UNIVERSITI TEKNOLOGI MARA

COMPARATIVE STUDY OF TRAPEZOIDAL RULE (TR), CIRCULAR-BASED MODIFIED TRAPEZOIDAL RULE (CMTR) AND TRIANGULAR-BASED MODIFIED TRAPEZOIDAL RULE (TMTR) FOR UNDERESTIMATED CURVATURE

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ABSTRACT

This study presents a comparative performance analysis of the standard Trapezoidal Rule (TR), the Triangular-based Modified Trapezoidal Rule (TMTR), and the Circular-based Modified Trapezoidal Rule (CMTR) for numerical integration of functions exhibiting downward curvature (underestimation). The modified techniques (TMTR and CMTR) aim to enhance the traditional linear approximation by incorporating geometric correction terms that compensate for inherent underestimation errors. Using Maple, the methods were evaluated across ten benchmark functions with various categories, utilizing subinterval ranges of n = 12, 24, 48, 96, 192, and 384. The comparison was based on numerical accuracy, percentage relative error, CPU computation time, and performance profile analysis. The results show that all three methods converged towards the exact solution as the number of subintervals increased. CMTR achieved the lowest percentage relative error for many individual test functions, demonstrating high accuracy for suitable function characteristics. However, TMTR provided more consistent performance across the complete set of test functions, maintaining stable and low errors while avoiding the large error variations observed in some CMTR results. Although the modified methods required slightly more computation time than TR, the increase was small compared to the improvement in accuracy. Overall, both TMTR and CMTR are effective alternatives to the classical Trapezoidal Rule, with TMTR offering the best overall balance between accuracy, consistency, and computational efficiency for functions with downward curvature.