UNIVERSITI TEKNOLOGI MARA

MILNE-SIMPSON PREDICTOR-CORRECTOR METHOD FOR SOLVING ORDINARY DIFFERENTIAL EQUATIONS

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ABSTRACT

Ordinary Differential Equations (ODEs) are widely used to model phenomena in engineering, physics, and finance. ODEs could be solved theoretically and numerically. However, for complicated ODE problems the theoretical approach might be difficult, lengthy, and time-consuming. Alternatively, researchers tend to use numerical methods in the form of approximation to find the solution. This research compares the performance of four numerical methods, namely Euler's method, Heun's method, the Fourth-Order Runge-Kutta (RK4) method, and the Milne-Simpson Predictor-Corrector (MSPC) method in solving first-order ODEs. As for comparison, ten ODE problems consisting of Separable, Linear, and Bernoulli differential equations are solved theoretically and numerically using *Maple* software with different step sizes of $h = 0.1$, $h = 0.01$, and $h = 0.001$. The performance of the methods is analysed based on percentage of relative error and CPU time using performance profiles. The results show that RK4 produced the highest accuracy, followed by MSPC-RK4, while Euler's method yields the lowest CPU time but generated the largest errors.