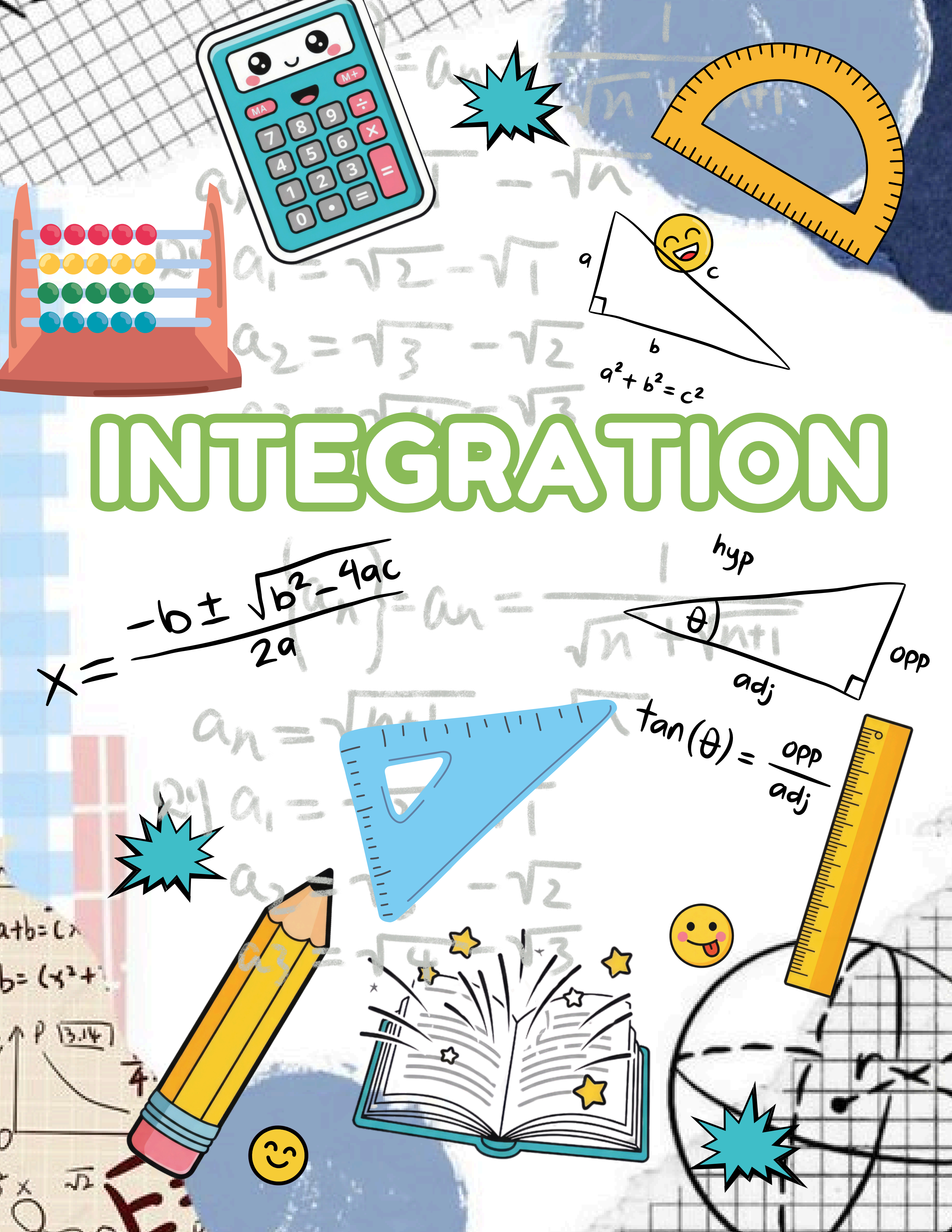
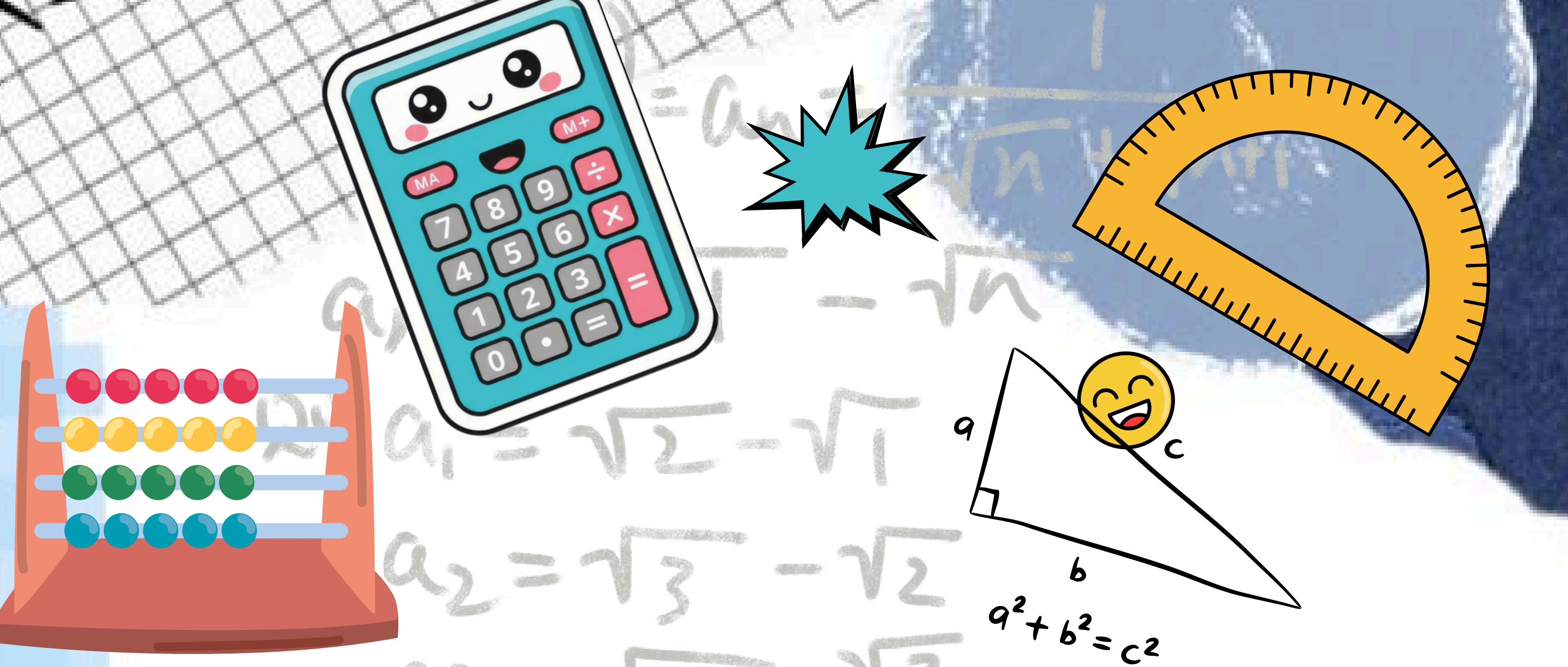
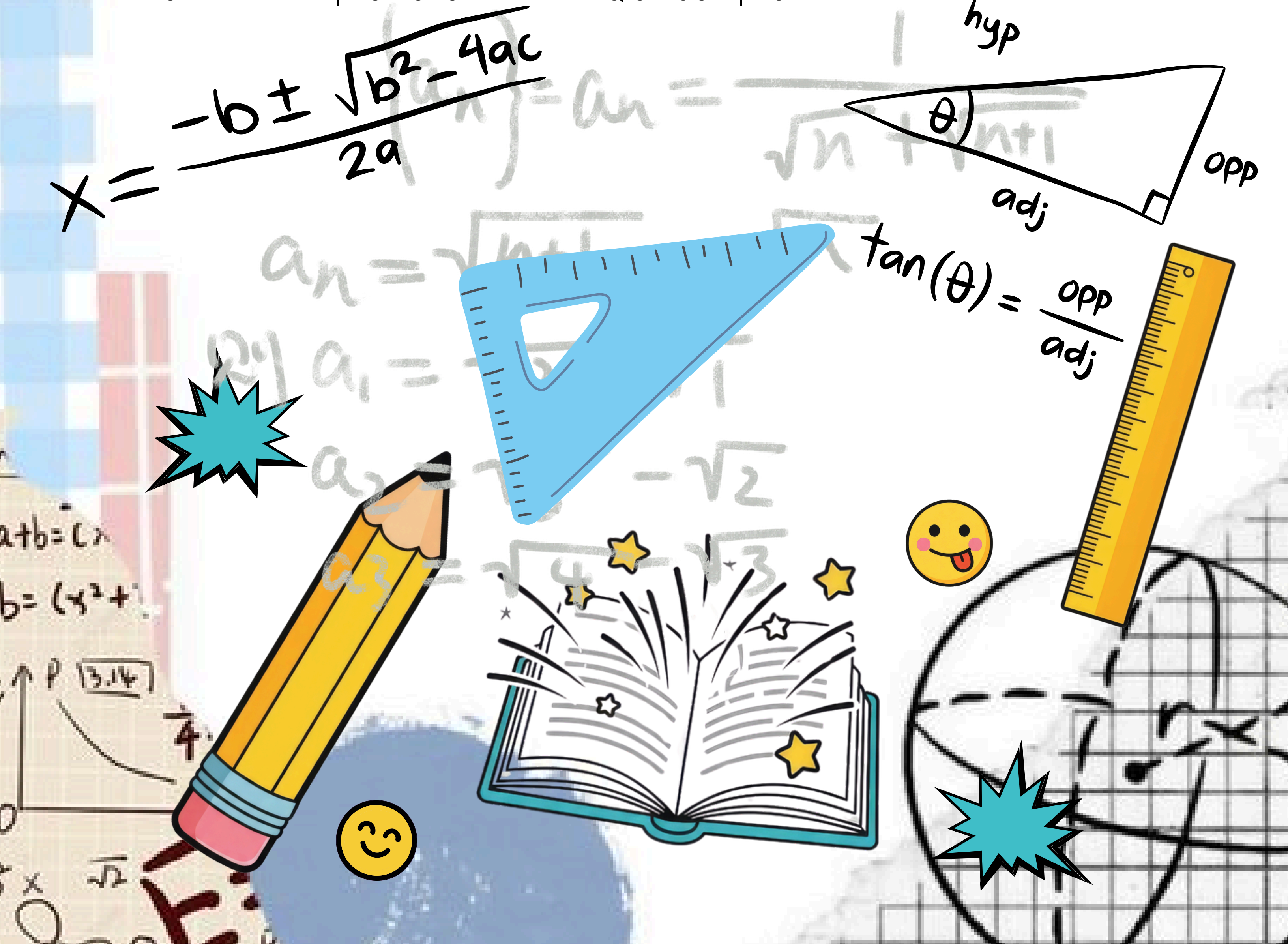
INTEGRATION





INTEGRATION

AISHAH MAHAT | NUR SYUHADAH BALQIS ROSLI | NUR KYRA ADRIENNA FADLY AMIR



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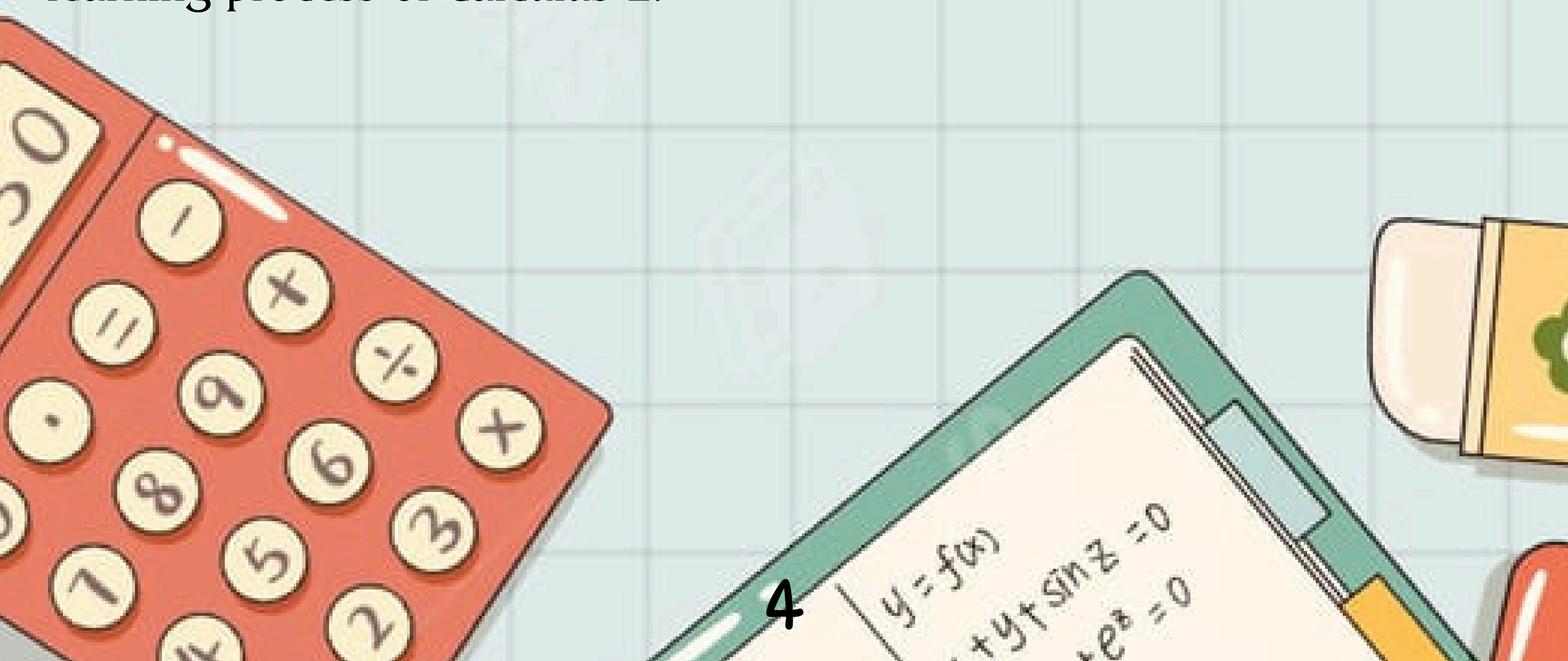
PREFACE

This e-book , Integration, is prepared to assist students in understanding the fundamental concepts of Calculus 1, particularly the topic of Integration. The targeted users for this module are students who are enrolled in the **MAT 183 -CALCULUS 1** course.

This e-book focuses on several important topics related to integration,including basic concepts of integration, indefinite integrals, definite integrals, and basic techniques of integration. These topics are essential in helping students develop a clear understanding of calculus and its applications in solving mathematical and engineering-related problems.

The contents of this e-book are arranged according to the syllabus and lesson plan of the MAT183 course.Mathematical formulas, explanations, and examples are presented based on the subtopics covered during lectures.In addition, several practice questions are provided to help students strengthen their understanding and improve their problem-solving skills.

It is hoped that this e-book will serve as a helpful reference for students in revising important concepts and supporting the teaching and learning process of Calculus 1.



BIOGRAPHY

Aishah Mahat is a lecturer at Universiti Teknologi MARA, Johor. Interests include optimisation and management mathematics. She graduated with second degree from Universiti Kebangsaan Malaysia in Mathematics.

Nur Syuhadah Balqis Binti Rosli is a Diploma student in Civil Engineering who is currently pursuing her studies in engineering and mathematical analysis.She has a strong interest in understanding fundamental concepts in mathematics, particularly calculus, which is essential in solving engineering problems.

Nur Kyra Adrienna Binti Fadly Amir is also a Diploma student in Civil Engineering.She is interested in developing her analytical and problem-solving skills through mathematics subjects such as Calculus.She believes that a strong mathematical foundation is important for future engineering applications.

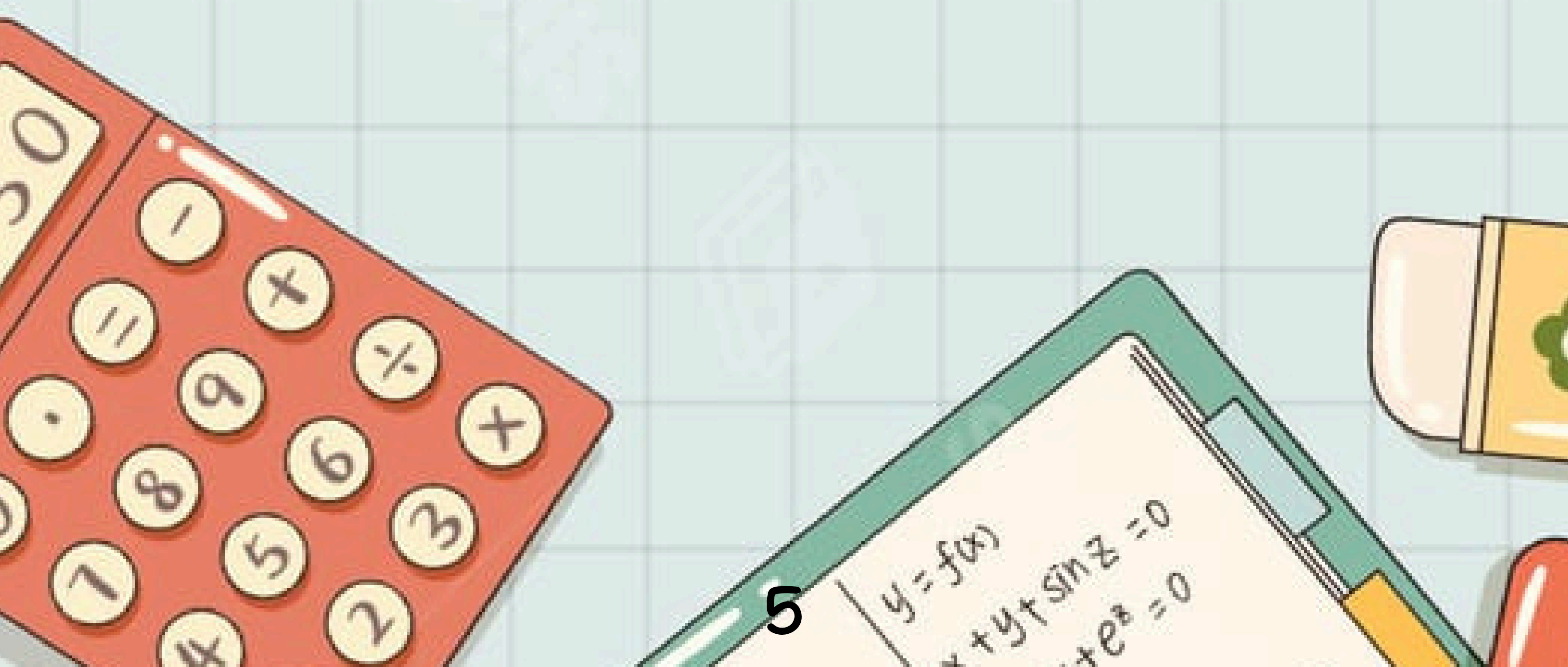


Table Of Content

INDEFINITE INTEGRAL

Integration is the reverse process of differentiation

$$y \rightleftarrows \frac{dy}{dx}$$

power rule :

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

logarithmic rule :

$$\int \frac{1}{x}\,dx = \ln x + c$$

$$\int \frac{1}{x+b}\,dx = \ln x + b + c$$

$$\int \frac{1}{ax+b}\,dx = \frac{\ln ax+b}{a} + c$$

exponential rule :

$$\int e^x\,dx = e^x + c$$

$$\int e^{x+b}\,dx = e^{x+b} + c$$

$$\int e^{ax+b}\,dx = \frac{e^{ax+b}}{a} + c$$

trigonometric rule :

$$\int \sin x\,dx = -\cos x + c$$

$$\int \sin ax\,dx = \frac{-\cos ax}{a} + c$$

$$\int \cos x\,dx = \sin x + c$$

$$\int \cos ax\,dx = \frac{\sin ax}{a} + c$$

$$\int sec^2 x\,dx = \tan x + c$$

$$\int sec^2 ax\,dx = \frac{\tan ax}{a} + c$$

$$\int sec\,x \tan x\,dx = \sec x + c$$

$$\int sec\,ax \tan ax\,dx = \frac{\sec ax}{a} + c$$

power rule

example 1 : $\int_1^4 \sqrt{x+3}\,dx$

Step 1 : Change the square root to a power

$$\int_1^4 (x+3)^{\frac{1}{2}}\,dx$$

Step 2 : Apply the power rule

REMEMBER!!

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)}$$

$$= \left[\frac{(x+3)^{\frac{3}{2}}}{\frac{3}{2}}\right]$$

$$= \left[\frac{2}{3}(x+3)^{\frac{3}{2}}\right]$$

Step 3 : Substitute the limits :

$$= \frac{2}{3}(4+3)^{\frac{3}{2}} - \frac{2}{3}(1+3)^{\frac{3}{2}}$$

$$= \frac{2}{3}(7)^{\frac{3}{2}} - \frac{2}{3}(4)^{\frac{3}{2}}$$

$$= \frac{2}{3}\,(7\sqrt{7}-8)$$

---

example 2 : $\int (3x+2)^4\,dx$

Step 1 : Apply the power rule

The power is 4, so : $4+1=5$

Because the coefficient of x is 3, divide by 3 :

$$= \frac{(3x+2)^5}{3(5)} + C$$

Evaluate the following using power rule.

1. $\int_0^3 \frac{1}{\sqrt{5x+1}}\,dx$	2. $\int_{-2}^{3} (2x+1)^3\,dx$
3. $\int \sqrt{x+5}\,dx$	4. $\int \frac{1}{\sqrt{2x+5}}\,dx$

Logarithmic rule

example 1 : $\int \frac{5x^4}{x^5+3}\,dx$

$$Step\ 1: Identify\ the\ denominator$$

$$f(x) = x^5 + 3$$

$$f'(x) = 5x^4$$

$$Step\ 2: Apply\ logarithmic\ rule$$

$$= ln|x^5 + 3| + C$$

---

example 2 : $\int \frac{3x^2+4x}{x^3+2x^2+5}\,dx$

$$Step\ 1: Identify\ the\ denominator$$

$$f(x) = x^3 + 2x^2 + 5$$

$$f'(x) = 3x^2 + 4x$$

$$Step\ 2: Apply\ logarithmic\ rule$$

$$= ln|x^3 + 2x^2 + 5| + C$$

Evaluate the following using logarithmic rule.

1. $\int \frac{2}{1-3x}\,dx$	2. $\int \frac{4x}{2x^2+5}\,dx$
3. $\int \frac{8x^5+3x^{\frac{1}{3}}-x^2}{4x^3}\,dx$	4. $\int \frac{1}{\sqrt{x}}\left(\frac{3}{\sqrt{x}}+4\sqrt{x}\right)dx$

exponential rule

example 1 : $\int (e^{2x}+3)^2 e^{2x}\,dx$

Step 1 : Expand the bracket

$$(e^{2x}+3)^2 = e^{4x}+6e^{2x}+9$$

$$= \int (e^{4x}+6e^{2x}+9)e^{2x}$$

Step 2 : Multiply e^{2x}

$$= \int (e^{6x}+6e^{4x}+9e^{2x})dx$$

$$= \frac{1}{6}e^{6x}+\frac{6}{4}e^{4x}+\frac{9}{2}e^{2x}+C$$

---

example 2 : $\int \frac{e^{3x+2}}{5}\,dx$

Step 1 : Move the constant $\frac{1}{5}$ outside

$$= \frac{1}{5}\int e^{3x+2}\,dx$$

Step 2 : Apply exponential rule

$$= \frac{1}{5}\left(\frac{1}{3}e^{3x+2}\right)+C$$

$$= \frac{1}{15}e^{3x+2}+C$$

Evaluate the following using exponential rule.

1. $\int_{-1}^{1}(e^{2x+1})dx$	2. $\int_{0}^{1}\left[\frac{e^{3x+1}}{3}\right]dx$
3. $\int \frac{(e^{2x}+1)^2}{e^{2x}}dx$	4. $\int \frac{(\sqrt{2}+e^{3x})^2}{e^{3x}}dx$

trigonometric rule

example 1 : Find $\int \frac{6cos^3x - 8}{cos^2x} dx.$

$$= \int \left(6cosx - \frac{8}{cos^2x}\right) dx$$

$$= \int (6cosx - 8sec^2x) dx$$

$$= 6\sin x - 8\tan x + C$$

---

example 2 : Evaluate $\int_0^{\frac{\pi}{6}} (\sec x \tan x + \sin 3x)\, dx$

Step 1 : Integrate each term

$$= \left[\sec x - \frac{\cos 3x}{3}\right]_0^{\frac{\pi}{6}}$$

Step 2: Substitute the upper limit

$$= \left[\sec\frac{\pi}{6} - \frac{\cos(3\frac{\pi}{6})}{3}\right]$$

Step 3 : Substract the lower limit

$$= \left[\sec 0 - \frac{\cos(3(0))}{3}\right]$$

$$= \left[\sec\frac{\pi}{6} - \frac{\cos(3\frac{\pi}{6})}{3}\right] - \left[\sec 0 - \frac{\cos(3(0))}{3}\right]$$

TAKE NOTE !!

$$\sec\frac{\pi}{6} = \frac{2}{\sqrt{3}}, \cos\frac{\pi}{2} = 0, \sec 0 = 1, \cos 0 = 1$$

$$= \left[\frac{2}{\sqrt{3}} - 0\right] - \left[1 - \frac{1}{3}\right]$$

$$= 0.4880$$

Evaluate the following using trigonometric rule.

1. $\int (\sin 8x - e^{5x} + 1)$

2. $\int \frac{sec^2 x}{(3+\tan x)^2}\, dx$

3. $\int_0^{\frac{\pi}{6}} \sec(2x)\tan(2x)\, dx$

4. $\int [e^{3x} + sec^2(2x+1)]\, dx$

second fundamental theorem of calculus

$$f'(x)=fg(x)\,g'(x)$$

Example:

Given $f(x)=\int_0^{\frac{1}{x}}\sqrt{1+t^2}\,dt$. *Use tje seond fundamental theorem of calculus to find* $f'(x)$

(1) $g(x)=\frac{1}{x}$ *or* x^{-1}

(2) $g'(x)=-1x^{-2}$

$$=\frac{-1}{x^2}$$

(3) $f(t)=\sqrt{1+t^2}$

(4) $fg(x)=\sqrt{1+(\frac{1}{x})^2}$

(5) $f'(x)=fg(x)g'(x)$

$$=\sqrt{1+\frac{1}{x^2}}\left(-\frac{1}{x^2}\right)$$

Exercise 1:

Given $f(x) = \int_0^{4x^2} (3t^2 + 5)\,dt$.

(a) Find $f'(x)$ by using 2^{nd} Fundamental Theorem of Calculus.

(b) Hence, evaluate $f'(1)$.

Exercise 2:

Let $f(x) = \int_0^{x^2-4} e^{\sin t}(2-t)\,dt$. Use the 2^{nd} Fundamental Theorem of Calculus to find :

$(a) f'(x)$

$(b) f'(1)$

Exercise 3:

Let $f(x) = \int_0^{2x+1} \frac{t^2 - 1}{5 + \sqrt{3t}} dt.$:

(a) *Find* $f'(x)$ *by using* 2^{nd} *Fundamental Theorem of Calculus.*

(b) *Find* $f'(x)$ *when* $x = 1$.

(c) *Hence, find the equation of tangent line at the point* $(1,1)$.

properties of integrals

EXAMPLE 1: $\int_a^b f(x)dx = -\int_b^a f(x)dx$

Given $\int_2^5 f(x)dx = 7$,find $\int_5^2 f(x)dx$

SOLUTION;

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$\int_2^5 f(x)dx = 7$$

$$\int_5^2 f(x)dx = -7$$

NOTES
If the upper and lower limits are switched,change the sign of the integrals.

EXAMPLE 2: $\int_a^a f(x)dx = 0$

Given $\int_3^3 (5x^2+2x-7)dx$,find the value.

SOLUTION;

prove; $\int_3^3 (5x^2+2x-7)dx$

$$= \left[\frac{5x^3}{3} + \frac{2x^2}{2} - 7x\right]_3^3$$

$$= \left[\frac{5(3)^3}{3} + \frac{2(3)^2}{2} - 7(3)\right] - \left[\frac{5(3)^3}{3} + \frac{2(3)^2}{2} - 7(3)\right]$$

$$= 0$$

NOTES
If the upper and lower limits are the same,the value of integral is always zero.

EXAMPLE 3: $\int^{b} kf(x)dx = k\int^{b} f(x)dx$

NOTES

A constant multiplied by the function can be taken outside the integral.

Given $\int_{1}^{5} f(x)dx = 6$, find the value of $\int_{1}^{5} 4f(x)dx$.

SOLUTION;

$$\int_{1}^{5} 4f(x)dx = 4\int_{1}^{5} f(x)dx$$

$$= 4(6)$$

$$= 24$$

EXAMPLE 4: $\int_{a}^{b} [f(x)dx + g(x)dx] = \int_{a}^{b} f(x)dx + \int_{a}^{b} g(x)dx$

NOTES

Separate the integral of a sum into the sum of the individual integrals.

Given $\int_{1}^{4} f(x)dx = 5$ and $\int_{1}^{4} g(x)dx = 8$, find the value of

$\int_{1}^{4} [f(x) + g(x)]dx.$

SOLUTION;

$$\int_{1}^{4} [f(x) + g(x)]dx = \int_{1}^{4} f(x)dx + \int_{1}^{4} g(x)dx$$

$$= 5 + 8$$

$$= 13$$

EXAMPLE 5: $\int_a^b [f(x)dx - g(x)dx] = \int_a^b f(x)dx - \int_a^b g(x)dx$

NOTES

Separate the integral of a difference into the difference of the individual integrals.

Given $\int_0^2 f(x)dx = 9$ and $\int_0^2 g(x)dx = 4$, find the value of

$\int_0^2 [f(x) - g(x)]dx.$

SOLUTION;

$$\int_0^2 [f(x) - g(x)]dx = \int_0^2 f(x)dx - \int_0^2 g(x)dx$$

$$= 9 - 4$$

$$= 5$$

EXAMPLE 6: $\int_a^c f(x)dx = \int_b^c f(x)dx + \int_a^b f(x)dx$

NOTES

You can split a definite integral into two or more integrals at any point within the interval.

Given $\int_0^4 f(x)dx = 10$ and $\int_4^7 f(x)dx = 6$, find the value of

$\int_0^7 f(x)dx.$

SOLUTION;

$$\int_0^7 f(x)dx = \int_0^4 f(x)dx + \int_4^7 f(x)dx$$

$$= 10 + 6$$

$$= 16$$

Evaluate the following :

1.Given $\int_{2}^{5} f(x)dx = 8,$ find $\int_{5}^{2} f(x)dx.$	2.Find $4+\int_{-5}^{-5} (x^4 - 3x + 10)dx.$
3.Given $\int_{0}^{3} f(x)dx = -4,$ find $-2\int_{0}^{3} 3f(x)dx.$	4.Given $\int_{0}^{8} f(x)dx = 20,$ and $\int_{0}^{3} f(x)dx = 7$ find $\int_{3}^{8} f(x)dx.$

Evaluate the following :

5.Given $\int_0^4 f(x)dx = 7$, and $\int_0^4 g(x)dx = -3$, find $3\int_0^4 [2f(x) - g(x)]dx$.

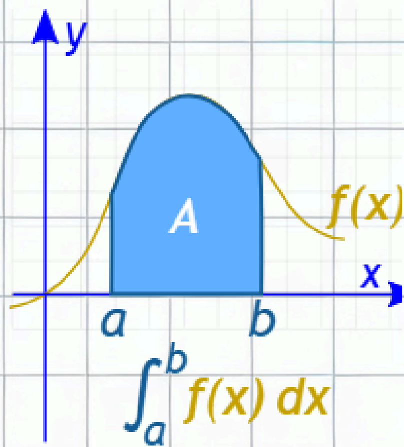
6.Given $\int_0^8 f(x)dx = 12$, $\int_0^3 f(x)dx = 5$, and $\int_3^8 g(x)dx = -4$, find $\int_3^8 [f(x) - g(x)]dx$.

7.Given $\int_0^5 f(x)dx = 8$, $\int_0^2 f(x)dx = 3$, $\int_2^5 g(x)dx = 6$, find $\int_5^2 [f(x) + g(x)]dx$.

8.Given $\int_1^4 f(x)dx = 6$, $\int_4^7 f(x)dx = -2$, $\int_1^7 g(x)dx = 5$, find $2\int_7^1 [3f(x) + 2g(x)]dx$.

DEFINITE INTEGRAL

A **definite integral** has start and end values,in other words there is an interval (a,b)



EXAMPLE 1:

Evaluate the following :

Given $f(x) = \begin{cases} 2x+3; x \leqslant 0 \\ x^3; x > 0 \end{cases}$. Find the value of k if $\int_{-1}^{4} kf(x)dx = 33.$

SOLUTION:

$$\int_{-1}^{4} kf(x)dx = 33$$

$$k\left(\int_{-1}^{0} f(x)dx + \int_{0}^{4} f(x)dx\right) = 33$$

$$k\left(\int_{-1}^{0} (2x+3)dx + \int_{0}^{4} x^3dx\right) = 33$$

$$k\left(\left[x^2+3x+c\right]_{-1}^{0} + \left[\frac{x^4}{4}+c\right]_{0}^{4}\right) = 33$$

$$k\left(0-[1-3]+64-0\right) = 33$$

$$66k = 33$$

$$k = \frac{1}{2}$$

Evaluate the following :

1. $\int_0^6 f(x)dx$ if $f(x) = \begin{cases} x^2; x \leqslant 2 \\ 3x - 2; x > 2 \end{cases}$

2. $\int_{-\frac{\pi}{4}}^{2} f(x)dx$ if $f(x) = \begin{cases} cos2x; x < 0 \\ e^{2x}; x \geqslant 0 \end{cases}$

Evaluate the following :

3. Given $\int_{0}^{5} f(x)dx = -5$ and $\int_{0}^{5} g(x)dx = \begin{cases} x^2 - 1; & 0 \leqslant x < 3 \\ 3x + 2; & 3 \leqslant x \leqslant 5 \end{cases}$

Using the properties of definite integral, find the value of k that satisfies

$\int_{5}^{0} [3g(x) - kf(x)]dx = -2.$

Evaluate the following :

4. Given $\int_0^{\pi} f(x)dx = -4$ and $g(x) = \begin{cases} 2sinx + 1; & 0 \leqslant x < \frac{\pi}{2} \\ 3cosx + 2; & \frac{\pi}{2} \leqslant x \leqslant \pi \end{cases}$ .Using the properties of definite integral, find the value of k that satisfies

$$\int_{\pi}^{0} [2g(x) - kf(x)]dx = -2.$$

5. Given $f(x) = \begin{cases} x - 2; & x \leqslant 1 \\ kx^2; & x > 1 \end{cases}$, Find the value of k if $\int_0^3 \frac{f(x)}{3} dx = 6.$

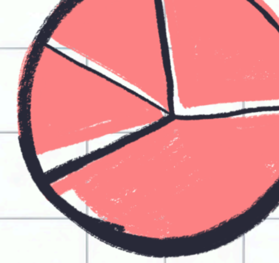
Evaluate the following :

6. Given $\int_0^{\pi} f(x)dx = 5$, and $g(x) = \begin{cases} 4sin^2x - 1; & 0 \leqslant x < \dfrac{\pi}{2} \\ 2cosx + 3; & \dfrac{\pi}{2} \leqslant x \leqslant \pi \end{cases}$

.Using the properties of definite integral, find the value of k that satisfies $\displaystyle\int_{\pi}^{0} [3g(x) - kf(x)]dx = 4.$

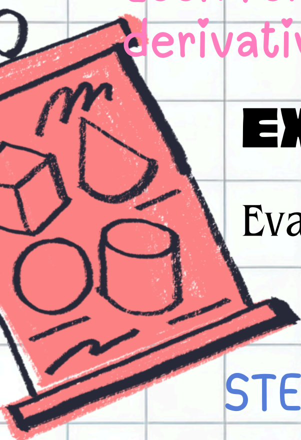
7. Given $\int_0^{4} f(x)dx = -5,$, and $g(x) = \begin{cases} x + 2; 0 \leqslant x < 2 \\ 2x - 1; 2 \leqslant x \leqslant 4 \end{cases}$

.Using the properties of definite integral, find the value of k that satisfies $\displaystyle\int_{4}^{0} [10g(x) - kf(x)]dx = 6.$



integration by Substitution

NOTES
Look for the inner function and check whether its derivative is also present in the integral.



EXAMPLE 1:

Evaluate the following : $\int 2x(x^2+3)^4dx$

SOLUTION;

STEP 1:let $u = x^2 + 3$

STEP 2:differentiate $du = 2xdx$

STEP 3:substitute $\int u^4du$

STEP 4:integrate $= \frac{u^5}{5} + C$

STEP 5:substitute back

$$= \frac{(x^2+3)^5}{5} + C$$

Evaluate the following :

1. $\int 6x^2(2x^3+1)^5 dx$

2. $\int \frac{2x}{x^2+5} dx$

3. $\int xe^{x^2} dx$

4. $\int_0^2 x(x^2+1)^3 dx$

Evaluate the following :

5. $\int_{1}^{3} (x^2+6)^3 x dx$

6. $\int \frac{cos3x}{(4+sin3x)^4} dx$

7. $\int \sqrt{7x-6} dx$

8. $\int (1-5x)^4 dx$

Evaluate the following :

9. $\int (2x+1)(x^2+x+3)^5 dx$

10. $\int \frac{x}{\sqrt{x^2+9}} dx$

11. $\int \frac{(2x+1)}{(x^2+x+4)^4} dx$

12. $\int sinx(cosx+2)^6 dx$

ANSWER :

INDEFINITE INTEGRAL

POWER RULE

1. $\frac{6}{5}$
2. 290
3. $\frac{2}{3}(x+5)^{\frac{3}{2}}$
4. $\sqrt{2x+5}+C$

LOGARITHMIC RULE

1. $-\frac{2}{3}\ln|1-3x|+C$
2. $\ln|2x^2+5|+C$
3. $\frac{2}{3}x^3-\frac{9}{20}x^{-\frac{5}{3}}-\frac{1}{4}\ln|x|+C$
4. $3\ln|x|+4x+C$

EXPONENTIAL RULE

1. $\frac{e^3-e^{-1}}{2}$
2. $\frac{e^4-e}{9}$
3. $\frac{1}{2}e^{2x}+2x-\frac{1}{2}e^{-2x}+C$
4. $-\frac{2}{3}e^{-3x}+2\sqrt{2}x+\frac{1}{3}e^{3x}+C$

TRIGONOMETRIC RULE

1. $-\frac{1}{8}\cos 8x-\frac{1}{5}e^{5x}+x+C$
2. $-\frac{1}{3+\tan x}+C$
3. 0.5
4. $\frac{1}{3}e^{3x}+\frac{1}{2}\tan(2x+1)+C$

SECOND FUNDAMENTAL OF THEOREM CALCULUS

1. a) $f'(x)=384x^5+40x$
   b) 424

2. a) $f'(x)=2x(6-x^2)e^{\sin(x^2-4)}$
   b) $10e^{\sin(-3)}$

3. a) $f'(x)=\frac{2[(2x+1)^2-1]}{5+\sqrt{3(2x+1)}}$
   b) 2
   c) $y=2x-1$

ANSWER :

PROPERTIES OF INTEGRAL

1. -8
2. 4
3. 24
4. 13
5. 51
6. 11
7. - 11
8. -44

DEFINITE INTEGRAL

1. $\frac{128}{3}$
2. $\frac{e^4}{2}$
3. $k = -20$
4. $k = \frac{3(2-\pi)}{4}$
5. $k = \frac{9}{4}$
6. $k = \frac{(6\pi - 2)}{5}$
7. $k = \frac{-166}{5}$

INTEGRATION BY SUBSTITUTION

1. $\frac{(2x^3+1)^6}{6} + C$
2. $\ln(x^2+5) + C$
3. $\frac{1}{2}e^{x^2} + C$
4. 78
5. 6028
6. $-\frac{1}{9}(4+sin3x)^3 + C$
7. $\frac{2}{21}(7x-6)^{\frac{3}{2}} + C$
8. $\frac{-(1-5x)^5}{25} + C$
9. $\frac{(x^2+x+3)^6}{6} + C$
10. $\sqrt{(x^2+9)} + C$
11. $\frac{-1}{3(x^2+x+4)^3} + C$
12. $\frac{-(cosx+2)^7}{7} + C$

REFERENCES

Ismail, A., & Mohd Nor, N. A. (n.d.). MAT183 Calculus 1: Concise notes, worked examples, tutorials & sample questions. Universiti Teknologi MARA.

"IT'S OKAY IF YOU DON'T UNDERSTAND IT TODAY.KEEP TRYING UNTIL THE THINGS THAT ONCE CONFUSED YOU BECOME THE THINGS YOU'RE PROUD OF."

$$\int hardwork dx = success + C$$

"ONE PROBLEM AT A TIME, ONE STEP CLOSER TO SUCCESS."

INTEGRATION

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