

Research Article

Prototype of Approximating Area by using Newton Forward Divided Difference Interpolation Approach

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Abstract: Newton's Forward Divided Difference in estimating the area of an item, approximation calculation is performed to assist in the construction of a prototype. Nevertheless, there is no particular application that can precisely calculate an object's area to serve as a main guide for the upcoming software model, making development difficult. Plus, there are insufficient resources, such as workforce, money, and experience, to successfully optimise an algorithm of the prototype version of the software, which requires sufficient time to correctly make the application prototyping proceed. There are a pair of answers to these challenges: the Newton Forward Divided Difference Interpolation computation is a numerical analytic technique that can be utilised to help in constructing of a prototype in determining the measurement area of an object. Not only that, but designers can refer to or use any advanced graphic calculator from any available platform, digital or genuine, as an indicator to reference for the software design prototype to be successful. By utilising this existing prototype, many places across Malaysia will be able to gain from the establishment of this software prototype for everyday aims, such as a group of individuals studying mathematics curriculum can benefit from this sort of technological innovation for educational purposes. Furthermore, this potential research software prototype may be useful in allowing any industrial area to experiment with more information and performing better in genuine real-world settings. Therefore, this study will broaden the Newton Forward Divided Difference Interpolation framework to include other numerical analytic methodologies and implementations for the objective of educating and acquiring knowledge.

Keywords: Newton Forward Divided Difference Interpolation; prototype; measuring object's area; numerical analytic; technological innovation; academic purposes.



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1. INTRODUCTION

In this research paper, we intend to concentrate on mathematical analytical approaches for calculating the area of an object. A fruit such as a pear shall be applied as an example in this study assessment. Pears (*genus Pyrus*) display an asymmetrical form, exemplified due to their curvy bottom, broadened edges, and pointed tip. Pears can be found in an array of varieties, notably Asian Pear,

Seckel, Barlett, and many others. Through its various characteristics, not all pears will have identical area according to the differing curves. Thus, an early prototype of a software called **NFD-CALC** will be created to allow users associated with the architectural and mechanical engineering industries for instance, to properly estimate the area of an object. Users can perform the calculations necessary by this framework using the Newton Forward Divided Difference Interpolation rather than manually determining an object's area with the Discrete Least Squares method to accommodate educational reasons as well as learning practises with the existence of this prospective software prototype design.

2. METHOD & MATERIAL

The lecture materials from the Introduction to Numerical Analysis lessons were the primary source of knowledge that students required to get a greater understanding on creating prototypes of estimating areas. The prototype computations are covered in the sixth chapter of the course, *Newton Divided Difference Interpolation*. Going over this foundational topic is one of the crucial pieces of knowledge required to construct a good sample. Aside from that, the usefulness of Internet access as a networking material applied to obtain additional support in the prototype's estimations is included as well. Apart from that, the two-dimensional (2D) representation of a pear is used for illustrating into graphed paper. Hence, the coordinate of a drawn pear is obtained from a graphic image of 2cmX2cm scales. The NFD-CALC software prototyping interface was designed using Canva, a virtual tool for graphic design which enables us to view the visualised sequence of the design prototype.

In terms of the mathematical approach applied to calculate the prototype's region, Newton Forward Divided Difference Interpolation is a method used to determine an object's area independent of its distinctive form. the Newton's Divided Difference is a technique for obtaining an interpolated equation which additionally employs the Finite Divided Difference algorithm (Biswajit Das & Dhritikesh Chakrabarty, 2016). Therefore, Newton Forward Divided Difference Interpolation represents one of the estimation methods that uses polynomial equations $P(x)$, to express a given collection of data points on an array of the variables. Zou, L. et al. in the article state that the mathematical and description and the construction method of curves and surfaces are critical issues in computer-aided geometric design. In terms of mathematical expressions, the Newton Forward Divided Difference Interpolation formula is $P_n(x) = f[x_0] + f[x_0, x_1](x - x_0) + \dots + f[x_0, x_1, \dots, x_n](x - x_0) \dots (x - x_{n-1})$ where the k^{th} divided difference of the function f with respect to $x_i, x_{i+1}, \dots, x_{i+k}$ denoted by $f[x_i, x_{i+1}, \dots, x_{i+k-1}, x_{i+k}]$ is defined by
$$\frac{f[x_{i+1}, x_{i+2}, \dots, x_{i+k}] - f[x_i, x_{i+1}, \dots, x_{i+k-1}]}{x_{i+k} - x_i}.$$

Figure 1 shows the flow and interfaces of NFD-CALC software prototype:

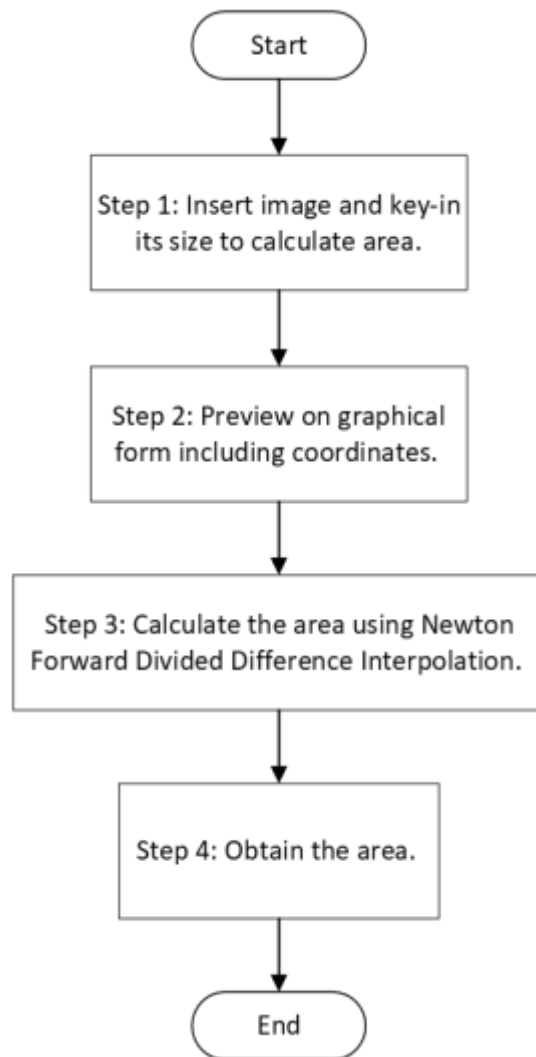


Figure 1. Flowchart of NFD-CALC.

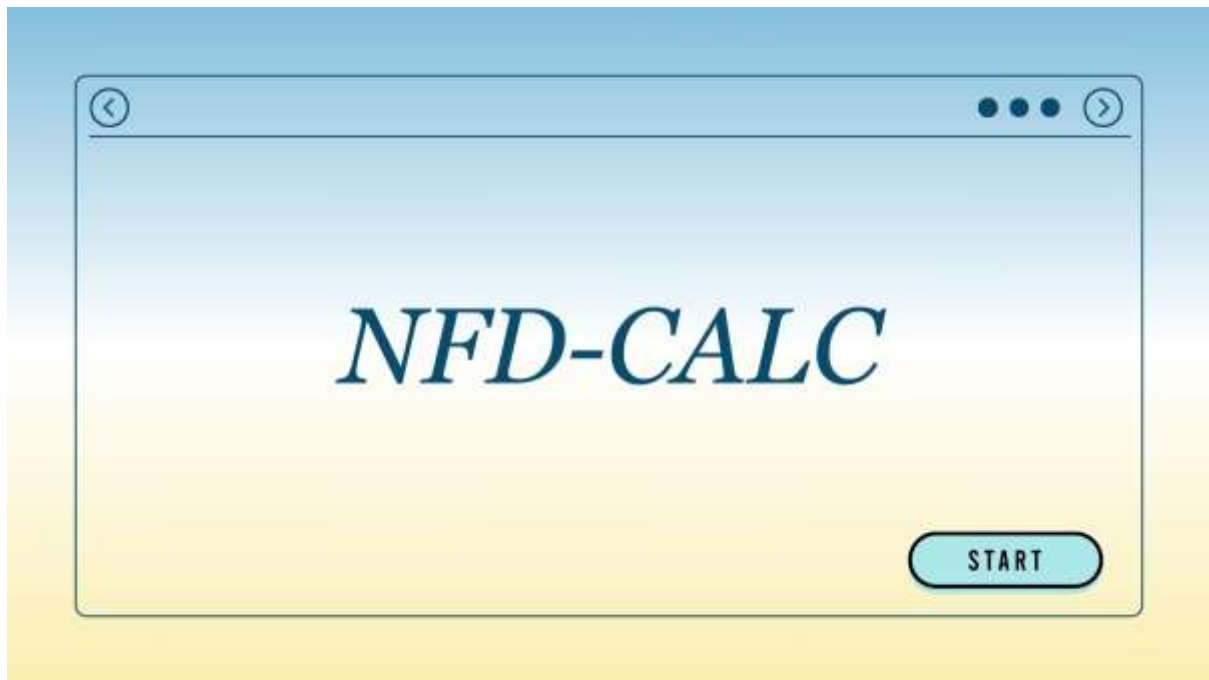


Figure 2. First interface: Press “Start” to begin.

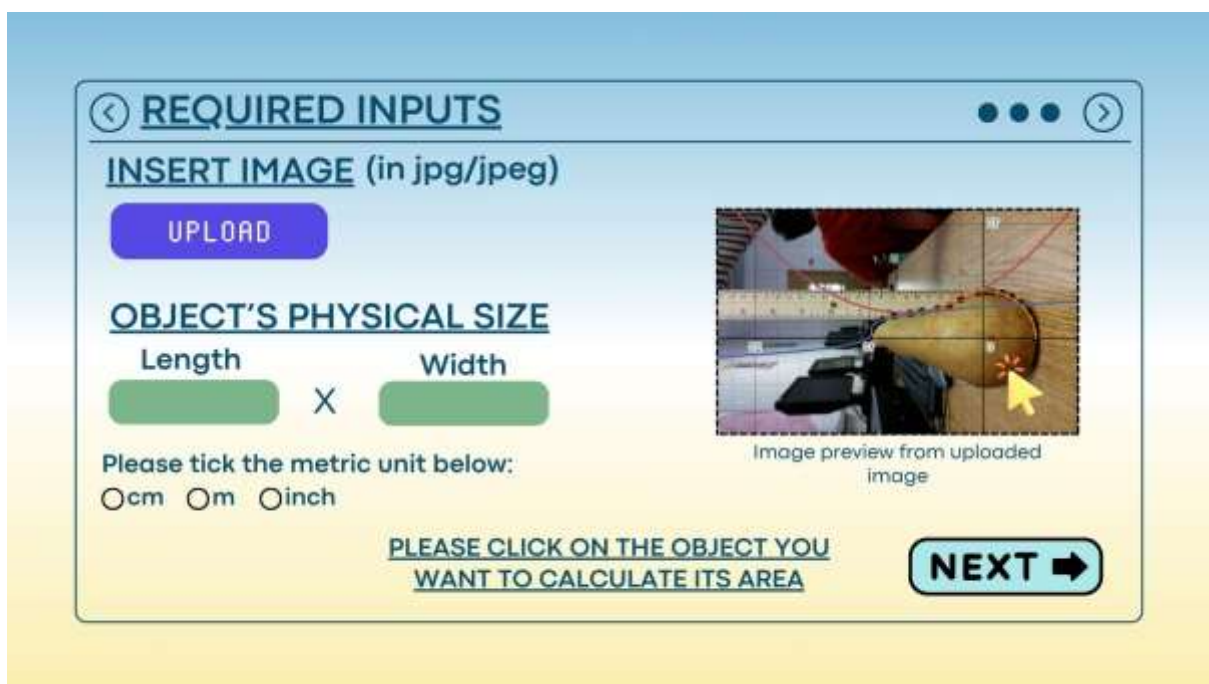


Figure 3. Second interface: User key-in the required inputs, select the metric unit and click on the object to calculate its area then press “Next”.

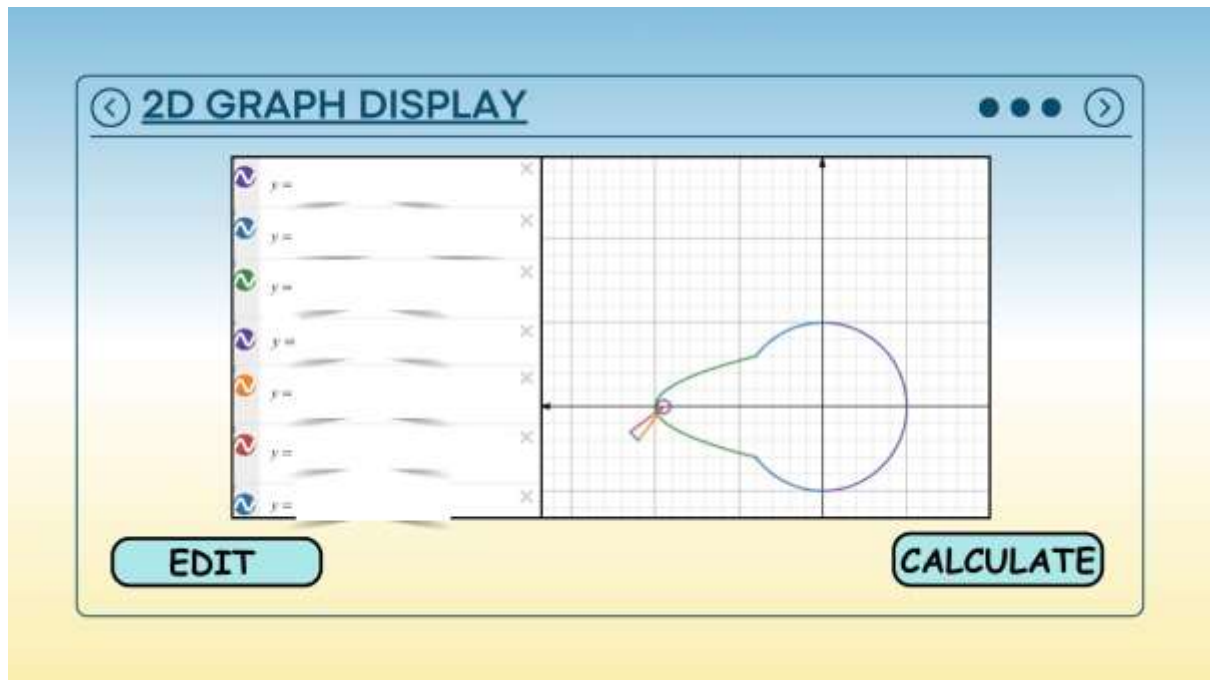


Figure 4. Third interface: Preview on the object in graphical form.

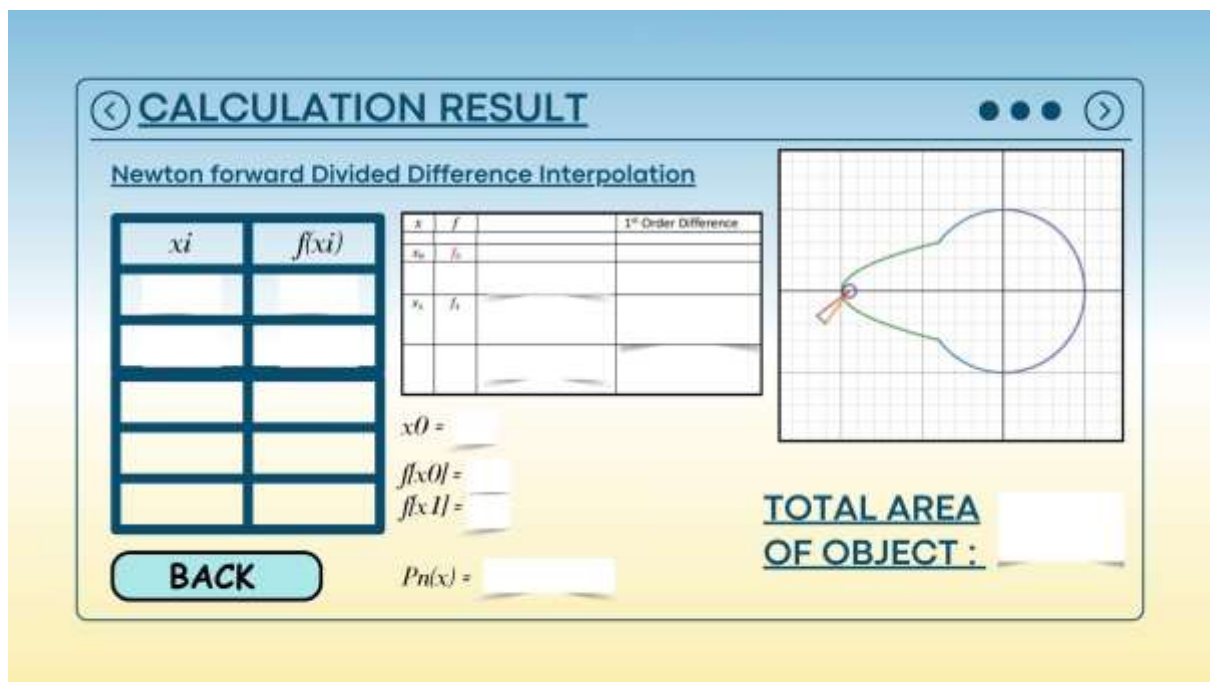


Figure 5. Fourth interface: Calculation result using Newton Forward Divided Difference Interpolation for its area.

3. FINDINGS

3.1 The figures and data of the prototype: A pear.

This section will focus on one object and then employ it as a working example for demonstrating the area computations (using *Newton Forward Divided Difference Interpolation*). The illustration below depicts an image of a pear (**Figure 6**), which will subsequently be drawn onto graph paper (**Figure 7**) to acquire its coordinate values at a scale of 2cmX2cm (refer **Table 1**).



Figure 6. A pear and a half of a pear.

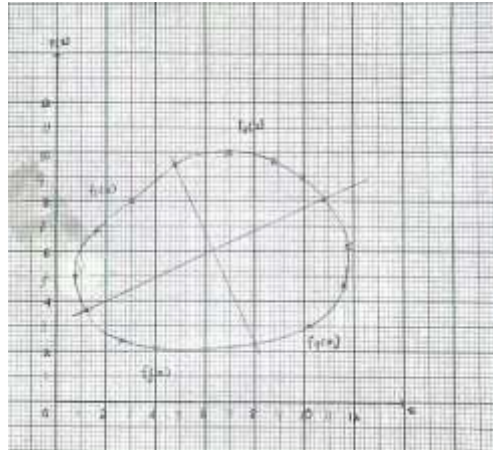


Figure 7. The outline sketched of a half of a pear on a graphing paper.

Table 1. The coordinates for the outline sketched of a half of a pear on a graphing paper.

x	1.2	0.8	1.6	3.0	4.8	2.6	4.0	6.0	8.0
$F(x)$	3.6	5.0	6.8	8.0	9.5	2.4	2.1	2.0	2.3

x	7.0	8.8	10.0	10.8	10.2	11.6	11.8
$F(x)$	10.0	9.6	9.0	8.1	3.0	4.6	6.2

3.2 The calculations of the prototype: Newton Forward Divided Difference Interpolation approach.

The next section of findings will provide the complete computations for calculating the area of a pear through the Newton Forward Divided Difference Interpolation methodology. Before doing so, separated the contour of a half-pear on graph paper into four parts of functions that correspond to the data provided below:

Table 2. The coordinates for the first part sketched of a half of a pear, $f_1(x)$.

x	1.2	0.8	1.6	3.0	4.8
$f_1(x)$	3.6	5.0	6.8	8.0	9.5

Table 3. The coordinates for the second part sketched of a half of a pear, $f_2(x)$.

x	4.8	7.0	8.8	10.0	10.8
$f_2(x)$	9.5	10.0	9.6	9.0	8.1

Table 4. The coordinates for the third part sketched of a half of a pear, $f_3(x)$.

x	10.8	11.8	11.6	10.2	8.0
$f_3(x)$	8.1	6.2	4.6	3.0	2.3

Table 5. The coordinates for the last (fourth) part sketched of a half of a pear, $f_4(x)$.

x	8.0	6.0	4.0	2.6	1.2
$f_4(x)$	2.3	2.0	2.1	2.4	3.6

The calculations following describe the working procedures in estimating the polynomial expression of the form drew of a half of a pear on a piece of graph paper, divided into four parts:

3.2.1 Polynomial equation for region $f_1(x)$.

Table 6. 1 Polynomial equation for region $f_1(x)$..

x	1.2	0.8	1.6	3.0	4.8
$f_1(x)$	3.6	5.0	6.8	8.0	9.5

Table 7. The Finite Divided Difference for $f_1(x)$.

x_i	$f(x_i)$	$f[x_i, x_{i+1}]$	$f[x_i, \dots, x_{i+2}]$	$f[x_i, \dots, x_{i+3}]$	$f[x_i, \dots, x_{i+4}]$
1.2	3.6				
		-3.5000			
0.8	5.0		14.3750		
		2.2500		-8.3378	
1.6	6.8		-0.6331		2.3595
		0.8571		0.1564	
3.0	8.0		-0.0074		
		0.8333			
4.8	9.5				

Let $x_0 = 1.2$

$$f[x_0] = 3.6$$

$$f[x_0, x_1] = -3.5000$$

$$f[x_0, x_1, x_2] = 14.3750$$

$$f[x_0, x_1, x_2, x_3] = -8.3378$$

$$f[x_0, x_1, x_2, x_3, x_4] = 2.3595$$

$$P_n(x) = 3.6 + (-3.5000)(x-1.2) + 14.3750(x-1.2)(x-0.8) + (-8.3378)(x-1.2)(x-0.8)(x-1.6) + 2.3595(x-1.2)(x-0.8)(x-1.6)(x-3.0)$$

$$P_n(x) = 2.3595x^4 - 23.9105x^3 + 79.6892x^2 - 100.006x + 45.2794$$

3.2.2 Polynomial equation for region $f_2(x)$.

Table 8. Polynomial equation for region $f_2(x)$.

x	4.8	7.0	8.8	10.0	10.8
$f_2(x)$	9.5	10.0	9.6	9.0	8.1

Table 9. The Finite Divided Difference for $f_2(x)$.

x_i	$f(x_i)$	$f[x_i, x_{i+1}]$	$f[x_i, \dots, x_{i+2}]$	$f[x_i, \dots, x_{i+3}]$	$f[x_i, \dots, x_{i+4}]$
4.8	9.5				
		0.2273			
7.0	10.0		-0.1124		
		-0.2222		0.0038	
8.8	9.6		-0.0926		-0.0102
		-0.5		-0.0579	
10.0	9.0		-0.3125		
		-1.1250			
10.8	8.1				

Let $x_0 = 4.8$

$$f[x_0] = 9.5$$

$$f[x_0, x_1] = 0.2273$$

$$f[x_0, x_1, x_2] = -0.1124$$

$$f[x_0, x_1, x_2, x_3] = 0.0038$$

$$f[x_0, x_1, x_2, x_3, x_4] = -0.0102$$

$$P_n(x) = 9.5 + 0.2273(x-4.8) + (-0.1124)(x-4.8)(x-7.0) + 0.0038(x-4.8)(x-7.0)(x-8.8) + (-0.0102)(x-4.8)(x-7.0)(x-8.8)(x-10.0)$$

$$P_n(x) = -0.0103x^4 + 0.3190x^3 - 3.7281x^2 + 19.2777x - 26.9463$$

3.2.3 Polynomial equation for region $f_3(x)$.

Table 10. Polynomial equation for region $f_3(x)$.

x	10.8	11.8	11.6	10.2	8.0
$f_3(x)$	8.1	6.2	4.6	3.0	2.3

Table 11. The Finite Divided Difference for $f_3(x)$.

x_i	$f(x_i)$	$f[x_i, x_{i+1}]$	$f[x_i, \dots, x_{i+2}]$	$f[x_i, \dots, x_{i+3}]$	$f[x_i, \dots, x_{i+4}]$
10.8	8.1				
		-1.9000			
11.8	6.2		12.3750		
		8.0000		13.4821	
11.6	4.6		4.2857		4.4337
		1.1429		1.0675	
10.2	3.0		0.2290		
		0.3182			
8.0	2.3				

Let $x_0 = 10.8$

$$f[x_0] = 8.1$$

$$f[x_0, x_1] = -1.9000$$

$$f[x_0, x_1, x_2] = 12.3750$$

$$f[x_0, x_1, x_2, x_3] = 13.4821$$

$$f[x_0, x_1, x_2, x_3, x_4] = 4.4337$$

$$P_n(x) = 8.1 + (-1.9000)(x-10.8) + 12.3750(x-10.8)(x-11.8) + 13.4821(x-10.8)(x-11.8)(x-11.6) + 4.4337(x-10.8)(x-11.8)(x-11.6)(x-10.2)$$

$$P_n(x) = 4.4337x^4 - 183.3742x^3 + 2825.3086x^2 - 19202.4744x + 48529.4834$$

3.2.4 Polynomial equation for region $f_4(x)$.

Table 12. Polynomial equation for region $f_4(x)$.

x	8.0	6.0	4.0	2.6	1.2
$f_4(x)$	2.3	2.0	2.1	2.4	3.6

Table 13. The Finite Divided Difference for $f_4(x)$.

x_i	$f(x_i)$	$f[x_i, x_{i+1}]$	$f[x_i, \dots, x_{i+2}]$	$f[x_i, \dots, x_{i+3}]$	$f[x_i, \dots, x_{i+4}]$
8.0	2.3				
		0.1500			
6.0	2.0		0.0500		
		-0.0500		0.0003	
4.0	2.1		0.0483		0.0056
		-0.2143		-0.0378	
2.6	2.4		0.2296		
		-0.8571			
1.2	3.6				

Let $x_0 = 8.0$

$$f[x_0] = 2.3$$

$$f[x_0, x_1] = 0.1500$$

$$f[x_0, x_1, x_2] = 0.0500$$

$$f[x_0, x_1, x_2, x_3] = 0.0003$$

$$f[x_0, x_1, x_2, x_3, x_4] = 0.0056$$

$$P_n(x) = 2.3 + 0.1500 (x-8.0) + 0.0500 (x-8.0) (x-6.0) + 0.0003 (x-8.0) (x-6.0) (x-4.0) + 0.0056 (x-8.0) (x-6.0) (x-4.0) (x-2.6)$$

$$P_n(x) = 0.0056x^4 - 0.1151x^3 + 0.8891x^2 - 3.1082x + 6.2379$$

3.2.5 Polynomial equation for the shape of a half-pear sketched on graph paper region, $F(x)$.

After deriving the polynomial equations for each section, the entire expression of the shape of a half-pear sketched on graph paper in polynomial notation is:

$$F(x) = 6.7885x^4 - 207.0808x^3 + 2902.1588x^2 - 19286.3109x + 48554.0544$$

4. DISCUSSION

After calculating the equation for each curve by using Newton Forward Divided Difference Interpolation method, all equations are added together to form only one equation. This equation then will be used to determine the area under the graph of an object such as a pear in this study by integrating the equation. Zou, L. et al. in the article state that the interpolant function is straightforward to compute because it simply depends on the values of the function being interpolated and the (partial) divided differences. The formula to calculate the area is as follow:

$$\begin{aligned}
 Area &= \int_a^b f(x) dx \\
 Area &= \int_{0.8}^{11.8} 6.7885x^4 - 207.0808x^3 + 2902.1588x^2 - 19286.3109x + 48554.0544 dx \\
 &= \left[\frac{6.7885x^5}{5} - \frac{207.0808x^4}{4} + \frac{2902.1588x^3}{3} - \frac{19286.3109x^2}{2} + 48554.0544x \right]_{0.8}^{11.8} \\
 &= 126571.2163 - 33146.1656 \\
 &= 93425.0507 \text{ cm}^2
 \end{aligned}$$

5. CONCLUSION

In summary, the **NFD-CALC** software prototype will provide an organised result from a graphic calculation of an object's area without considering its structure through the Newton Forward Divided Difference Interpolation process. The software prototype will additionally function as an inspiration to make further improvements to various types of numerical computation in regard to practical applications problem responses, such as enhancing quantitative analysis methodologies for both educational and practical uses. Hence, this research intends to broaden the Newton Forward Divided Difference Interpolation system to encompass more mathematical analytical methods and applications in future studies with the ultimate objective of improving knowledge and acquiring understanding.

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