\right)^{\frac{1}{\lambda}}} \right)^{\frac{1}{q}}, \left(1 - \frac{1}{1 + \left(b\, g(\nu_j)\right)^{\frac{1}{\lambda}}} \right)^{\frac{1}{q}} \right), \tag{4.29}$$

where

$$f(\mu_i) = \left(\frac{1 - \mu_i^q}{\mu_i^q} \right)^{\lambda}, \qquad g(\nu_i) = \left(\frac{\nu_i^q}{1 - \nu_i^q} \right)^{\lambda}, \qquad \lambda > 0.$$

Next, applying the Dombi product $\otimes$ using Definition 4.2.9 to Equation (4.28) and (4.29), we obtain

$$(\chi_i)^a \otimes (\chi_j)^b = \left(\left(1 - \frac{1}{1 + \left(\frac{1}{af(\mu_i) + bf(\mu_j)} \right)^{\frac{1}{\lambda}}} \right)^{\frac{1}{q}}, \left(\frac{1}{1 + \left(\frac{1}{ag(\nu_i) + bg(\nu_j)} \right)^{\frac{1}{\lambda}}} \right)^{\frac{1}{q}} \right). \tag{4.30}$$

From Equation (4.30), the membership component of each pair is

$$\mu_{ij} = \left(1 - \frac{1}{1 + \left(\frac{1}{af(\mu_i) + bf(\mu_j)} \right)^{1/\lambda}} \right)^{1/q}. \tag{4.31}$$

Applying the Dombi sum $\oplus$ (Definition 4.2.9) to two such terms adds their g-generator values; since the membership generator satisfies $g(\mu_{ij}) = \dfrac{1}{af(\mu_i) + bf(\mu_j)}$, the Dombi sum of all $\frac{n(n+1)}{2}$ pairwise terms gives

$$\bigoplus_{i \le j} (\mu_{ij}, \cdot) \implies g\text{-sum} = \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{af(\mu_i) + bf(\mu_j)}. \tag{4.32}$$

Multiplying by the Heronian normalisation scalar $\frac{2}{n(n+1)}$ (a Dombi scalar multiplication that multiplies the g-sum by that factor) and then applying the outer Dombi power $\frac{1}{a+b}$ (which multiplies the accumulated generator value by $a + b$) replaces the bare sum by

$$\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{af(\mu_i) + bf(\mu_j)}. \tag{4.33}$$

By the associativity of the Dombi-based aggregation over the collection $\{(\mu_{ij}, \nu_{ij}) : 1 \leq i \leq j \leq n\}$, the normalization coefficient $\frac{2}{n(n+1)}$ yields

$$\mu^* = \left(1 - \frac{1}{1 + \left(\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{af(\mu_i)+bf(\mu_j)} \right)^{\frac{1}{\lambda}}} \right)^{\frac{1}{q}}, \tag{4.34}$$

$$\nu^* = \left(\frac{1}{1 + \left(\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{ag(\nu_i)+bg(\nu_j)} \right)^{\frac{1}{\lambda}}} \right)^{\frac{1}{q}}. \tag{4.35}$$

Here the aggregated membership degree μ^* is the q-rung membership of the single qROFN that represents the Dombi–Heronian consensus of all inputs; the inner double sum accumulates the pairwise interactions, the $1/\lambda$ exponent applies the Dombi curvature, and the outer $1/q$ root returns the value to the membership scale. The non-membership degree ν^* is obtained by the dual construction using the decreasing generator g. Therefore,

$$\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \left((\chi_i)^a \otimes (\chi_j)^b \right) = (\mu^*, \nu^*), \tag{4.36}$$

and substituting this result into Equation (4.27) gives

$$qROFDGHM^{a,b}(\chi_1, \chi_2, \ldots, \chi_n) = \chi^* = (\mu^*, \nu^*), \tag{4.37}$$

where μ^* and ν^* are exactly given by Equation (4.34) and (4.35). Thus, the proof is completed. □

The proposed operator is now shown to satisfy the standard desirable properties of an aggregation function. Idempotency, monotonicity, and boundedness are the three properties required of any well-defined aggregation operator (Beliakov et al., 2016), and they have been established for related q-rung orthopair fuzzy Dombi and Heronian operators (Liu & Wang, 2018; Wei et al., 2018; Zhong et al., 2019). The following

theorems verify that the qROFDGHM operator possesses these properties.

Theorem 4.2 (Idempotency)**.**

Let $\{\chi_1, \chi_2, \ldots, \chi_n\}$ *be a collection of qROFNs where each* $\chi_i = (\mu_i, \nu_i)$ *is a qROFN satisfying* $\mu_i^q + \nu_i^q \leq 1$*. Let* $a, b \geq 0$ *with not both zero. If* $\chi_i = \chi = (\mu, \nu)$ *for all* $i = 1, 2, \ldots, n$*, then*

$$qROFDGHM^{a,b}(\chi_1, \chi_2, \ldots, \chi_n) = \chi. \tag{4.38}$$

Proof. Let

$$qROFDGHM^{a,b}(\chi_1, \chi_2, \ldots, \chi_n) = (\mu^*, \nu^*). \tag{4.39}$$

Since $\chi_i = \chi_j = \chi = (\mu, \nu)$ for all i, j, we have

$$f(\mu_i) = f(\mu_j) = f(\mu), \qquad g(\nu_i) = g(\nu_j) = g(\nu). \tag{4.40}$$

Hence,

$$af(\mu_i) + bf(\mu_j) = (a+b)f(\mu). \tag{4.41}$$

Therefore,

$$\sum_{i=1}^{n}\sum_{j=i}^{n} \frac{1}{af(\mu_i) + bf(\mu_j)} = \frac{n(n+1)}{2} \cdot \frac{1}{(a+b)f(\mu)}.$$

Substituting into the expression of μ^*, we obtain

$$\mu^* = \left(1 - \frac{1}{1 + \left(\frac{2(a+b)}{n(n+1)} \cdot \frac{n(n+1)}{2} \cdot \frac{1}{(a+b)f(\mu)}\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}} \tag{4.42}$$

$$= \left(1 - \frac{1}{1 + \left(\frac{1}{f(\mu)}\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}}. \tag{4.43}$$

Since

$$f(\mu) = \left(\frac{1-\mu^q}{\mu^q}\right)^{\lambda}, \tag{4.44}$$

we have

$$\left(\frac{1}{f(\mu)}\right)^{\frac{1}{\lambda}} = \frac{\mu^q}{1-\mu^q}. \tag{4.45}$$

Thus,

$$\mu^* = \left(1 - \frac{1}{1 + \frac{\mu^q}{1-\mu^q}}\right)^{\frac{1}{q}} \tag{4.46}$$

$$= \left(1 - \frac{1}{\frac{1}{1-\mu^q}}\right)^{\frac{1}{q}} \tag{4.47}$$

$$= (1 - (1 - \mu^q))^{\frac{1}{q}} \tag{4.48}$$

$$= (\mu^q)^{\frac{1}{q}} = \mu. \tag{4.49}$$

By an analogous argument for the non-membership component using

$$g(\nu) = \left(\frac{\nu^q}{1 - \nu^q}\right)^{\lambda}, \tag{4.50}$$

we similarly obtain

$$\nu^* = \nu. \tag{4.51}$$

Hence,

$$qROFDGHM^{a,b}(\chi, \ldots, \chi) = \chi. \tag{4.52}$$

Therefore, the idempotency property holds. □

Monotonicity guarantees that an improvement in any input qROFN cannot worsen the aggregated result; it is a fundamental requirement for aggregation operators (Beliakov et al., 2016) and has been verified for cognate Dombi–Heronian operators in the q-rung environment (Zhong et al., 2019).

Theorem 4.3 (Monotonicity)**.**

Let $\{\chi_1, \chi_2, \ldots, \chi_n\}$ *and* $\{\chi'_1, \chi'_2, \ldots, \chi'_n\}$ *be two collections of qROFNs, where* $\chi_i = (\mu_i, \nu_i)$ *and* $\chi'_i = (\mu'_i, \nu'_i)$ *for* $i = 1, 2, \ldots, n$. *Let* $a, b \geq 0$ *and* a *and* b *are not simultaneously zero. If* $\mu_i \leq \mu'_i$ *and* $\nu_i \leq \nu'_i$ *for all* $i = 1, 2, \ldots, n$, *then*

$$qROFDGHM^{a,b}(\chi_1, \chi_2, \ldots, \chi_n) \leq qROFDGHM^{a,b}(\chi'_1, \chi'_2, \ldots, \chi'_n) \tag{4.53}$$

Proof. Let

$$qROFDGHM^{a,b}(\chi_1, \chi_2, \ldots, \chi_n) = (\mu_\alpha, \nu_\alpha), \tag{4.54}$$

$$qROFDGHM^{a,b}(\chi'_1, \chi'_2, \ldots, \chi'_n) = (\mu', \nu'). \tag{4.55}$$

Since $\mu_i \leq \mu'_i$ and $\mu_j \leq \mu'_j$, and

$$f(\mu) = \left(\frac{1-\mu^q}{\mu^q}\right)^\lambda \tag{4.56}$$

is monotonically decreasing, it follows that

$$\frac{1-\mu_i^q}{\mu_i^q} \geq \frac{1-(\mu'_i)^q}{(\mu'_i)^q}, \qquad \frac{1-\mu_j^q}{\mu_j^q} \geq \frac{1-(\mu'_j)^q}{(\mu'_j)^q}, \tag{4.57}$$

and hence

$$af(\mu_i) + bf(\mu_j) \geq af(\mu'_i) + bf(\mu'_j). \tag{4.58}$$

Therefore,

$$\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{af(\mu_i) + bf(\mu_j)} \leq \frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{af(\mu'_i) + bf(\mu'_j)}.$$

Taking reciprocals reverses the inequality:

$$\frac{1}{\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{af(\mu_i)+bf(\mu_j)}} \geq \frac{1}{\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{af(\mu'_i)+bf(\mu'_j)}}. \tag{4.59}$$

Since $\lambda > 0$, raising both sides to the power $1/\lambda$ preserves the order:

$$\left(\frac{1}{\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{af(\mu_i)+bf(\mu_j)}}\right)^{1/\lambda} \geq \left(\frac{1}{\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{af(\mu'_i)+bf(\mu'_j)}}\right)^{1/\lambda}. \tag{4.60}$$

Adding 1 preserves the order, and taking reciprocals reverses it:

$$\frac{1}{1+\left(\frac{1}{\frac{2(a+b)}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\frac{1}{af(\mu_i)+bf(\mu_j)}}\right)^{1/\lambda}} \leq \frac{1}{1+\left(\frac{1}{\frac{2(a+b)}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\frac{1}{af(\mu_i')+bf(\mu_j')}}\right)^{1/\lambda}}. \tag{4.61}$$

Now consider the membership expression (as in Theorem 4.1):

$$\mu_\alpha = \left(1-\frac{1}{1+\left(\frac{2(a+b)}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\frac{1}{af(\mu_i)+bf(\mu_j)}\right)^{1/\lambda}}\right)^{1/q}. \tag{4.62}$$

The mapping

$$x \mapsto \left(1-\frac{1}{1+x}\right)^{1/q} \qquad (x>0) \tag{4.63}$$

is increasing for $q>0$. Hence,

$$\mu_\alpha \leq \mu'. \tag{4.64}$$

Similarly, since $\nu_i \leq \nu_i'$ and

$$g(\nu) = \left(\frac{\nu^q}{1-\nu^q}\right)^{\lambda} \tag{4.65}$$

is monotonically increasing, we have

$$g(\nu_i) \leq g(\nu_i'), \qquad g(\nu_j) \leq g(\nu_j'). \tag{4.66}$$

Repeating the same steps for the non-membership part yields

$$\nu_\alpha \leq \nu'. \tag{4.67}$$

Therefore,

$$qROFDGHM^{a,b}(\chi_1,\chi_2,\ldots,\chi_n) \leq qROFDGHM^{a,b}(\chi_1',\chi_2',\ldots,\chi_n') \tag{4.68}$$

This completes the proof. □

Theorem 4.4 (Boundedness)**.**

Let $\{\chi_1, \chi_2, \ldots, \chi_n\}$*, where* $\chi_i = (\mu_i, \nu_i)$ $(i = 1, 2, \ldots, n)$*, be a collection of qROFNs with* $q > 1$*. Let* $a, b \geq 0$ *with not both zero. Define*

$$\mu^+ = \max_{1 \leq i \leq n} \mu_i, \quad \mu^- = \min_{1 \leq i \leq n} \mu_i, \quad \nu^+ = \max_{1 \leq i \leq n} \nu_i, \quad \nu^- = \min_{1 \leq i \leq n} \nu_i, \tag{4.69}$$

and let

$$\chi_s = (\mu^+, \nu^-), \qquad \chi_t = (\mu^-, \nu^+). \tag{4.70}$$

Then

$$\chi_t \ \leq \ qROFDGHM^{a,b}(\chi_1, \chi_2, \ldots, \chi_n) \ \leq \ \chi_s \tag{4.71}$$

Proof. From the idempotency property (Theorem 4.2), we have

$$qROFDGHM^{a,b}(\chi_t, \chi_t, \ldots, \chi_t) = \chi_t, \tag{4.72}$$

$$qROFDGHM^{a,b}(\chi_s, \chi_s, \ldots, \chi_s) = \chi_s. \tag{4.73}$$

Since $\mu^- \leq \mu_i \leq \mu^+$ and $\nu^- \leq \nu_i \leq \nu^+$ for all i, it follows that

$$\chi_t \leq \chi_i \leq \chi_s \quad \text{for all } i. \tag{4.74}$$

By the monotonicity property (Theorem 4.3), we obtain

$$qROFDGHM^{a,b}(\chi_t, \chi_t, \ldots, \chi_t) \leq qROFDGHM^{a,b}(\chi_1, \chi_2, \ldots, \chi_n) \tag{4.75}$$

$$qROFDGHM^{a,b}(\chi_1, \chi_2, \ldots, \chi_n) \leq qROFDGHM^{a,b}(\chi_s, \chi_s, \ldots, \chi_s). \tag{4.76}$$

Therefore,

$$\chi_t \ \leq \ qROFDGHM^{a,b}(\chi_1, \chi_2, \ldots, \chi_n) \ \leq \ \chi_s, \tag{4.77}$$

which completes the proof. □

4.3.2 qROFDWGHM Operator

In practical MCDM problems, different criteria may possess different levels of importance. To incorporate the weight information into the aggregation process, we extend the proposed $qROFDGHM^{a,b}$ operator to its weighted version.

Definition 4.3.2.

Let $\{\chi_1, \chi_2, \ldots, \chi_n\}$ be a collection of qROFNs, where $\chi_i = (\mu_i, \nu_i)$ for $i = 1, 2, \ldots, n$, satisfying $\mu_i^q + \nu_i^q \leq 1$ with $q \geq 1$. Let $\mathbf{w} = (w_1, w_2, \ldots, w_n)^T$ be the weight vector such that $w_i \in [0, 1]$ and $\sum_{i=1}^n w_i = 1$. Let $a, b \geq 0$ and a and b are not simultaneously zero. Then, the qROFDWGHM operator is defined as Equation (4.78).

$$qROFDWGHM^{a,b}(\chi_1, \chi_2, \ldots, \chi_n) = \left[\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \left((w_i\chi_i)^a \otimes (w_j\chi_j)^b\right)\right]^{\frac{1}{a+b}}. \tag{4.78}$$

Theorem 4.5.

Let $\{\chi_1, \chi_2, \ldots, \chi_n\}$ be a collection of qROFNs with $\chi_i = (\mu_i, \nu_i)$ and $\mu_i^q + \nu_i^q \leq 1$. Let $\mathbf{w} = (w_1, \ldots, w_n)^T$ be the weight vector, $a, b \geq 0$ (not both zero), and $\lambda > 0$. Then

$$qROFDWGHM^{a,b}(\chi_1, \chi_2, \ldots, \chi_n) = \chi^* = (\mu^*, \nu^*), \tag{4.79}$$

where

$$\mu^* = \left(1 - \frac{1}{1 + \left(\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{aw_i f(\mu_i) + bw_j f(\mu_j)}\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}}, \tag{4.80}$$

$$\nu^* = \left(\frac{1}{1 + \left(\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{aw_i g(\nu_i) + bw_j g(\nu_j)}\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}}, \tag{4.81}$$

and

$$f(\mu_i) = \left(\frac{1 - \mu_i^q}{\mu_i^q}\right)^{\lambda}, \qquad g(\nu_i) = \left(\frac{\nu_i^q}{1 - \nu_i^q}\right)^{\lambda}. \tag{4.82}$$

Proof of Theorem 4.5. The proof parallels that of Theorem 4.1, with the criterion weight

w_i entering the Dombi scalar multiplication. By Definition 4.3.2,

$$qROFDWGHM^{a,b}(\chi_1,\ldots,\chi_n)=\left[\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\left((w_i\chi_i)^a\otimes(w_j\chi_j)^b\right)\right]^{\frac{1}{a+b}}. \tag{4.83}$$

Applying the Dombi scalar multiplication $w_i\chi_i$ followed by the power $(\cdot)^a$ to the membership component replaces the generator value $f(\mu_i)$ by $w_i f(\mu_i)$, and likewise the power $(\cdot)^b$ on $w_j\chi_j$ gives $w_j f(\mu_j)$. Hence the pairwise Dombi product yields the membership generator $aw_i f(\mu_i)+bw_j f(\mu_j)$. Summing over all pairs $i\le j$, applying the Heronian scalar $\frac{2}{n(n+1)}$ and the outer power $\frac{1}{a+b}$ exactly as in the proof of Theorem 4.1, we obtain

$$\mu^*=\left(1-\frac{1}{1+\left(\frac{2(a+b)}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\frac{1}{aw_i f(\mu_i)+bw_j f(\mu_j)}\right)^{1/\lambda}}\right)^{1/q}, \tag{4.84}$$

and, dually, the stated expression for ν^*. This completes the proof. □

Theorem 4.6 (Idempotency)**.**

Let $\{\chi_1,\chi_2,\ldots,\chi_n\}$ be a collection of qROFNs and let $a,b\ge 0$ (not both zero). If $\chi_i=\chi$ for all $i=1,2,\ldots,n$, then

$$qROFDWGHM^{a,b}(\chi_1,\chi_2,\ldots,\chi_n)=\chi. \tag{4.85}$$

Proof. Let $\chi_i=\chi=(\mu,\nu)$ for all i, so $f(\mu_i)=f(\mu)$ and $g(\nu_i)=g(\nu)$. With normalized weights $\sum_i w_i=1$, the membership denominator becomes $aw_i f(\mu)+bw_j f(\mu)=(aw_i+bw_j)f(\mu)$. A standard property of the generalized weighted Heronian mean under the normalization $\sum_i w_i=1$ gives

$$\frac{2(a+b)}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\frac{1}{(aw_i+bw_j)f(\mu)}=\frac{1}{f(\mu)}. \tag{4.86}$$

Substituting into μ^*,

$$\mu^* = \left(1 - \frac{1}{1 + (1/f(\mu))^{1/\lambda}}\right)^{1/q} = \left(1 - \frac{1}{1 + \frac{\mu^q}{1-\mu^q}}\right)^{1/q} = \left(1 - (1-\mu^q)\right)^{1/q} = \mu, \tag{4.87}$$

using $(1/f(\mu))^{1/\lambda} = \mu^q/(1-\mu^q)$. By the dual argument with $g(\nu)$, $\nu^* = \nu$. Hence the result. □

Theorem 4.7 (Monotonicity)**.**

Let $\{\chi_1, \ldots, \chi_n\}$ *and* $\{\chi'_1, \ldots, \chi'_n\}$ *be two collections of qROFNs. If* $\mu_i \leq \mu'_i$ *and* $\nu_i \leq \nu'_i$ *for all* i*, then*

$$qROFDWGHM^{a,b}(\chi_1, \ldots, \chi_n) \leq qROFDWGHM^{a,b}(\chi'_1, \ldots, \chi'_n). \tag{4.88}$$

Proof. Since $w_i > 0$ and f is decreasing while g is increasing, $\mu_i \leq \mu'_i$ implies $w_i f(\mu_i) \geq w_i f(\mu'_i)$ and $\nu_i \leq \nu'_i$ implies $w_i g(\nu_i) \leq w_i g(\nu'_i)$. These are exactly the monotonicity premises used in the proof of Theorem 4.3; repeating that argument with the weighted generators $w_i f(\mu_i)$ and $w_i g(\nu_i)$ in place of $f(\mu_i)$ and $g(\nu_i)$ yields $\mu^* \leq \mu'^*$ and $\nu^* \leq \nu'^*$ componentwise. Hence the operator is monotonic. □

Theorem 4.8 (Boundedness)**.**

Let

$$\chi_s = (\max_i \mu_i, \min_i \nu_i), \qquad \chi_t = (\min_i \mu_i, \max_i \nu_i). \tag{4.89}$$

Then

$$\chi_t \leq qROFDWGHM^{a,b}(\chi_1, \ldots, \chi_n) \leq \chi_s. \tag{4.90}$$

Proof. By idempotency (Theorem 4.6), $qROFDWGHM^{a,b}(\chi_t, \ldots, \chi_t) = \chi_t$ and $qROFDWGHM^{a,b}(\chi_s, \ldots, \chi_s) = \chi_s$. Since $\chi_t \leq \chi_i \leq \chi_s$ componentwise for every i, monotonicity (Theorem 4.7) gives

$$\chi_t = qROFDWGHM^{a,b}(\chi_t, \ldots, \chi_t) \tag{4.91}$$

$$\leq qROFDWGHM^{a,b}(\chi_1, \ldots, \chi_n) \tag{4.92}$$

$$\leq qROFDWGHM^{a,b}(\chi_s, \ldots, \chi_s) = \chi_s. \tag{4.93}$$

which is the assertion. □

4.4 Multi-Criteria Decision-Making Method based on the Presented Operator

This section presents a comprehensive MCDM framework developed under the q-rung orthopair fuzzy environment using the proposed qROFDWGHM operator. The primary objective of this framework is to determine the most appropriate alternative among a set of feasible options by systematically aggregating uncertain evaluation information provided by multiple decision makers. Consider an MCDM problem characterized by the following components:

- A finite set of alternatives $\mathcal{A} = \{A_1, A_2, \ldots, A_m\}$, where m denotes the number of competing alternatives.
- A finite set of evaluation criteria $\mathcal{C} = \{C_1, C_2, \ldots, C_n\}$, where n represents the number of attributes or criteria under consideration.
- A criteria weight vector $\boldsymbol{\omega} = (\omega_1, \omega_2, \ldots, \omega_n)$, where $\omega_j \in [0, 1]$ for $j = 1, 2, \ldots, n$ and $\sum_{j=1}^{n} \omega_j = 1$. These weights reflect the relative importance of each criterion in the decision-making process.
- A set of decision makers $\mathcal{D} = \{D_1, D_2, \ldots, D_K\}$, where K denotes the number of experts involved in the evaluation process.
- A decision-maker weight vector $\mathbf{w} = (w_1, w_2, \ldots, w_K)$, where $w_k \in [0, 1]$ and $\sum_{k=1}^{K} w_k = 1$. These weights represent the credibility, experience, or importance level of each decision maker.

Under this setting, each decision maker D_k provides an evaluation matrix in the form of a q-rung orthopair fuzzy decision matrix:

$$M_k = [\chi_{ij}^k]_{m \times n}, \tag{4.94}$$

where $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, and $k = 1, 2, \ldots, K$. Each element $\chi_{ij}^k = (\mu_{ij}^k, \nu_{ij}^k)$ is a q-rung orthopair fuzzy number satisfying $(\mu_{ij}^k)^q + (\nu_{ij}^k)^q \leq 1$, which represents the evaluation value assigned by decision maker D_k to alternative A_i with respect to criterion C_j.

Based on the above components, Figure 4.1 illustrates the overall procedure of the proposed qROFDWGHM-based MCDM framework and is described as follows.

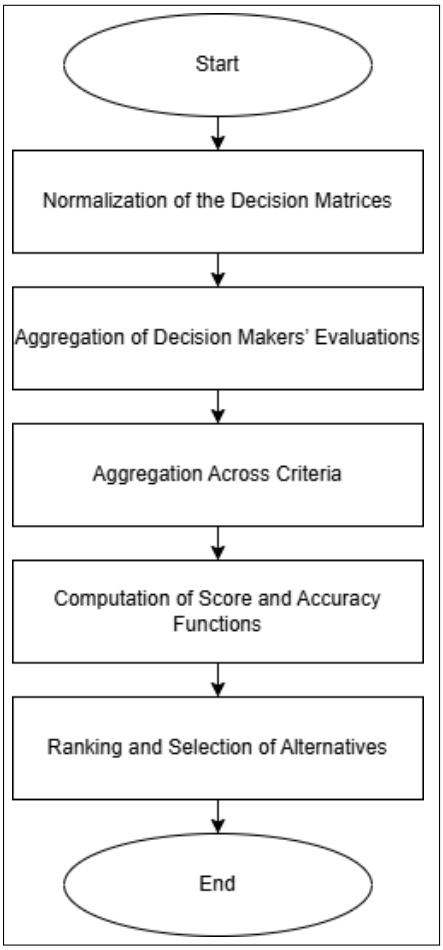


Figure 4.1 Flowchart of the Proposed qROFDWGHM-based MCDM Method

Step 1: Normalization of the Decision Matrices.

In practical decision-making problems, criteria are typically classified into two categories: benefit-type criteria and cost-type criteria. Benefit criteria contribute positively to the final decision outcome (i.e., higher values are preferred), whereas cost criteria contribute negatively (i.e., lower values are preferred).

To ensure consistency in the aggregation process, all criteria must be converted into a unified benefit-type structure. The normalized decision matrix M_k' for each decision maker D_k is defined as:

$$M_k' = \begin{cases} [\mu_{ij}^k, \nu_{ij}^k]_{m\times n}, & \text{if } C_j \text{ is a benefit criterion,} \\ [\nu_{ij}^k, \mu_{ij}^k]_{m\times n}, & \text{if } C_j \text{ is a cost criterion.} \end{cases} \tag{4.95}$$

This transformation ensures that higher membership values consistently indicate better performance.

Step 2: Aggregation of Decision Makers' Evaluations.

Since multiple decision makers are involved, their evaluations must be aggregated into a single collective decision matrix. Using the normalized matrices M'_k and the decision-maker weight vector **w**, the collective evaluation for alternative A_i under criterion C_j is computed as:

$$\chi_{ij} = qROFDWGHM^{a,b}(\chi_{ij}^1, \chi_{ij}^2, \ldots, \chi_{ij}^K). \tag{4.96}$$

This step integrates the subjective judgments of all decision makers while accounting for their respective importance weights.

Step 3: Aggregation Across Criteria.

After obtaining the collective decision matrix $[\chi_{ij}]_{m\times n}$, the overall evaluation of each alternative must be determined by aggregating across all criteria.

Using the criteria weight vector $\boldsymbol{\omega}$, the overall q-rung orthopair fuzzy evaluation value for alternative A_i is calculated as:

$$\chi_i = qROFDWGHM^{a,b}(\chi_{i1}, \chi_{i2}, \ldots, \chi_{in}). \tag{4.97}$$

This step incorporates the relative importance of each criterion into the final assessment.

Step 4: Computation of Score and Accuracy Functions.

For each aggregated qROFN $\chi_i = (\mu_i, \nu_i)$, the score function and accuracy function are computed according to Equation (4.4) and (4.5). The score function reflects the overall performance level of each alternative, while the accuracy function is used to resolve possible ranking ties.

Step 5: Ranking and Selection of Alternatives.

Based on the comparison rules defined in Definition 4.2.5, all alternatives are ranked in descending order according to their score (and accuracy when necessary). The alternative with the highest ranking is selected as the most appropriate decision.

4.5 Illustrative Example

This section demonstrates the applicability and effectiveness of the proposed qROFDWGHM-based MCDM framework through a practical numerical example. The purpose of this example is to illustrate the computational procedure of the developed method in a structured and transparent manner. Furthermore, a series of parameter analyses is conducted to examine the influence of different values of a, b, and λ on the aggregation results. The robustness and validity of the proposed approach are subsequently verified through comparisons with several well-established MCDM methods.

4.5.1 Numerical Example

Consider an investment enterprise that intends to allocate capital to one of several candidate companies. Five potential companies are under consideration, denoted by $\mathcal{A} = \{A_1, A_2, A_3, A_4, A_5\}$.

To ensure a comprehensive evaluation, the enterprise invites three domain experts, $\mathcal{D} = \{D_1, D_2, D_3\}$, to assess the companies under four evaluation criteria, defined as follows:

- C_1: Risk analysis,
- C_2: Growth analysis,
- C_3: Social-political impact analysis,
- C_4: Environmental impact analysis.

The importance of the four criteria is represented by the criteria weight vector

$$\boldsymbol{\omega} = (0.2,\ 0.1,\ 0.3,\ 0.4), \tag{4.98}$$

while the relative importance of the three decision makers is described by the decision-maker weight vector

$$\mathbf{w} = (0.35,\ 0.40,\ 0.25). \tag{4.99}$$

Each expert provides their evaluation in the form of a q-rung orthopair fuzzy decision matrix. The corresponding evaluation matrices supplied by D_1, D_2, and D_3 are presented in Tables 4.1,4.2, and 4.3, respectively.

Tables 4.1–4.3 present the individual q-rung orthopair fuzzy decision matrices used in this illustrative example, which are adopted from Zhong et al. (2019). Each entry is represented by a q-rung orthopair fuzzy number (μ, ν), where μ denotes the membership degree and ν denotes the non-membership degree associated with the evaluation of an alternative under a particular criterion. Since $q = 2$ is considered in this example, each assessment satisfies the condition $\mu^2 + \nu^2 \leq 1$.

For instance, the entry $(0.5, 0.4)$ indicates a membership degree of 0.5 and a non-membership degree of 0.4. Its corresponding hesitancy degree is given by $\pi = \sqrt{1 - 0.5^2 - 0.4^2} \approx 0.77$. These q-rung orthopair fuzzy numbers are directly used as the input values for the subsequent aggregation procedure.

The numerical example is evaluated using $q = 2$, $\lambda = 1$, and $a = b = 0.5$. The choice of $q = 2$ corresponds to the Pythagorean fuzzy case and represents the smallest rung that accommodates the experts' specified (μ, ν) pairs while maintaining the simple score function $S = \mu^2 - \nu^2$. The effect of different values of q, ranging from 2 to 8, is subsequently examined through sensitivity analysis. The Dombi parameter is set to $\lambda = 1$ as a balanced initial setting for the aggregation process. Its effect on the ranking results is further examined by varying λ from 0.5 to 100. The Heronian parameters are assigned as $a = b = 0.5$ to give equal and symmetric contributions to the two elements within each interacting pair. This setting is appropriate when no asymmetric interaction between the paired inputs is assumed, while alternative parameter settings are considered in the sensitivity analysis.

For the illustrative example, the criterion weights are specified as $\boldsymbol{\omega} = (0.2, 0.1, 0.3, 0.4)$, whereas the decision-maker weights are given by $\mathbf{w} = (0.35, 0.40, 0.25)$. The criterion weights assign greater importance to the environmental and social-political impact criteria, while the decision-maker weights reflect the relative seniority of the three experts. These values are selected solely for illustration, and the subsequent sensitivity analysis is used to examine whether changes in the operator parameters affect the final ranking.

In the following, the proposed qROFDWGHM operator is employed to solve this

MCDM problem. The selection procedure is implemented according to the following five systematic steps:

Step 1: Normalization of the decision matrices.

In practical multi-criteria decision-making problems, evaluation criteria are classified into benefit-type and cost-type attributes. To ensure consistency in the aggregation process, cost-type attributes must be converted into benefit-type form prior to computation.

The normalization rule is defined as

$$\chi_{ij}^{k'} = \begin{cases} (\mu_{ij}^k, \nu_{ij}^k), & \text{if } C_j \text{ is a benefit attribute,} \\ (\nu_{ij}^k, \mu_{ij}^k), & \text{if } C_j \text{ is a cost attribute.} \end{cases} \tag{4.100}$$

In this example, all four criteria (C_1, C_2, C_3, C_4) are benefit-type attributes. Therefore, no transformation is required and the normalized matrices remain unchanged, i.e.,

$$M_k' = M_k, \quad k = 1, 2, 3. \tag{4.101}$$

The decision matrices provided by D_1, D_2, and D_3 are given below.

Table 4.1
The q-rung Orthopair Fuzzy Decision Matrix M_1 Given by D_1.

	C_1	C_2	C_3	C_4
A_1	(0.5,0.4)	(0.5,0.4)	(0.2,0.6)	(0.4,0.4)
A_2	(0.7,0.3)	(0.7,0.3)	(0.6,0.2)	(0.6,0.2)
A_3	(0.5,0.4)	(0.6,0.4)	(0.6,0.2)	(0.5,0.3)
A_4	(0.8,0.2)	(0.7,0.2)	(0.4,0.2)	(0.5,0.2)
A_5	(0.4,0.3)	(0.4,0.2)	(0.4,0.5)	(0.4,0.6)

Table 4.2
The q-rung Orthopair Fuzzy Decision Matrix M_2 Given by D_2.

	C_1	C_2	C_3	C_4
A_1	(0.4,0.5)	(0.6,0.2)	(0.5,0.4)	(0.5,0.3)
A_2	(0.5,0.4)	(0.6,0.2)	(0.6,0.3)	(0.7,0.3)
A_3	(0.4,0.5)	(0.3,0.5)	(0.4,0.4)	(0.2,0.6)
A_4	(0.5,0.4)	(0.7,0.2)	(0.4,0.4)	(0.6,0.2)
A_5	(0.6,0.3)	(0.7,0.2)	(0.4,0.2)	(0.7,0.2)

Table 4.3
The q-rung Orthopair Fuzzy Decision Matrix M_3 Given by D_3.

	C_1	C_2	C_3	C_4
A_1	(0.4,0.2)	(0.5,0.2)	(0.5,0.3)	(0.5,0.2)
A_2	(0.5,0.3)	(0.5,0.3)	(0.6,0.2)	(0.7,0.2)
A_3	(0.4,0.4)	(0.3,0.4)	(0.4,0.3)	(0.3,0.3)
A_4	(0.5,0.3)	(0.5,0.3)	(0.3,0.5)	(0.5,0.2)
A_5	(0.6,0.2)	(0.6,0.4)	(0.4,0.4)	(0.6,0.3)

Step 2: Aggregation of decision makers.

In this step, the evaluation values provided by the three decision makers are integrated into a collective decision matrix using the proposed qROFDWGHM operator.

For this numerical illustration, the parameters are fixed as

$$q = 2, \qquad \lambda = 1, \qquad a = b = 0.5, \tag{4.102}$$

and the decision-maker weight vector is given by

$$\mathbf{w} = (0.35,\ 0.40,\ 0.25). \tag{4.103}$$

Let $K = 3$ denote the number of decision makers. For each alternative A_i and criterion C_j, the collective value is computed as

$$\chi_{ij} = qROFDWGHM^{a,b}(\chi_{ij}^1, \chi_{ij}^2, \chi_{ij}^3). \tag{4.104}$$

Illustrative computation for (A_1, C_1).

From Tables 4.1–4.3, we have

$$\chi_{11}^1 = (0.5, 0.4), \quad \chi_{11}^2 = (0.4, 0.5), \quad \chi_{11}^3 = (0.4, 0.2). \tag{4.105}$$

Since $q = 2$ and $\lambda = 1$, the additive generator functions are

$$f(\mu) = \frac{1-\mu^2}{\mu^2}, \qquad g(\nu) = \frac{\nu^2}{1-\nu^2}. \tag{4.106}$$

Thus,

$$f(0.5) = 3, \qquad f(0.4) = 5.25, \tag{4.107}$$

$$g(0.4) = 0.190476, \quad g(0.5) = 0.333333, \quad g(0.2) = 0.041667. \tag{4.108}$$

Since $a = b = 0.5$, we have $a + b = 1$, and

$$\frac{2(a+b)}{K(K+1)} = \frac{2}{3 \times 4} = \frac{1}{6}. \tag{4.109}$$

Define

$$X = \frac{1}{6} \sum_{i=1}^{3} \sum_{j=i}^{3} \frac{1}{a\, w_i\, f(\mu_i) + b\, w_j\, f(\mu_j)}, \tag{4.110}$$

$$Y = \frac{1}{6} \sum_{i=1}^{3} \sum_{j=i}^{3} \frac{1}{a\, w_i\, g(\nu_i) + b\, w_j\, g(\nu_j)}. \tag{4.111}$$

Substituting the numerical values yields

$$X = 0.709673, \qquad Y = 28.059832. \tag{4.112}$$

Therefore,

$$\mu_{11} = \left(\frac{X}{1+X} \right)^{1/2} = 0.6443, \tag{4.113}$$

$$\nu_{11} = \left(\frac{1}{1+Y} \right)^{1/2} = 0.1855. \tag{4.114}$$

Hence, the aggregated collective value for (A_1, C_1) is

$$\chi_{11} = (0.6443,\ 0.1855). \tag{4.115}$$

The remaining entries are obtained in the same manner. The complete collective decision matrix is presented in Table 4.4. All entries satisfy the q-rung orthopair fuzzy constraint $\mu_{ij}^2 + \nu_{ij}^2 \leq 1$.

Table 4.4
Collective q-rung Orthopair Fuzzy Decision Matrix Obtained by the qROFDWGHM Operator.

	C_1	C_2	C_3	C_4
A_1	(0.6443,0.1855)	(0.7410,0.1412)	(0.6196,0.2440)	(0.6803,0.1626)
A_2	(0.7748,0.1936)	(0.7978,0.1530)	(0.7959,0.1304)	(0.8452,0.1304)
A_3	(0.6443,0.2610)	(0.6133,0.2610)	(0.6814,0.1648)	(0.5316,0.2155)
A_4	(0.8144,0.1648)	(0.8225,0.1332)	(0.5651,0.1969)	(0.7410,0.1157)
A_5	(0.7474,0.1497)	(0.7794,0.1456)	(0.6076,0.1975)	(0.7794,0.1835)

Step 3: Aggregation across criteria (overall evaluation of alternatives).

After obtaining the collective q-rung orthopair fuzzy decision matrix $[\chi_{ij}]_{m\times n}$ in Table 4.4, the next step is to aggregate the evaluation information across the criteria in order to obtain a single comprehensive evaluation value for each alternative.

Let $n = 4$ denote the number of criteria and let the criteria weight vector be

$$\boldsymbol{\omega} = (\omega_1, \omega_2, \omega_3, \omega_4) = (0.2,\ 0.1,\ 0.3,\ 0.4), \quad \sum_{j=1}^{4} \omega_j = 1. \tag{4.116}$$

Using Equation (4.96), the overall evaluation value of each alternative A_i is computed by the proposed qROFDWGHM operator as

$$\chi_i = qROFDWGHM^{a,b}(\chi_{i1}, \chi_{i2}, \chi_{i3}, \chi_{i4}), \qquad i = 1, 2, \ldots, 5, \tag{4.117}$$

where the parameters are fixed at $q = 2$, $\lambda = 1$, and $a = b = 0.5$.

Illustrative computation for alternative A_1.

From Table 4.4, the collective evaluations of A_1 under the four criteria are

$$\chi_{11} = (0.6443, 0.1855), \quad \chi_{12} = (0.7410, 0.1412), \tag{4.118}$$

$$\chi_{13} = (0.6196, 0.2440), \quad \chi_{14} = (0.6803, 0.1626). \tag{4.119}$$

Since $q = 2$ and $\lambda = 1$, the generator functions become

$$f(\mu) = \frac{1-\mu^2}{\mu^2}, \qquad g(\nu) = \frac{\nu^2}{1-\nu^2}. \tag{4.120}$$

Because $a = b = 0.5$, we have $a + b = 1$ and the coefficient is

$$\frac{2(a+b)}{n(n+1)} = \frac{2}{4\times 5} = 0.1. \tag{4.121}$$

Define

$$X = 0.1\sum_{r=1}^{4}\sum_{s=r}^{4} \frac{1}{a\,\omega_r f(\mu_r) + b\,\omega_s f(\mu_s)}, \qquad Y = 0.1\sum_{r=1}^{4}\sum_{s=r}^{4} \frac{1}{a\,\omega_r g(\nu_r) + b\,\omega_s g(\nu_s)}, \tag{4.122}$$

where (μ_r, ν_r) corresponds to the r-th criterion value of A_1. Substituting the numerical values yields

$$X = 4.007750, \qquad Y = 149.982468. \tag{4.123}$$

Therefore, the comprehensive evaluation value of A_1 is obtained as

$$\mu_1 = \left(\frac{X}{1+X}\right)^{1/2} = 0.8946, \qquad \nu_1 = \left(\frac{1}{1+Y}\right)^{1/2} = 0.0814, \tag{4.124}$$

hence

$$\chi_1 = (0.8946,\ 0.0814). \tag{4.125}$$

Similarly, the comprehensive evaluation values of the remaining alternatives are obtained as

$$\chi_1 = (0.8946, 0.0814), \quad \chi_2 = (0.9458, 0.0703), \quad \chi_3 = (0.8650, 0.1056),$$

$$\chi_4 = (0.9323, 0.0689), \quad \chi_5 = (0.9219, 0.0765).$$

Step 4: Calculate the score and accuracy values.

After obtaining the comprehensive evaluation value $\chi_i = (\mu_i, \nu_i)$ for each alternative, the corresponding score and accuracy values are computed in order to perform ranking. Since the q-rung parameter is set to $q = 2$, the score function and accuracy function in Equations (4.4) and (4.5) are applied as

$$S(\chi_i) = \mu_i^2 - \nu_i^2, \qquad A(\chi_i) = \mu_i^2 + \nu_i^2.$$

Accordingly, the score and accuracy values for all alternatives are calculated and summarized in Table 4.5.

Table 4.5
Score and Accuracy Values of the Comprehensive Evaluation Results

Indicator	A_1	A_2	A_3	A_4	A_5
Score	0.7937	0.8895	0.7370	0.8644	0.8441
Accuracy	0.8069	0.8994	0.7594	0.8739	0.8558

Step 5: Rank the alternatives and select the optimal company.

Based on the comparison rule in Definition 4.2.5, the alternatives are first ranked according to their score values in descending order. If any tie occurs, the accuracy values are used as a secondary comparison index. From Table 4.5, the ranking order of the five companies is obtained as $A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$. Therefore, the investment enterprise should select company A_2 as the most desirable alternative under the given criteria weights and decision-maker weights.

4.5.2 Sensitivity Analysis

To further examine the robustness and reliability of the proposed qROFDWGHM-based decision framework, a comprehensive sensitivity analysis is conducted. The objective of this analysis is to investigate how variations in the parameters q, (a, b), and λ influence the score values and the resulting ranking order of the alternatives.

In each experiment, only one parameter is varied while the remaining parameters are kept fixed. For each parameter configuration, Steps 3–5 (aggregation across criteria, scoring, and ranking) of the proposed MCDM procedure are repeated to obtain the comprehensive evaluation values, score values, and final ranking results.

Experiment 1: Influence of the q-Rung Parameter

In this experiment, the q-rung parameter q is varied from 2 to 8, while fixing $a = b = 0.5$ and $\lambda = 1$. The resulting score values and ranking orders are presented in Table 4.6.

Table 4.6
Sensitivity Analysis with Respect to q

q	A_1	A_2	A_3	A_4	A_5	Ranking
2	0.7937	0.8895	0.7371	0.8644	0.8441	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
3	0.6800	0.8329	0.5969	0.7921	0.7603	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
4	0.5588	0.7684	0.4543	0.7123	0.6674	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
5	0.4429	0.7002	0.3287	0.6308	0.5732	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
6	0.3400	0.6305	0.2285	0.5512	0.4825	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
7	0.2543	0.5609	0.1543	0.4759	0.3988	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
8	0.1864	0.4931	0.1021	0.4067	0.3246	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$

From Table 4.6, it can be clearly observed that increasing the value of q leads

to a gradual reduction in the score magnitudes for all alternatives. This phenomenon is mathematically explained by the score function $S(\chi) = \mu^q - \nu^q$.

As q increases, both μ^q and ν^q decrease (for $0 < \mu, \nu < 1$), which results in a compression effect. Consequently, the differences among alternatives become less pronounced. In other words, larger values of q reduce the dominance of extreme membership values and produce a smoothing effect on the evaluation outcomes. Despite these numerical changes in score magnitudes, the ranking order remains completely stable across all tested values of q, namely:

$$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3. \tag{4.126}$$

This stability indicates that the relative preference structure among alternatives is not sensitive to the choice of q. Therefore, although q affects the discrimination intensity of the score function, it does not alter the final decision.

From a practical perspective, this implies that decision-makers can adjust the q-rung parameter to control the degree of uncertainty representation without affecting the overall ranking results. Such robustness enhances the credibility and reliability of the proposed method.

Experiment 2: Influence of the Heronian Parameters (a, b)

In this experiment, different combinations of (a, b) are tested while fixing $q = 2$ and $\lambda = 1$. The full results are shown in Table 4.7.

Table 4.7
Sensitivity Analysis with Respect to (a, b).

a	b	A_1	A_2	A_3	A_4	A_5	Ranking
1	1	0.7937	0.8895	0.7371	0.8644	0.8441	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
1	2	0.7915	0.8881	0.7314	0.8629	0.8422	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
3	1	0.8072	0.8947	0.7524	0.8745	0.8544	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
1	3	0.7922	0.8879	0.7295	0.8632	0.8424	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
3	3	0.7937	0.8895	0.7371	0.8644	0.8441	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
2	1	0.8010	0.8925	0.7461	0.8697	0.8497	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
2	3	0.7918	0.8885	0.7333	0.8631	0.8425	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
2	2	0.7937	0.8895	0.7371	0.8644	0.8441	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
1	5	0.7943	0.8881	0.7282	0.8642	0.8435	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
5	1	0.8158	0.8974	0.7605	0.8817	0.8609	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
5	5	0.7937	0.8895	0.7371	0.8644	0.8441	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$

Table 4.7 demonstrates that varying the Heronian parameters (a, b) influences the numerical score values but does not affect the ranking order. The parameters (a, b) control the interaction intensity between paired criteria within the generalized Heronian structure. When a and b increase simultaneously (e.g., $(5, 5)$), the aggregation process gives stronger emphasis to inter-attribute interactions. Conversely, smaller values (e.g., $(1, 1)$) yield a more moderate interaction effect.

Furthermore, asymmetric combinations such as $(3, 1)$ and $(1, 3)$ allow the model to emphasize one component more than the other. Although these asymmetric settings slightly modify the score magnitudes, the final ranking remains unchanged. For all tested combinations, the ranking order is:

$$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3. \tag{4.127}$$

This consistency indicates that the proposed model is insensitive to the choice of Heronian parameters. Therefore, decision-makers may flexibly adjust (a, b) to reflect different interaction assumptions without risking instability in the final ranking outcome.

Experiment 3: Influence of the Dombi Parameter λ

In this experiment, the Dombi parameter λ is varied over a wide range:

$$\lambda = \{0.5, 1, 1.5, 2, 3, 4, 5, 6, 10, 20, 50, 100\}, \tag{4.128}$$

while fixing $q = 2$ and $a = b = 0.5$. The results are presented in Table 4.8.

Table 4.8
Sensitivity Analysis with Respect to λ.

λ	A_1	A_2	A_3	A_4	A_5	Ranking
0.5	0.9474	0.9742	0.9298	0.9663	0.9617	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
1	0.7937	0.8895	0.7371	0.8644	0.8441	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
1.5	0.6969	0.8275	0.6236	0.7981	0.7655	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
2	0.6341	0.7848	0.5517	0.7527	0.7128	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
3	0.5676	0.7345	0.4709	0.7008	0.6469	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
4	0.5409	0.7145	0.4444	0.6804	0.6208	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
5	0.5273	0.7045	0.4320	0.6703	0.6077	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
6	0.5192	0.6985	0.4249	0.6643	0.5999	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
10	0.5081	0.6903	0.4159	0.6560	0.5892	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
20	0.5042	0.6881	0.4124	0.6538	0.5863	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
50	0.5030	0.6875	0.4112	0.6532	0.5854	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
100	0.5291	0.6945	0.4341	0.6629	0.5875	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$

The results reveal that λ significantly affects the magnitude of the score values. When λ is small (e.g., $\lambda = 0.5$), the Dombi operator exhibits a sharper aggregation behaviour, leading to larger score magnitudes and stronger discrimination among alternatives. As $\lambda \to 0$, the Dombi generator $f(t) = \big((1-t^q)/t^q\big)^{\lambda} \to 1$ for every interior $t \in (0,1)$, so all generator values collapse to a common constant and the operator degenerates toward a drastic (non-discriminating) aggregation in which the membership and non-membership contributions are no longer distinguished. Consequently $\lambda = 0$ is excluded from the admissible range ($\lambda > 0$ is required in Definition 4.2.8); the analysis therefore begins at $\lambda = 0.5$, the smallest practically meaningful value, where discrimination is strongest but the operator is still well defined. As λ increases, the score values gradually converge. For large λ (e.g., $\lambda = 50$ and $\lambda = 100$), the scores approach a limiting region, indicating that the aggregation process becomes more linear and less extreme. This behaviour suggests that λ can be interpreted as reflecting the decision-maker's attitude toward interaction intensity:

- Smaller λ values represent a stricter or more contrast-sensitive aggregation.
- Larger λ values correspond to a more balanced and moderate aggregation behaviour.

Nevertheless, despite substantial numerical variation in score magnitudes across

different λ values, the ranking order remains completely unchanged:

$$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3.$$

This confirms that the proposed decision framework is highly robust with respect to the Dombi parameter. The invariance of ranking across a wide range of λ values strengthens the practical applicability of the model.

Overall, the sensitivity analysis confirms that the proposed qROFDWGHM-based MCDM framework exhibits strong robustness with respect to variations in q, (a, b), and λ. Although these parameters influence the magnitude and dispersion of score values, the final ranking order remains completely stable in all experiments. This invariance indicates that the decision-making results are not sensitive to parameter adjustments, thereby enhancing the reliability, credibility, and practical applicability of the proposed method in complex decision environments.

4.5.3 Comparative Analysis

To further evaluate the effectiveness and rationality of the proposed qROFDWGHM operator, a comparative analysis is conducted with the qROFDWPPHM operator proposed by Zhong et al. (2019). Both methods are implemented under identical parameter settings, namely $q = 2$, $a = 1$, $b = 2$, and $\lambda = 1.5$, to ensure fairness and consistency in comparison.

The resulting score values, accuracy values, and ranking orders are presented in Table 4.9.

Table 4.9
Comparison of qROFDWGHM and qROFDWPPHM under Identical Parameters

Operator	Indicator	A_1	A_2	A_3	A_4	A_5	Ranking
$qROFDWGHM$	Score	0.6960	0.8269	0.6197	0.7981	0.7648	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
	Accuracy	0.7175	0.8432	0.6560	0.8135	0.7844	
$qROFDWPPHM$	Score	-0.5456	-0.2298	-0.5127	-0.2579	-0.4925	$A_2 \succ A_4 \succ A_5 \succ A_3 \succ A_1$
	Accuracy	0.8686	0.8598	0.8989	0.7945	0.8493	

For comparison purposes, the qROFDWPPHM operator proposed by Zhong et al. (2019) is used as the benchmark method. The comparison results under identical parameter settings are presented in Table 4.9.

Table 4.9 shows that both qROFDWGHM and qROFDWPPHM identify A_2 as the most preferred alternative. This consistent result indicates that the proposed qROFDWGHM operator produces a ranking outcome that is compatible with the benchmark operator under the selected parameter settings.

However, differences are observed in the intermediate ranking positions. The proposed qROFDWGHM operator produces the ranking order $A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$, whereas the qROFDWPPHM operator yields $A_2 \succ A_4 \succ A_5 \succ A_3 \succ A_1$. The difference occurs only between A_1 and A_3. Under the proposed qROFDWGHM operator, $S(A_1) = 0.6960$ is greater than $S(A_3) = 0.6197$, and thus A_1 is preferred to A_3. In contrast, under the qROFDWPPHM operator, $S(A_3) = -0.5127$ is greater than $S(A_1) = -0.5456$, resulting in A_3 being preferred to A_1.

This difference can be attributed to the distinct aggregation structures of the two operators. The qROFDWPPHM operator employs a partitioned power aggregation structure, in which attributes are grouped before aggregation. Therefore, its results may be influenced by the distribution of performance values across the specified attribute groups. In contrast, the proposed qROFDWGHM operator aggregates the criteria through its weighted Dombi–Heronian structure without partitioning. Consequently, it gives greater recognition to the more consistent performance profile of A_1 across the criteria, whereas the qROFDWPPHM operator gives relatively greater influence to the stronger individual evaluations of A_3. Despite this difference in intermediate positions, both methods consistently identify A_2 as the most preferred alternative.

In contrast, the proposed qROFDWGHM operator performs a unified aggregation process without partitioning. By integrating q-rung orthopair fuzzy sets with Dombi operations and the generalized Heronian mean in a continuous framework, the proposed method captures attribute interactions more coherently and avoids dependency on subgroup configurations. Furthermore, the score values obtained by qROFDWGHM are strictly positive and more dispersed, indicating stronger discrimination among alternatives. For instance, the score difference between the best alternative (A_2) and the worst alternative (A_3) is larger under the proposed method compared to qROFDWPPHM, demonstrating improved separation capability.

Although the accuracy values of qROFDWPPHM appear slightly higher for certain alternatives, the overall ranking structure remains more consistent under

qROFDWGHM. In particular, the proposed method provides a stable and interpretable ordering that aligns with the evaluation tendencies observed in the sensitivity analysis.

Overall, the comparative analysis confirms that the qROFDWGHM operator not only preserves rational decision outcomes but also enhances aggregation coherence and discrimination ability in complex q-rung orthopair fuzzy environments. In summary, the proposed qROFDWGHM operator demonstrates competitive and stable performance compared to the qROFDWPPHM method. While both operators identify the same optimal alternative, the proposed method provides a more consistent and structurally unified aggregation mechanism. These results further validate the feasibility, effectiveness, and practical superiority of the proposed decision-making framework.

4.6 Conclusions

This chapter developed the qROFDGHM and qROFDWGHM aggregation operators within the framework of q-rung orthopair fuzzy sets by integrating Dombi t-norm and t-conorm operations with the generalized Heronian mean structure. The fundamental properties of the proposed operators, including idempotency, monotonicity, boundedness, and closure, were established to ensure their mathematical validity and consistency. The weighted formulation further enhances their applicability in MCDM environments.

An MCDM model based on the proposed qROFDWGHM operator was formulated and illustrated through a numerical example. Sensitivity analyses with respect to parameters q, a, b, and λ demonstrated the robustness and stability of the ranking results under varying conditions. A comparative analysis with an existing operator further confirmed the effectiveness and strong discrimination capability of the proposed approach. Overall, the developed framework provides a reliable and flexible tool for handling uncertain and complex decision-making problems.

CHAPTER 5
INTEGRATION OF Q-RUNG ORTHOPAIR FUZZY DOMBI GENERALIZED HERONIAN MEAN–ADAPTIVE UTILITY RANKING ALGORITM FRAMEWORK

5.1 Introduction

This chapter demonstrates the practical application of the proposed q-Rung Orthopair Fuzzy Dombi Generalized Heronian Mean (qROFDGHM) integrated with the Adaptive Utility Ranking Algorithm (AURA) in evaluating educational technologies for English language learning. Ten experts provided linguistic assessments across six main criteria, each consisting of three sub-criteria. The linguistic data were converted into q-rung orthopair fuzzy numbers (q-ROFNs) and aggregated using the qROFDGHM operator to obtain a collective decision matrix. The aggregated values were then transformed into crisp scores using the q-rung score function and further analyzed through the AURA method to derive the final ranking of alternatives. This integration effectively handles uncertainty and subjectivity in expert judgments while producing stable and interpretable rankings for educational decision-making.

5.2 Framework of Integration of qROFDGHM and AURA

The integration of the qROFDGHM operator with the AURA forms a structured multi-criteria decision-making (MCDM) framework designed to evaluate educational technologies for English language learning. The process begins with data collection from domain experts through a structured questionnaire using a 5-point Likert scale. The numerical responses are subsequently converted by the researcher into linguistic terms and further transformed into q-ROFNs to represent uncertainty and hesitation in expert judgment.

The framework then aggregates multiple expert evaluations into a collective decision matrix using the qROFDGHM operator, which preserves interdependencies and the non-linear relationship between membership and non-membership degrees through Dombi operations. Finally, the crisp scores derived from the aggregated

fuzzy evaluations are analyzed using the AURA method to produce final rankings of educational technology alternatives. The overall integration process is illustrated in Figure 5.1.

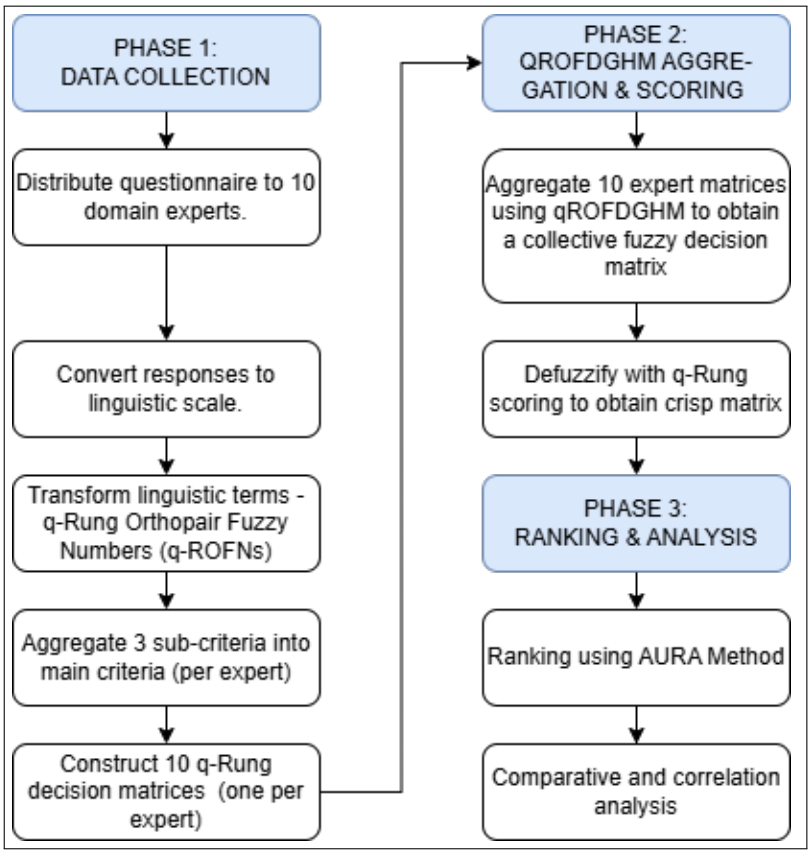


Figure 5.1 Framework for Integrating qROFDGHM and AURA in Educational Technology Evaluation

For clarity, Figure 5.1 illustrates the qROFDGHM–AURA framework in three sequential phases, which correspond to the numbered steps described below. The first phase involves data preparation and covers the collection of evaluation data and its conversion into linguistic terms and q-ROFNs. The second phase involves fuzzy aggregation, including the aggregation of sub-criteria and the application of the qROFDGHM operator to construct the collective decision matrix. The third phase involves ranking and validation, beginning with the transformation of the aggregated fuzzy values into crisp values, followed by ranking using the AURA method and validation through comparative and correlation analyses. The figure is presented at full text width to ensure that all step labels are clearly legible.

The implementation of the qROFDGHM–AURA framework follows the steps

below:

1. **Data Collection:** Ten domain experts complete the questionnaire using a 5-point Likert scale to evaluate six educational technology alternatives across six main criteria, each with three sub-criteria.

2. **Conversion to Linguistic and qROF Scale:** The researcher transforms the numerical responses into linguistic variables (e.g., Strongly Disagree to Strongly Agree) and subsequently into q-ROFNs based on predefined membership and non-membership functions.

3. **Sub-Criteria Aggregation:** Three sub-criteria under each main criterion are aggregated into a single representative value, ensuring that the analysis focuses on six major evaluation criteria.

4. **Aggregation using qROFDGHM:** The qROFDGHM operator integrates all ten expert decision matrices into one collective fuzzy decision matrix that captures expert consensus while maintaining uncertainty characteristics.

5. **Defuzzification (Scoring):** Each fuzzy value A_{ij} is transformed into a crisp score using the q-rung score function $S(A_{ij}) = \mu_{ij}^q - \nu_{ij}^q$, producing a clear and interpretable numerical matrix.

6. **Ranking using AURA:** The crisp matrix is processed through normalization, weighting, and utility-based ranking in the AURA method to determine the most suitable educational technology alternatives.

7. **Comparative and Correlation Analysis:** The results are compared with those obtained from classical MCDM methods such as TOPSIS, VIKOR, SAW, and COBRA. Correlation coefficients (e.g., Spearman) are computed to assess ranking consistency.

This framework ensures that expert evaluations collected through conventional Likert-based questionnaires can be systematically transformed, aggregated, and ranked within a fuzzy MCDM environment, resulting in a transparent and reliable decision-making process.

5.3 Linguistic Variables and q-Rung Orthopair Fuzzy Representation

The evaluations collected from experts through the 5-point Likert questionnaire were first expressed in linguistic terms and then converted into q-ROFNs. This two-stage transformation enables the inclusion of human subjectivity and uncertainty within the analytical process while preserving mathematical operability. Each linguistic term is associated with a pair of membership (μ) and non-membership (ν) degrees that satisfy the q-rung condition $(\mu^q + \nu^q \leq 1)$.

Table 5.1 presents the mapping between the Likert-scale values, their corresponding linguistic labels, and the q-rung orthopair fuzzy representations adopted in this study (for $q = 2$). The selected scale reflects balanced positivity and negativity toward each evaluative statement.

Table 5.1
Mapping of 5-point Likert Scale to Linguistic Variables and q-ROFNs

Likert Scale	**Linguistic Term**	**Membership Degree** (μ)	**Non-Membership Degree** (ν)
1	Strongly Disagree	0.10	0.90
2	Disagree	0.30	0.70
3	Neutral	0.50	0.50
4	Agree	0.70	0.30
5	Strongly Agree	0.90	0.10

The 5-point Likert scale was adopted in this study because of its simplicity, clarity, and balanced midpoint that effectively captures neutral perceptions (Al-Qarni et al., 2024). It reduces cognitive load on experts compared with more complex seven- or nine-point scales while maintaining sufficient granularity to differentiate evaluative intensity. When combined with q-rung orthopair fuzzy modelling, this scale allows richer representation of uncertainty without requiring additional response categories from the participants. The transformation of the Likert scale into q-rung orthopair fuzzy form therefore enhances the interpretability and realism of expert assessments by accommodating hesitation and imprecision inherent in human judgment.

For example, consider a case in which an expert rates the alternative "AI-based Writing Assistant" with a score of four under the criterion "Engagement and Motivation." The score of four corresponds to the linguistic term "Agree," represented by the q-rung orthopair fuzzy pair $(\mu, \nu) = (0.7, 0.3)$ when $q = 2$. The pair satisfies the

condition $\mu^2 + \nu^2 = 0.58 < 1$, confirming its validity within the q-rung domain. This fuzzy value will later be used in the aggregation stage of the qROFDGHM operator to compute the collective decision matrix.

The flexibility of q-rung orthopair fuzzy sets lies in the choice of the parameter q, which determines how strictly the sum of membership and non-membership degrees is constrained. Table 5.2 illustrates how increasing q expands the feasible region for (μ, ν), allowing greater expressiveness in representing uncertainty. As q increases, experts can assign higher degrees of both agreement and disagreement simultaneously without violating the q-rung condition.

Table 5.2
Effect of Different q-values on Permissible (μ, ν) pairs

q Value	**Constraint**	**Example Valid Pair** (μ, ν)	**Interpretation**
1	$\mu + \nu \leq 1$	(0.6, 0.4)	Intuitionistic fuzzy set
2	$\mu^2 + \nu^2 \leq 1$	(0.8, 0.6)	Pythagorean fuzzy set
3	$\mu^3 + \nu^3 \leq 1$	(0.9, 0.7)	q-rung orthopair fuzzy set

This comparison highlights that q-rung orthopair fuzzy sets generalize both intuitionistic and Pythagorean fuzzy models, providing additional flexibility in capturing ambiguous human perceptions. The adoption of $q = 2$ in this study offers a balance between computational simplicity and sufficient tolerance for uncertainty, making it appropriate for expert-based evaluations derived from linguistic data.

5.4 Data Description and Structure

This section describes the data used to apply the integrated qROFDGHM-AURA framework in educational technology evaluation. It presents the respondents and sampling approach, the selected criteria and sub-criteria, and the construction of the individual q-rung orthopair fuzzy decision matrices provided by the experts.

5.4.1 Respondents and Sampling

A total of ten experts participated in this study as evaluators, representing academic professionals and instructors with experience in English language teaching and educational technology. All respondents hold postgraduate qualifications, either a Master's or a Doctoral degree, and possess at least five years of experience in

implementing digital learning tools in language education. The use of domain experts ensures that the collected evaluations are informed, reliable, and reflective of real classroom applications rather than general user perceptions.

Each expert was provided with a questionnaire consisting of two sections: demographic information and a multi-criteria evaluation table. The evaluation section employed a 5-point Likert scale ranging from "Strongly Disagree" to "Strongly Agree" for six categories of educational technologies. The experts independently rated the performance of each alternative across six major criteria, each comprising three sub-criteria. The data collected from these questionnaires form the foundation for constructing ten individual decision matrices, denoted as DM_1, DM_2, …, DM_{10}, corresponding to each expert's evaluations.

The responses from the experts are subsequently transformed by the researcher into linguistic terms and further converted into q-ROFNs to represent uncertainty and hesitation within human judgment. These fuzzy decision matrices are then used for aggregation in the qROFDGHM–AURA framework.

5.4.2 Criteria and Sub-Criteria

The evaluation of educational technologies in English language learning was structured around six primary criteria, each representing a key dimension of learning effectiveness, engagement, usability, interactivity, reliability, and cost. Each criterion includes three sub-criteria that further refine the scope of assessment. This hierarchical structure ensures that both pedagogical and technological aspects are comprehensively represented in the analysis. Table 5.3 presents the detailed structure of the criteria and sub-criteria used in this study.

Table 5.3
Criteria and Sub-criteria for Evaluating Educational Technologies in English Language Learning

Criterion	Description and Sub-Criteria
C_1: Effectiveness in Language Acquisition	C_{11}: Improves vocabulary and grammar learning

Criterion	Description and Sub-Criteria
	C_{12}: Enhances speaking, listening, reading, and writing skills
	C_{13}: Provides personalized learning experiences
C_2: Engagement and Motivation	C_{21}: Encourages active student participation
	C_{22}: Increases motivation to learn English
	C_{23}: Provides interactive and gamified learning experiences
C_3: Usability and Accessibility	C_{31}: Easy to navigate and use
	C_{32}: Accessible across multiple devices (mobile, tablet, computer)
	C_{33}: Available for students with disabilities (e.g., text-to-speech, voice recognition)
C_4: Interactivity and Features	C_{41}: Includes AI-based feedback and assessment
	C_{42}: Provides real-time feedback on language usage
	C_{43}: Supports collaborative learning (e.g., discussion forums, group projects)
C_5: Technical Reliability and Stability	C_{51}: Runs smoothly without crashes or technical issues
	C_{52}: Ensures data privacy and security
	C_{53}: Offers technical support and troubleshooting
C_6: Cost-Effectiveness	C_{61}: Provides good value for money

Criterion	Description and Sub-Criteria
	C_{62}: Affordable for students and institutions
	C_{63}: Offers sufficient free features for quality learning

Each of the six criteria represents a core aspect of evaluating educational technologies. Effectiveness in Language Acquisition (C_1) focuses on measurable learning improvements, while Engagement and Motivation (C_2) addresses affective and participatory aspects of learning. Usability and Accessibility (C_3) ensures inclusivity and ease of implementation across platforms, whereas Interactivity and Features (C_4) captures the degree of technological advancement and user collaboration. Technical Reliability and Stability (C_5) guarantees that platforms perform consistently without major disruptions, and Cost-Effectiveness (C_6) evaluates the economic practicality of adopting each tool within educational settings.

The alternatives assessed under these criteria include six main categories of educational technologies: Learning Management Systems (A_1), AI-based Writing Assistants (A_2), Gamification and Quizzing Platforms (A_3), Mobile Language Learning Applications (A_4), Virtual and Augmented Reality Tools (A_5), and Automated Speech Recognition (ASR) Tools (A_6). Each alternative is evaluated across all six criteria, producing ten individual decision matrices (DM_1–DM_{10}) that are subsequently aggregated using the qROFDGHM operator to form the collective decision matrix for AURA ranking.

5.4.3 Individual Decision Matrices of Experts

Each expert evaluated the six educational technology alternatives under six main criteria. Each main criterion consists of three sub-criteria. The response for each sub-criterion was first converted into a q-rung orthopair fuzzy number (q-ROFN) based on Table 5.1. The membership and non-membership values of the three sub-criteria were then averaged separately to obtain one overall q-ROFN for each main criterion. For alternative A_i under criterion C_k, the membership value is calculated as $\mu_{ik} = (\mu_{ik1} + \mu_{ik2} + \mu_{ik3})/3$, while the non-membership value is calculated as $\nu_{ik} = (\nu_{ik1} + \nu_{ik2} + \nu_{ik3})/3$.

For example, consider Expert 1's evaluation of alternative A_2, namely the AI-based Assistant, under criterion C_1, Effectiveness in Language Acquisition. The three sub-criteria, C_{11}, C_{12}, and C_{13}, are represented by $(0.7, 0.3)$, $(0.7, 0.3)$, and $(0.9, 0.1)$, respectively. The membership value is calculated as $\mu_{21} = (0.7 + 0.7 + 0.9)/3 = 2.3/3 = 0.7667$. The non-membership value is calculated as $\nu_{21} = (0.3 + 0.3 + 0.1)/3 = 0.7/3 = 0.2333$.

Therefore, the three sub-criteria evaluations for alternative A_2 under criterion C_1 are represented by the aggregated q-rung orthopair fuzzy number $(0.7667, 0.2333)$. The same procedure is applied to all alternatives and main criteria to construct the individual decision matrices (DM_1–DM_{10}).

Table 5.4
Decision Matrix DM_1 (Expert 1) in q-ROF Form

Alternatives	$\mathbf{C}_1$	$\mathbf{C}_2$	$\mathbf{C}_3$	$\mathbf{C}_4$	$\mathbf{C}_5$	$\mathbf{C}_6$
LMS	(0.5000, 0.5000)	(0.7667, 0.2333)	(0.7000, 0.3000)	(0.7000, 0.3000)	(0.6333, 0.3667)	(0.5667, 0.4333)
AI Writing Assistants	(0.7667, 0.2333)	(0.8333, 0.1667)	(0.7667, 0.2333)	(0.8333, 0.1667)	(0.5667, 0.4333)	(0.7667, 0.2333)
Gamification & Quizzing	(0.7000, 0.3000)	(0.8333, 0.1667)	(0.7667, 0.2333)	(0.9000, 0.1000)	(0.5000, 0.5000)	(0.6333, 0.3667)
Mobile Apps	(0.7000, 0.3000)	(0.7000, 0.3000)	(0.7000, 0.3000)	(0.9000, 0.1000)	(0.7000, 0.3000)	(0.7000, 0.3000)
VR/AR Tools	(0.7000, 0.3000)	(0.7667, 0.2333)	(0.7000, 0.3000)	(0.7667, 0.2333)	(0.3000, 0.7000)	(0.5000, 0.5000)
ASR Tools	(0.6333, 0.3667)	(0.7000, 0.3000)	(0.7667, 0.2333)	(0.5000, 0.5000)	(0.7000, 0.3000)	(0.7667, 0.2333)

Table 5.5
Decision Matrix DM_2 (Expert 2) in q-ROF Form

Alternatives	$\mathbf{C}_1$	$\mathbf{C}_2$	$\mathbf{C}_3$	$\mathbf{C}_4$	$\mathbf{C}_5$	$\mathbf{C}_6$
LMS	(0.7000, 0.3000)	(0.3000, 0.7000)	(0.7000, 0.3000)	(0.7667, 0.2333)	(0.3000, 0.7000)	(0.9000, 0.1000)
AI Writing Assistants	(0.4333, 0.5667)	(0.6333, 0.3667)	(0.7667, 0.2333)	(0.7000, 0.3000)	(0.5000, 0.5000)	(0.4333, 0.5667)
Gamification & Quizzing	(0.8333, 0.1667)	(0.9000, 0.1000)	(0.9000, 0.1000)	(0.5667, 0.4333)	(0.5667, 0.4333)	(0.5667, 0.4333)
Mobile Apps	(0.7000, 0.3000)	(0.7667, 0.2333)	(0.8333, 0.1667)	(0.7667, 0.2333)	(0.5667, 0.4333)	(0.4333, 0.5667)
VR/AR Tools	(0.4333, 0.5667)	(0.5000, 0.5000)	(0.5000, 0.5000)	(0.6333, 0.3667)	(0.5000, 0.5000)	(0.5000, 0.5000)
ASR Tools	(0.4333, 0.5667)	(0.5000, 0.5000)	(0.5000, 0.5000)	(0.5000, 0.5000)	(0.5000, 0.5000)	(0.5000, 0.5000)

Table 5.6
Decision Matrix DM_3 (Expert 3) in q-ROF Form

Alternatives	$\mathbf{C}_1$	$\mathbf{C}_2$	$\mathbf{C}_3$	$\mathbf{C}_4$	$\mathbf{C}_5$	$\mathbf{C}_6$
LMS	(0.6333, 0.3667)	(0.4333, 0.5667)	(0.6333, 0.3667)	(0.4333, 0.5667)	(0.8333, 0.1667)	(0.9000, 0.1000)
AI Writing Assistants	(0.8333, 0.1667)	(0.6333, 0.3667)	(0.6333, 0.3667)	(0.9000, 0.1000)	(0.7667, 0.2333)	(0.7667, 0.2333)
Gamification & Quizzing	(0.8333, 0.1667)	(0.9000, 0.1000)	(0.7667, 0.2333)	(0.6333, 0.3667)	(0.7000, 0.3000)	(0.8333, 0.1667)
Mobile Apps	(0.7000, 0.3000)	(0.5000, 0.5000)	(0.6333, 0.3667)	(0.7000, 0.3000)	(0.5000, 0.5000)	(0.7000, 0.3000)
VR/AR Tools	(0.5667, 0.4333)	(0.4333, 0.5667)	(0.5000, 0.5000)	(0.8333, 0.1667)	(0.5667, 0.4333)	(0.5000, 0.5000)
ASR Tools	(0.5667, 0.4333)	(0.6333, 0.3667)	(0.7667, 0.2333)	(0.9000, 0.1000)	(0.5667, 0.4333)	(0.5000, 0.5000)

Table 5.7
Decision Matrix DM_4 (Expert 4) in q-ROF Form

Alternatives	$\mathbf{C}_1$	$\mathbf{C}_2$	$\mathbf{C}_3$	$\mathbf{C}_4$	$\mathbf{C}_5$	$\mathbf{C}_6$
LMS	(0.7000, 0.3000)	(0.7000, 0.3000)	(0.6333, 0.3667)	(0.5667, 0.4333)	(0.5000, 0.5000)	(0.7000, 0.3000)
AI Writing Assistants	(0.6333, 0.3667)	(0.7000, 0.3000)	(0.7000, 0.3000)	(0.7000, 0.3000)	(0.7000, 0.3000)	(0.7000, 0.3000)
Gamification & Quizzing	(0.6333, 0.3667)	(0.7000, 0.3000)	(0.5000, 0.5000)	(0.7000, 0.3000)	(0.5000, 0.5000)	(0.7000, 0.3000)
Mobile Apps	(0.6333, 0.3667)	(0.7000, 0.3000)	(0.5667, 0.4333)	(0.6333, 0.3667)	(0.5000, 0.5000)	(0.5000, 0.5000)
VR/AR Tools	(0.6333, 0.3667)	(0.7000, 0.3000)	(0.5000, 0.5000)	(0.5000, 0.5000)	(0.7000, 0.3000)	(0.7000, 0.3000)
ASR Tools	(0.5000, 0.5000)	(0.5000, 0.5000)	(0.5000, 0.5000)	(0.5000, 0.5000)	(0.5000, 0.5000)	(0.5000, 0.5000)

Table 5.8
Decision Matrix DM_5 (Expert 5) in q-ROF Form

Alternatives	$\mathbf{C}_1$	$\mathbf{C}_2$	$\mathbf{C}_3$	$\mathbf{C}_4$	$\mathbf{C}_5$	$\mathbf{C}_6$
LMS	(0.7000, 0.3000)	(0.7000, 0.3000)	(0.4333, 0.5667)	(0.8333, 0.1667)	(0.6333, 0.3667)	(0.9000, 0.1000)
AI Writing Assistants	(0.8333, 0.1667)	(0.5667, 0.4333)	(0.9000, 0.1000)	(0.6333, 0.3667)	(0.5667, 0.4333)	(0.9000, 0.1000)
Gamification & Quizzing	(0.9000, 0.1000)	(0.9000, 0.1000)	(0.5667, 0.4333)	(0.7667, 0.2333)	(0.6333, 0.3667)	(0.9000, 0.1000)
Mobile Apps	(0.9000, 0.1000)	(0.9000, 0.1000)	(0.8333, 0.1667)	(0.7667, 0.2333)	(0.5667, 0.4333)	(0.9000, 0.1000)
VR/AR Tools	(0.3667, 0.6333)	(0.5000, 0.5000)	(0.5000, 0.5000)	(0.5000, 0.5000)	(0.5000, 0.5000)	(0.5000, 0.5000)
ASR Tools	(0.3667, 0.6333)	(0.5000, 0.5000)	(0.5000, 0.5000)	(0.5000, 0.5000)	(0.5000, 0.5000)	(0.1000, 0.9000)

Table 5.9
Decision Matrix DM_6 (Expert 6) in q-ROF Form

Alternatives	$\mathbf{C}_1$	$\mathbf{C}_2$	$\mathbf{C}_3$	$\mathbf{C}_4$	$\mathbf{C}_5$	$\mathbf{C}_6$
LMS	(0.6333, 0.3667)	(0.8333, 0.1667)	(0.8333, 0.1667)	(0.7667, 0.2333)	(0.5000, 0.5000)	(0.6333, 0.3667)
AI Writing Assistants	(0.7667, 0.2333)	(0.8333, 0.1667)	(0.7667, 0.2333)	(0.8333, 0.1667)	(0.5000, 0.5000)	(0.6333, 0.3667)
Gamification & Quizzing	(0.6333, 0.3667)	(0.9000, 0.1000)	(0.7667, 0.2333)	(0.8333, 0.1667)	(0.6333, 0.3667)	(0.6333, 0.3667)
Mobile Apps	(0.7667, 0.2333)	(0.9000, 0.1000)	(0.7667, 0.2333)	(0.9000, 0.1000)	(0.5000, 0.5000)	(0.5667, 0.4333)
VR/AR Tools	(0.7000, 0.3000)	(0.9000, 0.1000)	(0.7667, 0.2333)	(0.7000, 0.3000)	(0.5000, 0.5000)	(0.5667, 0.4333)
ASR Tools	(0.6333, 0.3667)	(0.7667, 0.2333)	(0.7667, 0.2333)	(0.7000, 0.3000)	(0.5000, 0.5000)	(0.6333, 0.3667)

Table 5.10
Decision Matrix DM_7 (Expert 7) in q-ROF Form

Alternatives	$\mathbf{C}_1$	$\mathbf{C}_2$	$\mathbf{C}_3$	$\mathbf{C}_4$	$\mathbf{C}_5$	$\mathbf{C}_6$
LMS	(0.9000, 0.1000)	(0.5667, 0.4333)	(0.5000, 0.5000)	(0.7000, 0.3000)	(0.6333, 0.3667)	(0.7000, 0.3000)
AI Writing Assistants	(0.9000, 0.1000)	(0.8333, 0.1667)	(0.7000, 0.3000)	(0.9000, 0.1000)	(0.5000, 0.5000)	(0.8333, 0.1667)
Gamification & Quizzing	(0.9000, 0.1000)	(0.9000, 0.1000)	(0.7000, 0.3000)	(0.7000, 0.3000)	(0.6333, 0.3667)	(0.9000, 0.1000)
Mobile Apps	(0.9000, 0.1000)	(0.9000, 0.1000)	(0.6333, 0.3667)	(0.7000, 0.3000)	(0.6333, 0.3667)	(0.5000, 0.5000)
VR/AR Tools	(0.9000, 0.1000)	(0.9000, 0.1000)	(0.5000, 0.5000)	(0.8333, 0.1667)	(0.7000, 0.3000)	(0.3000, 0.7000)
ASR Tools	(0.9000, 0.1000)	(0.5667, 0.4333)	(0.6333, 0.3667)	(0.7000, 0.3000)	(0.5000, 0.5000)	(0.4333, 0.5667)

Table 5.11
Decision Matrix DM_8 (Expert 8) in q-ROF Form

Alternatives	$\mathbf{C}_1$	$\mathbf{C}_2$	$\mathbf{C}_3$	$\mathbf{C}_4$	$\mathbf{C}_5$	$\mathbf{C}_6$
LMS	(0.8333, 0.1667)	(0.8333, 0.1667)	(0.9000, 0.1000)	(0.7000, 0.3000)	(0.7000, 0.3000)	(0.9000, 0.1000)
AI Writing Assistants	(0.9000, 0.1000)	(0.7667, 0.2333)	(0.8333, 0.1667)	(0.9000, 0.1000)	(0.9000, 0.1000)	(0.7667, 0.2333)
Gamification & Quizzing	(0.9000, 0.1000)	(0.8333, 0.1667)	(0.7667, 0.2333)	(0.9000, 0.1000)	(0.9000, 0.1000)	(0.9000, 0.1000)
Mobile Apps	(0.6333, 0.3667)	(0.7000, 0.3000)	(0.9000, 0.1000)	(0.9000, 0.1000)	(0.9000, 0.1000)	(0.8333, 0.1667)
VR/AR Tools	(0.6333, 0.3667)	(0.7000, 0.3000)	(0.3667, 0.6333)	(0.5000, 0.5000)	(0.5000, 0.5000)	(0.3667, 0.6333)
ASR Tools	(0.6333, 0.3667)	(0.7000, 0.3000)	(0.5000, 0.5000)	(0.5000, 0.5000)	(0.5000, 0.5000)	(0.7000, 0.3000)

Table 5.12
Decision Matrix DM_9 (Expert 9) in q-ROF Form

Alternatives	$\mathbf{C}_1$	$\mathbf{C}_2$	$\mathbf{C}_3$	$\mathbf{C}_4$	$\mathbf{C}_5$	$\mathbf{C}_6$
LMS	(0.5667, 0.4333)	(0.5000, 0.5000)	(0.6333, 0.3667)	(0.6333, 0.3667)	(0.5000, 0.5000)	(0.7000, 0.3000)
AI Writing Assistants	(0.8333, 0.1667)	(0.5667, 0.4333)	(0.6333, 0.3667)	(0.9000, 0.1000)	(0.5000, 0.5000)	(0.7000, 0.3000)
Gamification & Quizzing	(0.7667, 0.2333)	(0.9000, 0.1000)	(0.7000, 0.3000)	(0.5667, 0.4333)	(0.5000, 0.5000)	(0.7000, 0.3000)
Mobile Apps	(0.9000, 0.1000)	(0.9000, 0.1000)	(0.5667, 0.4333)	(0.6333, 0.3667)	(0.5000, 0.5000)	(0.7000, 0.3000)
VR/AR Tools	(0.9000, 0.1000)	(0.6333, 0.3667)	(0.3667, 0.6333)	(0.5000, 0.5000)	(0.5000, 0.5000)	(0.7000, 0.3000)
ASR Tools	(0.7000, 0.3000)	(0.5000, 0.5000)	(0.5667, 0.4333)	(0.4333, 0.5667)	(0.5000, 0.5000)	(0.7000, 0.3000)

Table 5.13
Decision Matrix DM_{10} (Expert 10) in q-ROF Form

Alternatives	$\mathbf{C}_1$	$\mathbf{C}_2$	$\mathbf{C}_3$	$\mathbf{C}_4$	$\mathbf{C}_5$	$\mathbf{C}_6$
LMS	(0.7667, 0.2333)	(0.9000, 0.1000)	(0.5667, 0.4333)	(0.7000, 0.3000)	(0.5667, 0.4333)	(0.5000, 0.5000)
AI Writing Assistants	(0.6333, 0.3667)	(0.5000, 0.5000)	(0.3000, 0.7000)	(0.7000, 0.5000)	(0.5000, 0.5000)	(0.3000, 0.7000)
Gamification & Quizzing	(0.7667, 0.2333)	(0.9000, 0.1000)	(0.5000, 0.5000)	(0.7000, 0.3000)	(0.6333, 0.3667)	(0.3000, 0.7000)
Mobile Apps	(0.6333, 0.3667)	(0.5000, 0.5000)	(0.5000, 0.5000)	(0.7000, 0.3000)	(0.5667, 0.3667)	(0.6333, 0.6333)
VR/AR Tools	(0.5000, 0.5000)	(0.3000, 0.7000)	(0.7000, 0.3000)	(0.7000, 0.3000)	(0.7000, 0.3000)	(0.6333, 0.6333)
ASR Tools	(0.3000, 0.7000)	(0.5000, 0.5000)	(0.3000, 0.7000)	(0.7000, 0.3000)	(0.4333, 0.5667)	(0.3000, 0.7000)

5.5 Application of qROFDGHM–AURA Framework

This section demonstrates the application of the integrated qROFDGHM–AURA framework using the data obtained from ten domain experts. Each expert evaluated six educational technology alternatives based on six major criteria after aggregating their respective sub-criteria. The evaluations were expressed as q-ROFNs to account for vagueness and hesitation in expert judgment.

5.5.1 Aggregation using the qROFDGHM Operator

The ten individual decision matrices (DM_1–DM_{10}) were aggregated using the qROFDGHM operator, as defined earlier in Definition 4.3.1. This operator integrates the nonlinear adjustment capability of Dombi operations with the relational averaging effect of the Heronian mean. In this study, the parameters employed during the aggregation process were fixed as follows:

$$\lambda = 1, \quad a = 0.5, \quad b = 0.5, \quad n = 10, \quad q = 2. \tag{5.1}$$

Here, λ represents the Dombi parameter controlling the level of flexibility, (a, b) are the Heronian coefficients balancing the interaction strength, n is the number of experts, and q denotes the degree of orthopair fuzziness. To initiate the aggregation process, the raw q-rung orthopair fuzzy values (μ_i, ν_i) from each expert were first transformed using the Dombi functions $f(\mu_i)$ and $g(\nu_i)$. This transformation stage allows the nonlinear mapping of membership and non-membership information prior to calculating the Heronian summations. The calculated Dombi transformation values for alternative A_1 under criterion C_1 are tabulated in Table 5.14.

Table 5.14
Calculated Dombi Transformation Values for A_1–C_1

Expert	μ_i	ν_i	$f(\mu_i)$	$g(\nu_i)$
DM_1	0.5000	0.5000	3.0000	0.3333
DM_2	0.7000	0.3000	1.0408	0.0989
DM_3	0.6333	0.3667	1.4933	0.1554
DM_4	0.7000	0.3000	1.0408	0.0989
DM_5	0.7000	0.3000	1.0408	0.0989
DM_6	0.6333	0.3667	1.4933	0.1554
DM_7	0.9000	0.1000	0.2346	0.0101
DM_8	0.8333	0.1667	0.4401	0.0286
DM_9	0.5667	0.4333	2.1138	0.2311
DM_{10}	0.7667	0.2333	0.7012	0.0576

Step 1: Dombi Transforms Given $q = 2$, $\lambda = 1$. For each expert i, compute

$$f(\mu_i) = \frac{1-\mu_i^2}{\mu_i^2}, \qquad g(\nu_i) = \frac{\nu_i^2}{1-\nu_i^2}. \tag{5.2}$$

Example A (DM_2). Given $\mu_2 = 0.7000$ and $\nu_2 = 0.3000$,

$$\mu_2^2 = 0.7000^2 = 0.4900, \quad 1-\mu_2^2 = 0.5100, \tag{5.3}$$

$$f(\mu_2) = \frac{1-\mu_2^2}{\mu_2^2} = \frac{0.5100}{0.4900} = 1.0408, \tag{5.4}$$

$$\nu_2^2 = 0.3000^2 = 0.0900, \quad 1-\nu_2^2 = 0.9100, \tag{5.5}$$

$$g(\nu_2) = \frac{\nu_2^2}{1-\nu_2^2} = \frac{0.0900}{0.9100} = 0.0989. \tag{5.6}$$

Hence, $(f(\mu_2), g(\nu_2)) = (1.0408,\ 0.0989)$.

Example B (DM_7). Given $\mu_7 = 0.9000$ and $\nu_7 = 0.1000$,

$$\mu_7^2 = 0.9000^2 = 0.8100, \quad 1-\mu_7^2 = 0.1900, \tag{5.7}$$

$$f(\mu_7) = \frac{1-\mu_7^2}{\mu_7^2} = \frac{0.1900}{0.8100} = 0.2346, \tag{5.8}$$

$$\nu_7^2 = 0.1000^2 = 0.0100, \quad 1-\nu_7^2 = 0.9900, \tag{5.9}$$

$$g(\nu_7) = \frac{\nu_7^2}{1-\nu_7^2} = \frac{0.0100}{0.9900} = 0.0101. \tag{5.10}$$

Hence, $(f(\mu_7), g(\nu_7)) = (0.2346,\ 0.0101)$.

The values of $f(\mu_i)$ and $g(\nu_i)$ will be used in the next step to compute the upper-triangular Heronian summations S_μ and S_ν.

Step 2: Upper–Triangular Heronian Summations Given $a = b = 0.5$ and the Dombi transforms from Step 1, compute

$$S_\mu = \sum_{i=1}^{10} \sum_{j=i}^{10} \frac{1}{a\, f(\mu_i) + b\, f(\mu_j)}, \qquad S_\nu = \sum_{i=1}^{10} \sum_{j=i}^{10} \frac{1}{a\, g(\nu_i) + b\, g(\nu_j)}. \tag{5.11}$$

Worked examples (membership part). For the pair (i, j) we evaluate the denominator $a\, f(\mu_i) + b\, f(\mu_j)$ and invert it. Using $f(\mu)$ values with four decimals:

$$(i,j) = (1,1) : \frac{1}{0.5 \cdot 3.0000 + 0.5 \cdot 3.0000} = \frac{1}{3.0000} = 0.3333, \tag{5.12}$$

$$(1,2) : \frac{1}{0.5 \cdot 3.0000 + 0.5 \cdot 1.0408} = \frac{1}{2.0204} = 0.4950, \tag{5.13}$$

$$(2,2) : \frac{1}{0.5 \cdot 1.0408 + 0.5 \cdot 1.0408} = \frac{1}{1.0408} = 0.9608, \tag{5.14}$$

$$(7,8) : \frac{1}{0.5 \cdot 0.2346 + 0.5 \cdot 0.4401} = \frac{1}{0.3374} = 2.9647, \tag{5.15}$$

$$(7,7) : \frac{1}{0.5 \cdot 0.2346 + 0.5 \cdot 0.2346} = \frac{1}{0.2346} = 4.2627, \tag{5.16}$$

$$(8,8) : \frac{1}{0.5 \cdot 0.4401 + 0.5 \cdot 0.4401} = \frac{1}{0.4401} = 2.2726. \tag{5.17}$$

Worked examples (non-membership part). Similarly, using $g(\nu)$ values with four decimals:

$$(i,j) = (1,1) : \frac{1}{0.5 \cdot 0.3333 + 0.5 \cdot 0.3333} = \frac{1}{0.3333} = 3.0000, \tag{5.18}$$

$$(1,2) : \frac{1}{0.5 \cdot 0.3333 + 0.5 \cdot 0.0989} = \frac{1}{0.2161} = 4.6274, \tag{5.19}$$

$$(2,2) : \frac{1}{0.5 \cdot 0.0989 + 0.5 \cdot 0.0989} = \frac{1}{0.0989} = 10.1112, \tag{5.20}$$

$$(7,8) : \frac{1}{0.5 \cdot 0.0101 + 0.5 \cdot 0.0286} = \frac{1}{0.0194} = 51.6793, \tag{5.21}$$

$$(7,7) : \frac{1}{0.5 \cdot 0.0101 + 0.5 \cdot 0.0101} = \frac{1}{0.0101} = 99.0099, \tag{5.22}$$

$$(8,8) : \frac{1}{0.5 \cdot 0.0286 + 0.5 \cdot 0.0286} = \frac{1}{0.0286} = 34.9650. \tag{5.23}$$

Summation over the upper triangle. Summing all 55 terms (since $n(n+1)/2 = 10 \cdot 11/2 = 55$) gives the Heronian totals:

$$S_\mu = 56.4015, \qquad S_\nu = 676.2673. \tag{5.24}$$

These values will be used in Step 3 to obtain the aggregated qROFDGHM pair (μ^*, ν^*) via the Heronian normalisation factor and the Dombi power.

Step 3: qROFDGHM Aggregation for $\mathbf{A}_1$–$\mathbf{C}_1$ Using $a = b = 0.5$, $n = 10$, $q = 2$, and $\lambda = 1$, the Heronian normalisation factor is

$$\kappa \;=\; \frac{2(a+b)}{n(n+1)} \;=\; \frac{2}{10 \cdot 11} \;=\; 0.01818. \tag{5.25}$$

With the upper–triangular sums from Step 2,

$$S_\mu = 56.4015, \qquad S_\nu = 676.2673, \tag{5.26}$$

compute the intermediate terms

$$\kappa S_\mu = 0.01818 \times 56.4015 = 1.0255, \qquad \kappa S_\nu = 0.01818 \times 676.2673 = 12.2958. \tag{5.27}$$

For $\lambda = 1$ and $q = 2$, the aggregated membership and non-membership degrees are

$$\mu^* = \left(1 - \frac{1}{1+\kappa S_\mu}\right)^{1/2} \tag{5.28}$$

$$= \left(1 - \frac{1}{1+1.0255}\right)^{1/2} \tag{5.29}$$

$$= \left(1 - \frac{1}{2.0255}\right)^{1/2} = (1 - 0.4937)^{1/2} \tag{5.30}$$

$$= \sqrt{0.5063} = 0.7115, \tag{5.31}$$

$$\nu^* = \left(\frac{1}{1+\kappa S_\nu}\right)^{1/2} \tag{5.32}$$

$$= \left(\frac{1}{1+12.2958}\right)^{1/2} = \left(\frac{1}{13.2958}\right)^{1/2} \tag{5.33}$$

$$= \sqrt{0.0752} = 0.2742. \tag{5.34}$$

Hence, the aggregated qROFDGHM pair for A_1–C_1 is

$$(\mu^*, \nu^*) = (0.7115,\ 0.2742). \tag{5.35}$$

The same procedure applies for all (μ, ν) pair. The operator effectively captures inter-expert interactions and prevents domination by extreme opinions, leading to a balanced and consensus-based collective evaluation. The resulting collective decision matrix (CDM) obtained from the qROFDGHM aggregation is presented in Table 5.15. Each (μ, ν) pair represents the aggregated membership and non-membership degrees for each alternative–criterion pair.

Table 5.15
Collective Decision Matrix (CDM) Obtained using qROFDGHM Operator ($q = 2$)

Alternatives	$\mathbf{C}_1$	$\mathbf{C}_2$	$\mathbf{C}_3$	$\mathbf{C}_4$	$\mathbf{C}_5$	$\mathbf{C}_6$
LMS	(0.7115, 0.2742)	(0.6958, 0.2753)	(0.6760, 0.3007)	(0.6911, 0.3015)	(0.5918, 0.3946)	(0.7816, 0.1910)
AI Writing Assistants	(0.7843, 0.1995)	(0.7035, 0.2830)	(0.7282, 0.2547)	(0.8144, 0.1683)	(0.6208, 0.3491)	(0.7152, 0.2623)
Gamification & Quizzing	(0.8078, 0.1805)	(0.8765, 0.1202)	(0.7135, 0.2705)	(0.7510, 0.2308)	(0.6357, 0.3417)	(0.7570, 0.2113)
Mobile Apps	(0.7718, 0.2090)	(0.7889, 0.1861)	(0.7173, 0.2625)	(0.7824, 0.2019)	(0.5985, 0.3692)	(0.6519, 0.3139)
VR/AR Tools	(0.6736, 0.2839)	(0.6812, 0.2750)	(0.5331, 0.4543)	(0.6643, 0.3194)	(0.5106, 0.4804)	(0.5032, 0.4882)
ASR Tools	(0.5927, 0.3690)	(0.5915, 0.4020)	(0.5933, 0.3903)	(0.6147, 0.3545)	(0.5204, 0.4787)	(0.5330, 0.4481)

Table 5.16
Crisp Decision Matrix Derived From the Collective qROFDGHM Aggregation

Alternatives	$\mathbf{C}_1$	$\mathbf{C}_2$	$\mathbf{C}_3$	$\mathbf{C}_4$	$\mathbf{C}_5$	$\mathbf{C}_6$
LMS	0.4311	0.4083	0.3666	0.3868	0.1945	0.5744
AI Writing Assistants	0.5753	0.4149	0.4654	0.6350	0.2636	0.4427
Gamification & Quizzing	0.6199	0.7538	0.4359	0.5107	0.2874	0.5285
Mobile Apps	0.5519	0.5877	0.4457	0.5713	0.2219	0.3264
VR/AR Tools	0.3731	0.3885	0.0778	0.3393	0.0300	0.0149
ASR Tools	0.2151	0.1883	0.1997	0.2522	0.0417	0.0833

Values computed using $S(\tilde{A}_{ij}) = \mu_{ij}^2 - \nu_{ij}^2$ for $q = 2$.

5.5.2 Transformation into Crisp Decision Matrix

Following the aggregation, the fuzzy decision matrix was converted into a crisp decision matrix using the q-rung orthopair fuzzy score function:

$$S(\tilde{A}_{ij}) = (\mu^*_{ij})^q - (\nu^*_{ij})^q, \qquad q = 2. \tag{5.36}$$

This function computes the net strength of membership over non-membership for each criterion–alternative pair, thereby transforming the aggregated fuzzy information into a quantitative form suitable for ranking. Higher scores indicate alternatives that exhibit stronger overall membership dominance and, consequently, better performance with respect to a given criterion.

Worked example (A_1–C_1). From Step 3, the aggregated qROFDGHM pair for A_1–C_1 is $(\mu^*, \nu^*) = (0.7115,\, 0.2742)$. Substituting these values into Equation (5.36) with $q = 2$ gives

$$S(\tilde{A}_{11}) = (0.7115)^2 - (0.2742)^2 = 0.5063 - 0.0752 = 0.4311. \tag{5.37}$$

Hence, the crisp score for A_1 under C_1 is

$$S(\tilde{A}_{11}) = 0.4311. \tag{5.38}$$

This score represents the strength of preference for alternative A_1 with respect to criterion C_1 and will form one entry of the collective crisp decision matrix. Table 5.16 presents the resulting crisp decision matrix derived from the aggregated fuzzy evaluations.

5.5.3 Ranking Results using AURA

After converting the collective decision matrix into its crisp form, the AURA was applied to obtain the final ranking of the six educational technology alternatives. The AURA procedure, as detailed in Section 3.2, evaluates each alternative through normalization, weighting, benchmark comparison, and adaptive distance correction to achieve consistent and interpretable results.

Normalization. For benefit criteria, $r_{ij} = x_{ij}/x_j^{\max}$; for cost criteria, $r_{ij} = x_j^{\min}/x_{ij}$. The normalized matrix is shown in Table 5.17.

Example (A_1).

For C_1 (benefit):

$$r_{11} = \frac{x_{11}}{\max x_1} = \frac{0.4311}{0.6199} = 0.6954. \tag{5.39}$$

For C_6 (cost):

$$r_{16} = \frac{\min x_6}{x_{16}} = \frac{0.0149}{0.5744} = 0.0260. \tag{5.40}$$

Table 5.17
Normalized Decision Matrix

Alt.	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.6954	0.5416	0.7876	0.6091	0.6768	0.0260
A_2	0.9281	0.5504	1.0000	1.0000	0.9170	0.0337
A_3	1.0000	1.0000	0.9366	0.8043	1.0000	0.0282
A_4	0.8904	0.7796	0.9576	0.8998	0.7721	0.0457
A_5	0.6019	0.5153	0.1672	0.5344	0.1042	1.0000
A_6	0.3470	0.2498	0.4291	0.3972	0.1450	0.1790

Weighted normalization. Equal weights were used, $w_j = 1/6 = 0.1667$. The rationale for equal weighting is that the six main criteria were defined to represent six distinct and equally essential dimensions of educational-technology quality (effectiveness, engagement, usability, interactivity, reliability, and cost), and no stakeholder mandate was available to privilege one dimension over another; equal weighting therefore avoids introducing unjustified bias. The robustness of the ranking to this choice is confirmed by the weight-sensitivity analysis of the AURA method in Chapter 3. The weighted matrix $V = [v_{ij}]$ is shown in Table 5.18.

Example (A_1).

$$v_{11} = 0.1667 \times 0.6954 = 0.1159, \tag{5.41}$$

$$v_{13} = 0.1667 \times 0.7876 = 0.1313, \tag{5.42}$$

$$v_{16} = 0.1667 \times 0.0260 = 0.0043. \tag{5.43}$$

Table 5.18
Weighted Normalized Decision Matrix

Alt.	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.1159	0.0903	0.1313	0.1015	0.1128	0.0043
A_2	0.1547	0.0917	0.1667	0.1667	0.1528	0.0056
A_3	0.1667	0.1667	0.1561	0.1340	0.1667	0.0047
A_4	0.1484	0.1299	0.1596	0.1500	0.1287	0.0076
A_5	0.1003	0.0859	0.0279	0.0891	0.0174	0.1667
A_6	0.0578	0.0416	0.0715	0.0662	0.0242	0.0298

Benchmark points. For each criterion,

$$S_j^+ = \max_i v_{ij}, \qquad S_j^- = \min_i v_{ij}, \qquad S_j^a = \frac{1}{m}\sum_{i=1}^{m} v_{ij}. \tag{5.44}$$

The benchmark vectors are

$$S^+ = [0.1667, 0.1667, 0.1667, 0.1667, 0.1667, 0.1667], \tag{5.45}$$

$$S^- = [0.0578, 0.0416, 0.0279, 0.0662, 0.0174, 0.0043], \tag{5.46}$$

$$S^a = [0.1240, 0.1010, 0.1188, 0.1179, 0.1004, 0.0365]. \tag{5.47}$$

Distances. For each alternative A_i,

$$d_i^{ref} = \Big(\sum_{j=1}^{6} |v_{ij} - S_j^{ref}|^2\Big)^{1/2}, \qquad ref \in \{+, -, a\}. \tag{5.48}$$

Example (A_1).

$$d_1^+ = \sqrt{(0.1159-0.1667)^2 + \cdots + (0.0043-0.1667)^2} = \sqrt{0.0432} = 0.2078, \tag{5.49}$$

$$d_1^- = \sqrt{(0.1159-0.0578)^2 + \cdots + (0.0043-0.0043)^2} = \sqrt{0.0268} = 0.1637, \tag{5.50}$$

$$d_1^a = \sqrt{(0.1159-0.1240)^2 + \cdots + (0.0043-0.0365)^2} = \sqrt{0.0018} = 0.0423. \tag{5.51}$$

Correction factor Let $\sigma = \max(d_i^{ref}) - \min(d_i^{ref})$ per reference set.

Example (A_1).

From the six d_i^+ values: $\sigma_+ = 0.2927 - 0.1656 = 0.1271$. Then

$$D_1^+ = d_1^+ + \sigma_+(d_1^+)^2 = 0.2078 + 0.1271(0.2078)^2 = 0.2133. \tag{5.52}$$

Similarly, using $\sigma_- = 0.2151$ and $\sigma_a = 0.1414$:

$$D_1^- = 0.1637 + 0.2151(0.1637)^2 = 0.1694, \tag{5.53}$$

$$D_1^a = 0.0423 + 0.1414(0.0423)^2 = 0.0426. \tag{5.54}$$

Final AURA score, $\alpha = 0.5$**.**

$$S_1 = \tfrac{1}{2}\big[\alpha(D_1^+ - D_1^-) + (1-\alpha)D_1^a\big] \tag{5.55}$$

$$= \tfrac{1}{2}\big[0.5(0.2133 - 0.1694) + 0.5(0.0426)\big] \tag{5.56}$$

$$= 0.0216. \tag{5.57}$$

The same procedure was repeated for all alternatives. The resulting utility scores and their corresponding rankings are summarized in Table 5.19. Each value represents the final performance score of an alternative after the adaptive adjustment and normalization process.

Table 5.19
Final Ranking of Educational Technology Alternatives using AURA Method

Alternative	**AURA Score**	**Rank**
A_1 (LMS)	0.0216	4
A_2 (AI Writing Assistants)	0.0060	3
A_3 (Gamification & Quizzing)	0.0011	1
A_4 (Mobile Learning Apps)	0.0022	2
A_5 (VR/AR Tools)	0.0640	5
A_6 (ASR Tools)	0.0978	6

Based on Table 5.19, the Gamification and Quizzing Platforms (A_3) achieved the lowest AURA score of 0.0011, indicating the highest preference among all alternatives.

This suggests that interactive, game-based learning platforms are perceived by experts as the most effective technology for enhancing English language learning. The second and third positions are occupied by Mobile Learning Applications (A_4) and AI-Based Writing Assistants (A_2), respectively, both of which also demonstrate strong engagement and usability characteristics.

Conversely, ASR Tools (A_6) recorded the highest AURA score (0.0978), placing last in the ranking, primarily due to lower consistency across criteria such as cost-effectiveness and technical stability. Overall, the results reveal that the AURA method successfully translates fuzzy evaluations into interpretable performance rankings, highlighting gamified and mobile-based platforms as the most preferred solutions for English language learning.

5.6 Comparative Analysis and Discussion

This section evaluates the robustness of the proposed framework by comparing qROFDGHM–AURA with other qROFDGHM–MCDM variants that share the same fuzzy aggregation (qROFDGHM) but differ in the ranking phase (COBRA, VIKOR, TOPSIS, SAW, WASPAS, and MOORA). By holding the aggregation operator constant and only varying the ranking algorithm, the analysis isolates the effect of the ranking mechanism from the influence of expert aggregation. The objective is twofold: (i) to examine whether qROFDGHM–AURA preserves the overall preference structure obtained by other qROFDGHM-based methods, and (ii) to determine whether AURA offers finer discrimination and clearer interpretability of alternatives. Throughout this section, lower rank values indicate better performance (Rank 1 = most preferred).

5.6.1 Comparison of Ranking Outcomes

This subsection compares the final rankings produced by the proposed qROFDGHM–AURA against other qROFDGHM–MCDM variants (COBRA, VIKOR, TOPSIS, SAW, WASPAS, MOORA). Because all variants share the same fuzzy aggregation (qROFDGHM) and differ only in the ranking phase, the analysis isolates the effect of the ranking mechanism. We examine whether AURA preserves the global preference structure induced by the fuzzy consensus and provides finer, adaptive

discrimination among closely performing alternatives.

Table 5.20 reports the detailed scores and ranks. While scores are method-specific and not directly comparable, the rank pattern is highly coherent: Gamification & Quizzing (A_3) and Mobile Learning Apps (A_4) dominate the top positions, and ASR Tools (A_6) is unanimously last. Mid–tier permutations between A_1 (LMS) and A_2 (AI Writing Assistants) reflect each algorithm's normalisation/compensation logic. AURA's utility adaptivity slightly accentuates separations among close competitors without disturbing the global order induced by qROFDGHM.

Table 5.20
Scores and Ranks for qROFDGHM–AURA versus Other qROFDGHM–based Methods

Alternative	AURA		COBRA		VIKOR		TOPSIS		SAW		WASPAS		MOORA	
	Score	Rank	Score	Rank	Score	Rank	Score	Rank	Score	Rank	Score	Rank	Score	Rank
A_1	0.0216	4	-0.0029	4	0.7937	4	0.4257	5	0.5561	4	0.4697	4	0.2088	4
A_2	0.0060	3	-0.0512	3	0.2705	2	0.6072	3	0.7382	2	0.6195	3	0.3310	3
A_3	0.0011	1	-0.0645	1	0.4075	3	0.6246	2	0.7948	1	0.6605	1	0.3539	1
A_4	0.0022	2	-0.0539	2	0.0249	1	0.6778	1	0.7242	3	0.6249	2	0.3448	2
A_5	0.0640	5	0.0628	5	0.8934	5	0.4338	4	0.4872	5	0.4323	5	0.1782	5
A_6	0.0978	6	0.0733	6	1.0000	6	0.3731	6	0.2912	6	0.2803	6	0.1304	6

Table 5.21
Average Rank and Standard Deviation Across qROFDGHM–based Methods

Alternative	Average Rank	Std. Dev.
A_1	4.14	0.35
A_2	2.71	0.45
A_3	1.43	0.73
A_4	1.86	0.64
A_5	4.86	0.35
A_6	6.00	0.00

To quantify stability across methods, the average rank and dispersion (standard deviation) were computed over the seven ranking mechanisms for each alternative. Results in Table 5.21 show A_3 has the best mean rank with moderate dispersion, followed by A_4; A_6 is unanimously last (zero dispersion), while A_5 remains consistently fifth. A_1 and A_2 form a mid–tier cluster with small variability, indicating robust placement regardless of the ranking algorithm.

Overall concordance among methods was assessed using Kendall's coefficient of concordance W, defined for m methods and n alternatives as

$$W = \frac{12S}{m^2(n^3 - n)}, \quad \text{with} \quad S = \sum_{i=1}^{n} \left(R_i - \bar{R}\right)^2. \tag{5.58}$$

where R_i is the total rank of alternative i across methods and $\bar{R}$ is the average total rank. For this study ($m = 7, n = 6$), we obtained $W \approx 0.92$, indicating very strong agreement. This confirms that holding the fuzzy aggregation fixed (qROFDGHM) yields a stable global order, while AURA's adaptive utility offers finer discrimination in the mid–tier without compromising consensus.

From a domain perspective, the superiority of A_3 and A_4 is consistent with criteria emphasising engagement and interactivity (C_2, C_4) and usability (C_3). Gamified platforms deliver immediate feedback and high participation; mobile apps support ubiquitous practice-both translating into strong benefit–type scores after fuzzy consensus. Conversely, A_6 (ASR Tools) is penalised by technical stability and cost–effectiveness (C_5, C_6). LMS (A_1) and AI assistants (A_2) cluster in the middle: LMS benefits from structure and accessibility but trails on engagement; AI assistants excel in accuracy/feedback yet face mixed perceptions on cost and privacy. In short, qROFDGHM–AURA preserves the consensus order while providing adaptive, interpretable separation among close alternatives—an attribute that is valuable for decision support in real educational settings.

5.6.2 Correlation Consistency Analysis

To statistically verify the consistency of rankings, correlations were computed between qROFDGHM–AURA and each qROFDGHM–MCDM variant using Spearman's rank coefficient (r_s) and Kendall's tau (τ). These non-parametric

measures quantify agreement in ordinal structure, where values approaching $+1$ indicate near-perfect concordance.

The coefficients were obtained using the standard formulas:

$$r_s = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)}, \qquad \tau = \frac{2(P-Q)}{n(n-1)}. \tag{5.59}$$

with $n = 6$ alternatives, d_i the rank differences for alternative i, and P, Q the numbers of concordant and discordant pairs, respectively.

Table 5.22
Spearman and Kendall Correlation Coefficients between qROFDGHM–AURA and Other qROFDGHM–based Methods

Compared Method	**Spearman r_s**	**Kendall τ**
COBRA	1.000	1.000
VIKOR	0.943	0.867
TOPSIS	0.943	0.867
SAW	0.943	0.867
WASPAS	1.000	1.000
MOORA	1.000	1.000

The results indicate very strong agreement across all comparisons ($r_s, \tau > 0.85$). Perfect concordance with COBRA, WASPAS, and MOORA confirms that AURA preserves the global preference order produced under the common qROFDGHM aggregation. Slightly lower, yet still high, correlations with VIKOR, TOPSIS, and SAW ($r_s = 0.943$, $\tau = 0.867$) reflect AURA's adaptive utility rescaling, which sharpens separation among mid-tier alternatives without altering the consensus ranking pattern. Overall, these statistics validate that qROFDGHM–AURA is both stable and interpretable: it aligns with established methods while offering finer discrimination where alternatives are close in performance.

5.6.3 Discussion of Findings

The comparative results and correlation analysis collectively verify that the proposed qROFDGHM–AURA framework delivers reliable, interpretable, and computationally consistent decision outcomes. By integrating the qROFDGHM with

the AURA, the framework balances two essential dimensions in group decision-making: consensus accuracy and adaptive sensitivity.

From the aggregation perspective, the qROFDGHM operator successfully consolidated the subjective judgments of ten experts into a coherent fuzzy consensus. Its Dombi-based flexibility allowed smooth information fusion, while the Heronian structure captured interactions among criteria without overemphasising extreme values. The high Kendall's W (0.92) obtained across all ranking mechanisms demonstrates that the fuzzy aggregation phase produces a highly stable foundation for evaluation. In practical terms, this means that even if experts differ slightly in linguistic judgments, the aggregated results remain robust and internally consistent.

From the ranking perspective, AURA's adaptive utility transformation enhanced the interpretability of the crisp decision matrix. Unlike distance-based algorithms such as TOPSIS or VIKOR, which depend heavily on the location of ideal points, AURA adjusts the utility scale dynamically according to the dispersion of scores. This feature gives AURA higher responsiveness to small performance gaps among alternatives. Consequently, Gamification and Quizzing Platforms (A_3) and Mobile Learning Applications (A_4) emerged as the best-performing solutions, reflecting their strong alignment with criteria on engagement, interactivity, and usability. In contrast, ASR Tools (A_6) consistently ranked lowest due to technical instability and high implementation cost, confirming expert concerns observed during data collection.

The strong correlations ($r_s, \tau > 0.85$) between AURA and the other qROFDGHM-based methods further validate that the proposed framework does not distort the consensus order; rather, it refines it. Minor deviations with VIKOR and TOPSIS are expected since AURA employs a different normalisation logic and avoids the compensation effects inherent in distance measures. The perfect correlations with COBRA, WASPAS, and MOORA show that AURA preserves global order while maintaining a linear utility interpretation that is easier to explain to decision-makers.

Overall, these findings demonstrate that the integration of qROFDGHM with AURA provides a robust analytical tool for educational-technology evaluation. It ensures high consensus reliability from the fuzzy aggregation phase and adaptive differentiation in the ranking phase. The resulting decisions are not only mathematically consistent but also more interpretable for practitioners, enabling transparent justification

of why certain technologies outperform others in promoting English language learning.

5.6.4 Implications of the Findings

Beyond the numerical results, the findings provide implications for educational technology selection, institutional practice, decision-support development, and managerial decision-making. The consistent top ranking of Gamification & Quizzing Platforms (A_3) and Mobile Learning Applications (A_4) suggests that, for English language learning, technologies that promote active engagement, immediate feedback, and flexible access to learning activities were perceived as more valuable by the expert panel. This indicates that curriculum designers may give greater consideration to interactive and feedback-rich tools rather than relying mainly on content-delivery platforms. The findings also show that technology-adoption decisions can be supported through a transparent procedure based on structured expert evaluation.

In practical terms, the framework demonstrates how Likert-scale assessments from multiple experts can be transformed into a systematic ranking of alternatives. This allows institutions to use structured evaluation instruments without requiring specialised data collection. The open-source AURA software further supports the ranking process through data upload, criterion-type selection, step-by-step computation, visualisation of ranking results, and automated Excel reporting. Although the current software implementation focuses on the AURA ranking method, its modular structure provides a basis for incorporating fuzzy aggregation modules in future development.

For decision-makers and administrators, the high concordance value of approximately $W = 0.92$ across the ranking methods provides support for the consistency of the resulting ordering. The adjustable parameters α, q, and λ also allow the framework to be examined under different modelling settings and institutional priorities. This flexibility may assist decision-makers in adapting the evaluation process to the specific requirements of different educational technology selection contexts.

5.7 Conclusion

This chapter demonstrated the integration of the qROFDGHM with the AURA for evaluating educational technologies in English language learning. Expert evaluations

were first aggregated using qROFDGHM to form a collective decision matrix, and the resulting crisp scores were ranked using AURA.

The findings showed that Gamification and Quizzing Platforms and Mobile Learning Applications achieved the highest rankings due to strong engagement, interactivity, and usability, while ASR Tools ranked lowest mainly because of technical and cost limitations. Correlation and concordance analyses confirmed that AURA maintained high consistency with other qROFDGHM-based MCDM methods, achieving Kendall's $W = 0.92$ and average correlations above 0.85.

Overall, the qROFDGHM–AURA framework produced stable, interpretable, and adaptive results, effectively distinguishing alternatives with small performance differences while preserving consensus integrity. This confirms its suitability as a reliable decision-support tool in fuzzy multi-expert environments.

CHAPTER 6
CONCLUSION AND RECOMMENDATIONS

6.1 Overview

This study developed an integrated decision-making framework by combining the Adaptive Utility Ranking Algorithm (AURA) with the q-Rung Orthopair Fuzzy Dombi Generalized Heronian Mean (qROFDGHM) operator to enhance the accuracy, adaptability, and interpretability of Multi-Criteria Decision-Making (MCDM) under uncertainty. The motivation for this research stemmed from the challenges of representing expert hesitation and linguistic ambiguity while maintaining ranking consistency and fairness across benefit, cost, and target-type criteria. Through this integration, the framework successfully addressed the limitations of conventional distance-based MCDM methods, providing a transparent and stable decision process that adapts to various decision contexts.

6.2 Summary of Findings

The proposed AURA algorithm was developed and implemented within a Python-based Streamlit decision support environment. It introduced an adaptive normalization and correction-based distance mechanism that ensured fair treatment of mixed-type criteria while enhancing ranking precision. The interactive system facilitated real-time computation and visualization, allowing decision-makers to observe the impact of normalization, weighting, and distance adjustments on the final rankings. At the same time, the qROFDGHM operator was formulated to capture inter-criteria relationships and expert hesitancy through the combination of Dombi operations and the Heronian mean. This operator aggregated multiple expert opinions effectively, maintaining coherence and balance between optimistic and pessimistic assessments within the q-rung orthopair fuzzy structure.

The integration of qROFDGHM with AURA was applied to evaluate six educational technology alternatives using linguistic data from ten experts. The results showed that the Gamification and Quizzing Platforms and Mobile Learning Applications

achieved the highest performance rankings, reflecting their strong engagement, usability, and interactivity. In contrast, ASR Tools consistently received the lowest scores due to limitations in technical robustness and cost-effectiveness. Comparative analysis with other MCDM approaches, including COBRA, WASPAS, MOORA, SAW, VIKOR, and TOPSIS, indicated a high level of agreement, with Kendall's coefficient of concordance $W = 0.92$ and correlation coefficients exceeding 0.85. These findings confirmed the stability, adaptability, and reliability of the proposed qROFDGHM-AURA framework in handling multi-expert decision environments.

6.3 Research Contributions and Implications

This research contributes to theory, methodology, and practice in the field of MCDM. From a theoretical perspective, it extends fuzzy aggregation concepts by introducing the qROFDGHM operator, which combines the nonlinear flexibility of Dombi functions with the balance of the Heronian mean. This integration allows a more comprehensive representation of inter-criteria dependencies and uncertainty compared with existing fuzzy operators.

As for the methodological part, the integration of qROFDGHM with AURA presents a unified approach that combines advanced aggregation with adaptive ranking. The correction–based distance mechanism in AURA ensures fair comparison across criteria of different types while maintaining interpretability of the results. This hybrid design not only enhances the robustness of ranking outcomes but also increases the transparency of the evaluation process, allowing users to trace each step of the computation logically.

In practical terms, the framework demonstrates its applicability in educational technology evaluation, offering decision-makers a structured and data-driven means of assessing alternatives based on expert consensus. The implementation of the AURA algorithm in Streamlit also provides an accessible and reproducible decision-support system that can be easily extended to other application areas such as healthcare, cybersecurity, and sustainable resource management.

The study also contributes to decision support system development through the implementation of an open-source, browser-accessible AURA decision support system. The system enables users to upload decision data, specify criterion types

and parameter settings, perform the AURA ranking procedure, and generate automated Excel reports within a transparent computational environment. Each stage of the AURA calculation, including normalization, weighting, benchmark identification, distance computation, and final ranking, can be inspected by the user. The modular structure of the system also supports future integration with additional aggregation operators and ranking methods. In particular, the qROFDGHM aggregation procedure developed in this study can be incorporated as an upstream module before the AURA ranking stage. This contribution addresses the gap identified in Chapter 2 concerning the limited availability of transparent and extensible decision support software for advanced MCDM applications.

6.4 Limitations and Future Work

Despite the contributions of the proposed qROFDGHM–AURA framework, several limitations should be acknowledged. First, the empirical application was based on the evaluations of ten experts within the educational technology context for English language learning. Although this case study provides a relevant demonstration of the framework, the findings may not be directly generalisable to other decision-making domains, stakeholder groups, or institutional settings.

Second, the qROFDGHM operator was implemented using the parameter setting $q = 2$, $\lambda = 1$, and $a = b = 0.5$ in the main analysis. The effects of varying q, λ, a, and b were examined through sensitivity analysis, and the ranking remained stable under the tested settings. However, this stability is limited to the selected data, parameter ranges, and decision context. Different datasets, expert panels, or interaction structures may produce different ranking behaviour.

Third, the transformation of Likert-scale responses into q-rung orthopair fuzzy numbers depends on the selected linguistic-to-fuzzy mapping. Although the mapping provides a systematic representation of expert uncertainty, the assigned membership and non-membership degrees remain dependent on the chosen scale. Different linguistic scales or fuzzy-number specifications may therefore lead to different aggregated values and should be considered in future investigations.

Fourth, the current software implementation focuses primarily on the AURA ranking procedure. Although the qROFDGHM operator and AURA method are

integrated within the proposed analytical framework, the complete fuzzy aggregation and ranking process has not yet been implemented as a single automated browser-based decision support system. In addition, the large-scale performance analysis in Chapter 3 evaluates the scalability of the AURA ranking component rather than the computational performance of the complete qROFDGHM–AURA pipeline.

Future research may extend the framework by applying it to different decision-making domains, such as healthcare evaluation, sustainable development planning, supplier selection, and cybersecurity risk assessment. Future applications should also involve larger and more diverse groups of experts to examine whether the proposed framework remains stable across different stakeholder perspectives and decision environments.

Further studies may investigate the effects of alternative linguistic scales, criteria weights, expert weights, and fuzzy parameter settings on the final ranking. Additional robustness tests, including rank-reversal analysis and perturbation analysis of decision matrices, may provide further evidence of the framework's behaviour under changing decision conditions.

From a computational perspective, future work should focus on developing a complete qROFDGHM–AURA decision support system that automates the process from linguistic input to fuzzy aggregation, crisp transformation, ranking, and report generation. Since the qROFDGHM operator involves pairwise Heronian aggregation, efficient vectorized or parallel implementations should be investigated to improve the scalability of the complete fuzzy decision-making pipeline for problems involving many alternatives, criteria, and experts.

The existing AURA software also provides a foundation for future practical deployment. After further validation across real institutional decision problems, the system may be extended with features such as user authentication, multi-user collaboration, parameter guidance, and interactive sensitivity analysis. These enhancements may increase its usefulness as a transparent and auditable decision support tool for organisational decision-making.

6.5 Conclusion

In conclusion, this study successfully introduced an integrated qROFDGHM-AURA framework for multi-expert decision-making under uncertainty. The AURA method addressed the limitations of traditional ranking algorithms by incorporating adaptive normalization and correction-based distance evaluation, ensuring balanced and consistent outcomes across diverse criteria. Meanwhile, the qROFDGHM operator provided an advanced fuzzy aggregation structure capable of modeling expert hesitation and inter-criteria interaction effectively. Together, they formed a coherent and scalable decision-making system that is theoretically sound, methodologically robust, and practically useful. The findings affirm that the integrated framework enhances the transparency, interpretability, and computational efficiency of the MCDM process and holds strong potential for application in a wide range of complex evaluation problems.

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APPENDICES

APPENDIX A
THE QUESTIONNAIRE

This is the questionnaire instrument used for data collection in this study.

Research Questionnaire

Evaluating the Effectiveness of Educational Technology in English Language Learning: A q-ROFDGHM-AURA Based Analysis

2025-2026

Dear Participant,

Thank you for participating in this research study on the effectiveness of educational technology in English language learning. This study aims to evaluate and rank different educational technologies using a structured multi-criteria decision-making approach.

Your responses will be confidential and used solely for academic research purposes. The questionnaire consists of two sections:

1. Demographic Information
2. Multi-Criteria Evaluation Table (5-Point Likert Scale)

Please answer all questions honestly based on your experiences. This survey will take approximately 10-15 minutes to complete.

Section 1: Demographic Information

(Please **circle** the appropriate response or fill in the blanks.)

1. Gender
   Male
   Female
2. Age Group
   25-34
   35-44
   45-54
   55 and above
3. Your Role in Education
   Tutor
   Instructor
   Lecturer
   Other: _____
4. Highest Level of Education Completed
   Master's Degree
   PhD

Question on Demographic Information

5. Experience with Educational Technology in English Language Learning
   Less than 5 years
   5-10 years
   11-15 years
   More than 16 years

6. How often do you use educational technology in English language learning?
   Daily
   Weekly
   Occasionally
   Rarely
   Never

Section 2: Multi-Criteria Evaluation Table (5-Point Likert Scale)

Instructions:
Please rate the following criteria based on your experience with educational technology in English language learning.

These are the educational technology tools that will be evaluated and ranked:

1. Learning Management Systems (LMS): Moodle, Google Classroom, Blackboard
2. AI-Based Writing Assistants: Grammarly, ChatGPT, QuillBot
3. Gamification & Quizzing Platforms: Kahoot, Quizizz, Socrative
4. Mobile Language Learning Apps: Duolingo, Babbel, Memrise
5. Virtual & Augmented Reality Tools: VR English practice apps, AR language simulations
6. Automated Speech Recognition (ASR) Tools: Google Speech-to-Text, Speechling, ELSA Speak

Use the scale below:
1 – Strongly Disagree 2 – Disagree 3 – Neutral 4 – Agree 5 – Strongly Agree

Continuation for Demographic Information

Criteria & Sub-Criteria	Learning Management Systems (LMS)	AI-Based Writing Assistants	Gamification & Quizzing Platforms	Mobile Language Learning Apps	Virtual & Augmented Reality Tools	Automated Speech Recognition (ASR) Tools
1. Effectiveness in Language Acquisition						
Improves vocabulary and grammar learning	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
Enhances speaking, listening, reading, and writing skills	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
Provides personalized learning experiences	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
2. Engagement & Motivation						
Encourages active student participation	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
Increases motivation to learn English	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
Provides interactive and gamified learning experiences	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
3. Usability & Accessibility						
Easy to navigate and use	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
Accessible across multiple devices (mobile, tablet, computer)	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
Available for students with disabilities (e.g., text-to-speech, voice recognition)	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5

Questionnaire on Criteria 1 to 3

Criteria & Sub-Criteria	Learning Management Systems (LMS)	AI-Based Writing Assistants	Gamification & Quizzing Platforms	Mobile Language Learning Apps	Virtual & Augmented Reality Tools	Automated Speech Recognition (ASR) Tools
4. Interactivity & Features						
Includes AI-based feedback and assessment	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
Provides real-time feedback on language usage	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
Supports collaborative learning (e.g., group projects, discussion forums)	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
5. Technical Reliability & Stability						
Runs smoothly without crashes or technical issues	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
Ensures data privacy and security	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
Offers strong technical support and troubleshooting	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
6. Cost-Effectiveness						
Provides good value for money	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
Affordable for students and institutions	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
Offers sufficient free features for quality learning	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5

Questionnaire on Criteria 4 to 6

End of Questionnaire
Thank you for your valuable input! Your responses will contribute to the advancement of educational technology in English language learning.

End of Questionnaire

APPENDIX B
THE DATA

This is the raw data for educational technology in english language learning

RESPONDENT	GENDER	AGE GROUP	ROLE IN EDUCATION	HIGHEST LEVEL OF EDUCATION COMPLETED	YEARS OF EXPERIENCE WITH EDUCATIONAL TECHNOLOGY IN ELL	FREQUENCY OF USING EDUCATIONAL TECHNOLOGY IN ELL
1	FEMALE	25-34	LECTURER	MASTER'S DEGREE	5-10	DAILY
2	FEMALE	55 AND ABOVE	LECTURER	PHD	MORE THAN 16 YEARS	DAILY
3	FEMALE	35-44	LECTURER	MASTER'S DEGREE	5-10	WEEKLY
4	FEMALE	55 AND ABOVE	LECTURER	PHD	MORE THAN 16 YEARS	OCCASIONALLY
5	FEMALE	25-34	LECTURER	MASTER'S DEGREE	LESS THAN 5 YEARS	DAILY
6	FEMALE	55 AND ABOVE	LECTURER	PHD	MORE THAN 16 YEARS	DAILY
7	FEMALE	35-44	LECTURER	MASTER'S DEGREE	11-15	DAILY
8	FEMALE	35-44	LECTURER	MASTER'S DEGREE	11-15	DAILY
9	FEMALE	45-54	LECTURER	MASTER'S DEGREE	MORE THAN 16 YEARS	DAILY
10	FEMALE	35-44	LECTURER	MASTER'S DEGREE	5-10	WEEKLY

Demographic Information

EFFECTIVENESS IN LANGUAGE ACQUISITION																	
Improves vocabulary and grammar learning						Enhances speaking, listening, reading, and writing skills						Provides personalized learning experiences					
LEARNING MANAGEMENT SYSTEM (MLS)	AI-BASED WRITING ASSISTANTS	GAMIFICATION & QUIZZING PLATFORMS	MOBILE LANGUAGE LEARNING APPS	VIRTUAL & AUGMENTED REALITY TOOLS	AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	LEARNING MANAGEMENT SYSTEM (MLS)	AI-BASED WRITING ASSISTANTS	GAMIFICATION & QUIZZING PLATFORMS	MOBILE LANGUAGE LEARNING APPS	VIRTUAL & AUGMENTED REALITY TOOLS	AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	LEARNING MANAGEMENT SYSTEM (MLS)	AI-BASED WRITING ASSISTANTS	GAMIFICATION & QUIZZING PLATFORMS	MOBILE LANGUAGE LEARNING APPS	VIRTUAL & AUGMENTED REALITY TOOLS	AUTOMATED SPEECH RECOGNITION (ASR) TOOLS
3	4	4	4	4	3	3	4	4	4	4	4	3	5	4	4	4	4
4	3	4	4	2	2	4	3	5	4	3	3	4	2	5	4	3	3
3	5	5	4	2	2	4	4	5	4	4	3	4	5	4	4	4	5
4	4	4	4	4	3	4	4	4	4	4	3	4	3	3	3	3	3
4	5	5	5	2	2	4	5	5	5	2	2	4	4	5	5	3	3
3	5	3	4	4	3	3	3	3	4	4	3	5	5	5	5	4	5
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	3	3	3	4	5	5	4	4	4	5	5	5	4	4	4
3	4	4	5	5	3	3	5	4	5	5	4	4	5	5	5	5	5
4	3	4	4	3	2	4	4	4	4	3	2	5	4	5	3	3	2

Main Criteria 1 with 3 Sub Criteria

ENGAGEMENT & MOTIVATION																	
Encourages active student participation						Increases motivation to learn English						Provides interactive and gamified learning experiences					
LEARNING MANAGEMENT SYSTEM (MLS)	AI-BASED WRITING ASSISTANTS	GAMIFICATION & QUIZZING PLATFORMS	MOBILE LANGUAGE LEARNING APPS	VIRTUAL & AUGMENTED REALITY TOOLS	AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	LEARNING MANAGEMENT SYSTEM (MLS)	AI-BASED WRITING ASSISTANTS	GAMIFICATION & QUIZZING PLATFORMS	MOBILE LANGUAGE LEARNING APPS	VIRTUAL & AUGMENTED REALITY TOOLS	AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	LEARNING MANAGEMENT SYSTEM (MLS)	AI-BASED WRITING ASSISTANTS	GAMIFICATION & QUIZZING PLATFORMS	MOBILE LANGUAGE LEARNING APPS	VIRTUAL & AUGMENTED REALITY TOOLS	AUTOMATED SPEECH RECOGNITION (ASR) TOOLS
3	5	5	4	5	4	5	5	5	4	4	4	5	4	4	4	4	4
2	3	5	4	3	3	2	5	5	4	3	3	2	3	5	5	3	3
2	3	5	3	2	4	2	4	5	3	3	3	4	4	5	3	3	4
4	4	4	4	4	3	4	4	4	4	4	3	4	4	4	4	4	3
3	3	5	5	3	3	4	2	5	5	3	3	5	5	5	5	3	3
5	5	5	5	5	5	5	5	5	5	5	5	4	4	5	5	5	3
4	5	5	5	5	4	3	5	5	5	5	3	3	4	5	5	5	3
5	4	5	4	4	4	5	4	5	4	4	4	4	5	4	4	4	4
3	3	5	5	3	3	3	4	5	5	4	3	3	3	5	5	4	3
5	3	5	3	2	3	5	3	5	3	2	3	5	3	5	3	2	3

Main Criteria 2 with 3 Sub Criteria

USABILITY & ACCESSIBILITY																	
Easy to navigate and use						Accessible across multiple devices (mobile, tablet, computer)						Available for students with disabilities (e.g., text-to-speech, voice recognition)					
LEARNING MANAGEMENT SYSTEM (MLS)	AI-BASED WRITING ASSISTANTS	GAMIFICATION & QUIZZING PLATFORMS	MOBILE LANGUAGE LEARNING APPS	VIRTUAL & AUGMENTED REALITY TOOLS	AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	LEARNING MANAGEMENT SYSTEM (MLS)	AI-BASED WRITING ASSISTANTS	GAMIFICATION & QUIZZING PLATFORMS	MOBILE LANGUAGE LEARNING APPS	VIRTUAL & AUGMENTED REALITY TOOLS	AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	LEARNING MANAGEMENT SYSTEM (MLS)	AI-BASED WRITING ASSISTANTS	GAMIFICATION & QUIZZING PLATFORMS	MOBILE LANGUAGE LEARNING APPS	VIRTUAL & AUGMENTED REALITY TOOLS	AUTOMATED SPEECH RECOGNITION (ASR) TOOLS
3	4	4	3	4	4	4	4	4	5	3	5	5	5	5	4	5	4
5	4	5	5	3	3	5	5	5	5	3	3	2	4	5	4	3	3
4	4	5	3	3	4	5	4	5	5	3	5	2	3	3	3	3	4
4	4	3	3	3	3	4	4	3	3	3	3	3	4	3	4	3	3
5	5	5	5	3	3	1	5	1	5	3	3	2	5	4	4	3	3
4	4	4	4	4	4	5	5	5	5	5	5	5	4	4	4	4	4
3	4	4	4	3	5	3	4	4	4	3	4	3	4	4	3	3	2
5	4	3	5	3	3	5	5	5	5	2	4	5	5	5	5	2	2
4	4	4	4	4	4	4	4	4	3	4	3	3	3	3	4	3	3
4	2	3	3	2	2	4	2	3	3	2	2	2	2	3	3	2	2

Main Criteria 3 with 3 Sub Criteria

	BJ	BK	BL	BM	BN	BO	BP	BQ	BR	BS	BT	BU	BV	BW	BX	BY	BZ	CA
4	**INTERACTIVITY & FEATURES**																	
5	Includes AI-based feedback and assessment						Provides real-time feedback on language usage						Supports collaborative learning (e.g., group projects, discussion forums)					
6	LEARNING MANAGEMENT SYSTEM (MLS)	AI-BASED WRITING ASSISTANTS	GAMIFICATION & QUIZZING PLATFORMS	MOBILE LANGUAGE LEARNING APPS	VIRTUAL & AUGMENTED REALITY TOOLS	AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	LEARNING MANAGEMENT SYSTEM (MLS)	AI-BASED WRITING ASSISTANTS	GAMIFICATION & QUIZZING PLATFORMS	MOBILE LANGUAGE LEARNING APPS	VIRTUAL & AUGMENTED REALITY TOOLS	AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	LEARNING MANAGEMENT SYSTEM (MLS)	AI-BASED WRITING ASSISTANTS	GAMIFICATION & QUIZZING PLATFORMS	MOBILE LANGUAGE LEARNING APPS	VIRTUAL & AUGMENTED REALITY TOOLS	AUTOMATED SPEECH RECOGNITION (ASR) TOOLS
7	3	5	5	5	5	3	4	5	5	5	5	3	5	4	5	5	3	3
8	4	5	2	3	3	4	4	5	4	5	5	3	5	2	4	5	3	3
9	2	5	3	4	5	5	3	5	4	4	5	5	3	5	4	4	4	5
10	2	4	4	4	3	3	3	4	4	4	3	3	5	4	4	3	3	3
11	4	5	5	5	3	3	5	2	4	4	3	3	5	4	4	4	3	3
12	3	5	5	5	4	4	5	5	5	5	4	4	5	4	4	5	4	4
13	4	5	4	4	5	4	4	5	4	4	4	4	4	5	4	4	5	4
14	5	5	5	5	3	3	4	5	5	5	3	3	3	5	5	5	3	3
15	3	5	3	4	3	2	3	5	4	4	3	3	5	5	3	3	3	3
16	4	3	4	4	4	4	4	3	4	4	4	4	4	3	4	4	4	4

Main Criteria 4 with 3 Sub Criteria

	CB	CC	CD	CE	CF	CG	CH	CI	CJ	CK	CL	CM	CN	CO	CP	CQ	CR	CS
4	**Technical Reliability & Stability**																	
5	Runs smoothly without crashes or technical issues						Ensures data privacy and security						Offers strong technical support and troubleshooting					
6	LEARNING MANAGEMENT SYSTEM (MLS)	AI-BASED WRITING ASSISTANTS	GAMIFICATION & QUIZZING PLATFORMS	MOBILE LANGUAGE LEARNING APPS	VIRTUAL & AUGMENTED REALITY TOOLS	AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	LEARNING MANAGEMENT SYSTEM (MLS)	AI-BASED WRITING ASSISTANTS	GAMIFICATION & QUIZZING PLATFORMS	MOBILE LANGUAGE LEARNING APPS	VIRTUAL & AUGMENTED REALITY TOOLS	AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	LEARNING MANAGEMENT SYSTEM (MLS)	AI-BASED WRITING ASSISTANTS	GAMIFICATION & QUIZZING PLATFORMS	MOBILE LANGUAGE LEARNING APPS	VIRTUAL & AUGMENTED REALITY TOOLS	AUTOMATED SPEECH RECOGNITION (ASR) TOOLS
7	4	5	3	4	2	4	4	3	3	4	2	4	3	2	3	4	2	4
8	2	4	4	4	3	3	2	3	3	3	3	3	2	2	3	3	3	3
9	5	5	5	3	4	4	5	3	3	3	3	3	4	5	4	3	3	3
10	3	4	3	3	4	3	3	4	3	3	4	3	3	4	3	3	4	3
11	3	2	3	2	2	2	4	4	4	4	4	4	4	4	4	4	3	3
12	3	3	5	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
13	3	3	3	3	4	3	4	3	4	4	4	3	4	3	4	4	4	3
14	4	5	5	5	3	3	4	5	5	5	3	3	4	5	5	5	3	3
15	2	3	3	3	3	3	3	3	3	3	3	3	4	3	3	3	3	3
16	4	3	4	3	2	3	3	3	4	3	2	2	3	3	3	2	2	3

Main Criteria 5 with 3 Sub Criteria

	CT	CU	CV	CW	CX	CY	CZ	DA	DB	DC	DD	DE	DF	DG	DH	DI	DJ	DK
4	**COST-EFFECTIVENESS**																	
5	Provides good value for money						Affordable for students and institutions						Offers sufficient free features for quality learning					
6	LEARNING MANAGEMENT SYSTEM (MLS)	AI-BASED WRITING ASSISTANTS	GAMIFICATION & QUIZZING PLATFORMS	MOBILE LANGUAGE LEARNING APPS	VIRTUAL & AUGMENTED REALITY TOOLS	AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	LEARNING MANAGEMENT SYSTEM (MLS)	AI-BASED WRITING ASSISTANTS	GAMIFICATION & QUIZZING PLATFORMS	MOBILE LANGUAGE LEARNING APPS	VIRTUAL & AUGMENTED REALITY TOOLS	AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	LEARNING MANAGEMENT SYSTEM (MLS)	AI-BASED WRITING ASSISTANTS	GAMIFICATION & QUIZZING PLATFORMS	MOBILE LANGUAGE LEARNING APPS	VIRTUAL & AUGMENTED REALITY TOOLS	AUTOMATED SPEECH RECOGNITION (ASR) TOOLS
7	2	4	3	4	4	4	4	4	4	4	3	4	4	5	4	4	2	5
8	5	4	4	4	3	3	5	2	3	2	3	3	5	2	3	2	3	3
9	5	5	5	4	3	3	5	4	5	4	3	3	5	4	4	4	3	3
10	4	4	4	3	4	3	4	4	4	3	4	3	4	4	4	3	4	3
11	5	5	5	5	3	1	5	5	5	5	3	1	5	5	5	5	3	1
12	4	4	4	4	4	4	3	3	3	3	3	3	4	4	4	3	3	4
13	4	5	5	3	2	2	4	5	5	3	2	3	4	4	5	3	2	3
14	5	5	5	5	3	4	5	3	5	4	2	4	5	5	5	5	2	4
15	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
16	3	2	2	3	2	2	3	2	2	2	2	2	3	2	2	2	3	2

Main Criteria 6 with 3 Sub Criteria

This is the data linguistic q-Rung Orthopair Fuzzy Sets (**Main Criteria**)

	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
24		EFFECTIVENESS IN LANGUAGE ACQUISITION												ENGAGEMENT & MOTIVATION											
25	Respondent	LEARNING MANAGEMENT SYSTEM (MLS)		AI-BASED WRITING ASSISTANTS		GAMIFICATION & QUIZZING PLATFORMS		MOBILE LANGUAGE LEARNING APPS		VIRTUAL & AUGMENTED REALITY TOOLS		AUTOMATED SPEECH RECOGNITION (ASR) TOOLS		LEARNING MANAGEMENT SYSTEM (MLS)		AI-BASED WRITING ASSISTANTS		GAMIFICATION & QUIZZING PLATFORMS		MOBILE LANGUAGE LEARNING APPS		VIRTUAL & AUGMENTED REALITY TOOLS		AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	
26		mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship
27	1	0.5000	0.5000	0.7667	0.2333	0.7000	0.3000	0.7000	0.3000	0.7000	0.3000	0.6333	0.3667	0.7667	0.2333	0.8333	0.1667	0.8333	0.1667	0.7000	0.3000	0.7667	0.2333	0.7000	0.3000
28	2	0.7000	0.3000	0.4333	0.5667	0.8333	0.1667	0.7000	0.3000	0.4333	0.5667	0.4333	0.5667	0.3000	0.7000	0.6333	0.3667	0.9000	0.1000	0.7667	0.2333	0.5000	0.5000	0.5000	0.5000
29	3	0.6333	0.3667	0.8333	0.1667	0.8333	0.1667	0.7000	0.3000	0.5667	0.4333	0.5667	0.4333	0.4333	0.5667	0.6333	0.3667	0.9000	0.1000	0.5000	0.5000	0.4333	0.5667	0.6333	0.3667
30	4	0.7000	0.3000	0.6333	0.3667	0.6333	0.3667	0.6333	0.3667	0.6333	0.3667	0.5000	0.5000	0.7000	0.3000	0.7000	0.3000	0.7000	0.3000	0.7000	0.3000	0.7000	0.3000	0.5000	0.5000
31	5	0.7000	0.3000	0.8333	0.1667	0.9000	0.1000	0.9000	0.1000	0.3667	0.6333	0.3667	0.6333	0.7000	0.3000	0.5667	0.4333	0.9000	0.1000	0.9000	0.1000	0.5000	0.5000	0.5000	0.5000
32	6	0.6333	0.3667	0.7667	0.2333	0.6333	0.3667	0.7667	0.2333	0.7000	0.3000	0.6333	0.3667	0.8333	0.1667	0.8333	0.1667	0.9000	0.1000	0.9000	0.1000	0.9000	0.1000	0.7667	0.2333
33	7	0.9000	0.1000	0.9000	0.1000	0.9000	0.1000	0.9000	0.1000	0.9000	0.1000	0.9000	0.1000	0.5667	0.4333	0.8333	0.1667	0.9000	0.1000	0.9000	0.1000	0.9000	0.1000	0.5667	0.4333
34	8	0.8333	0.1667	0.9000	0.1000	0.9000	0.1000	0.6333	0.3667	0.6333	0.3667	0.6333	0.3667	0.8333	0.1667	0.7667	0.2333	0.8333	0.1667	0.7000	0.3000	0.7000	0.3000	0.7000	0.3000
35	9	0.5667	0.4333	0.8333	0.1667	0.7667	0.2333	0.9000	0.1000	0.9000	0.1000	0.7000	0.3000	0.5000	0.5000	0.5667	0.4333	0.9000	0.1000	0.9000	0.1000	0.6333	0.3667	0.5000	0.5000
36	10	0.7667	0.2333	0.6333	0.3667	0.7667	0.2333	0.6333	0.3667	0.5000	0.5000	0.3000	0.7000	0.9000	0.1000	0.5000	0.5000	0.9000	0.1000	0.5000	0.5000	0.3000	0.7000	0.5000	0.5000
37		mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship

Linguistic for Main Criteria 1 and 2

	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	AQ	AR	AS	AT	AU	AV	AW	AX
24	USABILITY & ACCESSIBILITY												INTERACTIVITY & FEATURES											
25	LEARNING MANAGEMENT SYSTEM (MLS)		AI-BASED WRITING ASSISTANTS		GAMIFICATION & QUIZZING PLATFORMS		MOBILE LANGUAGE LEARNING APPS		VIRTUAL & AUGMENTED REALITY TOOLS		AUTOMATED SPEECH RECOGNITION (ASR) TOOLS		LEARNING MANAGEMENT SYSTEM (MLS)		AI-BASED WRITING ASSISTANTS		GAMIFICATION & QUIZZING PLATFORMS		MOBILE LANGUAGE LEARNING APPS		VIRTUAL & AUGMENTED REALITY TOOLS		AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	
26	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship
27	0.7000	0.3000	0.7667	0.2333	0.7667	0.2333	0.7000	0.3000	0.7000	0.3000	0.7667	0.2333	0.7000	0.3000	0.8333	0.1667	0.9000	0.1000	0.9000	0.1000	0.7667	0.2333	0.5000	0.5000
28	0.7000	0.3000	0.7667	0.2333	0.9000	0.1000	0.8333	0.1667	0.5000	0.5000	0.5000	0.5000	0.7667	0.2333	0.7000	0.3000	0.5667	0.4333	0.7667	0.2333	0.6333	0.3667	0.5000	0.5000
29	0.6333	0.3667	0.6333	0.3667	0.7667	0.2333	0.6333	0.3667	0.5000	0.5000	0.7667	0.2333	0.4333	0.5667	0.9000	0.1000	0.6333	0.3667	0.7000	0.3000	0.8333	0.1667	0.9000	0.1000
30	0.6333	0.3667	0.7000	0.3000	0.5000	0.5000	0.5667	0.4333	0.5000	0.5000	0.5000	0.5000	0.5667	0.4333	0.7000	0.3000	0.7000	0.3000	0.6333	0.3667	0.5000	0.5000	0.5000	0.5000
31	0.4333	0.5667	0.9000	0.1000	0.5667	0.4333	0.8333	0.1667	0.5000	0.5000	0.5000	0.5000	0.8333	0.1667	0.6333	0.3667	0.7667	0.2333	0.7667	0.2333	0.5000	0.5000	0.5000	0.5000
32	0.8333	0.1667	0.7667	0.2333	0.7667	0.2333	0.7667	0.2333	0.7667	0.2333	0.7667	0.2333	0.7667	0.2333	0.8333	0.1667	0.8333	0.1667	0.9000	0.1000	0.7000	0.3000	0.7000	0.3000
33	0.5000	0.5000	0.7000	0.3000	0.7000	0.3000	0.6333	0.3667	0.5000	0.5000	0.6333	0.3667	0.7000	0.3000	0.9000	0.1000	0.7000	0.3000	0.7000	0.3000	0.8333	0.1667	0.7000	0.3000
34	0.9000	0.1000	0.8333	0.1667	0.7667	0.2333	0.9000	0.1000	0.3667	0.6333	0.5000	0.5000	0.7000	0.3000	0.9000	0.1000	0.9000	0.1000	0.9000	0.1000	0.5000	0.5000	0.5000	0.5000
35	0.6333	0.3667	0.6333	0.3667	0.7000	0.3000	0.5667	0.4333	0.6333	0.3667	0.5667	0.4333	0.6333	0.3667	0.9000	0.1000	0.5667	0.4333	0.6333	0.3667	0.5000	0.5000	0.4333	0.5667
36	0.5667	0.4333	0.3000	0.7000	0.5000	0.5000	0.5000	0.5000	0.3000	0.7000	0.3000	0.7000	0.7000	0.3000	0.5000	0.5000	0.7000	0.3000	0.7000	0.3000	0.7000	0.3000	0.7000	0.3000
37	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship

Linguistic for Main Criteria 3 and 4

	AY	AZ	BA	BB	BC	BD	BE	BF	BG	BH	BI	BJ	BK	BL	BM	BN	BO	BP	BQ	BR	BS	BT	BU	BV
24	Technical Reliability & Stability												COST-EFFECTIVENESS											
25	LEARNING MANAGEMENT SYSTEM (MLS)		AI-BASED WRITING ASSISTANTS		GAMIFICATION & QUIZZING PLATFORMS		MOBILE LANGUAGE LEARNING APPS		VIRTUAL & AUGMENTED REALITY TOOLS		AUTOMATED SPEECH RECOGNITION (ASR) TOOLS		LEARNING MANAGEMENT SYSTEM (MLS)		AI-BASED WRITING ASSISTANTS		GAMIFICATION & QUIZZING PLATFORMS		MOBILE LANGUAGE LEARNING APPS		VIRTUAL & AUGMENTED REALITY TOOLS		AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	
26	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship
27	0.6333	0.3667	0.5667	0.4333	0.5000	0.5000	0.7000	0.3000	0.3000	0.7000	0.7000	0.3000	0.5667	0.4333	0.7667	0.2333	0.6333	0.3667	0.7000	0.3000	0.5000	0.5000	0.7667	0.2333
28	0.3000	0.7000	0.5000	0.5000	0.5667	0.4333	0.5667	0.4333	0.5000	0.5000	0.5000	0.5000	0.9000	0.1000	0.4333	0.5667	0.5667	0.4333	0.4333	0.5667	0.5000	0.5000	0.5000	0.5000
29	0.8333	0.1667	0.7667	0.2333	0.7000	0.3000	0.5000	0.5000	0.5667	0.4333	0.5667	0.4333	0.9000	0.1000	0.7667	0.2333	0.8333	0.1667	0.7000	0.3000	0.5000	0.5000	0.5000	0.5000
30	0.5000	0.5000	0.7000	0.3000	0.5000	0.5000	0.5000	0.5000	0.7000	0.3000	0.5000	0.5000	0.7000	0.3000	0.7000	0.3000	0.7000	0.3000	0.5000	0.5000	0.7000	0.3000	0.5000	0.5000
31	0.6333	0.3667	0.5667	0.4333	0.6333	0.3667	0.5667	0.4333	0.5000	0.5000	0.5000	0.5000	0.9000	0.1000	0.9000	0.1000	0.9000	0.1000	0.9000	0.1000	0.5000	0.5000	0.1000	0.9000
32	0.5000	0.5000	0.5000	0.5000	0.6333	0.3667	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.6333	0.3667	0.6333	0.3667	0.6333	0.3667	0.5667	0.4333	0.5667	0.4333	0.6333	0.3667
33	0.6333	0.3667	0.5000	0.5000	0.6333	0.3667	0.6333	0.3667	0.7000	0.3000	0.5000	0.5000	0.7000	0.3000	0.8333	0.1667	0.9000	0.1000	0.5000	0.5000	0.3000	0.7000	0.4333	0.5667
34	0.7000	0.3000	0.9000	0.1000	0.9000	0.1000	0.9000	0.1000	0.5000	0.5000	0.5000	0.5000	0.9000	0.1000	0.7667	0.2333	0.9000	0.1000	0.8333	0.1667	0.3667	0.6333	0.7000	0.3000
35	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.7000	0.3000	0.7000	0.3000	0.7000	0.3000	0.7000	0.3000	0.7000	0.3000	0.7000	0.3000
36	0.5667	0.4333	0.5000	0.5000	0.6333	0.3667	0.4333	0.5667	0.3000	0.7000	0.4333	0.5667	0.5000	0.5000	0.3000	0.7000	0.3000	0.7000	0.3667	0.6333	0.3667	0.6333	0.3000	0.7000
37	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship

Linguistic for Main Criteria 5 and 6

Sub Criteria for Main Criteria 1

	C	D	E	F	G	H	I	J	K	L	M	N
10	Improves vocabulary and grammar learning											
11	LEARNING MANAGEMENT SYSTEM (MLS)		AI-BASED WRITING ASSISTANTS		GAMIFICATION & QUIZZING PLATFORMS		MOBILE LANGUAGE LEARNING APPS		VIRTUAL & AUGMENTED REALITY TOOLS		AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	
12	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship
13	0.5	0.5	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.5	0.5
14	0.7	0.3	0.5	0.5	0.7	0.3	0.7	0.3	0.3	0.7	0.3	0.7
15	0.5	0.5	0.9	0.1	0.9	0.1	0.7	0.3	0.3	0.7	0.3	0.7
16	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.5	0.5
17	0.7	0.3	0.9	0.1	0.9	0.1	0.9	0.1	0.3	0.7	0.3	0.7
18	0.5	0.5	0.9	0.1	0.5	0.5	0.7	0.3	0.7	0.3	0.5	0.5
19	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1
20	0.9	0.1	0.9	0.1	0.9	0.1	0.5	0.5	0.5	0.5	0.5	0.5
21	0.5	0.5	0.7	0.3	0.7	0.3	0.9	0.1	0.9	0.1	0.5	0.5
22	0.7	0.3	0.5	0.5	0.7	0.3	0.7	0.3	0.5	0.5	0.3	0.7

Linguistic for Sub Criteria 1 Main Criteria 1

	O	P	Q	R	S	T	U	V	W	X	Y	Z
10	Enhances speaking, listening, reading, and writing skills											
11	LEARNING MANAGEMENT SYSTEM (MLS)		AI-BASED WRITING ASSISTANTS		GAMIFICATION & QUIZZING PLATFORMS		MOBILE LANGUAGE LEARNING APPS		VIRTUAL & AUGMENTED REALITY TOOLS		AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	
12	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship
13	0.5	0.5	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3
14	0.7	0.3	0.5	0.5	0.9	0.1	0.7	0.3	0.5	0.5	0.5	0.5
15	0.7	0.3	0.7	0.3	0.9	0.1	0.7	0.3	0.7	0.3	0.5	0.5
16	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.5	0.5
17	0.7	0.3	0.9	0.1	0.9	0.1	0.9	0.1	0.3	0.7	0.3	0.7
18	0.5	0.5	0.5	0.5	0.5	0.5	0.7	0.3	0.7	0.3	0.5	0.5
19	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1
20	0.7	0.3	0.9	0.1	0.9	0.1	0.7	0.3	0.7	0.3	0.7	0.3
21	0.5	0.5	0.9	0.1	0.7	0.3	0.9	0.1	0.9	0.1	0.7	0.3
22	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.5	0.5	0.3	0.7

Linguistic for Sub Criteria 2 Main Criteria 1

	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL
10	Provides personalized learning experiences											
11	LEARNING MANAGEMENT SYSTEM (MLS)		AI-BASED WRITING ASSISTANTS		GAMIFICATION & QUIZZING PLATFORMS		MOBILE LANGUAGE LEARNING APPS		VIRTUAL & AUGMENTED REALITY TOOLS		AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	
12	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship
13	0.5	0.5	0.9	0.1	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3
14	0.7	0.3	0.3	0.7	0.9	0.1	0.7	0.3	0.5	0.5	0.5	0.5
15	0.7	0.3	0.9	0.1	0.7	0.3	0.7	0.3	0.7	0.3	0.9	0.1
16	0.7	0.3	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
17	0.7	0.3	0.7	0.3	0.9	0.1	0.9	0.1	0.5	0.5	0.5	0.5
18	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.7	0.3	0.9	0.1
19	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1
20	0.9	0.1	0.9	0.1	0.9	0.1	0.7	0.3	0.7	0.3	0.7	0.3
21	0.7	0.3	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1
22	0.9	0.1	0.7	0.3	0.9	0.1	0.5	0.5	0.5	0.5	0.3	0.7

Linguistic for Sub Criteria 3 Main Criteria 1

Sub Criteria for Main Criteria 2

	AM	AN	AO	AP	AQ	AR	AS	AT	AU	AV	AW	AX
10	Encourages active student participation											
11	LEARNING MANAGEMENT SYSTEM (MLS)		AI-BASED WRITING ASSISTANTS		GAMIFICATION & QUIZZING PLATFORMS		MOBILE LANGUAGE LEARNING APPS		VIRTUAL & AUGMENTED REALITY TOOLS		AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	
12	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship
13	0.5	0.5	0.9	0.1	0.9	0.1	0.7	0.3	0.9	0.1	0.7	0.3
14	0.3	0.7	0.5	0.5	0.9	0.1	0.7	0.3	0.5	0.5	0.5	0.5
15	0.3	0.7	0.5	0.5	0.9	0.1	0.5	0.5	0.3	0.7	0.7	0.3
16	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.5	0.5
17	0.5	0.5	0.5	0.5	0.9	0.1	0.9	0.1	0.5	0.5	0.5	0.5
18	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1
19	0.7	0.3	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.7	0.3
20	0.9	0.1	0.7	0.3	0.9	0.1	0.7	0.3	0.7	0.3	0.7	0.3
21	0.5	0.5	0.5	0.5	0.9	0.1	0.9	0.1	0.5	0.5	0.5	0.5
22	0.9	0.1	0.5	0.5	0.9	0.1	0.5	0.5	0.3	0.7	0.5	0.5

Linguistic for Sub Criteria 1 Main Criteria 2

	AY	AZ	BA	BB	BC	BD	BE	BF	BG	BH	BI	BJ
10	Increases motivation to learn English											
11	LEARNING MANAGEMENT SYSTEM (MLS)		AI-BASED WRITING ASSISTANTS		GAMIFICATION & QUIZZING PLATFORMS		MOBILE LANGUAGE LEARNING APPS		VIRTUAL & AUGMENTED REALITY TOOLS		AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	
12	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship
13	0.9	0.1	0.9	0.1	0.9	0.1	0.7	0.3	0.7	0.3	0.7	0.3
14	0.3	0.7	0.9	0.1	0.9	0.1	0.7	0.3	0.5	0.5	0.5	0.5
15	0.3	0.7	0.7	0.3	0.9	0.1	0.5	0.5	0.5	0.5	0.5	0.5
16	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.5	0.5
17	0.7	0.3	0.3	0.7	0.9	0.1	0.9	0.1	0.5	0.5	0.5	0.5
18	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1
19	0.5	0.5	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.5	0.5
20	0.9	0.1	0.7	0.3	0.9	0.1	0.7	0.3	0.7	0.3	0.7	0.3
21	0.5	0.5	0.7	0.3	0.9	0.1	0.9	0.1	0.7	0.3	0.5	0.5
22	0.9	0.1	0.5	0.5	0.9	0.1	0.5	0.5	0.3	0.7	0.5	0.5

Linguistic for Sub Criteria 2 Main Criteria 2

	BK	BL	BM	BN	BO	BP	BQ	BR	BS	BT	BU	BV
10	Provides interactive and gamified learning experiences											
11	LEARNING MANAGEMENT SYSTEM (MLS)		AI-BASED WRITING ASSISTANTS		GAMIFICATION & QUIZZING PLATFORMS		MOBILE LANGUAGE LEARNING APPS		VIRTUAL & AUGMENTED REALITY TOOLS		AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	
12	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship
13	0.9	0.1	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3
14	0.3	0.7	0.5	0.5	0.9	0.1	0.9	0.1	0.5	0.5	0.5	0.5
15	0.7	0.3	0.7	0.3	0.9	0.1	0.5	0.5	0.5	0.5	0.7	0.3
16	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.5	0.5
17	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.5	0.5	0.5	0.5
18	0.7	0.3	0.7	0.3	0.9	0.1	0.9	0.1	0.9	0.1	0.5	0.5
19	0.5	0.5	0.7	0.3	0.9	0.1	0.9	0.1	0.9	0.1	0.5	0.5
20	0.7	0.3	0.9	0.1	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3
21	0.5	0.5	0.5	0.5	0.9	0.1	0.9	0.1	0.7	0.3	0.5	0.5
22	0.9	0.1	0.5	0.5	0.9	0.1	0.5	0.5	0.3	0.7	0.5	0.5

Linguistic for Sub Criteria 3 Main Criteria 2

Sub Criteria for Main Criteria 3

	BW	BX	BY	BZ	CA	CB	CC	CD	CE	CF	CG	CH
10	Easy to navigate and use											
11	LEARNING MANAGEMENT SYSTEM (MLS)		AI-BASED WRITING ASSISTANTS		GAMIFICATION & QUIZZING PLATFORMS		MOBILE LANGUAGE LEARNING APPS		VIRTUAL & AUGMENTED REALITY TOOLS		AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	
12	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship
13	0.5	0.5	0.7	0.3	0.7	0.3	0.5	0.5	0.7	0.3	0.7	0.3
14	0.9	0.1	0.7	0.3	0.9	0.1	0.9	0.1	0.5	0.5	0.5	0.5
15	0.7	0.3	0.7	0.3	0.9	0.1	0.5	0.5	0.5	0.5	0.7	0.3
16	0.7	0.3	0.7	0.3	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
17	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.5	0.5	0.5	0.5
18	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3
19	0.5	0.5	0.7	0.3	0.7	0.3	0.7	0.3	0.5	0.5	0.9	0.1
20	0.9	0.1	0.7	0.3	0.5	0.5	0.9	0.1	0.5	0.5	0.5	0.5
21	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3
22	0.7	0.3	0.3	0.7	0.5	0.5	0.5	0.5	0.3	0.7	0.3	0.7

Linguistic for Sub Criteria 1 Main Criteria 3

	CI	CJ	CK	CL	CM	CN	CO	CP	CQ	CR	CS	CT
10	Accessible across multiple devices (mobile, tablet, computer)											
11	LEARNING MANAGEMENT SYSTEM (MLS)		AI-BASED WRITING ASSISTANTS		GAMIFICATION & QUIZZING PLATFORMS		MOBILE LANGUAGE LEARNING APPS		VIRTUAL & AUGMENTED REALITY TOOLS		AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	
12	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship
13	0.7	0.3	0.7	0.3	0.7	0.3	0.9	0.1	0.5	0.5	0.9	0.1
14	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.5	0.5	0.5	0.5
15	0.9	0.1	0.7	0.3	0.9	0.1	0.9	0.1	0.5	0.5	0.9	0.1
16	0.7	0.3	0.7	0.3	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
17	0.1	0.9	0.9	0.1	0.1	0.9	0.9	0.1	0.5	0.5	0.5	0.5
18	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1
19	0.5	0.5	0.7	0.3	0.7	0.3	0.7	0.3	0.5	0.5	0.7	0.3
20	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.3	0.7	0.7	0.3
21	0.7	0.3	0.7	0.3	0.7	0.3	0.5	0.5	0.7	0.3	0.5	0.5
22	0.7	0.3	0.3	0.7	0.5	0.5	0.5	0.5	0.3	0.7	0.3	0.7

Linguistic for Sub Criteria 2 Main Criteria 3

	CU	CV	CW	CX	CY	CZ	DA	DB	DC	DD	DE	DF
10	Available for students with disabilities (e.g., text-to-speech, voice recognition)											
11	LEARNING MANAGEMENT SYSTEM (MLS)		AI-BASED WRITING ASSISTANTS		GAMIFICATION & QUIZZING PLATFORMS		MOBILE LANGUAGE LEARNING APPS		VIRTUAL & AUGMENTED REALITY TOOLS		AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	
12	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship
13	0.9	0.1	0.9	0.1	0.9	0.1	0.7	0.3	0.9	0.1	0.7	0.3
14	0.3	0.7	0.7	0.3	0.9	0.1	0.7	0.3	0.5	0.5	0.5	0.5
15	0.3	0.7	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.7	0.3
16	0.5	0.5	0.7	0.3	0.5	0.5	0.7	0.3	0.5	0.5	0.5	0.5
17	0.3	0.7	0.9	0.1	0.7	0.3	0.7	0.3	0.5	0.5	0.5	0.5
18	0.9	0.1	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3
19	0.5	0.5	0.7	0.3	0.7	0.3	0.5	0.5	0.5	0.5	0.3	0.7
20	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.3	0.7	0.3	0.7
21	0.5	0.5	0.5	0.5	0.7	0.3	0.5	0.5	0.5	0.5	0.5	0.5
22	0.3	0.7	0.3	0.7	0.5	0.5	0.5	0.5	0.3	0.7	0.3	0.7

Linguistic for Sub Criteria 3 Main Criteria 3

Sub Criteria for Main Criteria 4

	DG	DH	DI	DJ	DK	DL	DM	DN	DO	DP	DQ	DR
10	Includes AI-based feedback and assessment											
11	LEARNING MANAGEMENT SYSTEM (MLS)		AI-BASED WRITING ASSISTANTS		GAMIFICATION & QUIZZING PLATFORMS		MOBILE LANGUAGE LEARNING APPS		VIRTUAL & AUGMENTED REALITY TOOLS		AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	
12	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship
13	0.5	0.5	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.5	0.5
14	0.7	0.3	0.9	0.1	0.3	0.7	0.5	0.5	0.5	0.5	0.5	0.5
15	0.3	0.7	0.9	0.1	0.5	0.5	0.7	0.3	0.9	0.1	0.9	0.1
16	0.3	0.7	0.7	0.3	0.7	0.3	0.7	0.3	0.5	0.5	0.5	0.5
17	0.7	0.3	0.9	0.1	0.9	0.1	0.9	0.1	0.5	0.5	0.5	0.5
18	0.5	0.5	0.9	0.1	0.9	0.1	0.9	0.1	0.7	0.3	0.7	0.3
19	0.7	0.3	0.9	0.1	0.7	0.3	0.7	0.3	0.9	0.1	0.7	0.3
20	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.5	0.5	0.5	0.5
21	0.5	0.5	0.9	0.1	0.5	0.5	0.7	0.3	0.5	0.5	0.3	0.7
22	0.7	0.3	0.5	0.5	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3

Linguistic for Sub Criteria 1 Main Criteria 4

	DS	DT	DU	DV	DW	DX	DY	DZ	EA	EB	EC	ED
10	Provides real-time feedback on language usage											
11	LEARNING MANAGEMENT SYSTEM (MLS)		AI-BASED WRITING ASSISTANTS		GAMIFICATION & QUIZZING PLATFORMS		MOBILE LANGUAGE LEARNING APPS		VIRTUAL & AUGMENTED REALITY TOOLS		AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	
12	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship
13	0.7	0.3	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.5	0.5
14	0.7	0.3	0.9	0.1	0.7	0.3	0.9	0.1	0.9	0.1	0.5	0.5
15	0.5	0.5	0.9	0.1	0.7	0.3	0.7	0.3	0.9	0.1	0.9	0.1
16	0.5	0.5	0.7	0.3	0.7	0.3	0.7	0.3	0.5	0.5	0.5	0.5
17	0.9	0.1	0.3	0.7	0.7	0.3	0.7	0.3	0.5	0.5	0.5	0.5
18	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.7	0.3	0.7	0.3
19	0.7	0.3	0.9	0.1	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3
20	0.7	0.3	0.9	0.1	0.9	0.1	0.9	0.1	0.5	0.5	0.5	0.5
21	0.5	0.5	0.9	0.1	0.7	0.3	0.7	0.3	0.5	0.5	0.5	0.5
22	0.7	0.3	0.5	0.5	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3

Linguistic for Sub Criteria 2 Main Criteria 4

	EE	EF	EG	EH	EI	EJ	EK	EL	EM	EN	EO	EP
10	Supports collaborative learning (e.g., group projects, discussion forums)											
11	LEARNING MANAGEMENT SYSTEM (MLS)		AI-BASED WRITING ASSISTANTS		GAMIFICATION & QUIZZING PLATFORMS		MOBILE LANGUAGE LEARNING APPS		VIRTUAL & AUGMENTED REALITY TOOLS		AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	
12	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship
13	0.9	0.1	0.7	0.3	0.9	0.1	0.9	0.1	0.5	0.5	0.5	0.5
14	0.9	0.1	0.3	0.7	0.7	0.3	0.9	0.1	0.5	0.5	0.5	0.5
15	0.5	0.5	0.9	0.1	0.7	0.3	0.7	0.3	0.7	0.3	0.9	0.1
16	0.9	0.1	0.7	0.3	0.7	0.3	0.5	0.5	0.5	0.5	0.5	0.5
17	0.9	0.1	0.7	0.3	0.7	0.3	0.7	0.3	0.5	0.5	0.5	0.5
18	0.9	0.1	0.7	0.3	0.7	0.3	0.9	0.1	0.7	0.3	0.7	0.3
19	0.7	0.3	0.9	0.1	0.7	0.3	0.7	0.3	0.9	0.1	0.7	0.3
20	0.5	0.5	0.9	0.1	0.9	0.1	0.9	0.1	0.5	0.5	0.5	0.5
21	0.9	0.1	0.9	0.1	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
22	0.7	0.3	0.5	0.5	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3

Linguistic for Sub Criteria 3 Main Criteria 4

Sub Criteria for Main Criteria 5

	EQ	ER	ES	ET	EU	EV	EW	EX	EY	EZ	FA	FB
10	Runs smoothly without crashes or technical issues											
11	LEARNING MANAGEMENT SYSTEM (MLS)		AI-BASED WRITING ASSISTANTS		GAMIFICATION & QUIZZING PLATFORMS		MOBILE LANGUAGE LEARNING APPS		VIRTUAL & AUGMENTED REALITY TOOLS		AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	
12	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship
13	0.7	0.3	0.9	0.1	0.5	0.5	0.7	0.3	0.3	0.7	0.7	0.3
14	0.3	0.7	0.7	0.3	0.7	0.3	0.7	0.3	0.5	0.5	0.5	0.5
15	0.9	0.1	0.9	0.1	0.9	0.1	0.5	0.5	0.7	0.3	0.7	0.3
16	0.5	0.5	0.7	0.3	0.5	0.5	0.5	0.5	0.7	0.3	0.5	0.5
17	0.5	0.5	0.3	0.7	0.5	0.5	0.3	0.7	0.3	0.7	0.3	0.7
18	0.5	0.5	0.5	0.5	0.9	0.1	0.5	0.5	0.5	0.5	0.5	0.5
19	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.7	0.3	0.5	0.5
20	0.7	0.3	0.9	0.1	0.9	0.1	0.9	0.1	0.5	0.5	0.5	0.5
21	0.3	0.7	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
22	0.7	0.3	0.5	0.5	0.7	0.3	0.5	0.5	0.3	0.7	0.5	0.5

Linguistic for Sub Criteria 1 Main Criteria 5

	FC	FD	FE	FF	FG	FH	FI	FJ	FK	FL	FM	FN
10	Ensures data privacy and security											
11	LEARNING MANAGEMENT SYSTEM (MLS)		AI-BASED WRITING ASSISTANTS		GAMIFICATION & QUIZZING PLATFORMS		MOBILE LANGUAGE LEARNING APPS		VIRTUAL & AUGMENTED REALITY TOOLS		AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	
12	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship
13	0.7	0.3	0.5	0.5	0.5	0.5	0.7	0.3	0.3	0.7	0.7	0.3
14	0.3	0.7	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
15	0.9	0.1	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
16	0.5	0.5	0.7	0.3	0.5	0.5	0.5	0.5	0.7	0.3	0.5	0.5
17	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3
18	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
19	0.7	0.3	0.5	0.5	0.7	0.3	0.7	0.3	0.7	0.3	0.5	0.5
20	0.7	0.3	0.9	0.1	0.9	0.1	0.9	0.1	0.5	0.5	0.5	0.5
21	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
22	0.5	0.5	0.5	0.5	0.7	0.3	0.5	0.5	0.3	0.7	0.3	0.7

Linguistic for Sub Criteria 2 Main Criteria 5

	FO	FP	FQ	FR	FS	FT	FU	FV	FW	FX	FY	FZ
10	Offers strong technical support and troubleshooting											
11	LEARNING MANAGEMENT SYSTEM (MLS)		AI-BASED WRITING ASSISTANTS		GAMIFICATION & QUIZZING PLATFORMS		MOBILE LANGUAGE LEARNING APPS		VIRTUAL & AUGMENTED REALITY TOOLS		AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	
12	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship
13	0.5	0.5	0.3	0.7	0.5	0.5	0.7	0.3	0.3	0.7	0.7	0.3
14	0.3	0.7	0.3	0.7	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
15	0.7	0.3	0.9	0.1	0.7	0.3	0.5	0.5	0.5	0.5	0.5	0.5
16	0.5	0.5	0.7	0.3	0.5	0.5	0.5	0.5	0.7	0.3	0.5	0.5
17	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.5	0.5	0.5	0.5
18	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
19	0.7	0.3	0.5	0.5	0.7	0.3	0.7	0.3	0.7	0.3	0.5	0.5
20	0.7	0.3	0.9	0.1	0.9	0.1	0.9	0.1	0.5	0.5	0.5	0.5
21	0.7	0.3	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
22	0.5	0.5	0.5	0.5	0.5	0.5	0.3	0.7	0.3	0.7	0.5	0.5

Linguistic for Sub Criteria 3 Main Criteria 5

Sub Criteria for Main Criteria 6

	GA	GB	GC	GD	GE	GF	GG	GH	GI	GJ	GK	GL
10	Provides good value for money											
11	LEARNING MANAGEMENT SYSTEM (MLS)		AI-BASED WRITING ASSISTANTS		GAMIFICATION & QUIZZING PLATFORMS		MOBILE LANGUAGE LEARNING APPS		VIRTUAL & AUGMENTED REALITY TOOLS		AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	
12	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship
13	0.3	0.7	0.7	0.3	0.5	0.5	0.7	0.3	0.7	0.3	0.7	0.3
14	0.9	0.1	0.7	0.3	0.7	0.3	0.7	0.3	0.5	0.5	0.5	0.5
15	0.9	0.1	0.9	0.1	0.9	0.1	0.7	0.3	0.5	0.5	0.5	0.5
16	0.7	0.3	0.7	0.3	0.7	0.3	0.5	0.5	0.7	0.3	0.5	0.5
17	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.5	0.5	0.1	0.9
18	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3
19	0.7	0.3	0.9	0.1	0.9	0.1	0.5	0.5	0.3	0.7	0.3	0.7
20	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.5	0.5	0.7	0.3
21	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3
22	0.5	0.5	0.3	0.7	0.3	0.7	0.5	0.5	0.3	0.7	0.3	0.7

Linguistic for Sub Criteria 1 Main Criteria 6

	GM	GN	GO	GP	GQ	GR	GS	GT	GU	GV	GW	GX
10	Affordable for students and institutions											
11	LEARNING MANAGEMENT SYSTEM (MLS)		AI-BASED WRITING ASSISTANTS		GAMIFICATION & QUIZZING PLATFORMS		MOBILE LANGUAGE LEARNING APPS		VIRTUAL & AUGMENTED REALITY TOOLS		AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	
12	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship
13	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.5	0.5	0.7	0.3
14	0.9	0.1	0.3	0.7	0.5	0.5	0.3	0.7	0.5	0.5	0.5	0.5
15	0.9	0.1	0.7	0.3	0.9	0.1	0.7	0.3	0.5	0.5	0.5	0.5
16	0.7	0.3	0.7	0.3	0.7	0.3	0.5	0.5	0.7	0.3	0.5	0.5
17	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.5	0.5	0.1	0.9
18	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
19	0.7	0.3	0.9	0.1	0.9	0.1	0.5	0.5	0.3	0.7	0.5	0.5
20	0.9	0.1	0.5	0.5	0.9	0.1	0.7	0.3	0.3	0.7	0.7	0.3
21	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3
22	0.5	0.5	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.7

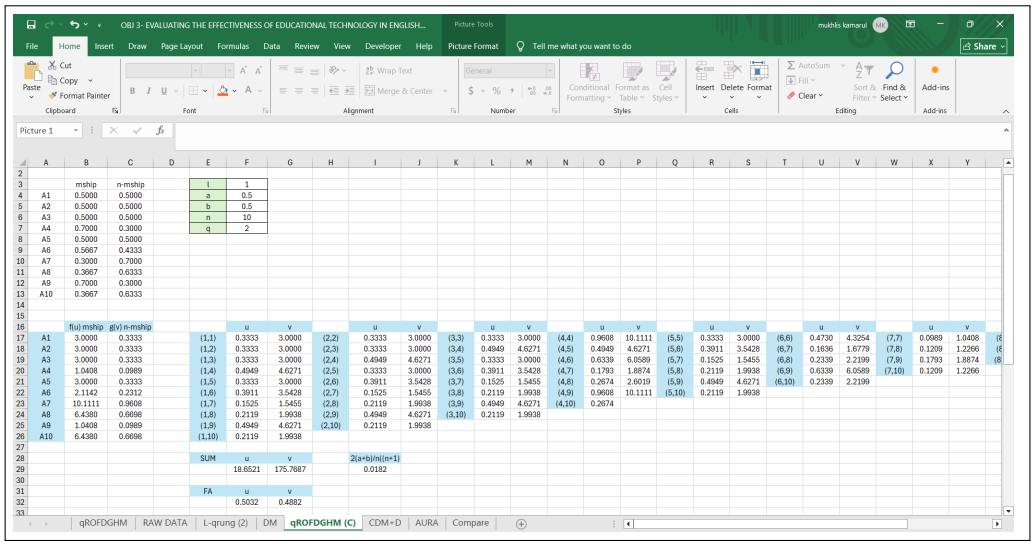
Linguistic for Sub Criteria 2 Main Criteria 6

	GY	GZ	HA	HB	HC	HD	HE	HF	HG	HH	HI	HJ
10	Offers sufficient free features for quality learning											
11	LEARNING MANAGEMENT SYSTEM (MLS)		AI-BASED WRITING ASSISTANTS		GAMIFICATION & QUIZZING PLATFORMS		MOBILE LANGUAGE LEARNING APPS		VIRTUAL & AUGMENTED REALITY TOOLS		AUTOMATED SPEECH RECOGNITION (ASR) TOOLS	
12	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship	mship	n-mship
13	0.7	0.3	0.9	0.1	0.7	0.3	0.7	0.3	0.3	0.7	0.9	0.1
14	0.9	0.1	0.3	0.7	0.5	0.5	0.3	0.7	0.5	0.5	0.5	0.5
15	0.9	0.1	0.7	0.3	0.7	0.3	0.7	0.3	0.5	0.5	0.5	0.5
16	0.7	0.3	0.7	0.3	0.7	0.3	0.5	0.5	0.7	0.3	0.5	0.5
17	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.5	0.5	0.1	0.9
18	0.7	0.3	0.7	0.3	0.7	0.3	0.5	0.5	0.5	0.5	0.7	0.3
19	0.7	0.3	0.7	0.3	0.9	0.1	0.5	0.5	0.3	0.7	0.5	0.5
20	0.9	0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.3	0.7	0.7	0.3
21	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.3
22	0.5	0.5	0.3	0.7	0.3	0.7	0.3	0.7	0.5	0.5	0.3	0.7

Linguistic for Sub Criteria 3 Main Criteria 6

APPENDIX C
AGGREGATION OPERATOR

This is the calculation for Aggregation Operator



| | | |

APPENDIX D
THE CODING

This is the Coding code for AURA Software.

Listing D.1: AURA (Streamlit) code

```
import streamlit as st
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from io import BytesIO

class AURA:
    def __init__(self, matrix, types, weights=None, alpha=0.5, p=2):
        self.matrix = np.array(matrix, dtype=float)
        self.types = types
        self.n_criteria = self.matrix.shape[1]
        self.weights = np.array(weights if weights else [1/self.n_criteria]*self.n_criteria,
        dtype=float)
        self.alpha = alpha
        self.p = p

    def normalize(self):
        m, n = self.matrix.shape
        norm = np.zeros_like(self.matrix, dtype=float)
        for j in range(n):
            col = self.matrix[:, j]
            h = np.max(col) - np.min(col)
            if h == 0:
                norm[:, j] = 1.0
                continue
            if self.types[j] == 'benefit':
                k = np.max(col)
            elif self.types[j] == 'cost':
                k = np.min(col)
            elif isinstance(self.types[j], (int, float)):
                k = float(self.types[j])  # target value
            else:
                raise ValueError(f"Invalid type for criterion {j}")
            norm[:, j] = 1 - np.abs(col - k) / h
        self.norm_matrix = norm

    def weighted(self):
        self.V = self.norm_matrix * self.weights
```

```
    def benchmarks(self):
        self.PIS = np.max(self.V, axis=0)
        self.NIS = np.min(self.V, axis=0)
        self.AVG = np.mean(self.V, axis=0)

    def distance(self, ref):
        return np.power(np.sum(np.abs(self.V - ref)**self.p, axis=1), 1/self.p)

    def correct(self, d):
        sigma = np.max(d) - np.min(d)
        return d + sigma * d**2

    def score(self):
        d_pos = self.correct(self.distance(self.PIS))
        d_neg = self.correct(self.distance(self.NIS))
        d_avg = self.correct(self.distance(self.AVG))
        return (self.alpha * (d_pos - d_neg) + (1 - self.alpha) * d_avg) / 2

    def rank(self):
        self.normalize(); self.weighted(); self.benchmarks()
        scores = self.score()
        return scores, np.argsort(scores) + 1

st.set_page_config(page_title="AURA-MCDM Web App", layout="wide")
st.title("AURA (Adaptive Utility Ranking Algorithm) for MCDM")

uploaded_file = st.file_uploader("Upload your Excel file (headers row 1, weights row 2)",
type=["xlsx"])
alpha = st.slider("Set Alpha (\texorpdfstring{$\alpha$}{alpha}): Trade-off between ideality and
average", 0.0, 1.0, 0.5, step=0.05)
p = st.selectbox("Select Distance Metric (p)", options=[1, 2], index=1)

def generate_excel_report(alternatives, criteria, weights, norm_matrix, weighted_matrix, pis,
nis, avg, distances, final_df):
    output = BytesIO()
    with pd.ExcelWriter(output, engine="xlsxwriter") as writer:
        df_raw = pd.DataFrame([[""] + criteria, [""] + weights] + [[a] + list(r) for a, r in
        zip(alternatives, weighted_matrix)])
        df_raw.to_excel(writer, sheet_name="Input Data", index=False, header=False)
        pd.DataFrame(norm_matrix, columns=criteria).to_excel(writer, sheet_name="Normalized
        Matrix", index=False)
        pd.DataFrame(weighted_matrix, columns=criteria).to_excel(writer, sheet_name="Weighted
        Matrix", index=False)
        pd.DataFrame({"PIS": pis, "NIS": nis, "AVG": avg}).to_excel(writer, sheet_name=
        "Benchmarks", index=False)
        pd.DataFrame(distances).to_excel(writer, sheet_name="Distances", index=False)
        final_df.to_excel(writer, sheet_name="Final Ranking", index=False)
    output.seek(0)
```

```
    return output

if uploaded_file:
    df_raw = pd.read_excel(uploaded_file, header=None)
    st.subheader("Raw Uploaded Data")
    st.dataframe(df_raw)

    try:
        criteria_columns = df_raw.iloc[0, 1:].tolist()
        weights = df_raw.iloc[1, 1:].astype(float).tolist()
        alternative_names = df_raw.iloc[2:, 0].tolist()
        matrix = df_raw.iloc[2:, 1:].astype(float).values

        st.subheader("Decision Matrix")
        df_matrix_display = pd.DataFrame(matrix, columns=criteria_columns)
        df_matrix_display.insert(0, "Alternative", alternative_names)
        st.dataframe(df_matrix_display)

        st.subheader("Define Criterion Type and (if applicable) Target Value")
        manual_types = []
        target_values = {}

        for i, col in enumerate(criteria_columns):
            c1, c2 = st.columns([2, 1])
            with c1:
                ctype = st.selectbox(f"Type for '{col}'", options=["Benefit", "Cost",
                "Target (custom)"], key=f"type_{i}")
            if ctype == "Target (custom)":
                with c2:
                    target = st.number_input(f"Target for '{col}'", key=f"target_{i}")
                    manual_types.append(target)
                    target_values[col] = target
            else:
                manual_types.append(ctype.lower())

        st.write("Finalized Criteria Types:")
        for col, t in zip(criteria_columns, manual_types):
            st.write(f"- {col}: {'Target (' + str(t) + ')' if isinstance(t, float) else
            t.capitalize()}")

        if st.button("Run AURA"):
            model = AURA(matrix, manual_types, weights, alpha=alpha, p=p)
            scores, _ = model.rank()

            sorted_indices = np.argsort(scores)
            ranks = np.empty_like(sorted_indices)
            ranks[sorted_indices] = np.arange(1, len(scores) + 1)
```

```
            df_result = pd.DataFrame({"Alternative": alternative_names, "Score": scores, "Rank":
            ranks}).sort_values("Rank").reset_index(drop=True)

            st.subheader("Step-by-Step Results")
            with st.expander("Step 1: Normalized Decision Matrix"):
                st.dataframe(pd.DataFrame(model.norm_matrix, columns=criteria_columns))
            with st.expander("Step 2: Weighted Normalized Matrix"):
                st.dataframe(pd.DataFrame(model.V, columns=criteria_columns))
            with st.expander("Step 3: Benchmark Values"):
                st.write("Positive Ideal (PIS):", model.PIS)
                st.write("Negative Ideal (NIS):", model.NIS)
                st.write("Average Solution (AVG):", model.AVG)
            with st.expander("Step 4: Distances to Benchmarks"):
                dp = model.correct(model.distance(model.PIS))
                dn = model.correct(model.distance(model.NIS))
                da = model.correct(model.distance(model.AVG))
                df_d = pd.DataFrame({"Alternative": alternative_names, "Dist to PIS": dp,
                "Dist to NIS": dn, "Dist to AVG": da})
                st.dataframe(df_d)

            st.subheader("Final AURA Ranking (Lower Score = Better)")
            st.dataframe(df_result)

            csv = df_result.to_csv(index=False).encode('utf-8')
            st.download_button("Download Result as CSV", csv, "aura_ranking.csv", "text/csv")

            excel_file = generate_excel_report(alternative_names, criteria_columns, weights,
            model.norm_matrix, model.V, model.PIS, model.NIS, model.AVG, df_d, df_result)
            st.download_button("Download Full Report as Excel", excel_file, "aura_full_report
            .xlsx",
            "application/vnd.openxmlformats-officedocument.spreadsheetml.sheet")

            fig, ax = plt.subplots()
            ax.barh(df_result['Alternative'], df_result['Score'])
            ax.invert_yaxis()
            ax.set_xlabel("Score"); ax.set_title("AURA Final Scores (Lower is Better)")
            st.pyplot(fig)

    except Exception as e:
        st.error(f"Failed to process file: {e}")
```

Listing D.2: Report generators

```
import io
import pandas as pd
from reportlab.pdfgen import canvas
from reportlab.lib.pagesizes import letter

def generate_excel_report(df_dict):
```

```
    """Accepts a dict of DataFrames with sheet names, returns Excel bytes."""
    buffer = io.BytesIO()
    with pd.ExcelWriter(buffer, engine="xlsxwriter") as writer:
        for sheet_name, df in df_dict.items():
            df.to_excel(writer, sheet_name=sheet_name, index=False)
    buffer.seek(0)
    return buffer.getvalue()

def generate_pdf_report(df, title="AURA Report"):
    """Creates a simple PDF report from one DataFrame."""
    buffer = io.BytesIO()
    pdf = canvas.Canvas(buffer, pagesize=letter)
    width, height = letter

    pdf.setFont("Helvetica-Bold", 14)
    pdf.drawString(40, height - 50, title)

    pdf.setFont("Helvetica", 10)
    y = height - 80

    for col in df.columns:
        pdf.drawString(40, y, f"{col}")
        y -= 15

    y -= 10
    for _, row in df.iterrows():
        row_str = " | ".join(str(v) for v in row)
        if y < 50:
            pdf.showPage()
            y = height - 50
        pdf.drawString(40, y, row_str)
        y -= 15

    pdf.showPage()
    pdf.save()
    buffer.seek(0)
    return buffer.getvalue()
```

AUTHOR'S PROFILE



Muhammad Mukhlis bin Kamarul Zaman obtained his Bachelor of Science (Hons.) in Management Mathematics from Universiti Teknologi MARA (UiTM) Seremban, Negeri Sembilan in 2024, after completing his Diploma in Mathematical Sciences at UiTM Tapah in 2022. He is currently pursuing a Master of Science (Mathematics) by Research at the Faculty of Computer and Mathematical Sciences (FSKM), Universiti Teknologi MARA (UiTM) Negeri Sembilan. His research focuses on the development of advanced decision-making frameworks using fuzzy logic and mathematical modelling. He has industrial experience in logistics during his internship at Exaira Sdn. Bhd., where he managed daily operational data and gained valuable analytical and organizational skills. His academic interests include applied mathematics, fuzzy systems, decision analysis, and computational intelligence.

LIST OF PUBLICATIONS

Zaman, M. M. K., Rodzi, Z. M., Andu, Y., Shafie, N. A., Sanusi, Z. M., Ghazali, A. W., & Mahyideen, J. M. (2025). Adaptive utility ranking algorithm for evaluating blockchain-enabled microfinance in emerging economies: A new MCDM perspective. *International Journal of Economic Sciences*, 14(1), 123–146.

Zaman, M. M. K., Rodzi, Z. M., Ghazali, A. W., Shafie, N. A., Sanusi, Z. M., & Al-Sharqi, F. (2025). AURA: An adaptive utility ranking algorithm for multi-criteria decision making with Python-based decision support interface. *SoftwareX*, 32, 102395.

Kamarul Zaman, M. M., Rodzi, Z. M., & Andu, Y. (2025). Enhancing MCDM with q-rung orthopair fuzzy Dombi generalized Heronian mean. *Mathematical Sciences and Informatics Journal (MIJ)*, 6(1), 65–81.

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Zaman, M. M. K., Rodzi, Z. M., Andu, Y., Bazlan, A. S., Anam, S., & Al-Sharqi, F. (2026). A q-Rung Orthopair Fuzzy Dombi Generalized Heronian Mean–AURA Decision-Making Framework for Educational Technology Evaluation. In *Proceedings of the International Conference on Intelligent and Fuzzy Systems (INFUS 2026)*. Springer, Lecture Notes in Networks and Systems (Accepted for publication).