

UNIVERSITI TEKNOLOGI MARA

**A NOVEL SIMPLIFIED YIELDED
AGGREGATION INDEX (SYAI)
METHOD INTEGRATED WITH
HESITANT TRIANGULAR FUZZY
GENERALIZED GEOMETRIC
HERONIAN MEAN FOR
ENHANCED
MCDM**

**WAN SYAIDATUL IZZATI WAN
ABDUL RAHMAN**

MSc

July 2026

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RAHMAN**

Thesis submitted in fulfilment
of the requirements for the degree of
**Master of Science
(Mathematics)**

Faculty of Computer and Mathematical Sciences

July 2026

CONFIRMATION BY PANEL OF EXAMINERS

I certify that a Panel of Examiners has met on May 12, 2026 to conduct the final examination of Wan Syaidatul Izzati Binti Wan Abdul Rahman on her Master of Science thesis entitled “A Novel Simplified Yielded Aggregation Index (SYAI) Method Integrated with Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean for Enhanced MCDM” in accordance with Universiti Teknologi MARA Act 1976 (Akta 173). The Panel of Examiners recommends that the student be awarded the relevant degree. The Panel of Examiners was as follows:

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ABSTRACT

This study addresses the problem of decision-making under linguistic uncertainty, expert hesitation, and interrelated criteria in Multi Criteria Decision Making (MCDM). Existing MCDM methods often assume precise numerical data and mainly focus on benefit and cost criteria, while limited attention is given to hesitant triangular fuzzy information and goal-type criteria. To overcome these issues, this study proposes the Simplified Yielded Aggregation Index (SYAI) integrated with the Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean (HTFGGHM) operator. The novelty of the proposed SYAI-HTFGGHM framework lies in its ability to combine a unified normalization structure for benefit, cost, and goal-type criteria with a hesitant triangular fuzzy aggregation operator that captures inter-criteria relationships. The mathematical properties of the HTFGGHM operator, including idempotency, monotonicity, and boundedness, are established to support its theoretical validity. The proposed framework is applied to evaluate humor-based teaching strategies in Arabic language learning at UiTM using expert evaluations from five Arabic language lecturers. The results show that verbal humor (A1) is the most preferred strategy, followed by non-verbal humor (A2), situational humor (A4), and technology-based humor (A3). Comparative analysis with TOPSIS, VIKOR, SAW, COBRA, WASPAS, and MOORA further confirms the consistency of the ranking results. Overall, the proposed SYAI-HTFGGHM framework provides a transparent and practical MCDM approach for handling hesitant linguistic evaluations, interrelated criteria, and mixed criterion types in uncertain decision making environments.

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LIST OF SYMBOLS

Symbols

X	Set of Alternatives
G	Set of Criteria
A_i	i^{th} Alternative
G_j	j^{th} Criterion
H_{ij}	HTF Evaluation of A_i Under G_j $H_i = (L_i, M_i, U_i)$
$h_i = (l_i, m_i, u_i)$	Triangular Fuzzy Number
l_{ij}^k	Lower Value of The K^{th} TFN
m_{ij}^k	Middle Value of The K^{th} TFN
u_{ij}^k	j^{th} Criterion
k	Index of Hesitant Evaluation
K	Number of Hesitant Values in H_{ij}
μ	Membership Degree
ν	Non-Membership Degree
p, q	Interaction Parameters
ω_i	Criterion Weight
n	Number of Criteria
$\oplus$	Fuzzy Addition Operator
$\otimes$	Fuzzy Multiplication Operator
$HTFGGHM^{p,q}$	HTF Generalized Geometric Heronian Mean Operator

$WHTFGGHM^{p,q}$	Weighted HTFGGHM Operator
$S(b_i)$	Score Function
β	SYAI Sensitivity Parameter
C	Normalization Constant
D_i	Distance Measure
N_{ij}	Normalized Value of A_i Under G_j
R_i	Final Ranking Index

LIST OF ABBREVIATIONS

Abbreviations

DM	Decision Maker
DM1-DM5	Expert Decision Matrices in the case study
GGHM	Generalized Geometric Heronian Mean
HFS	Hesitant Fuzzy Set
HM	Heronian Mean
HTF	Hesitant Triangular Fuzzy
HTFGGHM	Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean
IPSiS	Institute of Postgraduate Studies (UiTM)
MCDM	Multi-Criteria Decision-Making
SYAI	Simplified Yielded Aggregation Index
TFN	Triangular Fuzzy Number
UiTM	Universiti Teknologi MARA
UiTM Endowment	UiTM Endowment Scholarship
WHTFGGHM	Weighted Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean

CHAPTER 1

INTRODUCTION

1.1 Research Background

Nowadays, in the information-based world the issues are growing in complexity, and they are frequently created by many, competing standards. Multi Criteria Decision-Making (MCDM) methods have emerged as critical instruments to solving these problems in a wide range of disciplines, such as engineering, healthcare, environmental management, economics, and social sciences (Herrera-Viedma et al., 2020; Petrović et al., 2023). Effective strategies may be crucial when dealing with difficult decisions that carry a lot of stakes and uncertain outcomes. Multi-Criteria Decision-Making (MCDM) is concerned with the assessment and ranking of a small set of alternatives in terms of multiple decision criteria. This approach enables the adoption of both qualitative and quantitative factors, offering a more holistic method to the decision-making process (Raza et al., 2024). Through multiple lenses, individuals need to be ready to make more insightful choices; however, this journey will simultaneously boost one's assurance about the results. More conventional methods to MCDM like Simple Additive Weighting (SAW), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), and VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR), have shown a spectacular potential in systematic decision settings (Raza et al., 2024). Nevertheless, they tend to be restrictive in cases of uncertainty, vagueness, and hesitation that characterize real-world decision data (Kirişci, 2023).

To solve these problems, the Fuzzy Set (FS) theory developed by Zadeh (1965) transformed the representation of uncertainty with membership value instead of binary classification. Afterwards, Intuitionistic Fuzzy Sets (IFSs), Pythagorean Fuzzy Sets (PFSs) and q-Rung Orthopair Fuzzy Sets (q-ROFSs) extensions made it even more flexible by allowing more comprehensive expression of imprecision and hesitation. However, in most real decision situations, experts do not give their opinions as a single membership value but rather as a set of possible values because of hesitancy or lack of confidence. This is what led to the concept of developing the Hesitant Fuzzy Set (HFS) and its derivatives, such as the Hesitant Triangular Fuzzy Set (HTFS), which incorporates the power of the triangular fuzzy form with the capability to represent

various degrees of hesitant membership. The HTFS model works especially well in those problems where linguistic judgments are typically represented in ranges, as opposed to fixed points, and offer a more realistic and detailed representation of uncertainty.

Aggregation operators are essential in multi-criteria decision making (MCDM) because they help to summarize individual judgments to group judgments (Abdullah & Ahmad, 2024). The Heronian Mean (HM) and variants have received substantial academic interest because of their ability to reflect their dependence on one another to a set of decision criteria, unlike more basic arithmetic or geometric averages. The Generalized Geometric Heronian Mean (GGHM) extends this association and allows one to flexibly adjust the connection by controlling the parameters, which enhances the trade-off between sensitivity and strength. Nevertheless, existing operators of aggregation often have computational complexity or lack of interpretability in higher-order fuzzy settings, including hesitant triangular fuzzy settings. This therefore calls for an improved aggregation model that will be easier to compute but still preserve the geometry of the interaction of the criteria (Karaaslan & Al-Husseinawi, 2022).

In this sense, a framework to integrate the Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean (HTF-GGHM) with the new Simplified Yielded Aggregation Index (SYAI) implies an exciting research path. The SYAI method presents a simplified although powerful process of aggregation which gathers normalization and weighting processes in a single, flexible index. As opposed to traditional methods that are based on complex normalization schemes or domain-specific distributions of weights, the SYAI produces a balanced and interpretable decision index that can be used in both benefit and cost-type criteria. Its systematic hierarchy enables the decision-makers to control the aggregation sensitivity through a simplified parameter, eliminating bias and computation costs and maintaining stability in decisions.

The combination of SYAI framework and HTF-GGHM operator enhanced the improvements to the management of hesitant fuzzy environment. The HTF-GGHM helps in the effective capture of the interrelationships between multiple triangular fuzzy values and SYAI component ensures that the overall decision-making process is computationally light and conceptually sound at the same time. Together, these factors are forming a synergetic model that can treat multidimensional uncertainty, hesitation, and inter-criterion correlation in the decision issues in the real world. This integration,

therefore, aims to provide a powerful and flexible decision-support model which can be implemented across various domains, healthcare analysis, environmental sustainability, supplier evaluation, and policy development.

Despite the many improvements achieved in the fuzzy based MCDM frameworks, the current methods still lack the ability to balance out computational simplicity with overall representational ability. Expressive methods often have complex implementations, while simplified methods may lose precision or overlook interactions between criteria. The SYAI-HTFGGHM approach fills this gap by proposing a simplistic, but generalizable model that can process complex hesitant triangular fuzzy data and still be mathematically sound. Therefore, the method also leads to the methodological development as well as to the practical relevance of MCDM models in unpredictable and vague situations.

To summarize, the necessity to develop SYAI-HTFGGHM framework is driven by the need to have efficient, transparent, and balanced process of aggregation in hesitant fuzzy MCDM settings. The research will help to strengthen the robustness, versatility, and interpretability of MCDM decision frameworks because it will combine the interpretative power of the Generalized Geometric Heronian Mean and simplified structure of the SYAI method. It is expected that the proposed model will significantly enhance the quality and stability of decision outcomes and become a resourceful instrument to the researchers and practitioners of the field of decision science.

1.2 Problem Statement

Complex environments have several conflicting criteria, ambiguous information and reluctant expert opinions in decision making. The Multi-Criteria Decision Making (MCDM) approaches including Analytic Hierarchy Process (AHP), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) and ViseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) have been extensively employed to address such issues (Raza et al., 2024). Nevertheless, such classical techniques are normally based on sharp numerical information and presuppose a strict specification on the optimal and non-optimal options. This assumption becomes a major limitation when the evaluation data are obtained from expert judgments, linguistic assessments, and uncertain real-world conditions. At the realworld scenario like environmental management, healthcare and supply chain, often the decision-makers are subjected to

uncertain judgments which are unattainable to state accurately (Kirişci, 2023). Such ambiguity brings about indecisions within the process of evaluation and cannot be effectively realised in a traditional MCDM approach.

This issue is particularly relevant in the educational sector, especially in evaluating the effectiveness of language teaching. Language teaching effectiveness is not determined by a single measurable factor, but involves several qualitative aspects such as student engagement, language comprehension, motivation, teaching effectiveness, cultural appropriateness, and readiness for use. In Arabic language learning, these aspects are often evaluated through expert judgments and linguistic assessments rather than exact numerical data. Therefore, evaluating language teaching effectiveness requires a structured decision-making approach that can handle subjective opinions, uncertainty, and hesitation in expert evaluations.

In order to address this, the fuzzy set-based extensions were proposed in order to represent uncertainty with membership functions (Abdullah & Ahmad, 2024). One of them, hesitant triangular fuzzy sets (HTFS), are more realistic as they give experts an opportunity to describe multiple potential membership levels of each criterion (Gholizade et al., 2023). This enhancement, however, is accompanied by the inherent attributes of the existing fuzzy aggregation operators, which are marked by considerable computational intricacy and diminished interpretative clarity (Deli & Karaaslan, 2021; Karaaslan & Al-Husseinawi, 2022). In many existing fuzzy-based MCDM models, the aggregation process is either too complex for practical implementation or too simple to represent the interaction among criteria. They tend to apply all the criteria consistently and without looking at the fine differences between the benefits type, cost type and intermediate type criteria. As a result, the interaction among criteria and the different directions of criterion preference are not always represented clearly. This weakness limits the flexibility of the model in realistic decision-making settings whereby expert preferences can be inconsistent and inconsiderate in the direction and strength.

Moreover, the majority of existing aggregation techniques presuppose that the optimal level of the criterion can always be determined as the maximum (benefit type) or the minimum (cost type). This is not always the case in reality. There are numerous real-world criteria, especially concerning the operational efficiency and sustainability, which are optimum at the intermediate or goal-oriented level but not at the extremes (Hasnan et al., 2024). As an example, in the treatment of medical waste, excessive temperatures can lead to unnecessary energy use and emissions, and low temperatures

do not guarantee the sterilization (Kirişci, 2023). Therefore, a suitable MCDM method should not only consider benefit and cost criteria, but should also be able to handle goal-type criteria where the best value lies within a desired target range. Hence a good MCDM method should be able to accommodate the goal-type criteria in a flexible manner with the optimum value being between the max and minimum value, being more realistic and practical in decision making.

Based on these limitations, three main research gaps can be identified. First, existing MCDM methods still have limitations in handling hesitant and linguistic information provided by experts. Second, several fuzzy aggregation operators do not sufficiently model the interrelationships among criteria while maintaining computational simplicity. Third, most ranking procedures mainly focus on benefit and cost criteria, while the treatment of goal-type criteria is still limited. These gaps are directly related to the need to develop a transparent ranking method, formulate a suitable hesitant triangular fuzzy aggregation operator, and integrate both components into a practical decision-making framework.

Therefore, this study focuses on addressing these gaps by developing a decision-making framework that is able to manage hesitant triangular fuzzy information, model inter-criterion relationships, and rank alternatives under benefit, cost, and goal-type criteria. The proposed framework is expected to provide a more transparent, computationally efficient, and interpretable approach for solving complex MCDM problems under uncertainty.

1.3 Research Questions

- i) How can the Simplified Yielded Aggregation Index (SYAI) be systematically developed to provide a unified, transparent, and efficient aggregation mechanism capable of handling hesitant triangular fuzzy data and diverse criterion types (benefit, cost, and goal-type) in multi-criteria decision-making (MCDM)?
- ii) How can the Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean (HTFGGHM) operator be formulated to effectively capture the interaction and dependency among hesitant triangular fuzzy criteria for enhanced decision accuracy?

- iii) How can the integration of the SYAI method and HTFGGHM operator be applied and validated to produce consistent, interpretable, and reliable results across various real-world decision-making domains?

1.4 Research Objective

The research objectives of this study are:

- a) To develop the Simplified Yielded Aggregation Index (SYAI) method for Multi-Criteria Decision-Making (MCDM).
- b) To introduce the Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean (HTFGGHM) operator for modeling inter-criterion relationships under uncertain and hesitant environments.
- c) To apply and validate the integrated SYAI–HTFGGHM framework within multi criteria decision making settings, with particular emphasis on educational evaluation problems

1.5 Significance of Study

This research has theoretical value because it adds to the development of knowledge in Multi-Criteria Decision-Making (MCDM) with the fuzzy approach. The adoption of the Simplified Yielded Aggregation Index (SYAI) offers a fresh theoretical basis of integrating normalization, weighting, and aggregation into one, simplified model. In contrast to the standard aggregation techniques, SYAI is constructed to address properly various kinds of criteria, such as benefit, cost, and goal-type criteria, in hesitant triangular fuzzy setting. The conceptualization of the introduction of hesitation and fuzziness into decision models, which is facilitated by this integration, fills a significant gap in the literature so far, as aggregation mechanisms tend to focus on linear or extreme-value performance.

In terms of methodology, this study would increase the efficiency and flexibility of decision-making when faced with uncertainty. The proposed SYAI-HTFGGHM framework offers a carefully structured yet computationally efficient model, consequently streamlining the evaluation process while upholding the integrity of analytical thoroughness. The SYAI provides the transparent normalization and

aggregation of the hesitant fuzzy data and the Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean (HTFGGHM) operator models the interrelationships between the criteria using its geometric Heronian formulation. The combination of these elements forms a balanced approach to the methodology that breaks the shortcomings of the traditional operators by incorporating simplicity, precision, and flexibility into a single framework.

The research findings are very practical and can be utilized in a wide field of real-life decision-making processes, including the evaluation of the healthcare system, educational performance, environmental management, the optimization of industrial processes. Decision-makers are often confronted with cases of vague, unfinished or ambivalent information, and the suggested framework suggests a strong solution to generate reliable and interpretable outcomes. The SYAI-HTFGGHM framework facilitates more informed, transparent, and rational decision-making processes in the application of the framework in both the public and the private sector by allowing fair and balanced assessment of various criteria with varying directions of optimality.

1.6 Scope of Study

The scope of this study covers the theoretical development, methodological formulation, and empirical application of the Simplified Yielded Aggregation Index (SYAI) combined with the Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean (HTFGGHM) operator in Multi-Criteria Decision-Making (MCDM). The theoretical scope focuses on the formulation of mathematical definitions, properties, and proofs related to the HTFGGHM operator within the hesitant triangular fuzzy environment. The methodological scope focuses on the integration of SYAI with HTFGGHM to construct a decision-making framework that can handle benefit, cost, and goal-type criteria in a structured ranking process.

The specific application of this study is limited to educational decision-making, particularly the evaluation of humor-based teaching strategies in Arabic language learning at Universiti Teknologi MARA (UiTM), Kuala Pilah Campus. The alternatives considered in the application are verbal humor, non-verbal humor, technology-based or audiovisual humor, and situational humor. These alternatives are evaluated based on six main educational criteria, namely student engagement, language comprehension,

student motivation, teaching effectiveness, cultural appropriateness, and readiness for use.

The data used in the application are represented using hesitant triangular fuzzy numbers to capture hesitation, subjectivity, and linguistic uncertainty in expert judgments. The expert evaluations are obtained from Arabic language lecturers, and the proposed SYAI-HTFGGHM framework is applied to aggregate the hesitant fuzzy evaluations and rank the teaching strategies. This application is selected because the effectiveness of humor-based teaching cannot be measured only through exact numerical values, but depends on qualitative expert opinions related to classroom engagement, comprehension, motivation, cultural suitability, and practical readiness. Although the proposed framework is developed with flexibility for broader MCDM applications, the empirical validation in this study is specifically demonstrated in the educational sector. Other possible application areas, such as healthcare evaluation, environmental management, supplier selection, and industrial decision-making, are considered potential future applications rather than the main empirical scope of this study. Therefore, the findings of this study are interpreted within the context of educational decision-making and expert-based evaluation of Arabic language teaching strategies.

1.7 Limitation of the Study

Even with its benefits, the study does have certain limitations. One limitation of this study is that the decision makers are treated with equal importance during the fuzzy aggregation process. The proposed application does not assign different weights to the five expert lecturers because they are considered to have comparable expertise in Arabic language teaching. However, in real decision-making situations, experts may differ in terms of experience, specialization, or professional authority. Therefore, future research may improve the framework by incorporating decision-maker weights to better reflect the relative influence of each expert in the aggregation process. The findings rely on expert judgment for uncertain input, which can differ across various panels and fields. In addition, appropriate modifications to the variables p and q are necessary to effectively depict the interaction strengths among the criteria. Moreover, the current validation utilized a dataset of limited size; larger or cross-domain datasets could yield more profound insights into generalizability.

1.8 Research Contributions

This analysis highlights a robust integration of theoretical enhancements and practical utility within fuzzy multi-criteria decision-making (MCDM). On a theoretical level, it broadens the traditional Heronian mean framework to include hesitant triangular fuzzy sets (HTFSs), resulting in the development of the Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean (HTFGGHM) operator along with its weighted variant. The operators involved maintain key mathematical characteristics: idempotency, monotonicity, and boundedness that collectively support coherence, logical integrity, and stability in the outcomes of aggregation. By systematically revealing these traits, the study constructs a reliable mathematical support for navigating through uncertainty and confusion in decision-making processes. From a methodological perspective, the introduction of the Simplified Yielded Aggregation Index (SYAI) framework enhances this advancement by providing a computationally efficient approach that simplifies the aggregation and ranking of hesitant information. SYAI's framework expertly merges the demands of analytical precision and computational efficiency, empowering it to effectively process convoluted and ambiguous datasets with transparency and repeatability.

Considering practical aspects, merging SYAI and HTFGGHM results in a solid and flexible decision support system relevant to diverse domains where uncertainty, ambiguity, and subjective judgments frequently arise. This integrated model proficiently confronts practical decision-making obstacles, such as academic assessments, medical evaluations, and ecological governance, where doubts in communication and diverging expert perspectives commonly impede uniform decision-making results. By merging a mathematically rigorous operator (HTFGGHM) with a straightforward yet effective ranking mechanism (SYAI), the model equips decision-makers with a reproducible, comprehensible, and scalable instrument for assessing alternatives amid uncertainty. Ultimately, the SYAI–HTFGGHM framework bridges the divide between conceptual generalization and practical execution, contributing to both the theoretical progress of fuzzy aggregation methods and their real-world application in contemporary decision-support systems.

1.9 Thesis Outline

In this section, a concise overview of the overall structure of the thesis is presented. The thesis consists of six chapters, each corresponding to specific research objectives and systematically organized to ensure a logical flow of discussion. Figure 1.1 illustrates the overall structure of the thesis.

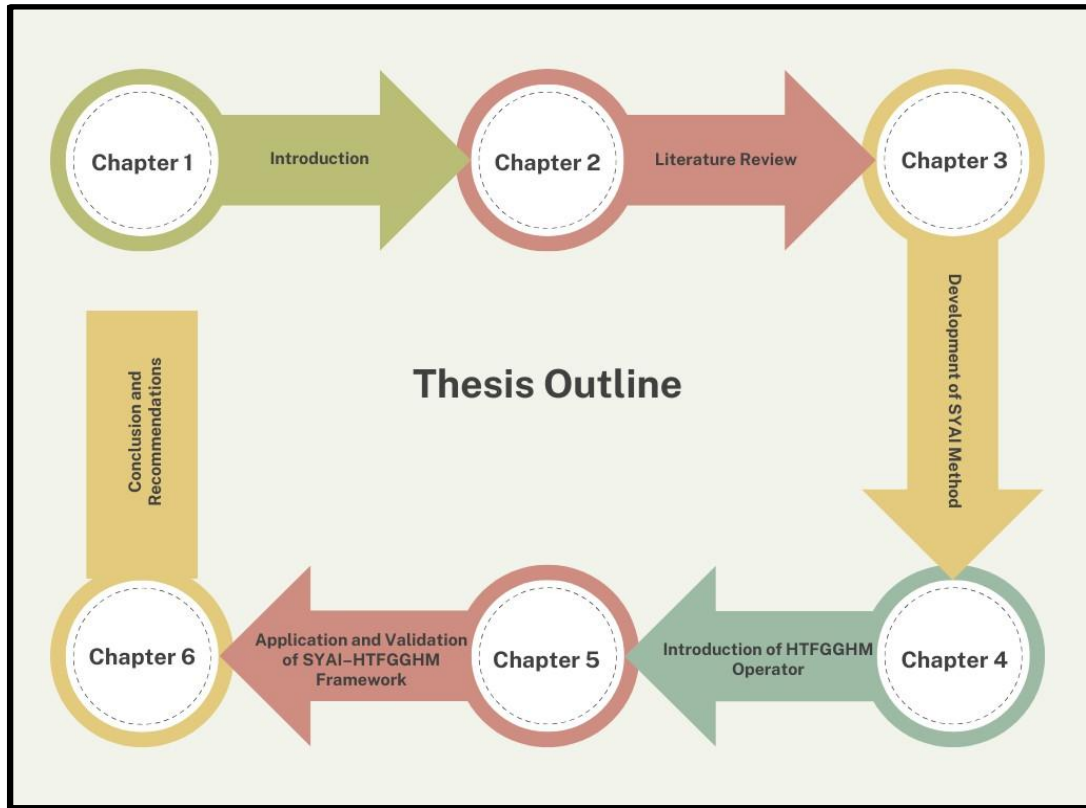


Figure 1.1 Thesis Outline

Chapter 1: Introduction

In this chapter the research context is presented including the background, problem statement, research questions, objectives, significance, and scope of the research. It ends with giving the general thesis outline.

Chapter 2: Literature Review

The chapter gives a comprehensive discussion of previous researches done on Multi-Criteria Decision-Making (MCDM), hesitant fuzzy environments, aggregation operators, and their extensions. This chapter determines the research gaps that drive the creation of the proposed method.

Chapter 3: Development of Simplified Yielded Aggregation Index (SYAI)

This chapter is devoted to the theoretical development of the SYAI approach to deal with the computation simplicity and flexibility in making decisions in hesitant triangular fuzzy environments. This chapter relates to Research Objective 1.

Chapter 4: Introduction of Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean (HTFGGHM)

In this chapter, the HTFGGHM operator is formulated, and it has the mathematical properties. It shows the relation of how the operator will capture relationships between decision criteria and this corresponds to Research Objective 2.

Chapter 5: Application and Validation of SYAI–HTFGGHM Framework

In this chapter, the proposed integrated model is applied and validated to different real-world decision-making fields including the sphere of healthcare, education and environment management. This chapter addresses Research Objective 3.

Chapter 6: Conclusion and Recommendations

This chapter highlights the findings and presents the theoretical, methodological, and practical contributions of the research and gives the recommendations on the future research directions.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter examines and integrates the literature that supports the proposed SYAI-HTFGGHM decision framework. It starts in Section 2.2 with the foundation of MCDM, explaining the necessity of structured, multi-criteria analysis. Section 2.3 subsequently evaluates traditional ranking methods and highlights the requirement for models capable of accommodating goal-type criteria, beyond just benefit/cost extremes. Section 2.4 assesses how uncertainty is managed in decision-making, paving the way for Section 2.5 on Hesitant Triangular Fuzzy Sets (HTFS) as a means to incorporate expert hesitation. Section 2.6 elaborates on the Heronian Mean and its extensions and positions the proposed HTFGGHM operator as an aggregator that is sensitive to interactions. Section 2.8 formalizes the linguistic-to-TFN mapping and its hesitant counterpart for effective expert input. Section 2.9 compiles the research gaps identified in the review and ties them directly to the objectives of this study. Lastly, Section 2.10 presents the mathematical definitions and terminologies employed throughout, ensuring consistent notation for the methodology detailed in the following chapter.

2.2 Foundation of MCDM

Decision-making within intricate environments frequently necessitates the assessment of numerous and at times contradictory criteria (Anand et al., 2022; Azzabi et al., 2020). In these circumstances, dependency on a singular attribute or instinctual judgment may result in biased or inferior outcomes (Seo & Sakawa, 1988; Zimmermann, 1987). The requirement for organized and procedural techniques in decision evaluation has spurred the development of Multi-Criteria Decision-Making (MCDM) as a recognized scholarly field (Majumder & Saha, 2016). Innovative thinkers like Zeleny and Cochrane (1982) and Saaty (2013) emphasized that real-world problems ranging from policy formulation to industrial planning require decision making frameworks proficient in merging both quantitative and qualitative aspects. Consequently, MCDM furnishes a mathematical and logical underpinning for the comparison of alternatives by evaluating trade-offs among competing objectives

(Majumder & Saha, 2016). This analytical system enhances clarity and the capacity for replication in decision-making undertakings, thereby making certain that elaborate concerns can be reviewed from assorted perspectives within a logical, data-centric framework (Nowaczyk, 2023).

As the years have passed, MCDM has become a vital asset for scholars and practitioners across multiple areas, spanning engineering, management, healthcare, and education (Kumar, 2024; Shaikh, 2024). Its fundamental adaptability promotes the combination with other analytical strategies such as optimization, simulation, and AI. This framework notably encompasses three significant phases: determining assessment criteria, evaluating options based on those criteria, and applying a ranking or aggregation technique to find the optimal choice (Josselin & Maux, 2017; Kabak & Ervural, 2017). The transformation of MCDM approaches has therefore modified the decision-making sphere shifting from choices informed by instinct to decisions firmly based on data-driven analysis (Kumar, 2024). By furnishing a synthesis between analytical precision and the preferences of decision-makers, MCDM persists as the cornerstone for contemporary decision-support systems and has catalyzed the development of numerous sophisticated extensions, including fuzzy, hesitant, and hybrid decision-making frameworks.

2.3 Limitation of Traditional Ranking Methods in MCDM

Leveraging the foundational principles of Multi-Criteria Decision Making (MCDM), several well-established ranking methodologies most notably the Analytic Hierarchy Process (AHP; Saaty (2013)) and the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS; Hwang and Yoon (1981)) have been widely applied due to their transparency, structured procedures, and ability to handle both qualitative and quantitative criteria.

Beyond AHP and TOPSIS, other conventional MCDM ranking techniques have also been extensively discussed in the literature. The Simple Additive Weighting (SAW) method aggregates alternative performances through a weighted summation process. Although recent hybrid extensions have improved its flexibility under fuzzy environments, SAW fundamentally assumes monotonic benefit–cost behaviour and does not explicitly support goal-type criteria, where optimal performance lies within a target range rather than at extreme values (Ciardiello & Genovese, 2023).

The PROMETHEE method ranks alternatives based on preference functions and pairwise dominance relations. However, its outcomes are highly sensitive to the selection of preference functions and threshold parameters, which may introduce subjectivity into the ranking results (Blouchoutzi et al., 2021). The MOORA method employs a ratio based multi-objective optimization framework that is computationally efficient, yet it remains dependent on benefit–cost classification and lacks mechanisms for modelling target-oriented preferences (Stanujkic et al., 2012).

Similarly, the WASPAS method combines weighted sum and weighted product models to enhance ranking robustness. Despite this improvement, it continues to rely on monotonic preference assumptions and does not naturally accommodate criteria with optimal target values (Bid & Siddique, 2019). The evolution of fuzzy-based adaptations, such as fuzzy TOPSIS (Chen, 2000), has further enhanced these methods by transforming linguistic assessments into fuzzy numerical representations to address uncertainty in expert evaluations.

Collectively, these traditional MCDM approaches provide a comprehensive toolkit for normalization, weighting, aggregation, and ranking, and have demonstrated effectiveness across various application domains, including engineering and education. Nevertheless, their common reliance on benefit–cost extremum assumptions highlights a limitation when decision-making problems involve goal-type criteria that require proximity-to-target evaluation rather than maximization or minimization.

In light of these benefits, the predominant number of typical Multi Criteria Decision-Making (MCDM) approaches are built upon a dualistic understanding of criterion orientation: criteria are regarded as either positive outcomes (to be augmented) or negative impacts (to be diminished) (Kumar, 2024; Temuçin, 2019). This monotonic assumption aids in modeling yet might fail to accurately portray genuine decision-making behaviors when optimal performance is neither at its peak nor its lowest point, but instead revolves around a mid-level 'goal' (Bonezzi et al., 2011). Illustrative examples involve class size (where extremes of being too small or too large are regarded as detrimental), cognitive load (which is most effective when kept at a moderate level), or the degree of humor in instructional delivery (where excessive humor might induce distraction, and insufficient humor could lead to disengagement). Within rigid benefit/cost frameworks, normalization and distance measures tend to drive alternatives toward extremes, potentially disadvantaging options that are nearest to the desired target in preference to those that represent the mathematical maximum or minimum

(Roszkowska, 2024). Consequently, rankings may exhibit a bias favoring extreme values, even when such extremes are pedagogically or operationally less than optimal.

These restrictions require the formulation of a ranking model that specifically includes target-focused benchmarks alongside value and expense classifications, while preserving straightforwardness, clarity, and computational productivity. The proposed Simplified Yielded Aggregation Index (SYAI) is formulated to address this deficiency by facilitating criterion-specific preference modeling that can prioritize nearness to a designated target, rather than merely progress toward an extremum. In this manner, SYAI preserves the functionality of established Multi-Criteria Decision-Making (MCDM) processes (including normalization, weighting, and aggregation) while relaxing the monotonicity condition, thus permitting decision models to align more closely with authentic preference frameworks particularly in educational contexts and analogous fields where achieving an optimal balance frequently supersedes the pursuit of maximizing outputs.

2.4 Addressing Uncertainty in Decision Making

Decision-making in practical settings usually occurs without comprehensive data. Experts commonly utilize subjective opinions, language studies, and vague data in their analysis of various choices (Aliev, 2013; Aminoroaya et al., 2025). Standard Multi-Criteria Decision-Making (MCDM) techniques, even with their mathematical exactness, often take for granted that every criterion value and weight is precisely known (Mahdavi et al., 2009; Zavadskas et al., 2020). This assumption frequently proves to be impractical across many real-life situations, like those found in educational environments, healthcare frameworks, or ecological stewardship, where evaluations by specialists can often be filled with uncertainty and hesitation (Aggarwal, 2022; Cohen et al., 2022). To overcome these issues, experts have taken steps to combine uncertainty modeling approaches that empower decision-makers to convey evaluations with enhanced versatility. Zadeh (1965) creation of fuzzy set theory represented one of the initial major breakthroughs in this domain, facilitating the expression of indistinct ideas using various degrees of membership rather than static numerical figures. This discovery illustrated a notable evolution in the paradigm of decision analysis by authorizing the mathematical representation of qualitative and linguistic insights.

As the layers of decision-making dilemmas deepened, following changes bolstered the fuzzy concept to more faithfully capture human vagueness (Al-Mutairi, 2010). In 1999, Dubois and Prade (1999) made groundbreaking contributions to the field by establishing possibility theory, which widened the fuzzy sets framework to allow for varying levels of plausibility, thus improving the representation of incomplete or subjective data. Innovative developments like intuitionistic fuzzy sets, Pythagorean fuzzy sets, and q-rung orthopair fuzzy sets brought forth more intricate frameworks to express ambiguity, discord, and uncertainty in expert evaluations (Katragunta et al., 2024; Li et al., 2019; Saha et al., 2020). Each of these advancements aimed to refine the articulation of uncertainty whether through the introduction of supplementary parameters (e.g., non-membership degrees) or through higher-order constructs that adeptly accommodate overlapping and ambiguous judgments (Ding et al., 2024).

These innovations empowered the more authentic modeling of decision-making processes led by experts in contexts where precise numerical data are either scarce or unreliable, thereby underscoring the necessity for adaptive fuzzy-based operators within Multi-Criteria Decision-Making (MCDM) frameworks.

Still, regardless of these improvements, one of the continuous issues in the realm of uncertainty modeling centers on the unification of the assorted and at times incongruous expert opinions into a singular representative evaluation (Chen et al., 2017; Farr et al., 2020). When experts present multiple possible membership values for a specific criterion, traditional fuzzy operators may resort to oversimplification through either averaging or the selection of a singular representative value, consequently undermining vital information regarding the dimension of hesitation (Zhang, 2017). To counter this deficiency, present-day aggregation frameworks, including hesitant fuzzy sets and their adaptations (such as hesitant triangular fuzzy numbers), have emerged (Xu & Zhu, 2013). These frameworks overtly promote the inclusion of multiple potential membership values in a unified evaluation, thus protecting the complete spectrum of expert indecision.

2.5 Hesitant Triangular Fuzzy Sets (HTFS)

The evolution of fuzzy set theory has sparked numerous developments focused on capturing the complex traits of human thought processes and the inherent ambiguity found in real-world decision-making scenarios (Herrera-Viedma et al., 2021). Among

these extensions, hesitant fuzzy sets (HFS) facilitate experts in articulating their uncertainty by allocating multiple potential membership degrees to a specific element rather than confining it to a singular deterministic value (Torra, 2010; Xu & Zhu, 2013). This conceptualization is particularly pertinent in scenarios where several experts furnish evaluations that exhibit slight variations, or when a solitary expert encounters indecision regarding linguistic classifications such as "High" or "Medium High" (Zou et al., 2024). To facilitate interpretability, triangular fuzzy numbers (TFNs) are frequently utilized to represent linguistic terms, as they adeptly encapsulate the gradual spectrum between qualitative assessments through three parameters: the lower, middle, and upper limits of membership (Gong et al., 2013; Pal, 2024).

By merging these concepts, the hesitant triangular fuzzy set (HTFS) advances conventional fuzzy representation by permitting multiple TFNs for each evaluation, thus preserving the comprehensive spectrum of uncertainty inherent in expert opinions (Gholizade et al., 2023). This amalgamated form has been extensively utilized in multi-criteria decision-making (MCDM) scenarios where precise numerical assessments are either impractical or unattainable. The addition of hesitance not only broadens the informational horizon but also decreases the likelihood of premature data loss that could stem from averaging or vagueness in the early parts of assessment (Zhang et al., 2022).

Yang et al. (2019) and Gholizade et al. (2023) pursuit focuses on the deployment of triangular hesitant fuzzy sets (THFS) to augment multi-criteria decision-making (MCDM) via the modeling of expert hesitancy and the integration of uncertain preferences, consequently leading to more consistent and trustworthy evaluations. Prakash and Vanathi (2014) employed hesitant triangular fuzzy sets (HTFS) within the context of multi-criteria decision making (MCDM) to integrate both hesitation and fuzziness, thereby facilitating a precise ranking of alternatives via expected values in scenarios where the weights of criteria are indeterminate, whereas Pu et al. (2022) advanced this theoretical framework by introducing the hesitant triangular fuzzy power average (HTFPA) operator, which effectively captures the interrelationships inherent in decision-making information and enhances the selection of alternatives amidst uncertainty. Hence, HTFS delivers a more genuine and adjustable model for merging expert wisdom in contexts distinguished by uncertainty and subjective opinions, including education, healthcare, and environmental oversight.

2.6 Heronian Mean Operator and Its Extensions

Within the domain of classical mathematics, the Heronian Mean (HM) operator was first introduced, and it has since attracted considerable academic curiosity in Multi-Criteria Decision Making (MCDM) for its proficiency in illustrating the connections between various decision criteria (Chumacero, 2022). In contrast to conventional arithmetic or geometric means that presuppose independence among attributes, the HM operator incorporates pairwise interactions between input values, thus facilitating a more comprehensive aggregation mechanism (Wang & Zhang, 2022; Xu et al., 2022). Various generalized and fuzzy adaptations have been formulated to address intricate decision-making environments, culminating in sophisticated iterations such as the Generalized Weighted Heronian Mean (GWHM) and the Generalized Geometric Heronian Mean (GGHM), both of which integrate weighting mechanisms and non-linear transformation properties to enhance aggregation flexibility. These improved versions of HM were developed to overcome the limitation of the classical HM, which mainly aggregates crisp numerical values and does not directly consider criterion weights, non-linear relationships, or fuzzy uncertainty. In particular, GWHM improves the classical HM by incorporating the relative importance of criteria, while GGHM extends the aggregation structure through a geometric formulation that is more suitable for modelling non-linear interrelationships among criteria.

In contemporary research, the application of the Heronian Mean within fuzzy and hesitant frameworks has empowered scholars to not only encapsulate inter-criteria dependencies but also to address the uncertainty inherent in subjective assessments (Chumacero, 2022). Chumacero (2022) presented Heronian mean-based aggregation operators within the framework of interval-valued dual hesitant q-rung orthopair fuzzy environments to elucidate the interrelations among various criteria and to effectively manage hesitation amidst uncertainty, while Wang et al. (2020) utilized the generalized Heronian mean (GHM) operator within q-rung orthopair hesitant fuzzy contexts to tackle inter-criteria dependencies and enhance the precision of decision-making. Together with these contributions, Mehmood et al. (2018) proposed cubic hesitant fuzzy Heronian mean operators that adeptly merge information and clarify relationships among criteria, consequently increasing the capacity of Heronian-oriented techniques to navigate uncertainty and intricate interrelations in multi-criteria decision-making frameworks. Therefore, the development of HM has gradually moved from a classical

crisp aggregation operator to weighted, generalized, geometric, fuzzy, and hesitant fuzzy versions. This progression shows that each improved version was introduced to strengthen the ability of HM-based operators to manage criterion importance, inter-criteria dependency, and uncertain expert evaluations.

However, despite these improvements, the application of the improved HM structure in hesitant triangular fuzzy environments remains limited, especially when linguistic hesitation and triangular fuzzy information need to be aggregated simultaneously. This motivates the extension of the GGHM structure into the hesitant triangular fuzzy setting in this study. The novel Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean (HTFGGHM) operator proposed in this investigation further advances this conceptual framework by amalgamating hesitant triangular fuzzy information with the geometric Heronian structure. This integration allows the HTFGGHM operator to keep both the uncertainty from expert inputs and the interrelation of criteria intact, thereby creating an aggregation framework that more adequately represents the complexities involved in real-world decision-making processes.

2.7 Humor-Based Teaching Strategies in Arabic Language Learning

The application context of this study focuses on the evaluation of humor based teaching strategies in Arabic language learning. In educational decision-making, the effectiveness of a teaching strategy is often difficult to measure using exact numerical values because it involves subjective perceptions, linguistic judgments, and different levels of expert hesitation (Mustapha et al., 2018; Saleh et al., 2025). For example, lecturers may evaluate a teaching strategy based on how well it improves student engagement, language comprehension, motivation, teaching effectiveness, cultural appropriateness, and readiness for use. These criteria are qualitative in nature and are usually expressed through linguistic terms such as low, medium, high, or very high. Therefore, this type of educational evaluation provides a suitable case study for the application of fuzzy-based MCDM methods.

Humor has been recognized as a teaching element that can support classroom interaction, reduce learning anxiety, and increase students' interest in the learning process (Saleh et al., 2025). In the context of Arabic language learning, humor may assist students in understanding vocabulary, grammar, and classroom expressions in a

more engaging manner (Alnazzawi & Yahya, 2025; Saleh et al., 2025). However, the use of humor in teaching must also be evaluated carefully because its effectiveness may depend on several factors, including the type of humor used, the students' background, the cultural context, and the lecturer's readiness to apply humor in the classroom. As a result, the selection of the most suitable humor-based teaching strategy requires a structured decision-making framework that can consider multiple criteria simultaneously.

In this study, four humor-based teaching strategies are considered as alternatives, namely verbal humor, non-verbal humor, technology-based or audiovisual humor, and situational humor. Verbal humor refers to the use of jokes, wordplay, or humorous explanations during teaching. Non-verbal humor involves gestures, facial expressions, and body language that support classroom communication. Technology-based humor includes the use of videos, memes, images, or audiovisual materials to make lessons more attractive. Situational humor refers to spontaneous humor that occurs naturally during classroom interaction. Each of these strategies may contribute differently to the learning process and therefore needs to be evaluated based on suitable educational criteria (Alnazzawi & Yahya, 2025; Saleh et al., 2025).

The case study was chosen because it reflects a real decision-making situation involving uncertainty, hesitation, and linguistic assessment (Mustapha et al., 2018). Expert evaluators may not always provide a single fixed judgment when assessing teaching strategies, especially when the criteria involve human perception and classroom experience. Instead, their opinions may fall between several linguistic levels. This condition is consistent with the use of Hesitant Triangular Fuzzy Sets (HTFS), which allow multiple possible triangular fuzzy values to represent expert hesitation. Furthermore, the relationships among the educational criteria should not be treated as fully independent because student engagement, motivation, comprehension, and teaching effectiveness may influence one another. Therefore, the Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean (HTFGGHM) operator is suitable for aggregating these interrelated and hesitant evaluations.

The integration of the Simplified Yielded Aggregation Index (SYAI) method with the HTFGGHM operator provides a practical framework for ranking humor-based teaching strategies under uncertain educational conditions. The HTFGGHM operator is used to aggregate hesitant triangular fuzzy expert evaluations, while the SYAI method is used to normalize, weight, and rank the alternatives in a transparent and interpretable

manner. Hence, the humor-based Arabic language teaching case study is appropriate for demonstrating the ability of the proposed SYAI-HTFGGHM framework to handle qualitative, hesitant, and multi-criteria decision-making problems in an educational setting.

2.8 Linguistic Representation for Hesitant Triangular Fuzzy Numbers

In scenarios of decision-making that necessitate human judgment, linguistic evaluations persist as some of the most instinctive and comprehensible modalities of assessment (Zeshui, 2012). Categories such as 'Very Low,' 'Medium,' and 'High' function as linguistic variables that furnish qualitative representations of perception, which can be systematically transformed into quantitative indices through the application of triangular fuzzy numbers (TFNs) (Prangishvili et al., 2015). Every linguistic classification is linked to a TFN described by a triplet that outlines its lower, middle, and upper membership thresholds (Pal, 2024). This transformation effectively bridges the gap between qualitative human perception and quantitative analysis, thereby facilitating fuzzy-based techniques in handling uncertain information.

When specialists face ambiguity among diverse linguistic expressions, the notion of a hesitant linguistic evaluation materializes. This scenario engenders the formation of Linguistic Hesitant Triangular Fuzzy Numbers (LHTFNs), which encapsulate multiple conceivable TFNs associated with a singular evaluative judgment. Prangishvili et al. (2015) focused on converting qualitative linguistic evaluations into triangular fuzzy numbers (TFNs) to transform expert insights into measurable metrics for group decision-making, while Karabasevic et al. (2022) employed linguistic variables translated into TFNs to tackle uncertainty and enhance the effectiveness of the Simple WISP approach. Likewise, Singh (2021) showcased the use of TFNs to convert qualitative linguistic expressions into quantitative indicators, thereby aiding in the evaluation of attribute significance and the comparison of alternatives in decision-making.

In the context of this research, these LHTFNs act as the core component for the aggregation process that employs the HTFGGHM operator. By directly incorporating linguistic hesitation into the fuzzy modeling framework, the SYAI-HTFGGHM paradigm ensures the retention of the intricacies of human uncertainty from the initial

appraisal to the final ranking phase, thus promoting a more lucid and genuine depiction of expert consensus.

Table 2.1
Literature Mapping for the Development of the SYAI-HTFGGHM Framework.

Author(s)	Focus / Model	Aggregation / Concept	Ranking / Method	Key contribution and limitation (gap)
Zeleny and Cochrane (1982)	MCDM foundation	General decision framework	General MCDM	Established foundational principles of multi-criteria decision-making; does not address uncertainty, hesitation, inter-criteria interaction, or goal-type preference modelling.
Saaty (2013)	Hierarchical structuring and weighting	Pairwise comparisons	AHP	Provides systematic weighting and consistency checking; limited in modelling linguistic hesitation, fuzzy uncertainty, and target-oriented criteria.
Hwang and Yoon (1981)	Distance-based evaluation	Ideal and anti-ideal solutions	TOPSIS	Widely applied and intuitive; fundamentally assumes benefit/cost criteria and monotonic preference behaviour; does not explicitly support goal-type (target) preferences.
Chu (2004)	Additive decision rule	Weighted summation	SAW	Simple and transparent ranking mechanism; sensitive to normalization and assumes monotonic benefit–cost orientation; lacks direct support for target-oriented criteria.
Blouchoutzi et al. (2021)	Preference-based outranking	Preference functions	PROMETHEE	Flexible in modelling pairwise preferences; highly sensitive to the choice of preference functions and thresholds, introducing potential subjectivity in rankings.

Stanujkic et al. (2012)	Ratio-based multi-objective optimization	Ratio system	MOORA	Computationally efficient and simple; relies on benefit/cost classification and does not incorporate explicit goal-type preference modelling.
Bid and Siddique (2019)	Hybrid additive–multiplicative ranking	Weighted sum and weighted product	WASPAS	Enhances robustness via hybrid aggregation; still assumes monotonic preference behaviour and lacks native handling of target-oriented criteria.
Zadeh (1965)	Uncertainty modelling	Fuzzy set theory	–	Introduced membership-based modelling for vagueness; does not explicitly represent hesitation with multiple possible membership values.
Chen (2000)	Fuzzy ranking under uncertainty	Fuzzy extension of TOPSIS	Fuzzy TOPSIS	Transforms linguistic evaluations into fuzzy values; still preserves monotonic benefit/cost structure and may lose detailed hesitation information.
Torra (2010)	Expert hesitation modelling	Hesitant fuzzy sets (HFS)	–	Allows multiple membership degrees per evaluation; requires suitable aggregation operators and ranking models to complete decision-making.
Zhao et al. (2014)	Hesitant triangular representation	HTFS / HTFEs	–	Extends hesitant modelling using triangular fuzzy numbers; lacks interaction-aware aggregation and a dedicated ranking mechanism.
Xia and Xu (2011)	Comparison mechanism	Score function for HFS	Score-based ranking	Provides a basic comparison rule for hesitant fuzzy elements; insufficient alone for full MCDM ranking after aggregation.

Beliakov et al. (2007)	Interaction-aware aggregation	Heronian Mean (HM)	–	Captures pairwise interactions among criteria; classical form is not applicable to hesitant triangular fuzzy environments.
Yu (2013)	Geometric interaction modelling	Geometric Heronian Mean (GHM)	–	Introduces nonlinear interaction aggregation; motivates extension to hesitant triangular fuzzy settings.
Shi et al. (2009)	Parameterized interaction aggregation	Generalized Geometric Heronian Mean	–	Introduces parameters (p,q) to control interaction strength; requires extension and justification for hesitant triangular fuzzy aggregation.

2.9 Gap Identified in the Literature

While extensive research has been conducted on fuzzy and hesitant-based MCDM models, there remain significant gaps that necessitate further analysis. Numerous established MCDM strategies like TOPSIS, VIKOR, and SAW tend to prioritize benefit and cost considerations, presuming that each decision is geared towards maximizing or minimizing effects (Ahmadi et al., 2010; Chu, 2004). This overly simplistic perspective overlooks the presence of goal-type criteria, where the optimal outcome is situated within a moderate range rather than at the extremes (Zopounidis & Doumpos, 2017). Thus, ranking methods are needed to manage multiple criteria preferences in a consistent system.

Secondly, although hesitant fuzzy sets and triangular fuzzy numbers have improved uncertainty modeling, current aggregation operators frequently fall short in adequately capturing inter-criterion dependencies (B, 2023). The prevailing fuzzy aggregation frameworks predominantly depend on weighted averaging or geometric means that treat each criterion in isolation, disregarding their possible interconnections. This shortcoming diminishes their representational strength in intricate decision-making situations. Therefore, a more comprehensive aggregation model like the proposed Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean (HTFGGHM) is essential to simultaneously accommodate both hesitation and interdependence among criteria.

Finally, a methodological gap still exists in creating efficient yet interpretable ranking mechanisms that can handle aggregated hesitant fuzzy data (Saha et al., 2022; Zeng et al., 2019). The majority of ranking models that exist now are either very complicated in terms of mathematics or demand a lot of computational effort, which limits their practicality for real-world use. This research introduces the Simplified Yielded Aggregation Index (SYAI) technique, which effectively fills this void by supplying a clear and computationally efficient structure for assessing alternatives. Combined with the HTFGGHM operator, it establishes a groundbreaking decision-support framework that harmonizes accuracy, simplicity, and interpretability addressing the existing research gap in linguistic hesitant fuzzy MCDM systems. Table 2.2 summarize the research gaps found in the literature.

Table 2.2
Summary of Gaps Identified in the Literature Review

Research Gap	Identified Issues / Opportunities
Handling of Goal-Type Criteria in MCDM	Traditional MCDM methods such as TOPSIS, VIKOR, and SAW mainly focus on benefit and cost criteria, where the best value is assumed to be either the maximum or minimum value . However, some real decision-making problems involve goal-type criteria, where the best value lies near a target or desired level rather than at the extremes . This creates a need for a ranking method that can handle benefit, cost, and goal-type criteria in a consistent framework.
Representation of Hesitant Linguistic Information	Expert evaluations are often expressed using linguistic terms and may involve hesitation, especially when experts are unsure between several possible judgments. Although hesitant fuzzy sets and triangular fuzzy numbers have been used to represent uncertainty, many studies still focus on single valued or simplified fuzzy information. This indicates the need for a hesitant triangular fuzzy representation that can preserve both linguistic uncertainty and expert hesitation.
Aggregation under Hesitant Triangular Fuzzy Environment	Existing fuzzy aggregation operators often aggregate criteria independently and may not sufficiently capture the interrelationship among criteria . In many decision making problems, criteria may influence one another. Therefore, a more suitable aggregation operator is needed to manage hesitant triangular fuzzy information while considering the interaction among criteria. This motivates the use of the Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean (HTFGGHM) operator.
Limited Use of Geometric Heronian Mean in Hesitant Triangular Fuzzy Setting	Previous studies have improved Heronian mean operators in several fuzzy environments, but the use of the generalized geometric Heronian mean structure in hesitant triangular fuzzy environments remains limited. The geometric Heronian structure is useful because it can reduce excessive compensation among criteria and capture pairwise relationships. This provides an opportunity to extend the generalized geometric Heronian mean into the hesitant triangular fuzzy setting.
Integration of Linguistic Hesitant Fuzzy Evaluation with Ranking Models	Although linguistic and hesitant fuzzy evaluations provide better representation of expert uncertainty, limited studies integrate them with simple and transparent ranking methods . The integration of hesitant triangular fuzzy aggregation with the Simplified

	Yielded Aggregation Index (SYAI) can improve the ranking process while maintaining interpretability and computational simplicity.
Need for a Simplified and Transparent MCDM Framework	Many fuzzy-based MCDM frameworks are mathematically complex and may be difficult for practitioners to apply . A simplified decision making framework is needed to reduce computational burden while still producing reliable and interpretable results. The SYAI-HTFGGHM framework addresses this need by combining fuzzy aggregation with a clear ranking mechanism.
Limited Application in Educational Decision-Making	Most fuzzy MCDM studies focus on areas such as supplier selection, healthcare, sustainability, and engineering problems. Limited attention has been given to educational decision-making, particularly in evaluating humor-based teaching strategies in language learning. This creates an opportunity to apply the proposed SYAI-HTFGGHM framework in an educational context involving subjective expert judgments, linguistic evaluations, and qualitative teaching criteria.
Decision-Maker Weight Consideration	Several existing MCDM studies assign different weights to decision makers based on experience, expertise, or authority. However, in some educational evaluation settings, experts may be treated with equal importance when they have comparable qualifications and teaching backgrounds. This study adopts equal decision-maker importance, but the absence of decision-maker weights is recognized as a limitation and can be extended in future research.

Therefore, there exists a need to develop a simplified and transparent decision making framework that can manage hesitant triangular fuzzy information, capture interrelationships among criteria, and rank alternatives under different criterion types. The integration of the Simplified Yielded Aggregation Index (SYAI) with the Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean (HTFGGHM) operator is proposed to address these gaps. The SYAI method provides a structured ranking mechanism, while the HTFGGHM operator supports the aggregation of hesitant linguistic evaluations by considering criterion interaction. Hence, the proposed SYAI-HTFGGHM framework is expected to offer a practical and interpretable approach for solving uncertain multi-criteria decision making problems, particularly in educational evaluation contexts.

2.10 Definitions and Terminology

This section defines mathematical framework that was related to HTFGGHM. The purpose of this section is to give a brief literature review of the concepts that have been developed previously in Hesitant Fuzzy, Triangular Fuzzy Numbers (TFNs), Heronian Mean (HM), Generalized Heronian Mean (GM).

2.10.1 Hesitant Fuzzy Set (HFS)

Definition 2.10.1. Fuzzy set (Zadeh, 1965). A fuzzy set A within a finite universe of discourse can be described as

$$A = \{\langle x, \mu_A(x) \rangle | x \in X\} \quad (2.1)$$

where $\mu_A(x) : X \rightarrow [0, 1]$ represents the membership function of A , where $\mu_A(x)$ specifies the degree of membership of an element $x \in X$ within A . Torra and Narukawa (2009) expanded on the concept of fuzzy sets by introducing hesitant fuzzy sets, which permit the membership degree of an element to include several possible values between 0 and 1. HFSs effectively capture situations where individuals express varying degrees of uncertainty in their preferences, enhancing decision-making by accommodating hesitancy.

Definition 2.10.2. Hesitant Fuzzy set: (Torra, 2010) Let $X = \{x_1, x_2, \dots, x_n\}$ be a reference set. A set E defined in X is represented as:

$$E = \{\langle x, h_E(x) \rangle | x \in X\}$$

where $h_E(x)$ is a set of various values within $[0, 1]$, representing the potential membership degrees of the element $x \in X$ in the set E . This is known as a hesitant fuzzy set. Additionally, Torra (2010) introduced the concepts of the 'empty hesitant fuzzy set' and the 'full hesitant fuzzy set' as follows:

$$\begin{aligned} E^\circ &= \{\langle x, h_{E^\circ}(x) \rangle | x \in X\} \text{ where } h_{E^\circ}(x) = \{0\} \forall x \in X, \\ E^* &= \{\langle x, h_{E^*}(x) \rangle | x \in X\} \text{ where } h_{E^*}(x) = \{1\} \forall x \in X \end{aligned} \quad (2.3)$$

2.10.2 Triangular Fuzzy Numbers

Definition 2.10.3. Triangular Fuzzy Numbers Fuzzy set: (Van Laarhoven & Pedrycz, 1983). A triangular fuzzy number $\tilde{a}$ can be defined by a triplet (a^L, a^M, a^U) . The membership function $u_{\tilde{a}}(x)$ is defined as:

$$u_{\tilde{a}}(x) = \begin{cases} 0, & x < a_L \\ \frac{x - a_L}{a_M - a_L}, & a_L \leq x \leq a_M \\ \frac{a_U - x}{a_U - a_M}, & a_M \leq x \leq a_U \\ 0, & x > a_U \end{cases} \quad (2.4)$$

where $0 < a_L \leq a_M \leq a_U$, a_L and a_U stand for the lower and upper values of the support of $\tilde{a}$, respectively, and a_M for the modal value.

Definition 2.10.4. Operational laws Triangular Numbers (Van Laarhoven & Pedrycz, 1983). Basic operational laws relating to fuzzy triangular numbers:

$$\tilde{a} \oplus \tilde{b} = [a_L, a_M, a_U] \oplus [b_L, b_M, b_U] = [a_L + b_L, a_M + b_M, a_U + b_U] \quad (2.5)$$

$$\tilde{a} \otimes \tilde{b} = [a_L, a_M, a_U] \otimes [b_L, b_M, b_U] = [a_L b_L, a_M b_M, a_U b_U] \quad (2.6)$$

$$\lambda \otimes \tilde{a} = \lambda \otimes [a_L, a_M, a_U] = [\lambda a_L, \lambda a_M, \lambda a_U], \lambda > 0 \quad (2.7)$$

Definition 2.10.5. Degree of possibility (Xu, 2009). Let $\tilde{b} = [b_L, b_M, b_U]$ and $\tilde{a} = [a_L, a_M, a_U]$ be two triangular fuzzy numbers, then the degree of possibility of $a \geq b$ is defined as:

$$\begin{aligned} p(a \geq b) = & \lambda \max \left\{ 1 - \max \left[\frac{b^M - a^L}{a^M - a^L + b^M - b^L}, 0 \right], 0 \right\} \\ & + (1 - \lambda) \max \left\{ 1 \right. \\ & \left. - \max \left[\frac{b^U - a^M}{a^U - a^M + b^U - b^M}, 0 \right], 0 \right\} \end{aligned} \quad (2.8)$$

where the value λ is an index of rating attitude. It reflects the decision maker's risk-bearing attitude. If $\lambda < 0.5$, the decision maker is risk averse.

From **Definition 2.10.5**, we can easily get the following results:

$$0 \leq p(\tilde{a} \geq \tilde{b}) \leq 1, 0 \leq p(\tilde{b} \geq \tilde{a}) \leq 1; \quad (2.9)$$

$$p(\tilde{a} \geq \tilde{b}) + p(\tilde{b} \geq \tilde{a}) = 1. \quad (2.10)$$

$$p(\tilde{a} \geq \tilde{a}) = p(\tilde{b} \geq \tilde{b}) = 0.5 \quad (2.11)$$

2.10.3 Hesitant Triangular Fuzzy Set (HTFS)

Definition 2.10.6. Hesitant Triangular Fuzzy Set (Zhao et al., 2014). Let X be a fixed set, a hesitant triangular fuzzy set (HTFS) on X is in terms of a function that, when applied to each x in X , returns a subset of values in $[0,1]$.

$$A = \{\langle x, \mu_A(x) \rangle \mid x \in X\} \quad (2.12)$$

where $\tilde{h}_E(x)$ is a set of some possible triangular fuzzy values in $[0,1]$, denoting the possible membership degrees of the element $x \in X$ to the set E . For convenience, we call $\tilde{h}_E(x) = \tilde{h}_1 = (\gamma_L, \gamma_M, \gamma_R)$ a hesitant triangular fuzzy element (HTFE) and $\tilde{h}$ the set of all HTFEs.

Given three HTFEs $\tilde{h} = (\gamma_L, \gamma_M, \gamma_R)$, $\tilde{h}_1 = (\gamma_1^L, \gamma_1^M, \gamma_1^R)$, and $\tilde{h}_2 = (\gamma_2^L, \gamma_2^M, \gamma_2^R)$, and $\lambda > 0$, we define their operations as follows:

$$h^\lambda = \bigcup_{x \in h} (\gamma^\lambda) \quad (2.13)$$

$$\lambda h = \bigcup_{x \in h} (1 - (1 - \gamma)^\lambda) \quad (2.14)$$

$$h_1 \oplus h_2 = \bigcup_{x_1 \in h_1, x_2 \in h_2} (\gamma_1 + \gamma_2 - \gamma_1 \gamma_2); \quad (2.15)$$

$$h_1 \otimes h_2 = \bigcup_{x_1 \in h_1, x_2 \in h_2} (\gamma_1 \gamma_2). \quad (2.16)$$

Definition 2.10.7. Score Function (Xia & Xu, 2011). Consider a hesitant fuzzy element h ; the score function S of an HFE is defined as:

$$S(h) = \frac{1}{\#h} \sum_{\gamma \in h} \gamma, \quad (2.17)$$

where $\#h$ represents the total number of elements in h .

Definition 2.10.8. (Xia & Xu, 2011) Let h_1 and h_2 be two hesitant fuzzy elements with scores $S(h_1)$ and $S(h_2)$, respectively. Then:

1. h_1 is larger than h_2 , denoted by $h_1 > h_2$, if $S(h_1) > S(h_2)$.
2. h_1 is considered equal to h_2 if $S(h_1) = S(h_2)$.

2.10.4 Heronian Mean

The Heronian Mean (HM) is an aggregation method notable for its capability to capture the relationships among the inputs. The HM is defined as follows:

Definition 2.10.9. Heronian Mean (Beliakov et al., 2007). For a set of nonnegative real numbers a_i where $i = 1, 2, \dots, n$, the Heronian mean is defined as:

$$HM(a_1, a_2, \dots, a_n) = \frac{2}{n(n+1)} \sum_{i,j=1}^n \sqrt{a_i a_j}. \quad (2.18)$$

Definition 2.10.10. Geometric Heronian Mean Building on Definition 2.10.9, Yu (2013) introduced the Geometric Heronian Mean (GHM) for a set of nonnegative real numbers a_i , where $i = 1, 2, \dots, n$, which is defined by:

$$GHM(a_1, a_2, \dots, a_n) = \prod_{i,j=1}^n \left(\frac{a_i + a_j}{2} \right)^{\frac{2}{n(n+1)}}. \quad (2.19)$$

Additionally, inspired by the geometric Bonferroni mean, proposed the Generalized Geometric Heronian Mean (GGHM) as follows

Definition 2.10.11. Generalized Geometric Heronian Mean .For nonnegative real numbers a_i where $i = 1, 2, \dots, n$ and parameters $p, q \geq 0$ (with p and q not both equal to 0), the Generalized Geometric Heronian Mean is defined as:

$$GGHM^{p,q}(a_1, a_2, \dots, a_n) = \frac{1}{p+q} \prod_{i,j=1}^n (p a_i + q a_j)^{\frac{2}{n(n+1)}}. \quad (2.20)$$

The properties of $GGHM^{p,q}$ are as follows:

1. $GGHM^{p,q}(0, 0, \dots, 0) = 0$ and $GGHM^{p,q}(1, 1, \dots, 1) = 1$;
2. $GGHM^{p,q}(a_1, a_2, \dots, a_n) = a$ if all $a_i = a$;
3. If $(a_i \leq b_i) \forall i$, then $GGHM^{p,q}(a_1, a_2, \dots, a_n) \leq GGHM^{p,q}(b_1, b_2, \dots, b_n)$, indicating $GGHM^{p,q}$ is monotonic;
4. $\min_i \{a_i\} \leq GGHM^{p,q}(a_1, a_2, \dots, a_n) \leq \max_i \{a_i\}$.

CHAPTER 3

SIMPLIFIED YIELDED AGGREGATION INDEX (SYAI)

METHOD FOR ENHANCED MULTI CRITERIA DECISION MAKING

3.1 Introduction

The Simplified Yielded Aggregation Index (SYAI) is crafted as an innovative yet clear decision-making framework, primarily aimed at integrating normalization, weighting, and aggregation into a cohesive, flexible format for Multi-Criteria Decision-Making (MCDM). A significant feature of SYAI is its integrated normalization rule that functions uniformly across various criterion types; benefit, cost, and the newly established goal-type criterion eliminating the need for users to change formulas or implement improvised transformations when the criteria vary in direction. Specifically, the goal-type criterion addresses realistic objectives where the optimal value is not simply the maximum or the minimum but rather a targeted intermediate level (for instance, target temperature, utilization, or dosage). In addition to normalization, SYAI offers a yielded aggregation index that yields bounded, interpretable scores within $(0,1)$, alongside a customizable sensitivity parameter β that clearly balances preference toward the ideal while mitigating aversion to the anti-ideal. Overall, these design attributes stress clarity, ease of understanding, and rapid computation, while staying firmly based on a strong mathematical structure.

This chapter addresses Research Objective 1 (RO1) by introducing, formalizing, and validating the SYAI methodology to enhance MCDM. Section 3.2 and Section 3.3 delineates the comprehensive framework and theoretical underpinnings, highlighting the single-rule normalization that effectively manages benefit, cost, and goal-type criteria. Section 3.5 outlines the important mathematical definitions, symbols, and initial concepts that are vital for creating the formal architecture of SYAI. Section 3.6 outlines the sequential computation process (normalization, weighting, distance assessment, and yielded aggregation with β), ensuring both reproducibility and clarity. Section 3.9 presents a practical numerical example for a mixed-criteria issue, illustrating the effective application of the unified normalization and goal-type processing. Lastly, Section 3.9.3 discusses validation and robustness analyses (sensitivity to β and weights,

scalability), confirming that SYAI yields consistent rankings, minimal computational burden, and applicability across various domains.

3.2 Inspired Foundation of the SYAI Framework

The Simplified Yielded Aggregation Index (SYAI) technique is conceptually motivated by the distance-based philosophy of decision making that is most often used in classical MCDM methods, specifically the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method. The main concept of this philosophy is that a desirable alternative must be both close to a positive ideal solution and distant to a negative ideal solution. This anti-ideal and ideal reasoning offers an intuitive and interpretive foundation on the ranking of alternatives, particularly in decision situations which have multiple and possibly conflicting criteria. This principle is the basic concept that SYAI uses as its fundamental guideline when it aimed to simplify and generalize the computational structure in a more accommodating way to practical decision-making cases.

The name Simplified Yielded Aggregation Index (SYAI) is established based on three main components of the proposed method. The term *Simplified* refers to the use of a single normalization structure that can handle benefit, cost, and goal-type criteria without requiring separate formulas. The term *Yielded* refers to the final score generated from the interaction between the ideal distance and the anti ideal distance through the sensitivity parameter β . The term *Aggregation Index* refers to the final bounded index value used to combine normalized and weighted criterion information into a single ranking score. Therefore, the name SYAI reflects the main purpose of the method, which is to provide a simplified, interpretable, and flexible aggregation index for MCDM problems.

While classical TOPSIS effectively applies distance measures to benefit and cost criteria, it often requires different normalization rules and additional transformations when mixed or non-extreme preferences are present. In real-world problems, however, decision criteria frequently include goal-type characteristics, where optimal performance is achieved at a specific target rather than at a maximum or minimum value. The SYAI framework is motivated by the need to retain the intuitive distance-to-ideal logic of TOPSIS while eliminating the complexity caused by multiple normalization schemes. To this end, SYAI introduces a unified normalization

mechanism that consistently processes benefit, cost, and goal-type criteria within a single mathematical expression, ensuring bounded and comparable normalized values across all criteria.

Furthermore, SYAI extends the traditional TOPSIS closeness concept by incorporating a sensitivity control parameter, denoted by β , which regulates the relative influence of proximity to the ideal solution and distance from the anti-ideal solution. Unlike the fixed closeness coefficient in TOPSIS, the yielded aggregation index in SYAI allows decision-makers to adjust the decision attitude according to the context, ranging from conservative (risk-averse) to performance-oriented preferences. This flexibility enables SYAI to adapt to diverse decision environments without altering its fundamental structure. Consequently, SYAI can be regarded as a TOPSIS-inspired framework that preserves the intuitive geometric interpretation of ideal and anti-ideal solutions while enhancing simplicity, adaptability, and applicability in mixed-criteria decision-making problems.

3.3 Research Design

This part focuses on the processes used to establish the Simplified Yielded Aggregation Index (SYAI) method for MCDM. The SYAI system was created to combine the processes of normalization, weighting, and aggregation into a holistic, logical framework that can fit various categories of criteria specifically including benefit, cost, and goal-type criteria. Common MCDM strategies, including TOPSIS, VIKOR, and WASPAS, usually implement multiple transformation rules that consider each criterion category independently, which can create inconsistencies and elevate computational loads. The SYAI method addresses this redundancy by presenting a single normalization expression that can be utilized across all criterion types at once. Also, a varied control parameter, β , is introduced to change the sensitivity of ranking results in relation to the decision-maker's preferences. SYAI's approach to design revolves around prioritizing straightforwardness, transparency, and computational effectiveness while upholding a strong mathematical basis.

The development of the SYAI method was guided by the following key considerations:

- (a) To design a single normalization rule that works consistently across all criterion types and preserves the original data structure.
- (b) To introduce a yielded aggregation index that produces bounded and interpretable results between 0 and 1.
- (c) To incorporate a parameter that allows adaptive control of ranking behaviour depending on the decision context.
- (d) To maintain linear computational complexity so that the method remains efficient for large-scale problems.

This methodological construction satisfies Research Objective 1 (RO1) of the study, which is to develop the SYAI method for MCDM.

3.4 Data Requirements and Decision Context

The SYAI framework operates on a decision matrix that represents the evaluation of a finite set of alternatives under several evaluation criteria. Let $A = \{A_1, A_2, \dots, A_m\}$ denote the set of alternatives and $C = \{C_1, C_2, \dots, C_n\}$ the set of criteria. Each criterion may belong to one of three categories:

- **Benefit criterion:** a higher value of the performance measure is considered better (e.g., profit, efficiency, accuracy).
- **Cost criterion:** a lower value is preferable (e.g., expenditure, error rate, energy consumption).
- **Goal-type criterion:** the best performance is achieved at an intermediate or target value rather than at the maximum or minimum (e.g., optimal temperature, moderate workload, target utilisation).

The method is domain-independent and can be applied in diverse fields including healthcare evaluation, educational programme selection, environmental sustainability assessment, and industrial process optimisation. The ability of SYAI

to process mixed criterion types ensures that decision makers can analyse real-world scenarios more accurately and flexibly.

3.5 Notation and Preliminaries

The decision matrix X elements are derived using observed data, expert analysis, or other data that has been reported before based on the application scenario. Here, the numerical data included in the chosen case study is used to obtain the matrix values that reflect the measured performance of each alternative on the basis of the corresponding criteria. The basic representation of the decision environment is given by the matrix:

$$X = [x_{ij}]_{m \times n}, \quad x_{ij} \in \mathbb{R}, \quad (3.1)$$

where x_{ij} denotes the performance value of alternative A_i with respect to criterion C_j . Each criterion is assigned an importance weight w_j such that:

$$w_j \geq 0, \quad \sum_{j=1}^n w_j = 1. \quad (3.2)$$

The weights may be elicited from experts or computed by an objective technique such as the entropy or standard-deviation method. The decision matrix serves as the foundation upon which all subsequent normalization and aggregation steps are performed.

3.6 The SYAI Procedure

The overall computational procedure of the SYAI method is developed based on a systematic decision making process consisting of decision matrix construction, normalization, weighting, ideal and anti-ideal determination, distance measurement, score computation, and final ranking. Since SYAI is inspired by the ideal-solution concept commonly used in TOPSIS, Figure 3.1 presents a side by side comparison between the computational procedures of TOPSIS and SYAI. This comparison highlights the main differences between both methods, particularly in the normalization process, distance measurement, and final scoring mechanism.

The comparison shows that TOPSIS and SYAI follow a similar general decision-making structure, but they differ in how criterion types are treated. In TOPSIS, vector normalization is applied to all criteria, while the difference between benefit and cost criteria is mainly handled during the selection of the positive ideal solution (PIS) and negative ideal solution (NIS). In contrast, SYAI handles the criterion direction earlier through its unified normalization formula, where the reference value x_j^* is defined based on the criterion type. For benefit criteria, x_j^* represents the maximum value; for cost criteria, it represents the minimum value; and for goal-type criteria, it represents the target value. Therefore, SYAI directly accommodates benefit, cost, and goal-type criteria within the normalization stage, while TOPSIS mainly distinguishes benefit and cost criteria during the ideal solution stage.

Each step of the SYAI method is then elaborated below in descriptive form.

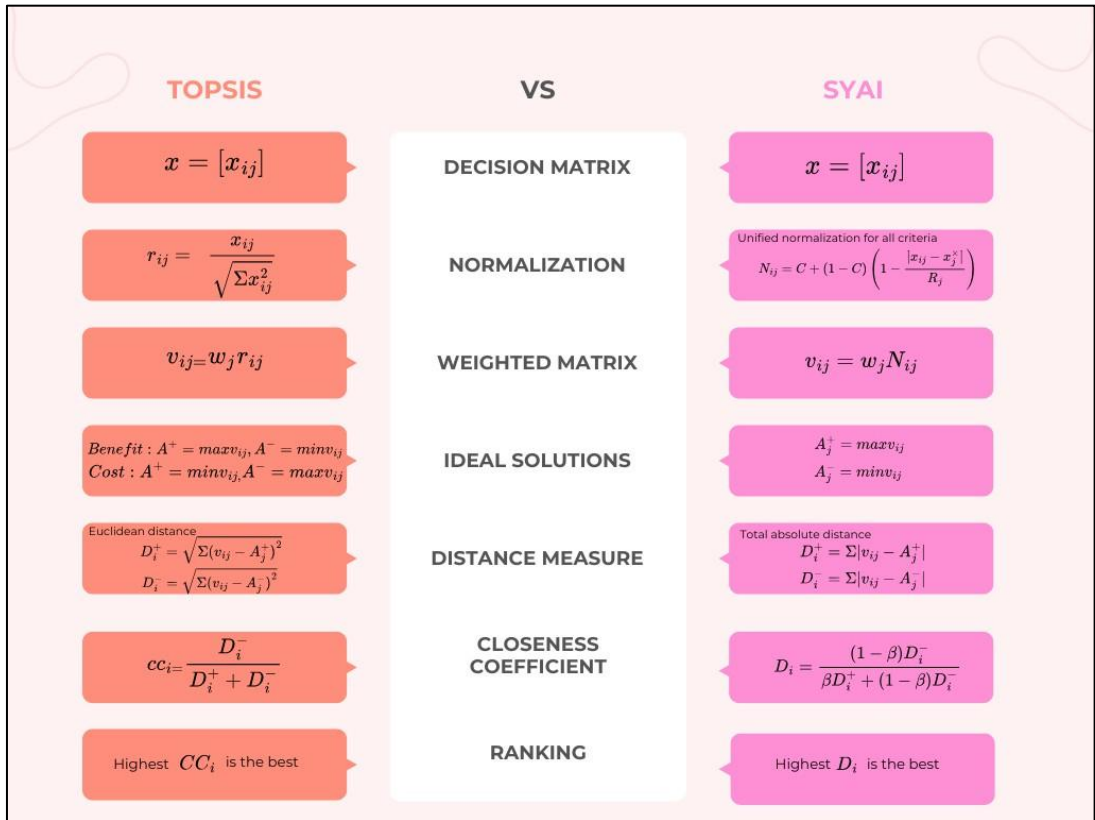


Figure 3.1 Side-by-Side Comparison Between TOPSIS and SYAI Computational Procedures

There are six (6) steps proposed under SYAI development as follows:

Step 1: Construction of the Decision Matrix

All relevant data are collected and organised into the decision matrix $X = [x_{ij}]$. Each row corresponds to an alternative, while each column represents a criterion. Data may originate from measurements, expert assessments, or aggregated survey responses. Before analysis, standard pre-processing methods like data imputation or exclusion of incomplete entries are used to handle missing or inconsistent data to ensure all x_{ij} values are valid for analysis. The SYAI method assumes a complete decision matrix and therefore requires all missing values to be resolved prior to implementation. Formally, the matrix is defined as:

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \quad (3.3)$$

Step 2: Unified Normalization

Normalization transforms heterogeneous raw data into dimensionless values so that criteria measured on different scales can be compared fairly. The SYAI method proposes a single normalization formula that accommodates benefit, cost, and goal-type criteria:

$$N_{ij} = C + (1 - C) \cdot \left(1 - \frac{|x_{ij} - x_j^*|}{R_j} \right), \quad (3.4)$$

where:

- C is a small positive additive constant (typically $C = 0.01$) that acts as a lower bound in the normalization process, ensuring numerical stability and guaranteeing that all normalized values remain strictly within the interval $(0,1]$.
- x_j^* denotes the reference or target value for criterion C_j : the maximum value for benefit criteria, the minimum value for cost criteria, and a specific target value for goal-type criteria.
- $R_j = \max_i x_{ij} - \min_i x_{ij}$ represents the range of observed values for criterion C_j .

Equation (3.4) guarantees that all normalized values fall within the interval (0,1]. The closer x_{ij} is to its ideal or target x_j^* , the larger the normalized value N_{ij} becomes. Unlike classical piecewise formulas, this expression uses a unified structure that reduces computational complexity and removes ambiguity when switching between criterion types.

Step 3: Weighted Normalized Matrix

The relative importance of each criterion is incorporated through weighting. The weighted normalized values are obtained by:

$$v_{ij} = w_j \times N_{ij}, \quad \sum_{j=1}^n w_j = 1. \quad (3.5)$$

This transformation ensures that criteria with higher importance contribute proportionally more to the overall assessment. The resulting matrix $V = [v_{ij}]$ captures both the normalized performance and the decision maker's priorities.

Step 4: Determination of Ideal and Anti-Ideal Values

After weighting, the best and worst achievable scores for each criterion are determined as:

$$A_j^+ = \max_i v_{ij}, \quad A_j^- = \min_i v_{ij}. \quad (3.6)$$

The positive ideal solution A^+ represents the hypothetical alternative that performs best on all criteria simultaneously, whereas the negative ideal solution A^- represents the least desirable combination. These two reference points serve as anchors for measuring how close each real alternative is to the ideal or away from the anti-ideal.

Step 5: Computation of Distances and Closeness Coefficient

The distance of each alternative from the ideal and anti-ideal points is computed using absolute differences:

$$D_i^+ = \sum_{j=1}^n |v_{ij} - A_j^+|, \quad D_i^- = \sum_{j=1}^n |v_{ij} - A_j^-|. \quad (3.7)$$

These values indicate how far an alternative is from the best and worst performance boundaries respectively.

Note that: D_i^+ should be minimised while D_i^- should be maximised for an alternative to be considered optimal.

The Simplified Yielded Aggregation Index (SYAI) for each alternative is then given by:

$$D_i = \frac{(1 - \beta) D_i^-}{\beta D_i^+ + (1 - \beta) D_i^-}, \quad 0 < \beta < 1. \quad (3.8)$$

Here, β is a control parameter that allows adjustment of the ranking emphasis:

- When $\beta > 0.5$, the method prioritises proximity to the ideal solution.
- When $\beta < 0.5$, it emphasises avoidance of the worst alternative.
- When $\beta = 0.5$, both tendencies are balanced, representing a neutral preference.

A practical explanation of the parameter β can be given by the following examples:

- **When $\beta > 0.5$:** this setting is appropriate to performance-based decisions like which system is fastest, which product is the best where the closeness to the ideal solution is valued.
- **When $\beta < 0.5$:** this setting is suitable in risk-averse or safety-critical contexts, such as deciding between the options of a delivery that does not incur too much

delay or choosing a supplier that does not incur too much of a risk of poor performance, even though it may not be the most performing option.

- **When $\beta = 0.5$:** this setting is a balanced decision scenario, like general supplier or project selection problems, in which performance optimisation and risk avoidance are treated as equally important.

The coefficient D_i always lies between 0 and 1, providing a clear and interpretable measure of performance. This boundedness follows directly from the structure of Equation (3.8) since $D_i^+ \geq 0$ and $D_i^- \geq 0$ and the parameter satisfies $0 < \beta < 1$, both the numerator $(1 - \beta)D_i^-$ and the denominator $\beta D_i^+ + (1 - \beta)D_i^-$ are strictly nonnegative. Furthermore, the denominator always dominates the numerator because it contains the additional nonnegative term βD_i^+ , which guarantees that the yielded aggregation index satisfies $0 \leq D_i \leq 1$ for all alternatives.

A larger D_i implies a better alternative. This simplicity and boundedness distinguish SYAI from other distance-based approaches that may yield unscaled or negative values.

Step 6: Ranking of Alternatives

Finally, all alternatives are ranked in descending order according to their D_i values. The alternative with the highest SYAI value is considered the most desirable, followed by others in decreasing order. Because all computations use normalized data, the ranking results are consistent even when criteria are expressed in different measurement units.

3.7 Parameterization and Weighting

The SYAI approach provides the flexibility of two variables under control: the weighting vector w and the sensitivity parameter beta. The weighting vector, which is denoted as w_j , denotes the subjective or objective weight of each criterion and can be calculated through a number of methods which include expert, pair-wise, and entropy weighting. The decision outcome will be representative because the right weights are assigned to it to mirror the priorities and preferences of the involved stakeholders.

At the same time, the parameter, β , acts as the regulator of the sensitivity of the decision and has an effect on the response of the model to the variations in the performance of some alternatives. It is a variable whose value can be altered with regard to the context of the decision. When dealing with situations that are high-risk or safety-critical, the value of 0.5 or less is appropriate, as it will eliminate alternatives that may have poor performance. On the other hand, when using performance-based or competitive testing, a greater value of a positive β , that is, when β is greater than 0.5, enhances the choice of alternatives being near to the optimal solution. A balanced weight to optimistic and conservative tendencies in decisions is given by a neutral value of 0.5 of the value of β . This flexibility makes SYAI method to reflect the broad decision attitudes without any changes in its basic structure.

Parameter β is a decision sensitivity controller that defines the way the SYAI approach reacts to changes in alternative performance. Its interpretive meaning may be summarized as follows:

- When $\beta < 0.5$, the result will be in a more conservative and cautious decision-making process, which focuses on avoiding low performing alternatives in high risk or safety critical situations.
- When $\beta > 0.5$, the model depicts a more positive or performance-based approach, encouraging solutions that are closely comparable to an ideal solution.
- When $\beta = 0.5$, the decision behaviour is in equilibrium, which allows a balanced weighing between optimistic and conservative behaviour.

This interpretation emphasizes the fact that the parameter β represents the attitude of the decision maker and therefore the SYAI framework can easily adjust to various decision-making contexts and does not have to change the structure formulation of the framework.

3.8 Validation and Robustness Analysis

To ensure the reliability of the SYAI framework, several validation analyses are proposed:

- (a) **Numerical validation:** Testing SYAI with benchmark datasets to confirm that all normalized values remain within the theoretical bounds $(0,1]$ and that ranking stability is maintained under moderate data perturbation.
- (b) **Sensitivity analysis:** Varying β and weights w_j to evaluate how ranking positions change when decision-maker preferences differ. Minor changes in β should not drastically alter the order of alternatives.
- (c) **Scalability test:** Applying SYAI to larger decision matrices (e.g., 10×10 or 15×12) to verify computational efficiency and consistency of results.

3.9 Numerical Example

In this section, a numerical example demonstrate the implementation of the Simplified Yielded Aggregation Index (SYAI) method for evaluating and ranking three alternatives (A_1, A_2, A_3) based on four criteria: Cost, Quality, Delivery Time, and Temperature. The objective is to identify the most suitable alternative by applying the SYAI framework.

Problem Context and Criteria

The problem considers the evaluation of three alternatives under the following criteria:

- Cost (C_1): Price of the alternative (Cost criterion).
- Quality (C_2): Performance quality score (Benefit criterion).
- Delivery Time (C_3): Number of days for delivery (Cost criterion).
- Temperature (C_4): Temperature level, aiming for a target of 60°C (Goal criterion).

Criteria Weights and Types

Table 3.1 summarizes the criteria, types, and assigned weights for criteria.

Table 3.1
Criteria, Types, and Weights of Criteria.

Criterion	Description	Type	Weight (w_j)
C1	Cost	Cost	0.25
C2	Quality	Benefit	0.25
C3	Delivery Time	Cost	0.25
C4	Temperature	Goal (60°C)	0.25

Initial Data and Decision Matrix

Table 3.2 shows the raw decision matrix between alternatives and criteria.

Table 3.2
Decision Matrix between Alternatives and Criteria.

Alternative	Cost	Quality	Delivery Time	Temperature
A1	200	8	4	30
A2	250	7	5	60
A3	300	9	6	85

3.9.1 Step by Step Procedure

This section shows the step-by-step procedure of SYAI method using the numerical example.

Step 1: Compute Decision Matrix

The SYAI method relies on the decision matrix to store the basic performance values for each alternative against various criteria. In this step, the rows display the alternatives and the columns represent the different evaluation criteria. Ratings in the matrix are made by experts, from earlier data or by measuring the alternatives, showing how much each alternative matches the different criteria.

This form of representation makes it easier to assess, standardize and compare data in the next phases of the SYAI process as shown in Table 3.2.

Step 2: Normalize the Decision Matrix

Normalization converts all criteria to a dimensionless scale so no single factor dominates. Using $C = 0.01$, apply the unified formula using Equation (3.4) Using $C = 0.01$, $\max \{300, 9, 6, 85\}$, $\min \{200, 7, 4, 30\}$, and ideal $\{200, 9, 4, 60\}$, the normalized matrix is shown in Table 3.3

Table 3.3
Normalized Decision Matrix.

Restaurant	C1	C2	C3	C4
A1	1	0.505	1	0.46
A2	0.505	0.01	0.505	1
A3	0.01	1	0.01	0.55

For illustration, two example calculations are shown below.

For C_1 (Cost criterion), the ideal value is $x_1^* = 200$ and the range is $R_1 = 300 - 200 = 100$. Thus, for A_2 ,

$$N_{21} = 0.01 + (1 - 0.01) \left(1 - \frac{|250 - 200|}{100} \right) = 0.505. \quad (3.9)$$

For C_2 (Quality criterion), the ideal value is $x_2^* = 9$ and the range is $R_2 = 9 - 7 = 2$. Thus, for A_1 ,

$$N_{12} = 0.01 + (1 - 0.01) \left(1 - \frac{|8 - 9|}{2} \right) = 0.505. \quad (3.10)$$

The remaining normalized values are computed using the same unified normalization formula. Normalization ensures that each criterion, regardless of its

original scale, contributes fairly to the final evaluation. This unified approach allows consistent comparison across cost, benefit, and target-type criteria, improving the robustness and interpretability of the decision-making process.

Step 3: Calculate Weighted Normalized Matrix

Weights $w_j = 0.25$ are applied uniformly by Equation (3.5) and the results is shown in Table 3.4

Example Calculations for Weighted Normalized Matrix

Below are example calculations for Step 3 using the normalized values from Table 3.3 and equal weights $w_j = 0.25$:

$$AI, CI (Cost) \quad v_{11} = r_{11} \times w_1 = 1.000 \times 0.25 = 0.25000 \quad (3.11)$$

and Table 3.4 illustrates weighted normalized decision matrix:

Table 3.4
Weighted Normalized Decision Matrix.

Alternative	Cost	Quality	Delivery Time	Temp
A1	0.25000	0.12625	0.25000	0.11500
A2	0.12625	0.00250	0.12625	0.25000
A3	0.00250	0.25000	0.00250	0.13750

Step 4: Identify Yielded Ideal and Anti-Ideal Solutions

The ideal and anti-ideal solutions are based on Equation (3.6)

$$A_j^+ = [0.25, 0.25, 0.25, 0.25], \quad A_j^- = [0.0025, 0.0025, 0.0025, 0.115] \quad (3.12)$$

Step 5: Compute Weighted Closeness Score

Let's define the closeness score D_i based on distances from the ideal and anti-ideal points, blended using the parameter β , which is set to 0.5 in this study. The calculation is performed using Equation (3.8)

Example Calculation for A1 under C1 (Cost)

From the weighted normalized matrix:

$$v_{A1,C1} = 0.25000 \quad (3.13)$$

From the ideal and anti-ideal values:

$$A_{C1}^+ = 0.25000, \quad A_{C1}^- = 0.00250 \quad (3.14)$$

Summing across all four criteria using Equation (3.7) (not shown here in detail), we get:

$$D_1^+ = 0.25875, \quad D_1^- = 0.61875 \quad (3.15)$$

Using the closeness score formula using Equation (3.8) with $\beta = 0.5$:

$$\begin{aligned} D_1 &= \frac{(1 - \beta) D_i^-}{\beta D_i^+ + (1 - \beta) D_i^-} = \frac{0.5 \cdot 0.61875}{0.5 \cdot 0.25875 + 0.5 \cdot 0.61875} \\ &= \frac{0.309375}{0.43875} = 0.705128 \end{aligned} \quad (3.16)$$

The corresponding distances and resulting closeness scores are illustrated in Tables 3.5 and Table 3.6.

Table 3.5
Distances to Ideal and Anti-Ideal Solutions.

Alternative	Ideal Distance (D_i^+)	Anti-Ideal Distance (D_i^-)
A1	0.25875	0.61875
A2	0.49500	0.38250
A3	0.60750	0.27000

Table 3.6
Closeness Scores of Alternatives.

Alternative	Closeness Score (D_i)
A1	0.705128
A2	0.435897
A3	0.307692

In Table 3.5, the absolute distances of each alternative from the ideal and anti-ideal solutions are provided. These values were then used to compute the closeness scores shown in Table 3.6 using the weighted formula in Equation 3.8. The closeness score reflects how close each alternative is to the ideal condition while factoring in its distance from the least desirable outcome. As shown, Alternative A1 has the highest closeness score, suggesting it is the most favorable option. A2 and A3 follow, with lower scores indicating comparatively reduced performance across the evaluated criteria.

Step 6: Rank the Alternatives

Finally, the alternatives are ranked in descending order based on their computed closeness scores D_i , as shown in Figure 3.2. The alternative with the highest score is considered the most favorable, as it indicates closer proximity to the ideal solution and greater distance from the anti-ideal.

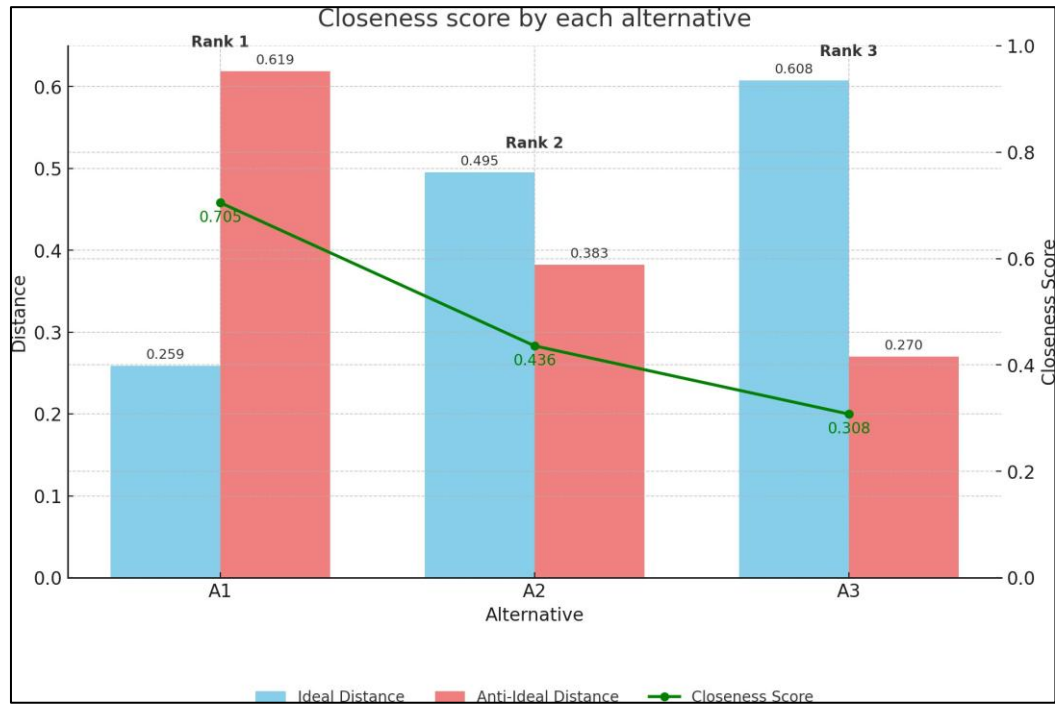


Figure 3.2 Closeness Score by Each Alternatives

From the Figure 3.2, Alternative A1 scores 0.705 for closeness, while A2 receives 0.436 and A3 gets 0.308. It is consistent with the expected distance relationships: A1 scores closest to the ideal, whereas A3 shows the greatest distance from the ideal. These results confirm that A1 is the optimal choice under the SYAI framework, supported by both the calculated scores and the graphical illustration.

3.9.2 Sensitivity Analysis

For multi-criteria decision-making, it is necessary to ensure that the final rankings remain solid when input conditions change. Sensitivity analysis is a useful way to check how changes in the decision variables affect the quality and stability of each option. For this study, both the weights given to the criteria and the β parameter are examined through sensitivity analysis in SYAI. This helps confirm that the closeness results are constant and the rankings of the alternatives are still secure if there are any changes in assumptions or preferences.

The analysis is performed in two stages. Weight sensitivity analysis means examining how the rankings alter when various weights are given to the different criteria. In β sensitivity analysis, discussed into how the importance given to both proximity options is reflected in the scores and places the locations receive. Altogether,

these analyses provide information on the model's sensitivity and steer decision-makers in choosing the appropriate parameters. Figure 3.3 shown the weight sensitivity for each alternatives.

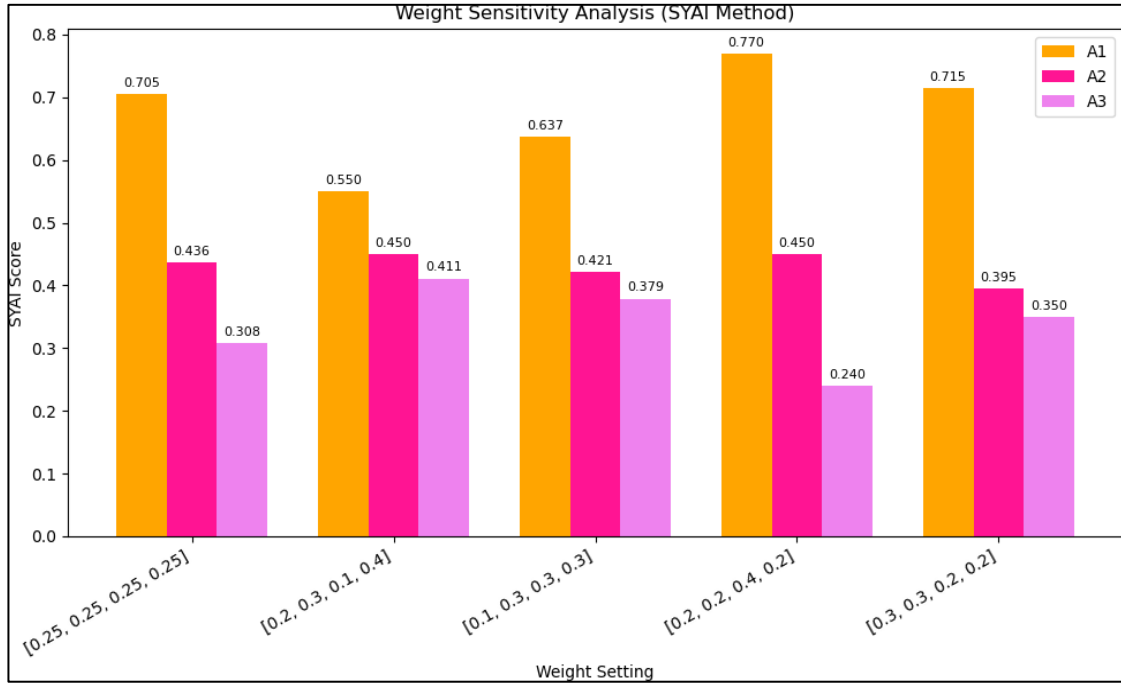


Figure 3.3 Weight Sensitivity Analysis (SYAI Method)

To test how sensitive the SYAI method is to weight adjustments, the evaluation considered how different weights were applied to the four decision criteria: Cost, Quality, Delivery Time, and Temperature as shown in Figure 3.3. In all cases, β was set to 0.5 so that both ideal and anti-ideal distances were given equal importance. All three alternatives (A1, A2, and A3) were compared across multiple settings, to reflect differences in the importance attached to each criterion.

Results show that the ranking of alternatives stays similar, regardless of the adjustments to the weights. Among all four scenarios, Alternative A1 always had the highest SYAI closeness, showing its high performance even with different weight priorities. Alternative A2 ranked second with scores between 0.395 and 0.45, and alternative A3 remained the least preferred with scores ranging from 0.24 to 0.4111. This repeated ranking clearly indicates that SYAI can handle different preferences among decision-makers.

In general, the SYAI with $\beta = 0.5$ shows that it can provide dependable conclusions, even when the strength of each factor changes. Because the average grades

remain stable, the approach can work well in settings where opinions on importance may be different between people or situations.

After examining weight sensitivity, the main goal in the following phase is to observe the impact of β sensitivity on the closeness scores and ranking results within the SYAI system. Figure 3.4 presents a line chart of β sensitivity.

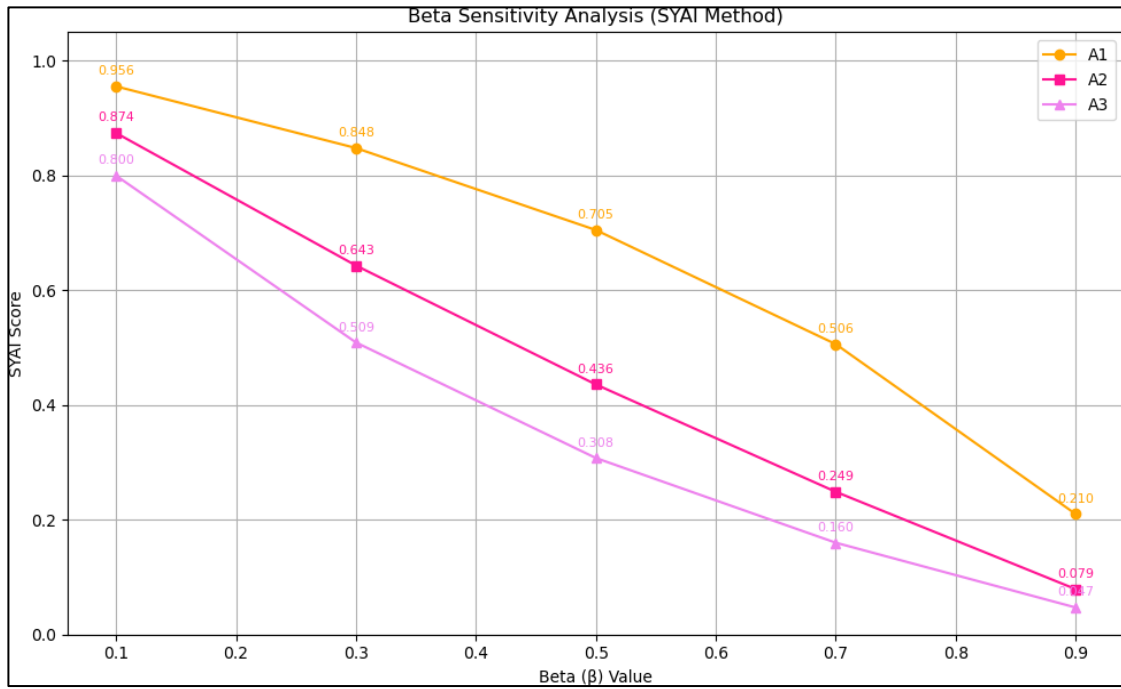


Figure 3.4 β Sensitivity Analysis (SYAI Method)

The line in the Figure 3.4 shows the impact of the β parameter on SYAI scores, but all criteria remain with the same weight of 0.25. The equal value given to each criterion makes the comparison fair and without bias. To control how close a solution is to the ideal and how far it is from the anti-ideal, four experts use the β parameter with five settings: 0.1, 0.3, 0.5, 0.7, and 0.9. Choosing a small β gives extra importance to avoiding the undesirable outcomes, whereas a large β focuses on selecting the very best (ideal) outcome.

According to the Figure 3.4, the order of alternatives does not change with different β values: A1 gets the highest SYAI score, A2 takes second place, and A3 comes third. For instance, A1 changes from a score of 0.9556 at $\beta = 0.1$ to 0.2099 at $\beta = 0.9$, while A2 goes from 0.8743 to 0.0791, and A3 falls from 0.8000 to 0.0471. Since β is paying less attention to anti-ideal closeness and more to ideal closeness, the drop

is reasonable. The fact that the SYAI rankings remain the same for all β values means the method is trustworthy for decision-making in any situation.

3.9.3 Comparative Analysis

In this comparative analysis, the Simplified Yielded Aggregation Index (SYAI) method is evaluated alongside other prominent Multi-Criteria Decision-Making (MCDM) methods such as TOPSIS, SAW, ARAS, WASPAS, VIKOR and MOORA. This evaluation utilizes the flotation machine selection problem as a study case by where five alternatives are analyzed according to ten criteria. There are three categories in which the criteria fall: constructional parameters economical parameters and technical parameters. These criteria are provided with specific weights depending on their relevance in making the decision and each one of the criteria has optimization directions issued to it (whether to maximize or minimize the value). The weights and optimization directions of the criteria are summarized in Table 3.7

Table 3.7
The Evolutional Criteria and Criteria Weights.

Criteria	Criteria Names	Opt.	Criteria Weights
C1	The size and the shape of the machine	max	0.070
C2	The volume or the capacity of the machine	min	0.070
C3	The construction of agitation and aeration system	max	0.070
C4	The number of the machines	max	0.140
C5	Investments	min	0.200
C6	Terms of payment and maintenance	max	0.080
C7	Operating costs	min	0.120
C8	Warranty period	max	0.125
C9	Delivery time	min	0.050
C10	Maintenance conditions	max	0.075

Table 3.7 summarizes the criteria names, their optimization directions, and the corresponding weights used in the SYAI method, along with the other MCDM methods for comparison. The decision making has five alternative that are analysed as follows: C1 to C4 are the constructional parameters with better being a higher

value for C1 and C3, respectively and vice versa for C2 and C4. C5, up to C7, are economical parameters, and the task is to minimize investments and operating costs and maximize terms of payment and maintenance. C8 to C10 are technical parameters and include both objectives of maximization and minimization. The outcome of this analysis indicates that the SYAI method has similar rankings to other methods especially in situations where the decision-makers have to be able to balance values and make wise decisions based on sturdy and interpretable rankings.

In this comparative study, the rating of alternatives according to the given criteria was based on the application of the SYAI method. The results of SYAI were compared with the rankings received from other MCDM methods. In this calculation, β is equal to 0.05 and C is 0.01. Python programming has been utilized to compute the analysis. Table 3.8 shown the comparison score values of the MCDM Method.

Table 3.8
Comparison of MCDM Method Scores and Ranks.

Alt.	TOPSIS	VIKOR	SAW	SYAI	COBRA	WASPAS	MOORA
A1	0.2160 (4)	0.883 (4)	0.6761 (4)	0.1975 (4)	0.0204 (4)	0.6679 (4)	-0.0024 (4)
A2	0.4546 (3)	0.413 (3)	0.7788 (3)	0.5375 (3)	0.0022 (3)	0.7739 (3)	0.0659 (3)
A3	0.7239 (2)	0.000 (1)	0.8867 (2)	0.7900 (1)	-0.0190 (1)	0.8831 (2)	0.1253 (1)
A4	0.7402 (1)	0.275 (2)	0.8951 (1)	0.7025 (2)	-0.0116 (2)	0.8903 (1)	0.1201 (2)
A5	0.0400 (5)	1.000 (5)	0.6063 (5)	0.0167 (5)	0.0340 (5)	0.5973 (5)	-0.0428 (5)

Table 3.8 compares five alternatives (A1 to A5) using several multi-criteria decision-making (MCDM) techniques, such as TOPSIS, VIKOR, SAW, SYAI, COBRA, WASPAS, and MOORA. Each column shows the score determined by a certain approach, while each row represents an alternative. The table shows how rankings change amongst methods showing how each method assesses choice factors in a different way. Figure 3.5 demonstrates the score comparison across each methods.

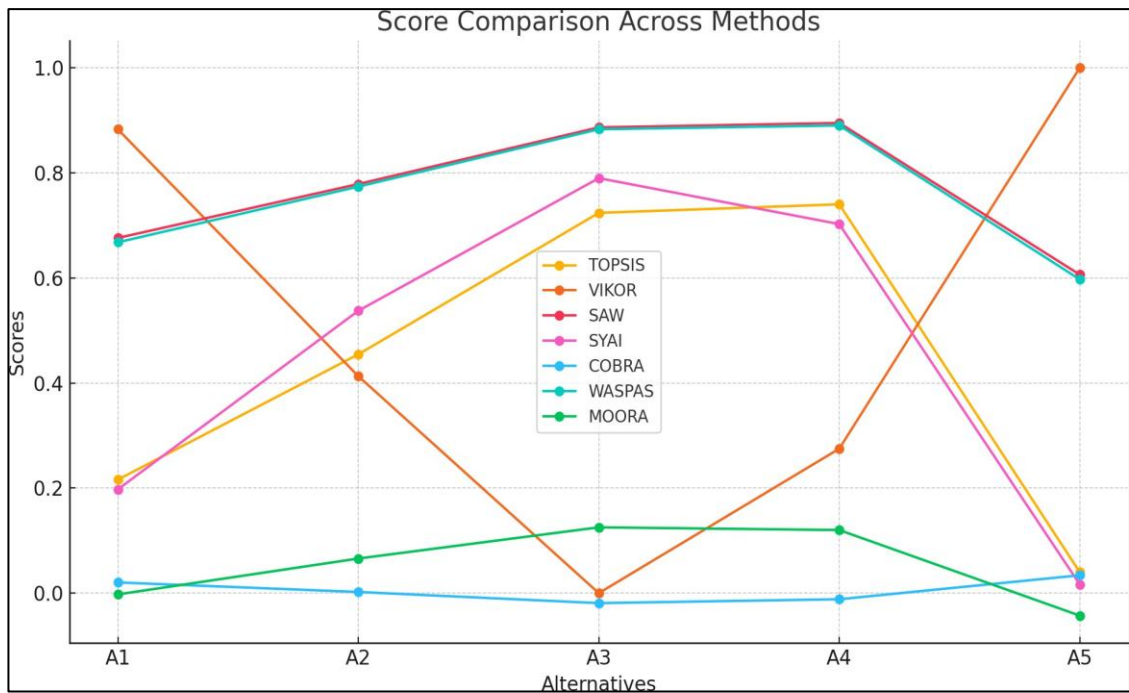


Figure 3.5 Scores of Five Alternatives Evaluated using Seven MCDM Methods

The line chart in Figure 3.5 compares five possible alternatives (A1–A5) that were evaluated using TOPSIS, VIKOR, SAW, SYAI, COBRA, WASPAS, and MOORA methods from Multi-Criteria Decision-Making (MCDM). Most of these techniques value higher scores as better, but both VIKOR and COBRA reverse this, valuing lower scores for better results. For instance, by using VIKOR, A3 is at the top with 0.000 as its score, and A5 comes in last with 1.000. COBRA also ranks A3 highest at -0.0190 and A5 lowest at 0.0340, proving again that these two have opposite preferences.

While the scoring procedures may differ, the majority of methods tend to reach fairly similar rankings. In all four methods, SAW, SYAI, WASPAS, and MOORA, A3 is consistently the best and often the top-ranked option. Meanwhile, A5 stays at the bottom with extremely low scores, including 0.0167 in SYAI, 0.0340 in COBRA, and -0.0428 in MOORA. Combining all the results from the different methods improves the trust in the decision outcome and highlights A3 as the best-chosen alternative.

The powerful agreement among the different methods shows that the ranking is both solid and reliable, especially indicating that A3 is better than A5. Despite the fact that these methods can be different in rules and how complex they are, they all lead to the same general preference ranking. Being consistent allows people to believe in the final recommendations because the outcome does not vary a lot from one method to

another. Numerous techniques such as TOPSIS and VIKOR are applied in MCDM to verify the dependability of the results.

Besides score and rank comparison, correlation based analyses are also used to conduct further analysis on the consistency and reliability of the obtained rankings. Spearman rank correlation coefficient and Pearson correlation coefficient are both employed to determine the level of agreement between various methods of MCDM using ordinal and linear views respectively. Also, heatmap displays are embraced to give a user-friendly view of the strength and direction of correlation among methods, and scatter plots are adopted to demonstrate pairwise correlation and identify similarities or an inverse ranking behaviour among various approaches. Looking at the correlation between their scores shows if the methods tend to be the same or vary because of their own approach. Figure 3.6 illustrates Spearman correlation coefficients matrix of each methods.

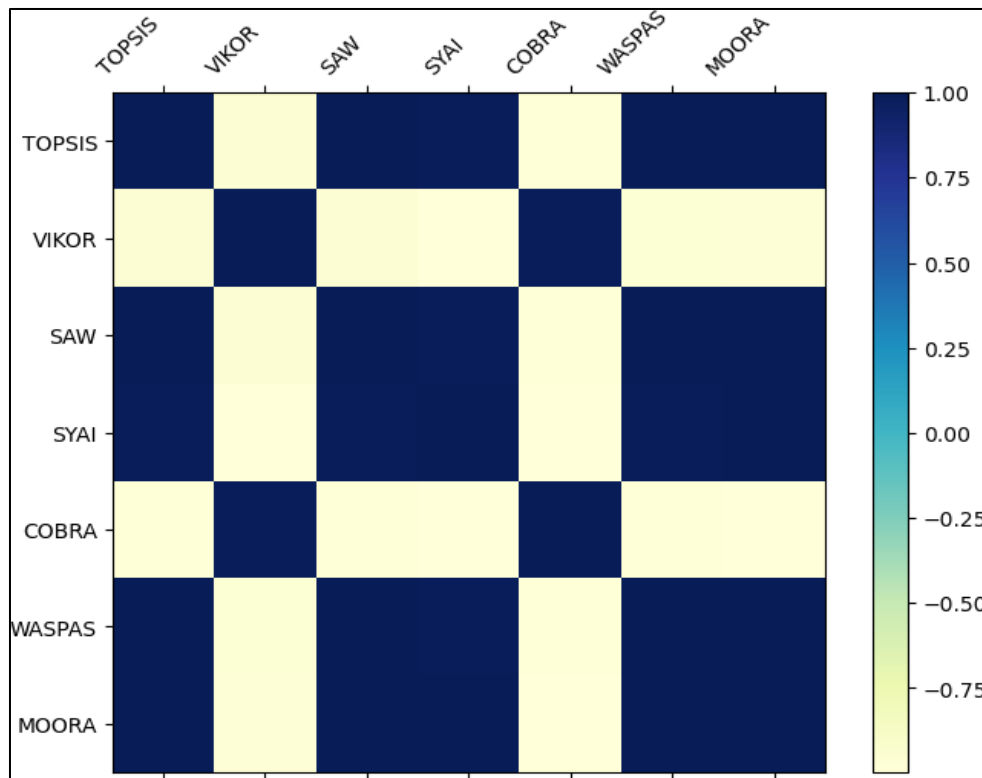


Figure 3.6 Correlation Matrix of MCDM Methods

The heatmap in Figure 3.6 represents the ways in which seven MCDM methods correlate with each other. It demonstrates that most methods are highly correlated in their results. For example, TOPSIS and SAW nearly perfectly match, with a Spearman correlation coefficients of 0.9999, suggesting they give almost the

same results. In addition, MOORA is highly similar to WASPAS (0.9980), and SYAI matches well with MOORA (0.9961) and SAW (0.9876). The fact that MOORA and WASPAS match very closely (0.9980) indicates that they tend to provide reliable and similar results.

On the other side, COBRA has strong negative relationships with methods like TOPSIS (-0.9895), SAW (-0.9890), and WASPAS (-0.9896), proving once more that its scores are opposite to what is expected. VIKOR is also designed so that a lower value is preferred, which can be seen in its strong correlation with COBRA (0.9847), meaning both models select the same alternatives based on lower numbers instead of higher ones. As a result, MCDM methods generally produce the same trends, yet it is important to carefully interpret or merge COBRA and VIKOR, given their unique inverted logic. Scatter matrix of each methods is shown in Figure 3.7.

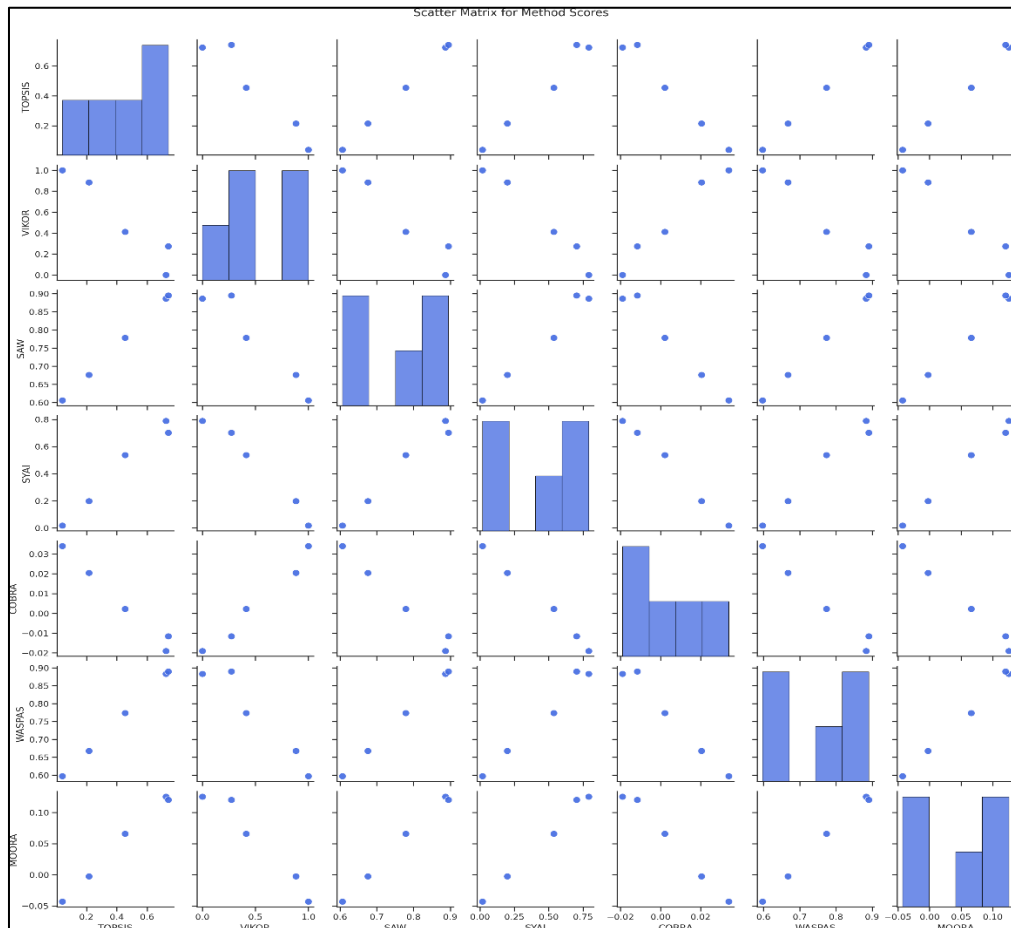


Figure 3.7 Scatter Matrix of MCDM Methods

Figure 3.7 presents a scatter plot that highlights the connections between the seven MCDM methods (TOPSIS, VIKOR, SAW, SYAI, COBRA, WASPAS, and

MOORA and five alternatives. Several methods in the scatter plots show a strong linear pattern, indicating a close relationship between them. It is evident from the plots and their high correlation values of 0.9999, 0.9999, and 0.9980 that TOPSIS and SAW, SAW and WASPAS, and MOORA and WASPAS display strong similarities. This means that the order in which each alternative is ranked is quite similar across different methods.

Furthermore, a strong inverse relationship is found between COBRA and SAW, TOPSIS, and WASPAS in the scatter plot. This is reflected in the correlation table, where COBRA has strong negative correlations with these methods, for instance, -0.9890 with SAW, -0.9895 with TOPSIS, and -0.9896 with WASPAS. The shape of the scatter plots makes it clear that COBRA ranks results in a way where lower scores are considered higher. Overall, the scatter matrix proves helpful for correlation analysis by highlighting how alike or different the outcomes are from one method to another.

3.10 Practical Software Implementation

To improve the practical usability of the SYAI framework, a user-friendly software application has been created using the Python-based Streamlit platform, which can be accessed at <https://wan-syaidatul-98syaimethod.streamlit.app/>. The SYAI web platform empowers individuals to perform detailed analyses across multiple criteria without requiring any coding expertise. To proceed, users simply upload their dataset, specify which criteria types they want (benefit, cost, or goal), and confirm or assign the weights for each of those criteria. After these inputs are entered, the system swiftly carries out all SYAI computations including normalization, weighting, distance evaluation, and final ranking in mere seconds. The system additionally allows users to measure the SYAI results against various established MCDM techniques such as TOPSIS, VIKOR, SAW, WASPAS, MOORA, and COBRA. Furthermore, the application produces downloadable analytical outputs, featuring tables, visual comparisons, and correlation analyses. This practical application guarantees that the SYAI framework remains not only theoretically sound but also transparent, reproducible, and easily accessible for decision-makers in both academic and professional environments. Figure 3.8 and Figure 3.9 displays the graphical user interface of the SYAI web-based system and displays the decision matrix insertion, type

of criteria selection, weighting setting, and the of outputs of ranking and comparative analysis.

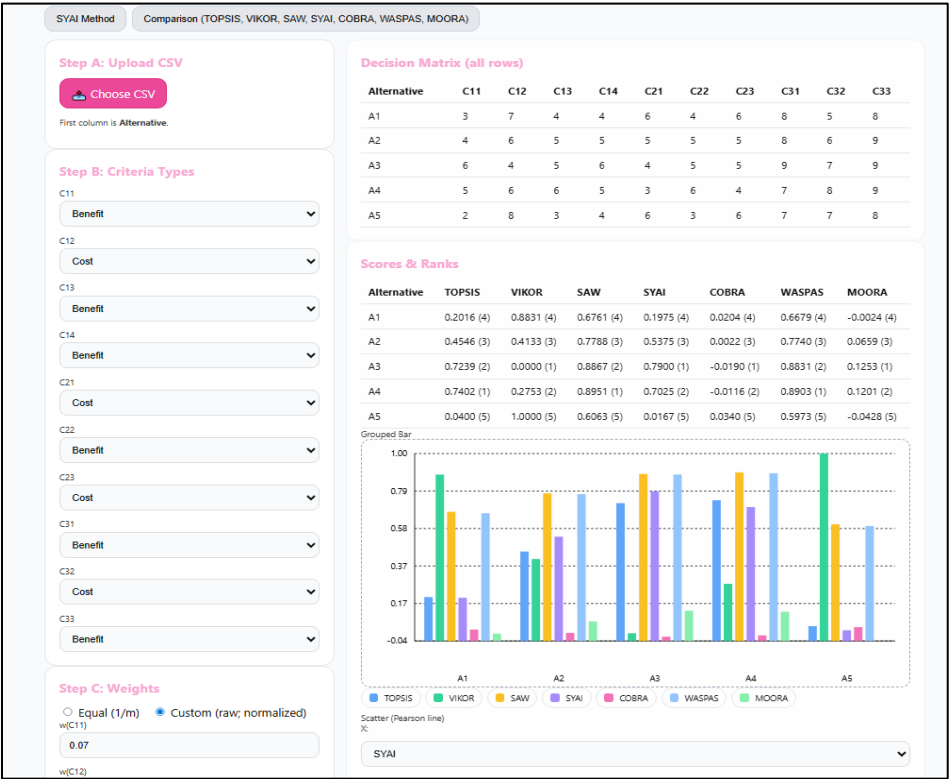


Figure 3.8 User Interface and Analytical Outputs of the SYAI Web-Based Decision Support System

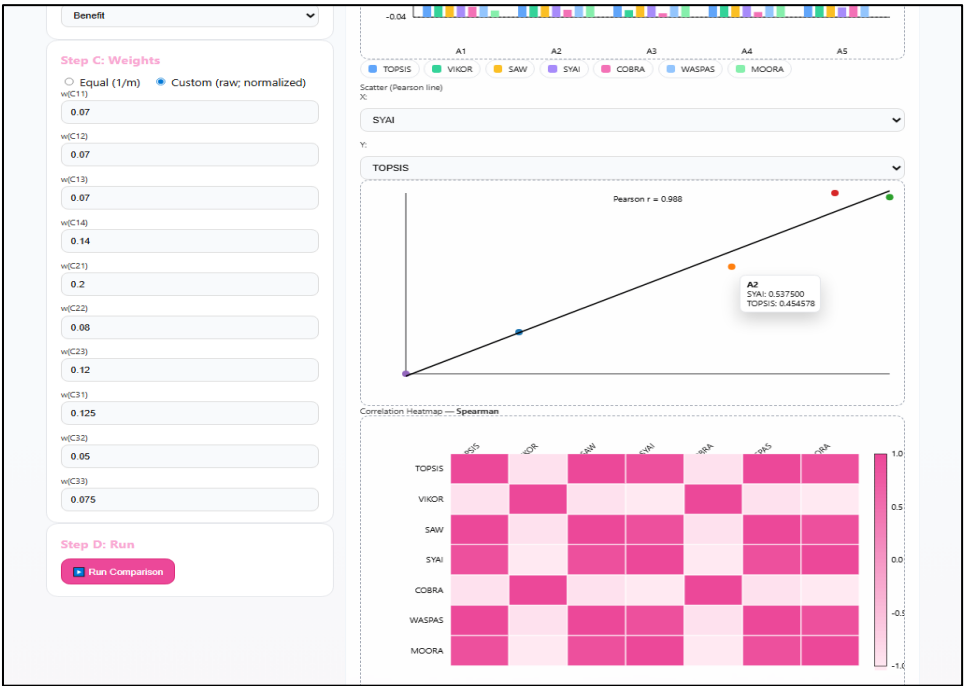


Figure 3.9 User Interface and Analytical Outputs of the SYAI Web-Based Decision Support System

CHAPTER 4

HESITANT TRIANGULAR FUZZY GENERALIZED GEOMETRIC HERONIAN MEAN (HTFGGHM) OPERATOR

4.1 Introduction

The Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean (HTFGGHM) operator is presented as a means to adeptly handle uncertainty, hesitation, and ambiguity in multi-criteria decision-making (MCDM) scenarios. By synthesizing hesitant triangular fuzzy representations with the inclusive geometric Heronian mean framework, this operator effectively illustrates both the ambiguity of human judgment and the dynamic interactions across various criteria. For validation, the weighted extension (WHTFGGHM) is utilized, as the reference dataset derived from Zhao et al. (2014) incorporates established criterion weights. This enables the approach to maintain the theoretical basis of HTFGGHM while being compatible with practical decision-making settings in which attributes possess differing significance. The chapter initiates with the conceptual structure of HTF with GGHM, followed by the formal definition and mathematical derivation of both the HTFGGHM operator and its weighted variation. The following sections outline theoretical properties including idempotency, monotonicity, and boundedness, concluding with a numerical example and sensitivity analysis that illustrate the robustness, dependability, and real-world applicability of the proposed method under uncertain and hesitant.

4.2 Framework of Hesitant Triangular Fuzzy with Generalized Geometric Heronian

The overall computational procedure of the proposed Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean (HTFGGHM) framework consists of five sequential stages, as illustrated in Figure 4.1. Each stage plays a crucial role in integrating hesitant triangular fuzzy representations with the generalized geometric Heronian mean operator for effective multi-criteria decision-making.

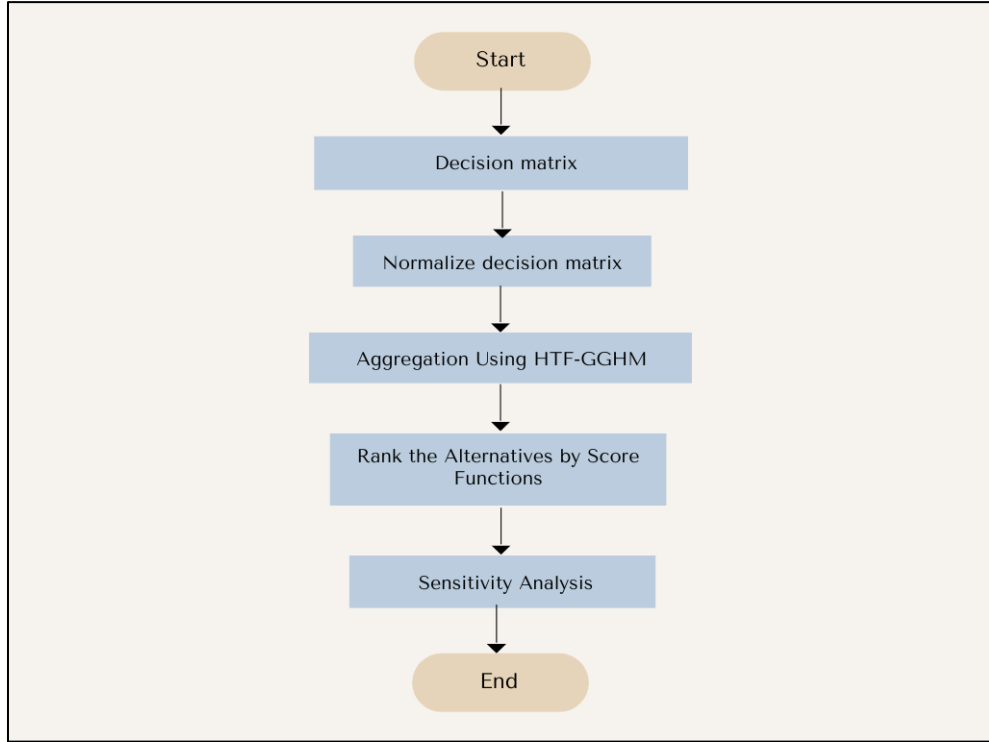


Figure 4.1 Framework for the Integration of Hesitant Triangular Fuzzy (HTF) and Generalized Geometric Heronian Mean (GGHM)

The detail of the following steps is discussed as follows:

Step 1: Construction of the Decision Matrix

In the first stage, a decision matrix is formulated to represent all alternatives and criteria in the hesitant triangular fuzzy environment. Each entry in the matrix is denoted as:

$$h_{ij} = \{(l_{ij}^k, m_{ij}^k, u_{ij}^k) \mid k = 1, 2, \dots, K\}, \quad (4.1)$$

where h_{ij} denotes the hesitant triangular fuzzy evaluation of alternative A_i under criterion G_j . The values l_{ij}^k , m_{ij}^k , and u_{ij}^k represent the lower, middle, and upper bounds of the k -th triangular fuzzy number, respectively. Here, k indicates a possible hesitant evaluation, while K is the total number of hesitant triangular fuzzy values in h_{ij} .

This representation captures the possible opinions or evaluations of experts in the form of hesitant triangular fuzzy numbers (HTFNs). The decision matrix can

therefore model hesitation and uncertainty more effectively than single-valued fuzzy sets, reflecting the diversity of human judgment across multiple criteria.

Step 2: Normalization of the Decision Matrix

Since the criteria may have different units or scales, normalization is performed to convert all hesitant triangular fuzzy values into a comparable domain. The normalization process ensures that benefit and cost criteria are adjusted accordingly while preserving the original shape and hesitation of each triangular fuzzy set. By maintaining fuzziness rather than defuzzifying early, this step safeguards the integrity and interpretability of expert uncertainty throughout the subsequent analysis.

Step 3: Aggregation Using HTFGGHM Operator

Once the normalized decision matrix is established, aggregation is carried out using the Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean (HTFGGHM) operator. This operator captures both the interaction among hesitant fuzzy criteria and the balance between geometric and arithmetic relationships. It integrates the pairwise correlation of hesitant triangular fuzzy values while retaining their underlying fuzzy nature.

The use of a geometric Heronian structure in hesitant fuzzy aggregation is motivated by previous work on hesitant fuzzy geometric Heronian mean operators in multi-criteria decision making . Although the proposed operator is conceptually related to the hesitant fuzzy geometric Heronian mean operator introduced by Verma and Sharma, it differs in terms of the fuzzy representation used. Their operator is formulated under hesitant fuzzy information, whereas the proposed HTFGGHM operator is developed under a hesitant triangular fuzzy environment. In the proposed approach, each hesitant evaluation is represented by triangular fuzzy numbers (l, m, u) , which allows the lower, middle, and upper uncertainty levels of expert judgments to be preserved during aggregation. In this study, the structure is extended into the hesitant triangular fuzzy environment through the proposed HTFGGHM operator.

Formally, the aggregation process is defined as :

$$HTFGGHM^{p,q}(h_1, h_2, \dots, h_n) = \frac{1}{p+q} \bigotimes_{i,j=1; i \leq j}^n [(ph_i \oplus qh_j) \otimes (ph_j \oplus qh_i)]^{\frac{2}{n(n+1)}}. \quad (4.2)$$

Where:

- h_i and h_j denote hesitant triangular fuzzy numbers (HTFNs);
- $\oplus$ and $\otimes$ represent the fuzzy addition and fuzzy multiplication operations under the hesitant triangular fuzzy environment;
- p and q are positive parameters that control the balance between the two interacting hesitant fuzzy elements;
- n represents the total number of criteria involved in the aggregation process.

Equation (4.2) ensures that the aggregated result remains within the hesitant triangular fuzzy domain, preserving both the hesitation and triangular structure of the original inputs. This approach allows interdependent criteria to be fused effectively, producing a comprehensive evaluation for each alternative.

Step 4: Ranking the Alternatives by Score Functions

The aggregated hesitant triangular fuzzy values are subsequently transformed into comparable crisp values through score and accuracy functions. These functions quantify both the central tendency and the uncertainty of each HTFN, enabling direct comparison among alternatives. An alternative with a higher score value is regarded as more desirable, leading to a clear and interpretable ranking outcome.

Step 5: Sensitivity Analysis

The final step involves sensitivity analysis to evaluate the robustness and consistency of the ranking results. By varying the decision parameters (p, q) or

adjusting the influence of criteria, the stability of the ranking order is examined. A consistent ranking under different settings confirms the reliability and resilience of the proposed HTFGGHM framework. This verification process enhances decision confidence and demonstrates the practical applicability of the method in handling complex and uncertain multi-criteria decision-making scenarios.

4.3 Justification for Using the Generalized Geometric Heronian Mean

The selection of the Generalized Geometric Heronian Mean (GGHM) operator in this study is motivated by its ability to model interrelationships among criteria while maintaining the multiplicative interaction structure of the input information. In MCDM problems, the criteria are often not fully independent. For example, in supplier selection, product quality, service performance, delivery performance, and price may influence one another. A supplier with high product quality but poor delivery performance may not be preferred, while a supplier with moderate price and reliable service may be more suitable. Therefore, an aggregation operator should be able to capture the interaction among criteria rather than treating each criterion as completely separate.

The Arithmetic Heronian Mean is useful for representing pairwise relationships among input values, but it is mainly based on additive aggregation. This means that a high value in one criterion may compensate for a low value in another criterion more directly. In contrast, the Geometric Heronian Mean is more suitable when the decision-making problem requires balanced performance among criteria because the geometric structure reduces the effect of extreme compensation. This is important in hesitant triangular fuzzy environments, where expert evaluations contain uncertainty and hesitation. If an arithmetic-based operator is used, a very strong evaluation in one criterion may dominate the final result even when other important criteria are weak. The geometric structure helps to preserve balance among criteria and produces a more cautious and interaction aware aggregation result.

A hybrid arithmetic geometric Heronian operator may also be considered, but it would introduce additional computational complexity and may require extra parameters to control the balance between arithmetic and geometric effects. Since this study aims to develop a framework that is both interpretable and computationally practical, the GGHM structure is selected as a more suitable foundation. It provides the advantage of pairwise interaction modelling through the Heronian structure while maintaining the

balanced aggregation behaviour of the geometric mean. This makes it appropriate for hesitant triangular fuzzy decision making problems, where the aggregation process must preserve uncertainty, hesitation, and inter-criterion dependency.

The Generalized Geometric Heronian Mean is improved in this study because the existing GGHM operator is originally designed for crisp or simpler fuzzy environments and cannot directly handle hesitant triangular fuzzy information. In real decision making settings, experts may provide more than one possible triangular fuzzy evaluation for the same criterion due to hesitation or uncertainty. Therefore, the current GGHM structure needs to be extended into the hesitant triangular fuzzy environment so that multiple triangular fuzzy values can be aggregated without losing the hesitation information. The proposed HTFGGHM operator improves the existing GGHM by incorporating hesitant triangular fuzzy operational laws into the generalized geometric Heronian structure.

To illustrate the interrelationship among criteria, consider the supplier selection problem used in the numerical example. The criteria include product quality G_1 , service G_2 , delivery performance G_3 , and price G_4 . These criteria are naturally connected. For instance, a supplier with high product quality may also require a higher price, while a supplier with faster delivery may affect service reliability or operational cost. Similarly, good service may support delivery performance because suppliers with better service systems are usually more responsive to customer needs. Hence, the final evaluation of a supplier should not be based only on independent criterion scores. The HTFGGHM operator is suitable in this situation because it aggregates the criteria by considering pairwise relationships among them, such as (G_1, G_2) , (G_1, G_3) , (G_1, G_4) , (G_2, G_3) , (G_2, G_4) , and (G_3, G_4) . This allows the proposed method to reflect the interaction structure of the decision problem more effectively.

4.4 Proposed Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean

Definition 4.4.1. Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean Let $p, q > 0$ and let $h_i = (l_i, m_i, u_i)$ be a collection of hesitant fuzzy triangular elements. The hesitant triangular fuzzy generalized geometric Heronian mean $HTFGGHM^{p,q}$ is defined as:

$$\begin{aligned}
& HTFGGHM^{p,q}(h_1, h_2, \dots, h_n) \\
&= \frac{1}{p+q} \bigotimes_{i,j=1; i \leq j}^n \left((ph_i \oplus qh_j) \otimes (ph_j \oplus qh_i) \right)^{\frac{2}{n(n+1)}}.
\end{aligned} \tag{4.3}$$

where $\oplus$ and $\otimes$ denote the operations applied according to the laws of hesitant fuzzy elements.

Theorem 4.1 Let $p, q > 0$ and let $h_i = (l_i, m_i, u_i)$ be a collection of hesitant triangular fuzzy elements. Then the aggregated value obtained using the $HTFGGHM^{p,q}$ operator is also a hesitant triangular fuzzy element, and:

$$\begin{aligned}
& HTFGGHM^{p,q}(h_1, h_2, \dots, h_n) \\
&= \frac{1}{p+q} \bigotimes_{i,j=1; i \leq j}^n \left((ph_i \oplus qh_j) \otimes (ph_j \oplus qh_i) \right)^{\frac{2}{n(n+1)}}.
\end{aligned} \tag{4.4}$$

which can also be expressed as:

$$\bigcup_{\eta_{i,j} \in \sigma_i, i \leq j} \left\{ 1 - \left(1 - \prod_{i,j=1; i \leq j}^n \eta_{i,j}^{\frac{2}{n(n+1)}} \right)^{\frac{1}{p+q}} \right\}. \tag{4.5}$$

Here, $\eta_{i,j}$ represents the individual elements within the hesitant fuzzy sets involved in the aggregation. To obtain the explicit form of the proposed operator, the scalar multiplication and addition operations for hesitant triangular fuzzy elements are first expanded. This expansion shows how each lower, middle, and upper component is transformed before the aggregation process is completed.

Using scalar multiplication in Equation (2.14);

$$ph_i \oplus qh_j = \tag{4.6}$$

By referring to Equation (2.14), we get:

$$\begin{aligned}
ph_i &= \{1 - (1 - l_i)^p, 1 - (1 - m_i)^p, 1 - (1 - u_i)^p\}, \\
qh_j &= \{1 - (1 - l_j)^q, 1 - (1 - m_j)^q, 1 - (1 - u_j)^q\}, \\
ph_j &= \{1 - (1 - l_j)^p, 1 - (1 - m_j)^p, 1 - (1 - u_j)^p\}, \\
qh_i &= \{1 - (1 - l_i)^q, 1 - (1 - m_i)^q, 1 - (1 - u_i)^q\}.
\end{aligned} \tag{4.7}$$

Proof. Using Equation (2.15), we get

$$\begin{aligned}
& ph_i \oplus qh_j \\
&= \begin{pmatrix} (1 - (1 - l_i)^p) + (1 - (1 - l_j)^q) - (1 - (1 - l_i)^p) \cdot (1 - (1 - l_j)^q), \\ (1 - (1 - m_i)^p) + (1 - (1 - m_j)^q) - (1 - (1 - m_i)^p) \cdot (1 - (1 - m_j)^q), \\ (1 - (1 - u_i)^p) + (1 - (1 - u_j)^q) - (1 - (1 - u_i)^p) \cdot (1 - (1 - u_j)^q) \end{pmatrix} \\
& ph_j \oplus qh_i \tag{4.8} \\
&= \begin{pmatrix} (1 - (1 - l_j)^p) + (1 - (1 - l_i)^q) - (1 - (1 - l_j)^p) \cdot (1 - (1 - l_i)^q), \\ (1 - (1 - m_j)^p) + (1 - (1 - m_i)^q) - (1 - (1 - m_j)^p) \cdot (1 - (1 - m_i)^q), \\ (1 - (1 - u_j)^p) + (1 - (1 - u_i)^q) - (1 - (1 - u_j)^p) \cdot (1 - (1 - u_i)^q) \end{pmatrix}
\end{aligned}$$

After obtaining $ph_i \oplus qh_j$ and $ph_j \oplus qh_i$, the next step is to apply the hesitant triangular fuzzy multiplication operation. This produces the pairwise product term used in the HTFGGHM operator.

The theoretical derivation of the proposed HTFGGHM operator is constructed from the hesitant triangular fuzzy operational laws introduced in the preliminary section. Specifically, the scalar multiplication, fuzzy addition, fuzzy multiplication, and power operations are applied step by step to the generalized geometric Heronian mean structure. Therefore, the new theoretical results are not derived independently, but are obtained by extending the existing operational rules into the hesitant triangular fuzzy generalized geometric Heronian mean framework.

By Equation (2.16), we get

$$\begin{aligned}
& (ph_i \oplus qh_j) \otimes (ph_j \oplus qh_i) = \\
& \left[\left((1 - (1 - l_i)^p) + (1 - (1 - l_j)^q) - (1 - (1 - l_i)^p) \cdot (1 - (1 - l_j)^q) \right) \cdot \right. \\
& \left((1 - (1 - l_j)^p) + (1 - (1 - l_i)^q) - (1 - (1 - l_j)^p) \cdot (1 - (1 - l_i)^q) \right), \\
& \left((1 - (1 - m_i)^p) + (1 - (1 - m_j)^q) - (1 - (1 - m_i)^p) \cdot (1 - (1 - m_j)^q) \right) \cdot \\
& \left((1 - (1 - m_j)^p) + (1 - (1 - m_i)^q) - (1 - (1 - m_j)^p) \cdot (1 - (1 - m_i)^q) \right), \\
& \left((1 - (1 - u_i)^p) + (1 - (1 - u_j)^q) - (1 - (1 - u_i)^p) \cdot (1 - (1 - u_j)^q) \right) \cdot \\
& \left((1 - (1 - u_j)^p) + (1 - (1 - u_i)^q) - (1 - (1 - u_j)^p) \cdot (1 - (1 - u_i)^q) \right) \left. \right] \tag{4.9}
\end{aligned}$$

From Equation (2.13),

$$\left((ph_i \oplus qh_j) \otimes (ph_j \oplus qh_i) \right)^{\frac{2}{n(n+1)}} = \quad (4.10)$$

$$\left[\left((1 - (1 - l_i)^p) + (1 - (1 - l_j)^q) - (1 - (1 - l_i)^p) \cdot (1 - (1 - l_j)^q) \right) \cdot \left((1 - (1 - l_j)^p) + (1 - (1 - l_i)^q) - (1 - (1 - l_j)^p) \cdot (1 - (1 - l_i)^q) \right)^{\frac{2}{n(n+1)}}, \left((1 - (1 - m_i)^p) + (1 - (1 - m_j)^q) - (1 - (1 - m_i)^p) \cdot (1 - (1 - m_j)^q) \right) \cdot \left((1 - (1 - m_j)^p) + (1 - (1 - m_i)^q) - (1 - (1 - m_j)^p) \cdot (1 - (1 - m_i)^q) \right)^{\frac{2}{n(n+1)}}, \left((1 - (1 - u_i)^p) + (1 - (1 - u_j)^q) - (1 - (1 - u_i)^p) \cdot (1 - (1 - u_j)^q) \right) \cdot \left((1 - (1 - u_j)^p) + (1 - (1 - u_i)^q) - (1 - (1 - u_j)^p) \cdot (1 - (1 - u_i)^q) \right)^{\frac{2}{n(n+1)}} \right]$$

$$\prod_{i,j=1; i \leq j}^n (ph_i \oplus qh_j) \otimes (ph_j \oplus qh_i)^{\frac{2}{n(n+1)}} = \quad (4.11)$$

$$\prod_{i,j=1; i \leq j}^n \left((1 - (1 - l_i)^p) + (1 - (1 - l_j)^q) - (1 - (1 - l_i)^p) \cdot (1 - (1 - l_j)^q) \right) \cdot \left((1 - (1 - l_j)^p) + (1 - (1 - l_i)^q) - (1 - (1 - l_j)^p) \cdot (1 - (1 - l_i)^q) \right)^{\frac{2}{n(n+1)}}, \prod_{i,j=1; i \leq j}^n \left((1 - (1 - m_i)^p) + (1 - (1 - m_j)^q) - (1 - (1 - m_i)^p) \cdot (1 - (1 - m_j)^q) \right) \cdot \left((1 - (1 - m_j)^p) + (1 - (1 - m_i)^q) - (1 - (1 - m_j)^p) \cdot (1 - (1 - m_i)^q) \right)^{\frac{2}{n(n+1)}}, \prod_{i,j=1; i \leq j}^n \left((1 - (1 - u_i)^p) + (1 - (1 - u_j)^q) - (1 - (1 - u_i)^p) \cdot (1 - (1 - u_j)^q) \right) \cdot \left((1 - (1 - u_j)^p) + (1 - (1 - u_i)^q) - (1 - (1 - u_j)^p) \cdot (1 - (1 - u_i)^q) \right)^{\frac{2}{n(n+1)}}$$

By referring to Equation (2.14), we get:

$$\frac{1}{p+q} \bigotimes_{i,j=1; i \leq j}^n \left((ph_i \oplus qh_j) \otimes (ph_j \oplus qh_i) \right)^{\frac{2}{n(n+1)}} \quad (4.12)$$

$$\begin{aligned}
&= 1 - \left(1 - \prod_{i,j=1; i \leq j}^n \eta_{i,j}^{\frac{2}{n(n+1)}} \right)^{\frac{1}{p+q}} \\
&= 1 - \left[1 - \prod_{i,j=1; i \leq j}^n \left((1 - (1 - l_i)^p) + (1 - (1 - l_j)^q) - (1 - (1 - l_i)^p) \cdot (1 - (1 - l_j)^q) \right) \cdot \left((1 - (1 - l_j)^p) + (1 - (1 - l_i)^q) - (1 - (1 - l_j)^p) \cdot (1 - (1 - l_i)^q) \right)^{\frac{2}{n(n+1)}} \right]^{\frac{1}{p+q}}, \\
&1 - \left[1 - \prod_{i,j=1; i \leq j}^n \left((1 - (1 - m_i)^p) + (1 - (1 - m_j)^q) - (1 - (1 - m_i)^p) \cdot (1 - (1 - m_j)^q) \right) \cdot \left((1 - (1 - m_j)^p) + (1 - (1 - m_i)^q) - (1 - (1 - m_j)^p) \cdot (1 - (1 - m_i)^q) \right)^{\frac{2}{n(n+1)}} \right]^{\frac{1}{p+q}}, \\
&1 - \left[1 - \prod_{i,j=1; i \leq j}^n \left((1 - (1 - u_i)^p) + (1 - (1 - u_j)^q) - (1 - (1 - u_i)^p) \cdot (1 - (1 - u_j)^q) \right) \cdot \left((1 - (1 - u_j)^p) + (1 - (1 - u_i)^q) - (1 - (1 - u_j)^p) \cdot (1 - (1 - u_i)^q) \right)^{\frac{2}{n(n+1)}} \right]^{\frac{1}{p+q}}
\end{aligned} \tag{4.13}$$

Equation (4.5) has been proven.°

To illustrate how to apply the proposed *HTFGGHM* operator, an example is given. $A_1 = 0.3, 0.5, 0.7$

Example 1. Let $A_1 = 0.3, 0.5, 0.7$, $A_2 = 0.2, 0.6, 0.7$, and $A_3 = 0.2, 0.4, 0.7$ be a collection of HTFEs, and let $p = 2, q = 1$

$$\begin{aligned}
&HTFGGHM^{p,q}(h_1, h_2, \dots, h_n) \\
&= \frac{1}{p+q} \bigotimes_{i,j=1; i \leq j}^n \left((ph_i \oplus qh_j) \otimes (ph_j \oplus qh_i) \right)^{\frac{2}{n(n+1)}}. \tag{4.14}
\end{aligned}$$

By using Equation (4.14), the l -component is calculated as follows. For simplicity, define

$$\begin{aligned} \eta_{ij}^{(l)} = & \left[\left(1 - (1 - l_i)^p + 1 - (1 - l_j)^q \right. \right. \\ & - \left. \left. (1 - (1 - l_i)^p)(1 - (1 - l_j)^q) \right) \right. \\ & \times \left(1 - (1 - l_j)^p + 1 - (1 - l_i)^q \right. \\ & \left. \left. - (1 - (1 - l_j)^p)(1 - (1 - l_i)^q) \right) \right]. \end{aligned} \quad (4.15)$$

Thus, the lower component can be written as

$$l\text{-part} = 1 - \left[1 - \prod_{i,j=1; i \leq j}^6 \left(\eta_{ij}^{(l)} \right)^{\frac{2}{6(6+1)}} \right]^{\frac{1}{2+1}}. \quad (4.16)$$

Substituting $p = 2$, $q = 1$, and $l \in \{0.3, 0.2, 0.2\}$, the product term gives

$$\prod_{i,j=1; i \leq j}^6 \left(\eta_{ij}^{(l)} \right)^{\frac{2}{6(6+1)}} = 0.879729. \quad (4.17)$$

Therefore,

$$l\text{-part} = 1 - [1 - 0.879729]^{\frac{1}{3}} = 1 - 0.493613 = 0.506387. \quad (4.18)$$

A similar procedure is used to obtain the m - and u -components. Therefore,

$$HTFGGHM^{p,q}(A_1, A_2, A_3) = (0.506387, 0.628647, 0.802096). \quad (4.19)$$

The example above illustrates the computational process of the proposed HTFGGHM operator for aggregating hesitant triangular fuzzy information. After obtaining the aggregated value, the next step is to verify that the proposed operator satisfies several fundamental mathematical properties. Therefore, the following part presents the idempotency, monotonicity, and boundedness properties of the HTFGGHM operator.

$$\bigcup_{\gamma \in h_i} \left(((1 - (1 - \gamma^L)^{p+q})^2)^{\frac{1}{p+q}}, ((1 - (1 - \gamma^M)^{p+q})^2)^{\frac{1}{p+q}}, ((1 - (1 - \gamma^U)^{p+q})^2)^{\frac{1}{p+q}} \right) \quad (4.20)$$

$$\leq 1 - \left(1 - \bigcup_{\gamma^+ \in h_i^+} ((1 - (1 - \gamma^L)^{p+q})^2)^{\frac{1}{p+q}} \right)$$

Then $h_i^- \leq h_i^+$, and

$$\bigcup_{\gamma^- \in h_i^-} \left((1 - (1 - \gamma^{(-L)})^{p+q})^2, (1 - (1 - \gamma^{(-M)})^{p+q})^2, (1 - (1 - \gamma^{(-U)})^{p+q})^2 \right) \quad (4.21)$$

$$\leq \bigcup_{\gamma^+ \in h_i^+} \left((1 - (1 - \gamma^{(+L)})^{p+q})^2, (1 - (1 - \gamma^{(+M)})^{p+q})^2, (1 - (1 - \gamma^{(+U)})^{p+q})^2 \right)$$

Theorem 4.2 (Idempotency). Let $A = (l_i, m_i, u_i) (i = 1, 2, \dots, n)$ represent a set of HTFEs. If $A_i = (l_i, m_i, u_i)$ for all i , then

$$HTF - GGHM^{p,q}(A_1, A_2, \dots, A_n) = a. \quad (4.22)$$

Proof. Since $A_i = (l_i, m_i, u_i)$ for $i = 1, 2, \dots, n$, then

$$GGHM^{p,q}(a_1, a_2, \dots, a_n) = \frac{1}{p+q} \prod_{i,j=1}^n (pa_i + qa_j)^{\frac{2}{n(n+1)}}. \quad (4.23)$$

Thus,

$$\frac{1}{p+q} \left[\prod_{i,j=1}^n (pa + qa) \right]^{\frac{2}{n(n+1)}} = \frac{1}{p+q} (a(p+q)) = a \quad (4.24)$$

Theorem 4.3 (Monotonicity). Let h_{α_i} and h_{β_i} ($i = 1, 2, \dots, n$) be two collections of HTFEs; if, for any $\gamma_{\alpha_i} \in h_{\alpha_i}$ and $\gamma_{\beta_j} \in h_{\beta_j}$ ($i, j = 1, 2, \dots, n; i \neq j$), one has $\gamma_{\alpha_i} \leq \gamma_{\beta_i}$ and $\gamma_{\alpha_j} \leq \gamma_{\beta_j}$, then

$$HTF - GGHM^{p,q}(h_{\alpha_1}, h_{\alpha_2}, \dots, h_{\alpha_n}) \leq HTF - GGHM^{p,q}(h_{\beta_1}, h_{\beta_2}, \dots, h_{\beta_n}). \quad (4.25)$$

Proof. By Proposition (4.25), we have $\eta_{\alpha_{ij}} \leq \eta_{\beta_{ij}}$ for all $i \neq j$. Hence,

$$1 - \left(1 - \prod_{i,j=1; i \leq j}^n \eta_{\alpha_{ij}}^{\frac{2}{n(n+1)}} \right)^{\frac{1}{p+q}} \leq 1 - \left(1 - \prod_{i,j=1; i \leq j}^n \eta_{\beta_{ij}}^{\frac{2}{n(n+1)}} \right)^{\frac{1}{p+q}}. \quad (4.26)$$

By **Definition 2.10.7** (HTF–GGHM), it follows that

$$\begin{aligned} HTF - GGHM^{p,q}(h_{\alpha_1}, \dots, h_{\alpha_n}) &= \frac{1}{p+q} \otimes_{i,j=1; i \leq j}^n \left(\eta_{\alpha_{ij}}^{\frac{2}{n(n+1)}} \right) \leq \\ \frac{1}{p+q} \otimes_{i,j=1; i \leq j}^n \left(\eta_{\beta_{ij}}^{\frac{2}{n(n+1)}} \right) &= HTF - GGHM^{p,q}(h_{\beta_1}, \dots, h_{\beta_n}). \end{aligned} \quad (4.27)$$

Theorem 4.4 (Boundedness). Let $h_i (i = 1, 2, \dots, n)$ be a collection of HTEs, where

$$h_i^- = \bigcup_{\gamma_i \in h_i} (\min\{\gamma_i^L\}, \min\{\gamma_i^M\}, \min\{\gamma_i^U\}), \quad (4.28)$$

$$h_i^+ = \bigcup_{\gamma_i \in h_i} (\max\{\gamma_i^L\}, \max\{\gamma_i^M\}, \max\{\gamma_i^U\})$$

$i, j \in \{1, 2, \dots, n\}$

Then,

For simplicity, Let

$$\begin{aligned} L_i^- &= \\ &= \bigcup_{\gamma^- \in h_i^-} \left(((1 - (1 - \gamma^{-L})^{p+q})^2)^{\frac{1}{p+q}}, ((1 - (1 - \gamma^{-M})^{p+q})^2)^{\frac{1}{p+q}}, ((1 - (1 - \gamma^{-U})^{p+q})^2)^{\frac{1}{p+q}} \right), \end{aligned} \quad (4.29)$$

and,

$$\begin{aligned} L_i^+ &= \bigcup_{\gamma^+ \in h_i^+} \left(((1 - (1 - \gamma^{+L})^{p+q})^2)^{\frac{1}{p+q}}, ((1 - (1 - \gamma^{+M})^{p+q})^2)^{\frac{1}{p+q}}, \right. \\ &\quad \left. ((1 - (1 - \gamma^{+U})^{p+q})^2)^{\frac{1}{p+q}} \right). \end{aligned} \quad (4.30)$$

Then, the boundedness property can be written as

$$L_i^- \leq HTFGGHM^{p,q}(h_1, \dots, h_n) \leq L_i^+. \quad (4.31)$$

Proof. By the boundedness of the hesitant triangular fuzzy elements, we have

$$\begin{aligned} & \bigcup_{\gamma^- \in h_i^-} ((1 - (1 - \gamma^{-L})^{p+q})^2, (1 - (1 - \gamma^{-M})^{p+q})^2, \\ & \quad (1 - (1 - \gamma^{-U})^{p+q})^2) \leq \eta_{ij}, \quad i < j, \\ & \eta_{ij} \leq \bigcup_{\gamma^+ \in h_i^+} ((1 - (1 - \gamma^{+L})^{p+q})^2, \\ & \quad (1 - (1 - \gamma^{+M})^{p+q})^2, (1 - (1 - \gamma^{+U})^{p+q})^2). \end{aligned} \quad (4.32)$$

Therefore,

$$\begin{aligned} L_i^- & \leq 1 - \left(1 - \prod_{i,j=1; i \leq j}^n \eta_{ij}^{\frac{2}{n(n+1)}} \right)^{\frac{1}{p+q}} \\ & \leq L_i^+. \end{aligned} \quad (4.33)$$

Since,

$$HTFGGHM^{p,q}(h_1, \dots, h_n) = 1 - \left(1 - \prod_{i,j=1; i \leq j}^n \eta_{ij}^{\frac{2}{n(n+1)}} \right)^{\frac{1}{p+q}}, \quad (4.34)$$

it follows that

$$L_i^- \leq HTFGGHM^{p,q}(h_1, \dots, h_n) \leq L_i^+. \quad (4.35)$$

This completes the proof.

4.5 Proposed Weighted Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean (WHTFGGHM)

Theorem 4.5. Suppose $p, q > 0$ and let $h_i = (l_i, m_i, u_i)$ represent a set of hesitant triangular fuzzy elements, accompanied by a weight vector $w = (w_1, w_2, \dots, w_n)^T$ where $w_i \geq 0, i = 1, 2, \dots, n$ and $\sum_{i=1}^n w_i = 1$. Then the aggregated value derived using the $WHTFGGHM^{p,q}$ operator is also a hesitant triangular fuzzy element, and

$$\begin{aligned}
 & WHTFGGHM^{p,q}(h_1, h_2, \dots, h_n) \\
 &= \frac{1}{p+q} \bigotimes_{i,j=1; i \leq j}^n \left((ph_i \oplus qh_j) \otimes (ph_j \oplus qh_i) \right)^{\frac{2w_i w_j}{1+w_i}}, \quad (4.36)
 \end{aligned}$$

Which can also be expressed as:

$$= \bigcup_{\eta_{ij} \in \sigma_{i,j}, i \leq j} \left\{ 1 - \left(1 - \prod_{i,j=1; i \leq j}^n \eta_{ij}^{\frac{2w_i w_j}{(1+w_i)}} \right)^{\frac{1}{p+q}} \right\}. \quad (4.37)$$

$$\begin{aligned} &= 1 - \left[1 - \prod_{i,j=1; i \leq j}^n \left(((1 - (1 - l_i)^p) + (1 - (1 - l_j)^q) - (1 - (1 - l_i)^p) \cdot (1 - (1 - l_j)^q)) \right. \right. \\ &\quad \cdot \left. \left((1 - (1 - l_j)^p) + (1 - (1 - l_i)^q) - (1 - (1 - l_j)^p) \cdot (1 - (1 - l_i)^q) \right) \right)^{\frac{2w_i w_j}{(1+w_i)}} \Big]^{\frac{1}{p+q}}, \\ &1 - \left[1 - \prod_{i,j=1; i \leq j}^n \left(((1 - (1 - m_i)^p) + (1 - (1 - m_j)^q) - (1 - (1 - m_i)^p) \cdot (1 - (1 - m_j)^q)) \right. \right. \\ &\quad \cdot \left. \left((1 - (1 - m_j)^p) + (1 - (1 - m_i)^q) - (1 - (1 - m_j)^p) \cdot (1 - (1 - m_i)^q) \right) \right)^{\frac{2w_i w_j}{(1+w_i)}} \Big]^{\frac{1}{p+q}}, \\ &1 - \left[1 - \prod_{i,j=1; i \leq j}^n \left(((1 - (1 - u_i)^p) + (1 - (1 - u_j)^q) - (1 - (1 - u_i)^p) \cdot (1 - (1 - u_j)^q)) \right. \right. \\ &\quad \cdot \left. \left((1 - (1 - u_j)^p) + (1 - (1 - u_i)^q) - (1 - (1 - u_j)^p) \cdot (1 - (1 - u_i)^q) \right) \right)^{\frac{2w_i w_j}{(1+w_i)}} \Big]^{\frac{1}{p+q}}. \end{aligned} \quad (4.38)$$

Here, $n_{i,j}$ represents the individual elements within the hesitant fuzzy sets involved in the aggregation. Using scalar multiplication in Equation (2.14)

$$ph_i \oplus qh_j = \quad (4.39)$$

By referring to Equation (2.13), we ge

$$ph_i = \{1 - (1 - l_i)^p, 1 - (1 - m_i)^p, 1 - (1 - u_i)^p\}. \quad (4.40)$$

$$qh_j = \{1 - (1 - l_j)^q, 1 - (1 - m_j)^q, 1 - (1 - u_j)^q\}, \quad (4.41)$$

$$ph_j = \{1 - (1 - l_j)^p, 1 - (1 - m_j)^p, 1 - (1 - u_j)^p\}, \quad (4.42)$$

$$qh_i = \{1 - (1 - l_i)^q, 1 - (1 - m_i)^q, 1 - (1 - u_i)^q\}. \quad (4.43)$$

Proof. Using Equation (2.15), we get

$$\begin{aligned} & \text{Addition}(\oplus): ph_i \oplus qh_j = \\ & \left(\begin{array}{l} (1 - (1 - l_i)^p) + (1 - (1 - l_j)^q) - (1 - (1 - l_i)^p) \cdot (1 - (1 - l_j)^q), \\ (1 - (1 - m_i)^p) + (1 - (1 - m_j)^q) - (1 - (1 - m_i)^p) \cdot (1 - (1 - m_j)^q), \\ (1 - (1 - u_i)^p) + (1 - (1 - u_j)^q) - (1 - (1 - u_i)^p) \cdot (1 - (1 - u_j)^q) \end{array} \right) \end{aligned} \quad (4.44)$$

$$\begin{aligned} & \text{Addition}(\oplus): ph_j \oplus qh_i = \\ & \left(\begin{array}{l} (1 - (1 - l_j)^p) + (1 - (1 - l_i)^q) - (1 - (1 - l_j)^p) \cdot (1 - (1 - l_i)^q), \\ (1 - (1 - m_j)^p) + (1 - (1 - m_i)^q) - (1 - (1 - m_j)^p) \cdot (1 - (1 - m_i)^q), \\ (1 - (1 - u_j)^p) + (1 - (1 - u_i)^q) - (1 - (1 - u_j)^p) \cdot (1 - (1 - u_i)^q) \end{array} \right) \end{aligned} \quad (4.45)$$

By Equation (2.16), we get

$$\begin{aligned}
& (ph_i \oplus qh_j) \otimes (ph_j \oplus qh_i) \\
& \quad ((1 - (1 - l_i)^p) + (1 - (1 - l_j)^q) \\
& \quad \quad - (1 - (1 - l_i)^p)(1 - (1 - l_j)^q)) \cdot ((1 - (1 - l_j)^p) + (1 - (1 - l_i)^q) \\
& \quad \quad - (1 - (1 - l_j)^p)(1 - (1 - l_i)^q)), \\
& \quad ((1 - (1 - m_i)^p) + (1 - (1 - m_j)^q) \\
= & \quad - (1 - (1 - m_i)^p)(1 - (1 - m_j)^q)) \cdot ((1 - (1 - m_j)^p) + (1 - (1 - m_i)^q) \\
& \quad - (1 - (1 - m_j)^p)(1 - (1 - m_i)^q)), \\
& \quad ((1 - (1 - u_i)^p) + (1 - (1 - u_j)^q) \\
& \quad - (1 - (1 - u_i)^p)(1 - (1 - u_j)^q)) \cdot ((1 - (1 - u_j)^p) + (1 - (1 - u_i)^q) \\
& \quad - (1 - (1 - u_j)^p)(1 - (1 - u_i)^q))
\end{aligned} \tag{4.46}$$

From Equation (2.13),

$$\begin{aligned}
& \left((ph_i \oplus qh_j) \otimes (ph_j \oplus qh_i) \right)^{\frac{2w_i w_j}{1+w_i}} \\
& \quad \left((((1 - (1 - l_i)^p) + (1 - (1 - l_j)^q) \right. \\
& \quad \quad \left. - (1 - (1 - l_i)^p)(1 - (1 - l_j)^q)) \cdot ((1 - (1 - l_j)^p) + (1 - (1 - l_i)^q) \right. \\
& \quad \quad \left. - (1 - (1 - l_j)^p)(1 - (1 - l_i)^q)) \right)^{\frac{2w_i w_j}{1+w_i}}, \\
& \quad \left((((1 - (1 - m_i)^p) + (1 - (1 - m_j)^q) \right. \\
& \quad \quad \left. - (1 - (1 - m_i)^p)(1 - (1 - m_j)^q)) \cdot ((1 - (1 - m_j)^p) + (1 - (1 - m_i)^q) \right. \\
& \quad \quad \left. - (1 - (1 - m_j)^p)(1 - (1 - m_i)^q)) \right)^{\frac{2w_i w_j}{1+w_i}}, \\
& \quad \left((((1 - (1 - u_i)^p) + (1 - (1 - u_j)^q) \right. \\
& \quad \quad \left. - (1 - (1 - u_i)^p)(1 - (1 - u_j)^q)) \cdot ((1 - (1 - u_j)^p) + (1 - (1 - u_i)^q) \right. \\
& \quad \quad \left. - (1 - (1 - u_j)^p)(1 - (1 - u_i)^q)) \right)^{\frac{2w_i w_j}{1+w_i}}
\end{aligned} \tag{4.47}$$

By referring to Equation (2.16),

$$\begin{aligned}
\prod_{i,j=1; i \leq j}^n \left((ph_i \oplus qh_j) \otimes (ph_j \oplus qh_i) \right)^{\frac{2w_i w_j}{1+w_i}} &= \prod_{i,j=1; i \leq j}^n \left[\left((1 - (1 - l_i)^p) + (1 - (1 - l_j)^q) - (1 - (1 - l_i)^p)(1 - (1 - l_j)^q) \right) \right. \\
&\quad \cdot \left. \left((1 - (1 - l_j)^p) + (1 - (1 - l_i)^q) - (1 - (1 - l_j)^p)(1 - (1 - l_i)^q) \right) \right]^{\frac{2w_i w_j}{1+w_i}} \\
&\times \prod_{i,j=1; i \leq j}^n \left[\left((1 - (1 - m_i)^p) + (1 - (1 - m_j)^q) - (1 - (1 - m_i)^p)(1 - (1 - m_j)^q) \right) \right. \\
&\quad \cdot \left. \left((1 - (1 - m_j)^p) + (1 - (1 - m_i)^q) - (1 - (1 - m_j)^p)(1 - (1 - m_i)^q) \right) \right]^{\frac{2w_i w_j}{1+w_i}} \quad (4.48) \\
&\times \prod_{i,j=1; i \leq j}^n \left[\left((1 - (1 - u_i)^p) + (1 - (1 - u_j)^q) - (1 - (1 - u_i)^p)(1 - (1 - u_j)^q) \right) \right. \\
&\quad \cdot \left. \left((1 - (1 - u_j)^p) + (1 - (1 - u_i)^q) - (1 - (1 - u_j)^p)(1 - (1 - u_i)^q) \right) \right]^{\frac{2w_i w_j}{1+w_i}}.
\end{aligned}$$

By referring to Equation (2.14), we get:

$$\begin{aligned}
& \frac{1}{p+q} \bigotimes_{i,j=1; i \leq j}^n \left((ph_i \oplus qh_j) \otimes (ph_j \oplus qh_i) \right)^{\frac{2w_i w_j}{1+w_i}} \\
&= 1 - \left(1 - \prod_{i,j=1; i \leq j}^n \eta_{i,j}^{\frac{2w_i w_j}{1+w_i}} \right)^{\frac{1}{p+q}} \\
&= 1 - \left[1 - \prod_{i,j=1; i \leq j}^n (\eta_{i,j})^{\frac{2w_i w_j}{1+w_i}} \right]^{\frac{1}{p+q}}
\end{aligned} \tag{4.49}$$

Equation (4.37) has been proven.

Theorem 4.6. Idempotency property $A = (l_i, m_i, u_i) (i = 1, 2, \dots, n)$ represent a set of HTFEs. If $A_i = (l_i, m_i, u_i)$ for all i , then,

$$HTF - GGHM^{p,q}(A_1, A_2, \dots, A_n) = a \tag{4.50}$$

Proof. Since $A_i = (l_i, m_i, u_i) (i = 1, 2, \dots, n)$, then,

$$\begin{aligned}
GGHM^{p,q}(a_1, a_2, \dots, a_n) &= \frac{1}{p+q} \left[\prod_{i,j=1}^n (pa_i + qa_j) \right]^{\frac{2}{n(n+1)}} \\
&= \frac{1}{p+q} \left[\prod_{i,j=1}^n (pa + qa) \right]^{\frac{2}{n(n+1)}} \\
&= \frac{1}{p+q} \left[(a(p+q))^{\frac{n(n+1)}{2}} \right]^{\frac{2}{n(n+1)}} \\
&= \frac{1}{p+q} (a(p+q)) \\
&= a.
\end{aligned} \tag{4.51}$$

Theorem 4.7. Monotonicity property Let h_{α_i} and h_{β_i} ($i = 1, 2, \dots, n$) be two collections of HTFEs; if, for any $\gamma_{\alpha_i} \leq h_{\alpha_i}$ and $\gamma_{\beta_j} \leq h_{\beta_j}$ ($i, j = 1, 2, \dots, n; i \neq j$), one has $\gamma_{\alpha_i} \leq \gamma_{\beta_i}$ and $\gamma_{\alpha_j} \leq \gamma_{\beta_j}$, then

$$HTF - GGHM^{p,q}(h_{\alpha_1}, h_{\alpha_2}, \dots, h_{\alpha_n}) \leq HTF - GGHM^{p,q}(h_{\beta_1}, h_{\beta_2}, \dots, h_{\beta_n}) \quad (4.52)$$

Proof. By proposition (4.20), we get $\eta_{\alpha_{ij}} \leq \eta_{\beta_{ij}}, i, j \in \{1, 2, \dots, n\}, i \neq j$. Then,

$$1 - \left(1 - \prod_{i,j=1; i \leq j}^n \eta_{\alpha_{ij}}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}} \leq 1 - \left(1 - \prod_{i,j=1; i \leq j}^n \eta_{\beta_{ij}}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}} \quad (4.53)$$

By Definition (4.4.1), we acquire

$$\begin{aligned} HTF - GGHM^{p,q}(h_{\alpha_1}, h_{\alpha_2}, \dots, h_{\alpha_n}) &= \frac{1}{p+q} \otimes_{i,j=1; i \leq j}^n \left(\eta_{\alpha_{ij}}^{\frac{2}{n(n+1)}} \right) \\ &= \frac{1}{p+q} \otimes_{i,j=1; i \leq j}^n \left(\eta_{\alpha_{ij}}^{\frac{2}{n(n+1)}} \right) \end{aligned} \quad (4.54)$$

$$\begin{aligned} &\bigcup_{\eta_{ij} \in \sigma_i, i \leq j} \left\{ 1 - \left(1 - \prod_{i,j=1; i \leq j}^n \eta_{\alpha_{ij}}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}} \right\} \\ &\leq \bigcup_{\eta_{ij} \in \sigma_i, i \leq j} \left\{ 1 - \left(1 - \prod_{i,j=1; i \leq j}^n \eta_{\beta_{ij}}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}} \right\} \end{aligned} \quad (4.55)$$

$$= \frac{1}{p+q} \bigotimes_{i,j=1; i \leq j}^n \left(\eta_{\beta_{ij}}^{\frac{2}{n(n+1)}} \right)$$

$$= HTF - GGHM^{p,q}(h_{\beta_1}, h_{\beta_2}, \dots, h_{\beta_n})$$

Theorem 4.8. Boundedness Property Let $h_i (i = 1, 2, \dots, n)$ be a collection of HTEs, where $h_i^- = \cup_{\gamma_i \in h_i} (\min\{\gamma_i^L\}, \min\{\gamma_i^M\}, \min\{\gamma_i^U\})$ and $h_i^+ = \cup_{\gamma_i \in h_i} (\max\{\gamma_i^L\}, \max\{\gamma_i^M\}, \max\{\gamma_i^U\})$, $i, j \in \{1, 2, \dots, n\}$. Then,

$$\begin{aligned}
& \bigcup_{\gamma^- \in h_i^-} \left(((1 - (1 - \gamma^{-L})^{p+q})^2)^{\frac{1}{p+q}}, ((1 - (1 - \gamma^{-M})^{p+q})^2)^{\frac{1}{p+q}}, ((1 - (1 - \gamma^{-U})^{p+q})^2)^{\frac{1}{p+q}} \right) \\
& \leq HTFGGHM^{p,q}(h_1, h_2, \dots, h_n) \\
& \leq \bigcup_{\gamma^+ \in h_i^+} \left(((1 - (1 - \gamma^{+L})^{p+q})^2)^{\frac{1}{p+q}}, ((1 - (1 - \gamma^{+M})^{p+q})^2)^{\frac{1}{p+q}}, ((1 - (1 - \gamma^{+U})^{p+q})^2)^{\frac{1}{p+q}} \right). \\
& \leq HTFGGHM^{p,q}(h_1, h_2, \dots, h_n)
\end{aligned} \tag{4.56}$$

Finally, the WHTFGGHM operator's properties are well-established

4.6 Numerical Example

To demonstrate how the proposed method works in practice, a numerical example inspired by Zhao et al. (2014) is used. Since the study by Zhao et al. (2014) has already established the criterion weights, the present analysis employs the WHTFGGHM operator. This example focuses on a supplier selection scenario within supply chain management, where hesitant triangular fuzzy information plays a central role. Specifically, it addresses a typical multiple attribute decision-making problem involving five candidate suppliers, $A_i (i = 1, 2, 3, 4, 5)$. These suppliers are evaluated based on four key attributes: product quality (G_1), service (G_2), delivery performance (G_3), and price (G_4).

To ensure fairness in the evaluation process, the suppliers are assessed anonymously by the decision makers. The evaluations are organized into a decision matrix $H = [h_{ij}]_{5 \times 4}$, where each element $h_{ij} (i = 1, 2, 3, 4, 5; j = 1, 2, 3, 4)$ represents a hesitant triangular fuzzy element (HTFE), as shown in Table 4.1. The importance of each attribute is reflected in the weight vector $\omega = (0.1, 0.3, 0.2, 0.4)$, where product quality G_1 , service G_2 , delivery performance G_3 , and price G_4 are assigned weights of 0.1, 0.3, 0.2, and 0.4, respectively. These weights indicate that price has the highest importance, followed by service, delivery performance, and product quality. Using this setup, the proposed method is applied to address the supplier selection problem under hesitant triangular fuzzy conditions.

Table 4.1
Hesitant Triangular Fuzzy Decision Matrix

	G_1	G_2	G_3	G_4
A_1	$\{(0.3, 0.4, 0.5)\}$	$\{(0.6, 0.7, 0.8), (0.7, 0.8, 0.9)\}$	$\{(0.2, 0.3, 0.5)\}$	$\{(0.2, 0.3, 0.4), (0.3, 0.4, 0.5)\}$
A_2	$\{(0.6, 0.7, 0.8)\}$	$\{(0.2, 0.4, 0.5)\}$	$\{(0.3, 0.4, 0.5), (0.6, 0.7, 0.8)\}$	$\{(0.2, 0.3, 0.5), (0.4, 0.5, 0.6), (0.5, 0.6, 0.7)\}$
A_3	$\{(0.5, 0.6, 0.7)\}$	$\{(0.7, 0.8, 0.9), (0.8, 0.9, 1.0)\}$	$\{(0.2, 0.3, 0.4), (0.4, 0.5, 0.6)\}$	$\{(0.5, 0.6, 0.7)\}$
A_4	$\{(0.7, 0.8, 0.9)\}$	$\{(0.6, 0.7, 0.8), (0.7, 0.8, 0.9)\}$	$\{(0.1, 0.2, 0.3)\}$	$\{(0.3, 0.4, 0.5), (0.6, 0.7, 0.8)\}$
A_5	$\{(0.6, 0.7, 0.8)\}$	$\{(0.2, 0.3, 0.4), (0.5, 0.6, 0.7)\}$	$\{(0.4, 0.5, 0.6)\}$	$\{(0.2, 0.3, 0.4)\}$

Step 1. Using the *WHTFGGHM* operator with $p = 1$ and $q = 1$, all preference values b_{ij} for $j = 1, 2, 3, 4$ in the i -th row are aggregated to obtain the overall performance value b_i corresponding to alternative A_i . The values $p = 1$ and $q = 1$ are selected as the baseline parameter setting because they assign equal interaction strength to the paired hesitant triangular fuzzy elements. This balanced setting is suitable for validating the proposed operator against the reference example without introducing additional bias from unequal parameter effects.

$$\begin{aligned}
 b_1 &= \{0.1653, 0.3556, 0.5084\}, b_2 = \{0.1699, 0.3708, 0.4953\}, \\
 b_3 &= \{0.3529, 0.5327, 0.6533\}, b_4 = \{0.2502, 0.4251, 0.5477\}, \\
 b_5 &= \{0.1404, 0.3325, 0.4457\}.
 \end{aligned} \tag{4.57}$$

Step 2. Compute the scores for all alternatives based on the values of b_i for $i = 1, 2, 3, 4, 5$.

$$\begin{aligned}
 S(b_1) &= 0.3431, S(b_2) = 0.3453, S(b_3) = 0.5130, \\
 S(b_4) &= 0.4077, S(b_5) = 0.3062.
 \end{aligned} \tag{4.58}$$

Step 3. Given that $S(b_3) > S(b_4) > S(b_2) > S(b_1) > S(b_5)$, and according to Definition (2.10.8), the ranking of the hesitant triangular fuzzy elements (b_i) is $b_3 > b_4 > b_2 > b_1 > b_5$. Consequently, the ranking of the alternatives A_i is $A_3 > A_4 > A_2 > A_1 > A_5$. Therefore, A_3 is identified as the best alternative.

Next, if the *WHTFGGHM* operator is applied with $p = 3$ and $q = 1$, then,

$$\begin{aligned} b_1 &= \{0.2012, 0.3632, 0.5079\}, b_2 = \{0.2100, 0.3998, 0.5177\}, \\ b_3 &= \{0.3884, 0.5214, 0.6216\}, b_4 = \{0.2570, 0.3905, 0.4913\}, \\ b_5 &= \{0.1803, 0.3595, 0.4664\}. \end{aligned} \quad (4.59)$$

and all of the alternatives' scores are

$$\begin{aligned} S(b_1) &= 0.3574, S(b_2) = 0.3758, S(b_3) = 0.5105, \\ S(b_4) &= 0.3796, S(b_5) = 0.3354. \end{aligned} \quad (4.60)$$

As a result, the ranking of the alternatives A_i ($i = 1, 2, 3, 4, 5$) is now

$$A_3 > A_4 > A_2 > A_1 > A_5. \quad (4.61)$$

Therefore, A_3 remains the best alternative. Figure 4.2 shows the scores for each alternative using the *WHTFGGHM* operator.

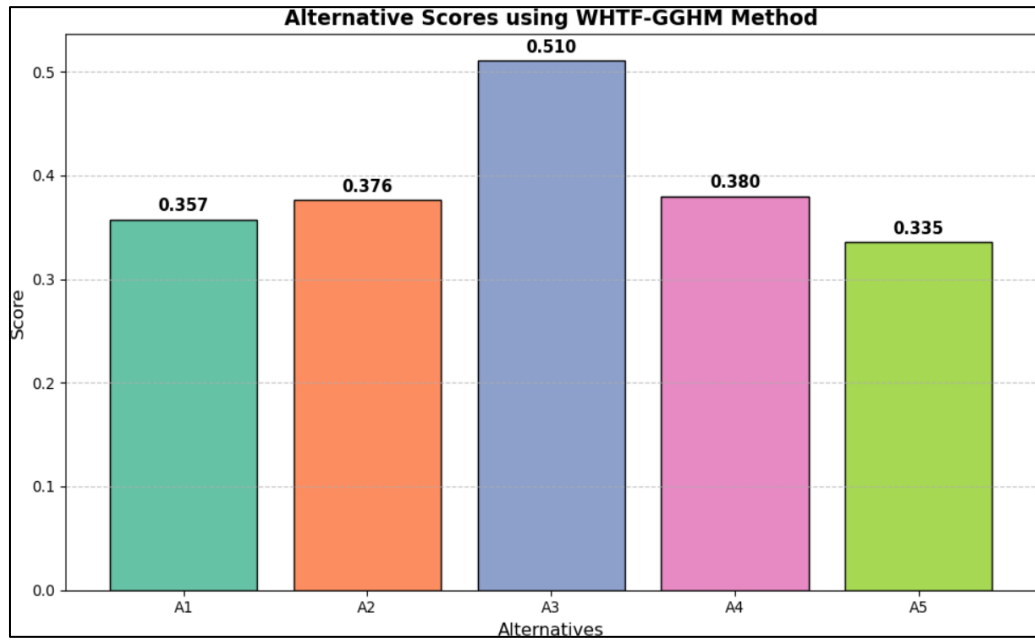


Figure 4.2 Score for Each Alternatives

From Figure 4.2, The WHTFGGHM operator produces rankings identical to Zhao et al. (2014)'s study ($A_3 > A_4 > A_2 > A_1 > A_5$) confirming its validity when parameter settings align with the original methodology. This consistency demonstrates its reliability in replicating rankings under similar conditions, making it a robust tool for handling hesitant triangular fuzzy information in multi criteria decision making.

Furthermore, as illustrated in Figure 4.2, A_3 achieves the highest score of 0.510, signifying its status as the most favored alternative based on the specified criteria. With scores of 0.380 and 0.376, respectively, A_4 and A_2 reflect a comparable performance while falling a bit short of A_3 's higher superiority. In the meantime, A_1 managed to achieve a decent score of 0.357, whereas A_5 fell behind with the lowest score of 0.335, suggesting it reflects the poorest overall performance. The variations in scores for the leading three alternatives (A_3 , A_4 , and A_2) signify a reliable ranking layout, underlining the technique's effectiveness in clarifying distinctions among similarly matched options. This outcome further affirms the consistency, accuracy, and appropriateness of the WHTFGGHM operator for practical decision-making scenarios that involve uncertainty and hesitation.

4.6.1 Sensitivity Analysis

This section covers sensitivity analysis, which evaluates the stability of a decision making model by adjusting input parameters like weights or membership values. It identifies the most influential factors and tests the model's performance under different conditions, improving reliability and real-world applicability. Figure 4.3 presents the sensitivity analysis results for each alternative.

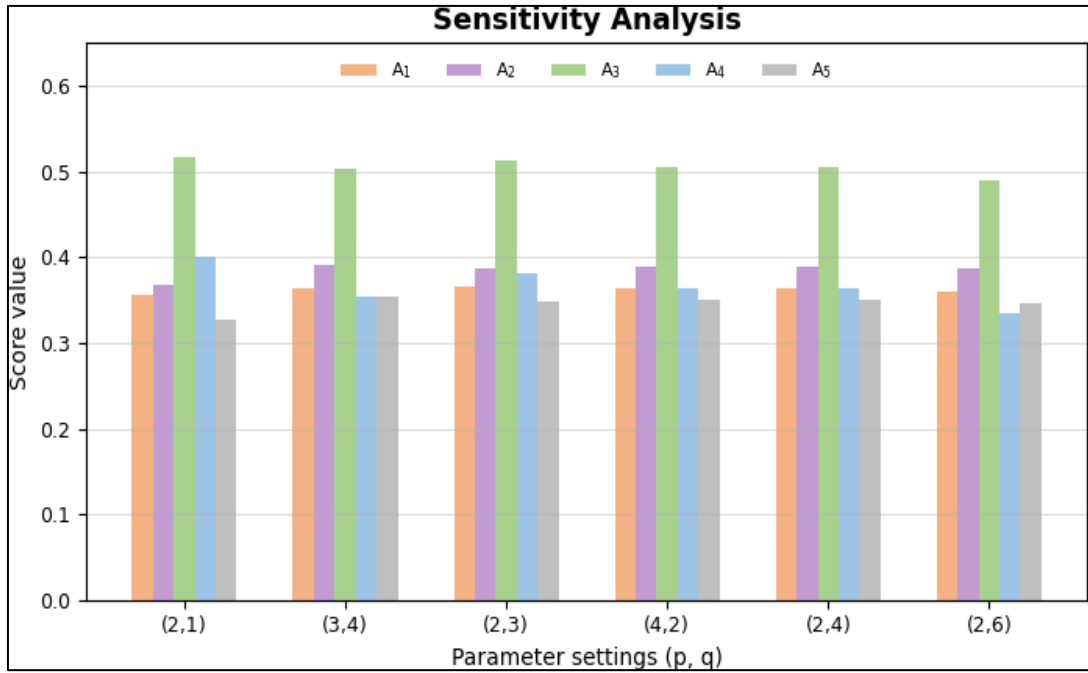


Figure 4.3 Sensitivity Analysis Results of The Alternatives Under Different Parameter Settings (p, q).

Figure 4.3 shows the sensitivity analysis results for five alternatives (A_1 to A_5) under different parameter settings (p and q). A_3 consistently ranks the highest, proving to be the most reliable choice, while A_5 performs the lowest, making it the least preferred option. Alternatives A_1 , A_2 , and A_4 show moderate sensitivity with minor score variations. A_3 's score slightly decreases as p and q shift from (2,1) to (2,6), while A_2 and A_5 remain stable or show slight improvement. This analysis highlights A_3 's dominance and underscores the importance of sensitivity evaluation in ensuring reliable decision making.

The parameter settings in Table 4.2 are selected to examine the effect of different interaction strengths between hesitant triangular fuzzy elements. The values p and q act as control parameters in the WHTFGGHM operator, where different

combinations reflect different levels of interaction between paired criteria. The setting $p = 2, q = 1$ is used as the baseline case, while the other combinations are included to test the stability of the ranking results when the influence of the two parameters changes. This standard procedure allows the sensitivity analysis to evaluate whether the proposed operator produces consistent rankings under various aggregation conditions.

Table 4.2
Sensitivity Analysis Under Different Parameter Settings (p, q) .for WHTFGGHM

p, q	score					Alternatives Ranking
	value					
	A ₁	A ₂	A ₃	A ₄	A ₅	
$p = 2, q = 1$	0.3557	0.3673	0.5169	0.3999	0.3276	$A_3 > A_4 > A_2 > A_1 >$
$p = 3, q = 4$	0.3647	0.3913	0.5028	0.3550	0.3545	A_5
						$A_3 > A_2 > A_1 > A_4 >$
$p = 2, q = 3$	0.3654	0.3868	0.5136	0.3820	0.3489	A_5
						$A_3 > A_4 > A_2 > A_1 >$
$p = 4, q = 2$	0.3638	0.3881	0.5057	0.3632	0.3497	A_5
						$A_3 > A_2 > A_1 > A_4 >$
$p = 2, q = 4$	0.3638	0.3881	0.5057	0.3632	0.3497	A_5
						$A_3 > A_2 > A_1 > A_4 >$
$p = 2, q = 6$	0.3595	0.3867	0.4903	0.3349	0.3459	A_5
						$A_3 > A_2 > A_1 > A_5 >$
						A_4

Table 4.2 presents the sensitivity analysis results for five alternatives (A_1 , to A_5) under different p and q values using the WHTFGGHM framework. The selected values of p and q test how well the WHTFGGHM method adapts to diverse aggregation influences and weighting preferences. In the first scenario ($p = 2, q = 1$), A_3 ranks highest with a score of 0.5169, followed by A_4 (0.3999), A_2 (0.3673), A_1 (0.3557), and A_5 (0.3276). In another scenario ($p = 3, q = 4$), A_3 remains the top choice with 0.5028, while A_2 (0.3913) and A_1 (0.3647) show slight improvements. A_4 (0.3550) and A_5 (0.3545) perform nearly the same, indicating similar effectiveness in this case.

As p and q shift to $p = 2, q = 3$, A_3 maintains its lead with a score of 0.5136. Other alternatives show minor changes, with A_2 at 0.3868, A_1 at 0.3654, A_4 at 0.3802,

and A_5 at 0.3489. For both $p = 4, q = 2$ and $p = 2, q = 4$, A_3 continues to dominate with a score of 0.5057, while the other alternatives remain relatively stable, with A_2 at 0.3881, A_1 at 0.3638, A_5 at 0.3497, and A_4 slightly lower. In the final scenario ($p = 2, q = 6$), A_3 sees a slight drop to 0.4903 but still holds the top spot, followed by A_2 (0.3867), A_1 (0.3595), A_5 (0.3459), and A_4 (0.3349).

The sensitivity analysis shows that A_3 consistently ranks highest across all scenarios, confirming the reliability and stability of the WHTFGGHM operator. While other alternatives shift slightly with changes in p and q , which affect criteria weights, A_3 remains on top. For example, with $p = 3$ and $q = 4$, the ranking is $A_3 > A_2 > A_1 > A_4 > A_5$ (as in Table 4.2). This proves the method's adaptability in handling uncertainty and hesitation. Figure 4.4 shows the radar plot scores supporting these findings.

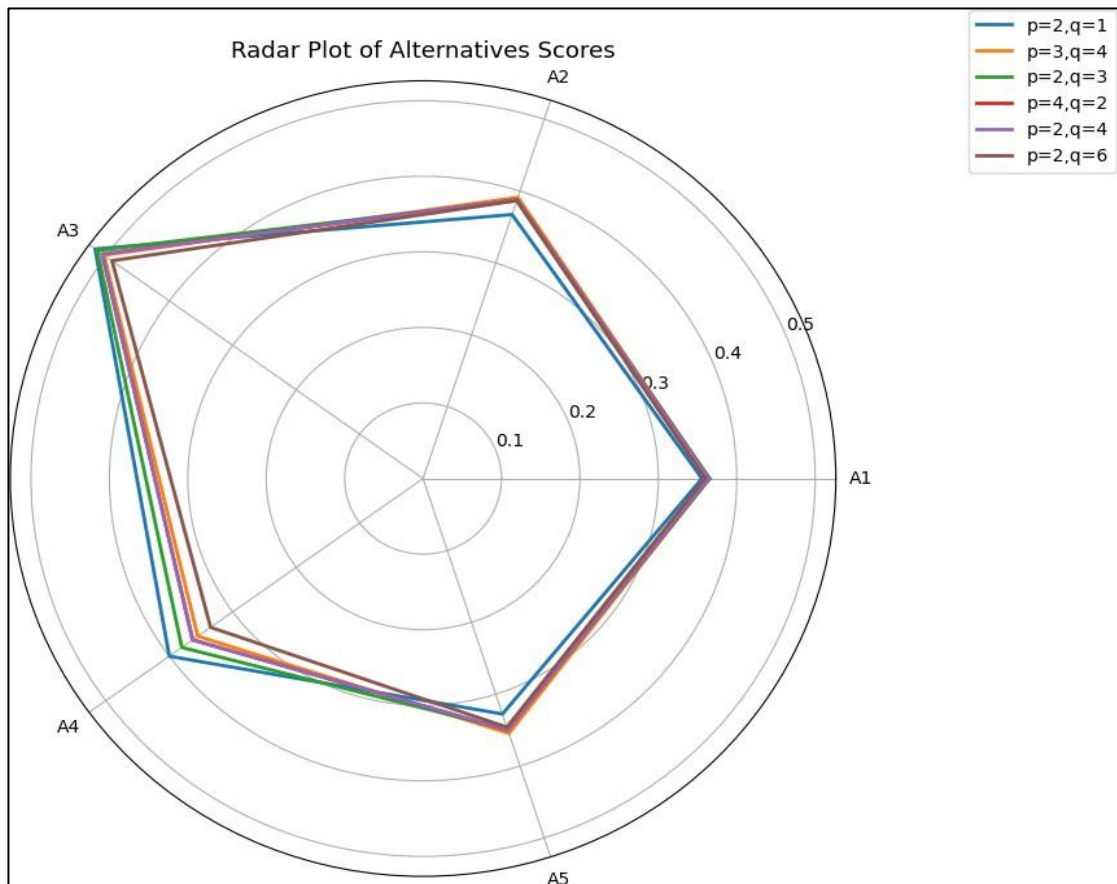


Figure 4.4 Radar Plot of Each Alternatives

The findings depicted in the Table 4.2 and radar plot Figure 4.4 demonstrate how fluctuations in the parameters p and q affect the combined scores and ranking results of the five alternatives ($A_1 > A_2 > A_3 > A_4 > A_5$). In every variation of p

and q considered, the third alternative (A_3) persistently earns the highest score, highlighting its status as the most favored selection no matter how the variables are tweaked. This resilience implies that A_3 showcases stable dominance across varying levels of fuzziness and interaction strengths represented by p and q . Conversely, A_5 often registers the lowest score in the majority of configurations, indicating that its performance is relatively inferior across all aggregation scenarios. The persistent order of preference across different parameter settings highlights the operator's consistency and dependability in yielding coherent decision results.

Examining the score values suggests that the variation across options is somewhat insignificant, with scores hovering around 0.33 to 0.52. This suggests that even though changes in parameters influence the sensitivity of the aggregation, they do not significantly alter the overall ranking framework. The scores for A_2 and A_4 show moderate fluctuations as p and q increase, suggesting that these alternatives are more responsive to alterations in the geometric Heronian mean parameters. As an illustration, when p is 3 and q is 4, A_2 hits its maximum score (0.3913), boosting its position to second place, whereas A_4 witnesses a small decrease. Such variations illustrate the impact of p and q in managing the interaction balance between optimistic and pessimistic assessments within the hesitant triangular fuzzy context.

The radar plot visually reinforces these observations. The alignment of the graphed lines for assorted (p, q) combinations indicates that parameter adjustments result in only minimal differences in the performance of the alternatives. Conversely, small differences emerge, especially along the axes associated with A_2 and A_4 , where the lines either extend or retract as per the parameter arrangement. This graphical consistency indicates that the WHTFGGHM operator maintains stability and convergence in aggregation, implying that it yields similar decision trends even under different parametric conditions. Such behavior is advantageous in practical MCDM scenarios, as it assures decision-makers of the reliability of outcomes without the danger of significant ranking shifts due to parameter adjustments.

In summary, the analysis verifies that the WHTFGGHM operator effectively maintains ranking consistency, sensitivity control, and interpretative reliability across varying parameter conditions. The parameters p and q function as adaptable control

factors that regulate the interaction level among hesitant fuzzy values, yet their impact remains within a confined range, ensuring decision stability. The combined findings from both the table and radar visualizations affirm the operator's ability to manage uncertain and hesitant information while sustaining a coherent and interpretable decision framework. This emphasizes how effective the method is in practical terms as a sturdy and well-balanced aggregation technique that fits intricate multi-criteria decision-making difficulties.

CHAPTER 5

APPLICATION OF HESITANT TRIANGULAR FUZZY GENERALIZED GEOMETRIC HERONIAN MEAN AND SYAI METHOD IN EDUCATIONAL DECISION-MAKING

5.1 Introduction

This chapter illustrates the application of the Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean (HTFGGHM) operator combined with the Simplified Yielded Aggregation Index (SYAI) method in an educational decision making setting. The HTFGGHM operator is employed in this chapter instead of the weighted HTFGGHM operator because the case study does not assign predetermined weights to the expert evaluations during the fuzzy aggregation stage. The five Arabic language lecturers are treated as expert evaluators with equal importance, and their hesitant triangular fuzzy judgments are aggregated without imposing additional weighting assumptions. This is suitable for the present educational decision-making context because the main objective is to preserve the hesitation and linguistic uncertainty of expert opinions while capturing the interrelationships among the evaluation criteria.

In this application, all five decision makers are assumed to have equal importance during the fuzzy aggregation stage. Therefore, no specific decision-maker weight is assigned to differentiate the influence of each expert. This assumption is reasonable in the present case study because all experts are Arabic language lecturers with relevant teaching experience and academic qualifications. However, the absence of decision-maker weights may limit the model when experts have different levels of experience, specialization, or authority. Future studies may extend the proposed framework by incorporating expert weight determination methods to reflect the relative importance of decision makers more explicitly.

Although weighting is later applied in the SYAI ranking stage, equal weights are assigned to the main evaluation criteria to indicate that each criterion is considered equally important in this case study. These weights are used only after the hesitant triangular fuzzy information has been converted into score values. Therefore, the use of HTFGGHM is appropriate at the aggregation stage, while SYAI handles the

subsequent weighting and ranking process. This separation avoids unnecessary assumptions about expert weights and maintains a clear methodological flow between fuzzy aggregation and final decision ranking.

The case study focuses on assessing the effectiveness of humor-based teaching strategies in Arabic language learning at Universiti Teknologi MARA (UiTM), Kuala Pilah Campus, where Arabic is offered as a third language. The expert respondents consist of five Arabic language lecturers who provided linguistic ratings based on several educational criteria, namely learner engagement, language comprehension, motivation, teaching effectiveness, cultural appropriateness, and readiness for use.

The collected data are first evaluated at the sub-criteria level and then aggregated to the main criteria level using the HTFGGHM operator. Next, the aggregated hesitant triangular fuzzy values are transformed into score values using the Hesitant Triangular Scoring (HTS) function and ranked using the SYAI method. The resulting rankings are compared with several classical MCDM methods, namely TOPSIS, VIKOR, SAW, COBRA, WASPAS, and MOORA, to examine the stability and robustness of the proposed framework.

The main contribution of this chapter is to demonstrate that the integration of HTFGGHM and SYAI can preserve hesitation, linguistic uncertainty, and expert subjectivity in educational evaluation. This makes the proposed framework suitable for assessing teaching effectiveness under fuzzy and uncertain decision-making conditions

5.2 Framework of Integration of HTFGGHM and SYAI

5.2.1 Conceptual Overview

The combination of HTFGGHM and SYAI allows systematizing the assessment of qualitative expert judgments. The proposed workflow is shown in Figure 5.1 as it operates with linguistic data collected and with comparative analysis as the final point.



Figure 5.1 Framework for Integrating HTFGGHM and SYAI in Multi-Criteria Decision-Making

There are seven procedural steps for enhanced the integrating of HTFGGHM and SYAI Method

5.2.2 Procedural Steps

Step 1: Data Collection: A panel of five experts was asked to evaluate how effective each humor-oriented teaching method was based on various detailed criteria. Every sub-criterion underwent evaluation with a 7-point linguistic scale that extends from '*Very Low*' to '*Very High*,' as illustrated in Table 5.1. The participation of various experts works to alleviate personal biases and offers a complete illustration of evaluative insights spanning linguistic, instructional, and cultural areas.

Step 2: Aggregation at Criteria Level: The responses corresponding to each primary criterion were systematically organized in accordance with their associated sub-criteria (SC_{ij}). Consequently, these sub-criteria values were integrated at the criteria level through the implementation of the Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean (HTFGGHM) operator. This operator skillfully brings together the connections between uncertain fuzzy assessments while maintaining a balance between multiplicative and additive aggregation techniques.

Step 3: Linguistic TFN Conversion: The assessments of language by each specialist were translated into related Triangular Fuzzy Numbers (TFNs) via the linguistic scale detailed in Table 5.1. This procedure guarantees that qualitative assessments articulated in linguistic terms can be quantitatively represented, while simultaneously maintaining the uncertainty and ambiguity that are intrinsic to expert judgments.

Step 4: HTFGGHM Aggregation: Following the conversion process, the disparate hesitant triangular fuzzy decision matrices (DM1–DM5) generated by all experts were amalgamated into a singular comprehensive decision matrix. The HTFGGHM operator was utilized to synthesize these evaluations, given its efficacy in managing hesitant and triangular fuzzy data. The aggregation formula guarantees that the opinions of all experts are proportionately incorporated in accordance with the geometric Heronian mean framework, thereby producing an all-encompassing hesitant triangular fuzzy decision matrix. The process of aggregation is represented mathematically as detailed in Equation (4.2), which delineates the Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean (HTFGGHM) operator.

Step 5: Hesitant Triangular Scoring (HTS): Upon gathering the triangular fuzzy matrix, every fuzzy element transitioned into a concrete numerical value by applying the hesitant fuzzy scoring technique. In accordance with Definition 2.10.7, the scoring function relevant to a hesitant fuzzy element h is formulated as:

$$S(h) = \frac{1}{\#h} \sum_{\gamma \in h} \gamma \quad (5.1)$$

where $\#h$ represents the total number of elements within h . This scoring process provides a single representative value for each hesitant fuzzy element, simplifying the subsequent normalization and ranking operations while maintaining the comparative significance among alternatives.

Step 6: SYAI Ranking: The processed HTS values, now normalized using Equation (3.2), utilize the Simplified Yielded Aggregation Index (SYAI) technique. This particular phase calculates the yielded index (YI) for each alternative, encapsulating the aggregated performance across all evaluative criteria. The SYAI methodology furnishes a transparent ranking framework by yielding consistent and interpretable outcomes while simultaneously mitigating computational complexity in comparison to other multi-criteria decision-making (MCDM) methodologies.

Step 7: Comparative Analysis: In order to validate the reliability and uniformity of the outcomes derived from the SYAI framework, a comparative assessment was performed in conjunction with traditional Multi-Criteria Decision-Making (MCDM) methods specifically, TOPSIS, VIKOR, SAW, COBRA, WASPAS, and MOORA. The rankings and score distributions were thoroughly analyzed to illuminate the correlation and consistency of SYAI in connection with established methodologies, thus guaranteeing that the proposed framework generates stable and theoretically consistent results across a range of evaluative contexts

5.3 Linguistic Variables and Triangular Fuzzy Representation

The evaluations provided by the five Arabic language lecturers were expressed linguistically using terms such as *Very Low*, *Low*, *Medium Low*, *Medium*, *Medium High*, *High*, and *Very High*. However, since linguistic data cannot be directly processed in numerical computation, these qualitative terms were converted into Triangular Fuzzy Numbers (TFNs) to represent the uncertainty and subjectivity inherent in human judgment.

Each linguistic variable corresponds to a triangular membership function, defined by three parameters: the *lower limit* (l), *most likely value* (m), and *upper limit* (u), which together describe the range and degree of confidence of each linguistic term. This transformation ensures that expert perceptions remain intact while enabling the fuzzy-based mathematical operations required in the HTFGGHM and SYAI frameworks.

Table 5.1 presents the linguistic–TFN conversion scale adapted from m Elkadry et al. (2023), while Figure 5.2 depicts the corresponding triangular membership functions.

Table 5.1
Linguistic Variables and Corresponding TFNs Adapted From Elkadry et al. (2023)

Linguistic Variable	Notation	TFN	Score
Very Low	VL	(0.0, 0.0, 0.1)	1
Low	L	(0.0, 0.1, 0.3)	2
Medium Low	ML	(0.1, 0.3, 0.5)	3
Medium	M	(0.3, 0.5, 0.7)	4
Medium High	MH	(0.5, 0.7, 0.9)	5
High	H	(0.7, 0.9, 1.0)	6
Very High	VH	(0.9, 0.9, 1.0)	7

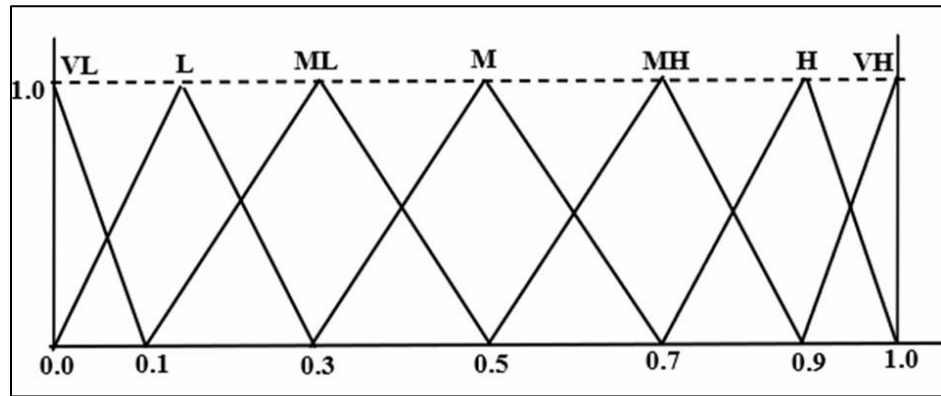


Figure 5.2 Triangular Fuzzy Membership Functions Representing Linguistic Variables.

This linguistic–TFN mapping plays a pivotal role in translating experts’ subjective evaluations into a fuzzy domain that retains the ambiguity of human reasoning. It allows the HTFGGHM operator to aggregate the evaluations while preserving the hesitancy of expert opinions rather than forcing early defuzzification. Consequently, the proposed framework maintains both precision and interpretability throughout the decision-making process.

5.4 Data Description and Structure

5.4.1 Respondent and Sampling

Five Arabic language lecturers from UiTM Kuala Pilah participated as expert evaluators. All respondents possess more than three years of teaching experience and

postgraduate academic qualifications (Master's or PhD). Their evaluations covered six major dimensions relevant to Arabic language teaching.

5.4.2 Criteria and Sub-Criteria Translation

Figure 5.3 presents a structured hierarchy for analyzing the effectiveness of humor-based teaching methodologies, connecting the central goal with six evaluative standards (C1–C6), their relevant sub-factors, along with four different alternatives (A1–A4).

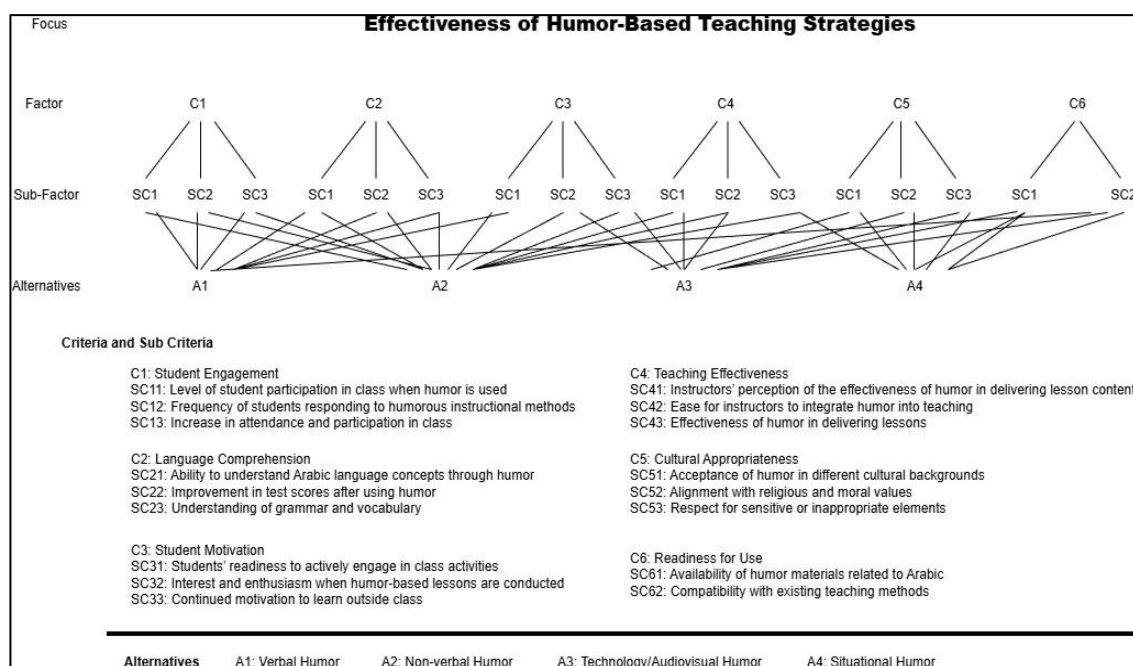


Figure 5.3 Evaluation Framework for Humor-Based Teaching: Factors, Sub-Factors, and Alternatives

Based on Figure 5.3, each sub-criterion was rated across four humor categories: Verbal Humor (A1), Non-Verbal Humor (A2), Technology-Based (Audiovisual) Humor (A3), and Situational Humor (A4). At the foundational level of the hierarchy, the four Alternatives delineate various methodologies through which humor may be employed as an instructional strategy. A1 (Verbal Humor) entails the application of jokes, puns, or lighthearted linguistic expressions that significantly enrich the language acquisition experience. A2 (Non-verbal Humor) involves the employment of gestures, body language, and facial expressions that can improve the classroom setting and aid comprehension in the lack of spoken communication. A3 (Technology/Audiovisual Humor) refers to the integration of tools such as memes, succinct video clips, or

auditory elements to bolster pedagogical ideas, particularly those that are elaborate. Ultimately, A4 (Situational Humor) surfaces spontaneously from various situations in the classroom, enhancing rapport and supporting student motivation.

The interconnecting relationships between Sub-Factors (SC_{ij}) and Alternatives (A1–A4) signify that each type of humor has the ability to influence multiple elements of the learning continuum. As an illustration, A2 may contribute to the enhancement of both C1 and C2 by keeping students alert and assisting their understanding of meanings that surpass verbal expression. In a similar manner, A3 could bolster C4 by making abstract concepts more comprehensible; nonetheless, it might also incite worries about C5 if the content does not reflect contextual nuance. A4 often serves to elevate C3 and C1, albeit its regularity may differ based on the teacher's improvisational capabilities and the nuances of the classroom atmosphere.

From a functional perspective, this hierarchical schema translates abstract judgments into discernible criteria. Each sub-factor represents a specific or measurable benchmark, including levels of involvement, improvements in examination results, or assessments of cultural relevance. This approach facilitates an impartial analysis of every Alternative. A Given that all six criteria (C1–C6) can be classified as benefits type and do not assume predetermined target values, no goal-based aspect of SYAI is used in the analysis, which guarantees its reasonable comparability with traditional MCDM practices. Hence, cost-type transformation will not be needed in the analysis, although intermediate numbers may occur naturally in the 7-point scale (e.g., 4.5 or 5.6). Prior to the allocation of weights, it is crucial to ensure that closely related Criteria, such as C1 and C3, are not redundantly counted and that all scales are standardized. Upon validation, the architecture establishes a coherent and rational basis for contrasting humor-infused instructional strategies, ultimately clarifying which Alternative most successfully realizes the principal goal of enhancing the Effectiveness of Humor-Based Teaching Strategies. Table 5.2 illustrates the six fundamental criteria utilized to evaluate the effectiveness of humor integration in educational settings, with each criterion clarifying a distinct aspect of how humor influences student engagement, understanding, motivation, instructional quality, cultural relevance, and practical readiness for application

Table 5.2
Criteria for Student Engagement Through Humor Integration

Criteria	Descriptions
Student Engagement (C1)	Refers to the level of students' active participation and involvement in classroom activities as influenced by humor-based teaching strategies. It reflects how humor encourages students to interact, respond, and participate more effectively during learning sessions.
Language Comprehension (C2)	Examines how humor contributes to students' understanding of Arabic language concepts, grammar, and vocabulary. It highlights how humor can simplify complex ideas and strengthen students' comprehension.
Student Motivation (C3)	Assesses the extent to which humor increases students' enthusiasm, interest, and willingness to learn. It indicates how humor promotes a more engaging and enjoyable classroom atmosphere that sustains learning motivation.
Teaching Effectiveness (C4)	Focuses on how effectively humor supports the teaching process and enhances instructors' ability to convey lessons. It considers how humor helps explain complex topics and improves classroom interaction.
Cultural Appropriateness (C5)	Ensures that humor used in teaching aligns with cultural, ethical, and religious norms. It emphasizes the use of appropriate humor that respects students' diverse backgrounds and values.
Readiness for Use (C6)	Evaluates instructors' preparedness and ability to apply humor-based approaches in their teaching. It also considers the availability of suitable materials and the compatibility of humor with existing teaching methods.

5.4.3 Experts' Evaluation

Each expert created a sub-criteria assessment matrix using linguistic expressions. Sub-criteria values were initially aggregated to their corresponding parent criterion using the HTFGGHM operator, thereby maintaining the inherently hesitant nature of expert judgment. Table 5.1–5.10 illustrates the five decision matrices (DM1–DM5) utilized in the current study, summarizing expert assessments of the four alternatives (A1–A4) across the extensive set of criteria (C1–C17). These preliminary ratings offer the necessary inputs for the upcoming normalization, weighting, and multi-method ranking. Criteria labels C1-C17 correspond to the following sub-criteria is illustrated in Table5.3:

Table 5.3
Description of Criteria Labels (C1–C17)

Criteria	Sub-Criteria Description
C1	SC11: Level of student participation in class when humor is used
C2	SC12: Frequency of students responding to humorous instructional methods
C3	SC13: Increase in attendance and participation in class
C4	SC21: Ability to understand Arabic language concepts through humor
C5	SC22: Improvement in test scores after using humor
C6	SC23: Understanding of grammar and vocabulary
C7	SC31: Students' readiness to actively engage in class activities
C8	SC32: Interest and enthusiasm during humor-based lessons
C9	SC33: Continued motivation to learn outside class
C10	SC41: Instructors' perception of humor effectiveness in delivering lesson content
C11	SC42: Ease for instructors to integrate humor into teaching
C12	SC43: Effectiveness of humor in delivering lessons
C13	SC51: Acceptance of humor in different cultural backgrounds
C14	SC52: Alignment with religious and moral values
C15	SC53: Respect for sensitive or inappropriate elements
C16	SC61: Availability of humor materials related to Arabic
C17	SC62: Compatibility with existing teaching methods

Table 5.4
Decision Matrices (DM1–DM5) for Alternatives A1–A4 Across Criteria C1–C17.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17
DM1																	
A1	7	7	4	5	4,5	5	6	7	5	6	6	4	6	5	5	7	7
A2	6	7	4	6	5	4,5	6	7	4,5	6	4,5	4	6	5	5	7	7
A3	6	5	4,5	6	5	5	6	6	6	5	5	4	6	6	5	4	5
A4	5,6	5	4	6	5	5	5	6,7	4	5	5	4	6	5	4,5	5	5
DM2																	
A1	5,7	5,6	6	6	7	7	7	7	5	7	6	6,7	6	7	6,7	7	6,7
A2	6,7	7	6	6	6	7	6,7	6	5	7	6,7	6	4,5	6	6	7	7
A3	6	6	5	5	4,5	4	5	4,5	4	4	5	4,5	5	4	6	4,5	6,7
A4	7	7	7	6	7	7	7	7	5	7	6	6	6	4,5	6	5	6
DM3																	
A1	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
A2	7	7	7	6,7	7	7	7	7	7	7	7	7	7	7	7	6	7

A3	6	4,5	6	6	6	6,7	6	6	6	6,7	6	6	6	6	6	6	5
A4	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7

DM4

A1	5,6	6	7	6	6	6	7	7	6	7	7	7	7	6	6	7	7
A2	6	6	6,7	6	6,7	6	7	6,7	6,7	7	7	7	7	6	6	7	7
A3	5,6	4	4	5	5	5	4,5	4	4	4,5	4	4	4	4	5	6,7	5
A4	5	6	6	6	6	6	6,7	6	6	6	6	6	6,7	6	6,7	7	7

DM5

A1	7	6,7	7	7	7	7	6	7	7	7	6	7	7	7	6	7	7
A2	4,5	6	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
A3	7	6	6	6	5	6	7	6	7	7	6	7	6	6	4	6	4,5
A4	7	6	7	7	7	7	7	7	7	7	7	6	7	6	6	7	7

Table 5.5

Experts' Evaluation in Hesitant Triangular Fuzzy Numbers for C1: Student Engagement

Alternatives	DM1	DM2	DM3	DM4	DM5
A1	(0.7,0.9,1.0)	(0.7,0.9,1.0)	(0.9,0.9,1.0)	(0.7,0.9,1.0)	(0.9,0.9,1.0)
A2	$\{(0.5,0.7,0.9),(0.7,0.9,1.0)\}$	$\{(0.7,0.9,1.0),(0.9,0.9,1.0)\}$	(0.9,0.9,1.0)	(0.7,0.9,1.0)	$\{(0.7,0.9,1.0),(0.9,0.9,1.0)\}$
A3	$\{(0.5,0.7,0.9),(0.7,0.9,1.0)\}$	$\{(0.5,0.7,0.9),(0.7,0.9,1.0)\}$	(0.7,0.9,1.0)	(0.3,0.5,0.7)	$\{(0.7,0.9,1.0),(0.9,0.9,1.0)\}$
A4	$\{(0.3,0.5,0.7),(0.5,0.7,0.9)\}$	(0.9,0.9,1.0)	(0.9,0.9,1.0)	$\{(0.5,0.7,0.9),(0.7,0.9,1.0)\}$	$\{(0.7,0.9,1.0),(0.9,0.9,1.0)\}$

Table 5.6

Experts' Evaluation in Hesitant Triangular Fuzzy Numbers for C2: Language Comprehension

Alternatives	DM1	DM2	DM3	DM4	DM5
A1	(0.5,0.7,0.9)	$\{(0.7,0.9,1.0),(0.9,0.9,1.0)\}$	(0.9,0.9,1.0)	(0.7,0.9,1.0)	(0.9,0.9,1.0)
A2	$\{(0.5,0.7,0.9),(0.7,0.9,1.0)\}$	$\{(0.7,0.9,1.0),(0.9,0.9,1.0)\}$	(0.9,0.9,1.0)	(0.7,0.9,1.0)	(0.9,0.9,1.0)
A3	$\{(0.5,0.7,0.9),(0.7,0.9,1.0)\}$	$\{(0.3,0.5,0.7),(0.5,0.7,0.9)\}$	(0.7,0.9,1.0)	(0.5,0.7,0.9)	$\{(0.5,0.7,0.9),(0.7,0.9,1.0)\}$
A4	$\{(0.5,0.7,0.9),(0.7,0.9,1.0)\}$	$\{(0.7,0.9,1.0),(0.9,0.9,1.0)\}$	(0.9,0.9,1.0)	(0.7,0.9,1.0)	(0.9,0.9,1.0)

Table 5.7

Experts' Evaluation in Hesitant Triangular Fuzzy Numbers for C3: Student Motivation

Alternatives	DM1	DM2	DM3	DM4	DM5
A1	(0.7,0.9,1.0)	{{(0.7,0.9,1.0),(0.9,0.9,1.0)}}	(0.9,0.9,1.0)	{{(0.7,0.9,1.0),(0.9,0.9,1.0)}}	{{(0.7,0.9,1.0),(0.9,0.9,1.0)}}
A2	{{(0.7,0.9,1.0),(0.9,0.9,1.0)}}	{{(0.5,0.7,0.9),(0.7,0.9,1.0)}}	(0.9,0.9,1.0)	(0.9,0.9,1.0)	(0.9,0.9,1.0)
A3	(0.7,0.9,1.0)	{{(0.3,0.5,0.7),(0.5,0.7,0.9)}}	(0.7,0.9,1.0)	(0.3,0.5,0.7)	(0.7,0.9,1.0)
A4	{{(0.3,0.5,0.7),(0.5,0.7,0.9)}}	{{(0.7,0.9,1.0),(0.9,0.9,1.0)}}	(0.9,0.9,1.0)	(0.7,0.9,1.0)	(0.9,0.9,1.0)

Table 5.8

Experts' Evaluation in Hesitant Triangular Fuzzy Numbers for C4: Teaching Effectiveness

Alternatives	DM1	DM2	DM3	DM4	DM5
A1	{{(0.5,0.7,0.9),(0.7,0.9,1.0)}}	{{(0.7,0.9,1.0),(0.9,0.9,1.0)}}	(0.9,0.9,1.0)	(0.9,0.9,1.0)	{{(0.7,0.9,1.0),(0.9,0.9,1.0)}}
A2	(0.5,0.7,0.9)	{{(0.7,0.9,1.0),(0.9,0.9,1.0)}}	(0.9,0.9,1.0)	(0.9,0.9,1.0)	(0.9,0.9,1.0)
A3	{{(0.3,0.5,0.7),(0.5,0.7,0.9)}}	{{(0.3,0.5,0.7),(0.5,0.7,0.9)}}	(0.7,0.9,1.0)	(0.3,0.5,0.7)	(0.7,0.9,1.0)
A4	{{(0.3,0.5,0.7),(0.5,0.7,0.9)}}	{{(0.7,0.9,1.0),(0.9,0.9,1.0)}}	(0.9,0.9,1.0)	(0.9,0.9,1.0)	{{(0.7,0.9,1.0),(0.9,0.9,1.0)}}

Table 5.9

Experts' Evaluation in Hesitant Triangular Fuzzy Numbers for C5: Cultural Appropriateness

Alternatives	DM1	DM2	DM3	DM4	DM5
A1	$\{(0.5,0.7,0.9),(0.7,0.9,1.0)\}$	$\{(0.7,0.9,1.0),(0.9,0.9,1.0)\}$	$(0.9,0.9,1.0)$	$\{(0.7,0.9,1.0),(0.9,0.9,1.0)\}$	$\{(0.7,0.9,1.0),(0.9,0.9,1.0)\}$
A2	$\{(0.5,0.7,0.9),(0.7,0.9,1.0)\}$	$(0.7,0.9,1.0)$	$(0.9,0.9,1.0)$	$\{(0.7,0.9,1.0),(0.9,0.9,1.0)\}$	$(0.9,0.9,1.0)$
A3	$\{(0.5,0.7,0.9),(0.7,0.9,1.0)\}$	$(0.5,0.7,0.9)$	$(0.7,0.9,1.0)$	$(0.3,0.5,0.7)$	$(0.5,0.7,0.9)$
A4	$\{(0.5,0.7,0.9),(0.7,0.9,1.0)\}$	$(0.7,0.9,1.0)$	$(0.9,0.9,1.0)$	$(0.7,0.9,1.0)$	$(0.7,0.9,1.0)$

Table 5.10

Experts' Evaluation in Hesitant Triangular Fuzzy Numbers for C6: Readiness for Use

Alternatives	DM1	DM2	DM3	DM4	DM5
A1	$(0.9,0.9,1.0)$	$(0.9,0.9,1.0)$	$(0.9,0.9,1.0)$	$(0.9,0.9,1.0)$	$(0.9,0.9,1.0)$
A2	$(0.9,0.9,1.0)$	$(0.9,0.9,1.0)$	$\{(0.7,0.9,1.0),(0.9,0.9,1.0)\}$	$(0.9,0.9,1.0)$	$(0.9,0.9,1.0)$
A3	$\{(0.3,0.5,0.7),(0.5,0.7,0.9)\}$	$(0.5,0.7,0.9)$	$\{(0.5,0.7,0.9),(0.7,0.9,1.0)\}$	$(0.5,0.7,0.9)$	$(0.7,0.9,1.0)$
A4	$(0.5,0.7,0.9)$	$\{(0.5,0.7,0.9),(0.7,0.9,1.0)\}$	$(0.9,0.9,1.0)$	$(0.9,0.9,1.0)$	$(0.9,0.9,1.0)$

5.4.4 Worked Example: A1 under C1

Step 1: From score $\rightarrow$ hesitant TFN. Suppose the linguistic score for A1 under C1 is mapped to the hesitant triangular fuzzy set. The five experts evaluated A1 under C1 using the following triangular fuzzy numbers:

$$H_{A1C1} = \{(0.7, 0.9, 1.0), (0.7, 0.9, 1.0), (0.9, 0.9, 1.0), (0.7, 0.9, 1.0), (0.9, 0.9, 1.0)\}. \quad (5.2)$$

Step 2: HTFGGHM aggregation ($p = q = 0.5$). The values $p = q = 0.5$ are used because they provide a balanced interaction between the paired hesitant triangular fuzzy elements without giving dominance to either side. Since the five expert evaluations are treated with equal importance, this setting provides a neutral aggregation condition for the HTFGGHM operator. Applying $HTFGGHM^{0.5, 0.5}$ to H_{A1C1} gives

$$\begin{aligned} & HTFGGHM^{0.5, 0.5}(H_{A1C1}) \\ &= \{(0.7, 0.9, 1.0), (0.7, 0.9, 1.0), (0.9, 0.9, 1.0), (0.7, 0.9, 1.0), (0.9, 0.9, 1.0)\}. \end{aligned} \quad (5.3)$$

Thus, the aggregated hesitant TFN for A1C1 is

$$h_{A1C1} = (l, m, u) = (0.7588, 0.8388, 1.0000). \quad (5.4)$$

Step 3: Hesitant triangular scoring (HTS). Using the scoring function for HTF, the score value is computed as

$$S(h_{A1C1}) = \frac{l + m + u}{3} = \frac{0.7588 + 0.8388 + 1.0000}{3} = 0.8659. \quad (5.5)$$

Step 4: Normalized values for all C_j of A1. Using the same formula column-wise, the normalized scores of A1 are

$$N_{ij_{j=1}}^6 = (1.0000, 0.8495, 1.0000, 1.0000, 0.9184, 1.0000) \quad (5.6)$$

Step 5: Weighted row of A1. With $w_j = \frac{1}{6}$,

$$W_{ij}^6 = \frac{1}{6}(N_{ij})^6 = (0.1667, 0.1416, 0.1667, 0.1667, 0.1531, 0.1667) \quad (5.7)$$

Step 6: Distances and closeness for A1. From the whole matrix involving all alternatives, for every criterion, we obtain

$$A_j^+ = \max W_{ij} = 0.1667, \quad A_j^- = \min W_{ij} = 0.0017, \quad j = 1, \dots, 6. \quad (5.8)$$

Hence,

$$\begin{aligned} D_1^+ &= \sum_{j=1}^6 |W_{1j} - A_j^+| \\ &= |0.1667 - 0.1667| + |0.1416 - 0.1667| + |0.1667 - 0.1667| \\ &\quad + |0.1667 - 0.1667| + |0.1531 - 0.1667| \\ &\quad + |0.1667 - 0.1667| \\ &= 0.0000 + 0.0251 + 0.0000 + 0.0000 + 0.0136 + 0.0000 \\ &= \mathbf{0.0387} \end{aligned} \quad (5.9)$$

$$\begin{aligned} D_1^- &= \sum_{j=1}^6 |W_{1j} - A_j^-| \\ &= |0.1667 - 0.0017| + |0.1416 - 0.0017| + |0.1667 - 0.0017| \\ &\quad + |0.1667 - 0.0017| + |0.1531 - 0.0017| \\ &\quad + |0.1667 - 0.0017| \\ &= \mathbf{0.9513} \end{aligned} \quad (5.10)$$

With $\beta = 0.5$, the SYAI closeness of A1 is

$$\begin{aligned} &\frac{(1 - \beta)D_1^-}{\beta D_1^+ + (1 - \beta)D_1^-} \\ &= \frac{0.5 \times 0.9513}{0.5 \times 0.0387 + 0.5 \times 0.9513} = \frac{0.4757}{0.4950} = \mathbf{0.9609} \end{aligned} \quad (5.11)$$

Thus, A1 receives a SYAI closeness score of 0.9609.

5.5 Comparative Analysis with Other MCDM Methods

5.5.1 Comparison Setup

The same dataset was analyzed using TOPSIS, VIKOR, SAW, COBRA, WASPAS and MOORA methods to validate the SYAI results as shown in Table 5.11. It should be noted that the score values produced by different MCDM methods are not directly comparable in terms of magnitude because each method applies a different normalization, aggregation, and ranking mechanism. For example, TOPSIS and SYAI are based on closeness to ideal solutions, SAW and WASPAS are based on weighted aggregation, while VIKOR and COBRA use compromise or distance-based ranking structures. Therefore, the comparison in this section focuses mainly on the consistency of the ranking order rather than the absolute score values. In addition, the interpretation of scores differs across methods. For VIKOR and COBRA, a lower score indicates better performance, whereas for TOPSIS, SAW, SYAI, WASPAS, and MOORA, a higher score indicates better performance.

Table 5.11
Scores and Ranks of the Four Alternatives Across Seven MCDM Methods.

Alt.	TOPSIS	VIKOR	SAW	SYAI	COBRA	WASPAS	MOORA
A1	0.9702 (1)	0.0000 (1)	0.9953 (1)	0.9609 (1)	-0.0329 (1)	0.9953 (1)	0.5307 (1)
A2	0.9635 (2)	0.0136 (2)	0.9924 (2)	0.9458 (2)	-0.0320 (2)	0.9924 (2)	0.5291 (2)
A3	0.5417 (3)	1.0000 (4)	0.8644 (4)	0.1139 (4)	0.0066 (3)	0.8642 (4)	0.4604 (4)
A4	0.2682 (4)	0.7043 (3)	0.8831 (3)	0.6148 (3)	0.0266 (4)	0.8694 (3)	0.4662 (3)

Note. In the case of VIKOR and COBRA, the lower the score, the better the performance; in the case of the other methods, the higher the score, the better the performance.

The relative performance of seven MCDM techniques shows that the ranking of the four alternatives is very consistent as shown in Table 5.11. The first alternative A1 performs the best among all the methods, with a steadily high score prevailing like 0.9702 in TOPSIS, 0.9953 in SAW, and 0.9609 in SYAI. In approaches, in which low scores reflect good performance as in VIKOR (0.0000) and COBRA (-0.0329), A1 is selected as the most effective. This uniformity of dominance between additive and multiplicative models and compromise-based models confirms that A1 has most desirable balance between evaluation criteria. The second alternative A2 is

almost behind A1, and scores are slightly lower than the first one, but they remain in the second place in all ranking methods, indicating high performance stability and little variation in scores between the methodological differences.

Conversely, the third A3 and A4 options are less stable with their rankings depending on the rule that is used to rank the options. A4 is better than A3 in compensatory approaches like SAW, SYAI and WASPAS, which means that it is more adaptable when there are trade-offs among criteria. On the other hand, non-compensatory or distance-based approaches such as TOPSIS and COBRA rank A3 a little higher, which indicates the sensitivity to the normalization and aggregation structures. On the whole, though A1 and A2 are at the top of the performance, A3 and A4 are located at the bottom, which shows the impact of various aggregation philosophies on the final rankings. All these findings indicate that there is a high degree of agreement on the superiority of Verbal Humor (A1), and this supports the strength of the decision result in a variety of MCDM frameworks. Figure 5.4 was a bar chart that showed Scores and ranks of each MCDM Method.

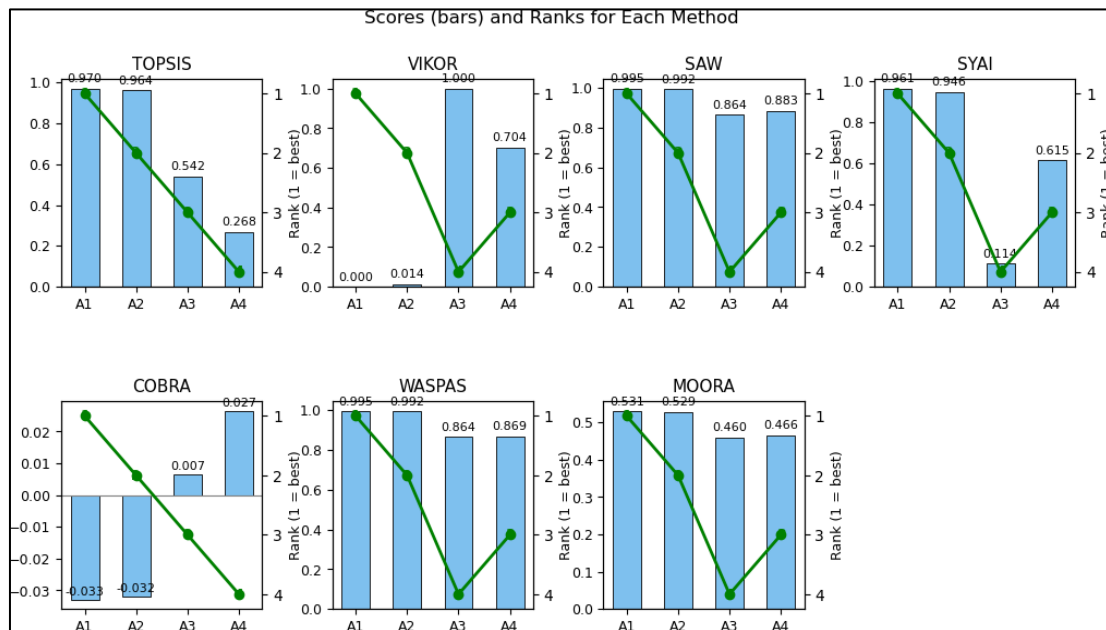


Figure 5.4 Scores and Ranks for Each MCDM Method

The comparative results in Figure 5.4 show how the normalized scores and ranks of each alternative (A1–A4) vary across the seven MCDM methods: TOPSIS, VIKOR, SAW, SYAI, COBRA, WASPAS, and MOORA. Across all methods, A1 stands out as the best-performing alternative, consistently taking the first rank with the highest

scores: 0.9702 (TOPSIS), 0.0000 (VIKOR), 0.9953 (SAW), 0.9609 (SYAI), 0.0329 (COBRA), 0.9953 (WASPAS), and 0.5307 (MOORA). This consistency shows that A1 performs strongly under different evaluation criteria and normalization techniques, suggesting that its superiority is not tied to any specific method. A2 also shows stable performance, maintaining the second rank across all methods with slightly lower yet still competitive scores: 0.9635 (TOPSIS), 0.0136 (VIKOR), 0.9924 (SAW), 0.9458 (SYAI), -0.0320 (COBRA), 0.9924 (WASPAS), and 0.5291 (MOORA). The smooth and parallel pattern between A1 and A2 in the figure suggests both are reliable options, with A1 having a slight but consistent advantage.

A notable discrepancy is apparent between A3 and A4, which demonstrate significantly greater variability in their rankings and scores. For A3, the scores of 0.5417 (TOPSIS), 1.0000 (VIKOR), 0.8644 (SAW), 0.1139 (SYAI), 0.0066 (COBRA), 0.8642 (WASPAS), and 0.4604 (MOORA) typically position it at Rank 3 or 4, contingent upon the specific methodology employed. The elevated score of 1.0000 in VIKOR is paradoxically interpreted as the most unfavorable outcome, given that VIKOR adheres to a lower-is-better criterion, resulting in a Rank of 4. A4 illustrates marginally improved metrics in comparison to A3 through several methodologies: 0.2682 (TOPSIS), 0.7043 (VIKOR), 0.8831 (SAW), 0.6148 (SYAI), 0.0266 (COBRA), 0.8694 (WASPAS), and 0.4662 (MOORA), yet primarily remains linked to Rank 3 or 4. The merging of their ranking trajectories, particularly noticeable in COBRA and WASPAS, emphasizes the sensitivity to discrepancies in scaling, distance metrics, or normalization techniques inherent to these methodologies.

In summary, the dual visualization of scores and ranks reinforces that $A1 > A2 > (A3, A4)$ establishes a coherent and interpretable hierarchy across all evaluative frameworks. The consistent ranking of A1 and A2 across various decision-making models signifies robust methodological concordance, thereby enhancing the reliability of their prioritization. Conversely, the diminished and more erratic scores of A3 and A4 signify alternatives with comparatively inferior performance or context-dependent advantages that manifest solely under specific formulations. From a decision-analytic perspective, the visualization further corroborates the resilience of the decision matrix, illustrating that the stability of the ranking of top alternatives remains impervious to the computational framework employed (distance-based, weighted-sum, or hybrid). Consequently, Figure 5.4 proficiently encapsulates both the preeminence of the leading

alternatives and the relative instability of the lower-ranked options across diverse MCDM methodologies

5.5.2 Ranking Consistency

Table 5.11 presents the ranking order from each method, showing that the SYAI-HTFGGHM integration produces consistent and interpretable outcomes. For the purpose of examining the reliability of the rankings obtained from different Multi-Criteria Decision-Making (MCDM) methodologies, a thorough rank consistency analysis was performed. Figure 5.5 illustrates the variations in the relative rankings of the four alternatives (A1–A4) across the seven distinct methodologies employed.

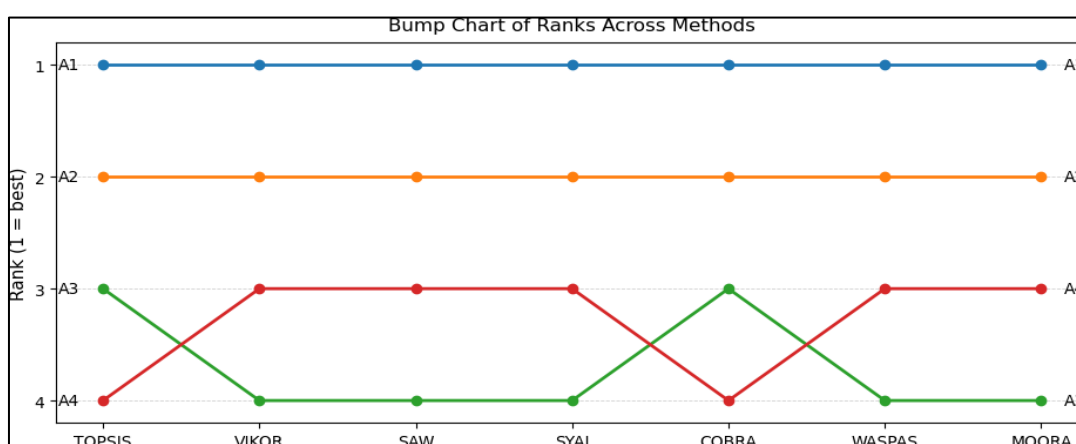


Figure 5.5 Rank Consistency of Alternatives Across MCDM Methods

The bump chart presented in Figure 5.5 reflects a marked level of rank consistency throughout all employed decision-making methodologies. Alternative A1 showcases a flawless stability pattern, consistently securing Rank 1 across each analytical method (TOPSIS, VIKOR, SAW, SYAI, COBRA, WASPAS, and MOORA), thus validating its eminent performance and strong preference independent of the foundational computational basis. In a comparable context, A2 consistently maintains Rank 2, signifying a considerable yet subordinate performance level and illustrating strong congruence among the analytical models. In contrast, A3 and A4 reveal some moderate shifts. A3 mainly ranks at Rank 3 when analyzed through distance-based models (TOPSIS and COBRA) yet falls to Rank 4 when examined by compensatory models like SAW, SYAI, WASPAS, and MOORA; in contrast, A4 shows a reverse trend attaining better rankings in compensatory frameworks. This variability implies

that A3's performance is more susceptible to the influences of normalization and distance metrics, in contrast to A4, which benefits from additive aggregation techniques. The combined stability of A1 and A2, in conjunction with the moderate variations exhibited between A3 and A4, indicates that the decision outcomes are both reliable and methodologically robust.

5.5.3 Rank Correlation and Stability Assessment

Spearman's rank correlation coefficient (ρ) was computed between SYAI and each comparative method to verify ranking stability. Figure 5.6 illustrates the pairwise Spearman rank correlation matrix, which evaluates the consistency of rankings among the seven Multi-Criteria Decision-Making (MCDM) methodologies TOPSIS, VIKOR, SAW, SYAI, COBRA, WASPAS, and MOORA were employed in the comparative study. The Spearman correlation acts as a measure based on ranks, analyzing how two approaches value the same selection of choices instead of evaluating their numeric amounts. This framework ensures that the analysis is purely focused on the uniformity of ranking results across methodologies, no matter the discrepancies in scale or directional scoring.

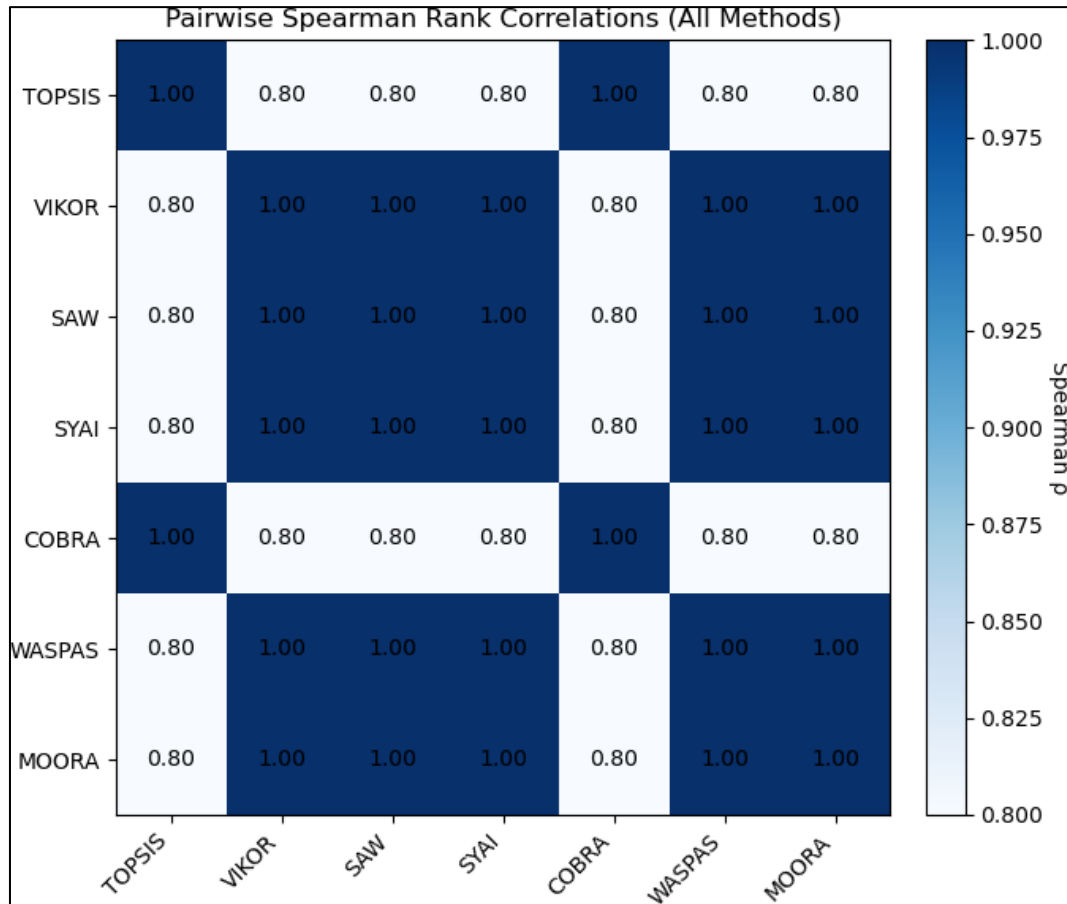


Figure 5.6 Pairwise Spearman Rank Correlations Among MCDM Methods

The heatmap presented in Figure 5.6 depicts the Spearman rank correlation coefficients among the seven Multi-Criteria Decision Making (MCDM) methodologies utilized in this research. In general, the correlations exhibit a strong congruence, with the majority of values falling between 0.80 and 1.00, which indicates a significant degree of consistency in the ranking patterns of alternatives across the different techniques. The SYAI method exhibits a perfect rank correlation ($\rho = 1.00$) with VIKOR, SAW, WASPAS, and MOORA, suggesting that it generates identical ranking sequences as these four established methodologies. This flawless agreement reinforces the reliability of SYAI's ranking mechanism, affirming that its results are both theoretically and practically compatible with other well-established aggregation frameworks. Simultaneously, its correlation of $\rho = 0.80$ with TOPSIS and COBRA reveals a slightly lower, yet still robust, connection, likely attributable to the distinct normalization and distance-based frameworks utilized by those methods.

From a methodological perspective, the findings validate that the SYAI-HTFGGHM integration exhibits substantial robustness and internal consistency when

assessed against multiple benchmark MCDM methodologies. The strong positive correlations suggest that the SYAI method effectively maintains the ranking integrity of alternatives, even when juxtaposed with models that employ different decision logics such as compensatory (SAW, WASPAS) and distance-based (TOPSIS, COBRA) approaches. This consistent trend indicates that the integrated model produces stable and interpretable decision outcomes across various analytical paradigms. In this way, the correlation heatmap supplies quantitative data that endorses the validity, flexibility, and comparative reliability of the suggested SYAI-HTFGGHM framework within language-oriented decision-making frameworks.

CHAPTER 6

CONCLUSION AND RECOMMENDATIONS

6.1 Overview

This research aimed to establish a cohesive multi-criteria decision-making framework that merges the Simplified Yielded Aggregation Index (SYAI) with the Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean (HTFGGHM). The aim of this integration is to confront indecision and ambiguity in linguistic evaluations while upholding computational efficiency in ranking selections. The study effectively illustrated that the SYAI–HTFGGHM framework can produce dependable and comprehensible rankings within hesitant triangular fuzzy contexts.

6.2 Summary of Findings

The findings of this study can be summarized according to three main aspects: the proposed aggregation operator, the SYAI ranking method, and the application results. First, the proposed HTFGGHM operator was able to aggregate hesitant triangular fuzzy information while considering the relationships among decision criteria. The geometric Heronian structure allowed the operator to capture the interaction between criteria, rather than treating each criterion as fully independent. The operator also satisfied the basic mathematical properties of idempotency, monotonicity, and boundedness, which confirms that the aggregation process is mathematically consistent and reliable.

Second, the SYAI method provided a simple and structured ranking procedure after the hesitant triangular fuzzy information was converted into score values. The method organized the normalization, weighting, distance calculation, and final ranking into a clear decision making process. In this study, equal weights were assigned to the main criteria to reflect that all criteria were considered equally important in the educational evaluation context.

Third, the application of the SYAI-HTFGGHM framework in educational decision-making showed that the proposed model can be used to evaluate humor-based teaching strategies in Arabic language learning. The results identified A1, which represents verbal humor, as the best alternative. This indicates that verbal humor was considered the most effective strategy based on the expert evaluations, followed by A2,

A4, and A3. The ranking result shows that the proposed framework can handle linguistic judgments and hesitation in expert opinions while producing an interpretable final ranking.

Finally, the comparative analysis with other MCDM methods, including TOPSIS, VIKOR, SAW, COBRA, WASPAS, and MOORA, showed that the ranking results were generally consistent. Although the score values differed across methods due to different normalization and ranking mechanisms, the top alternatives remained stable. The rank correlation analysis also supported the consistency of the SYAI result with several established MCDM methods. Therefore, the findings suggest that the proposed SYAI-HTFGGHM framework is suitable for solving educational decision-making problems involving uncertain and hesitant expert evaluations.

6.3 Recommendations for Future Work

Future studies may extend the SYAI-HTFGGHM framework to other fuzzy environments instead of focusing only on hesitant triangular fuzzy numbers. Possible extensions include q-rung orthopair fuzzy sets, Pythagorean fuzzy sets, Fermatean fuzzy sets, bipolar fuzzy sets, and neutrosophic sets. These extensions may help the framework handle more complex forms of uncertainty, hesitation, indeterminacy, and inconsistent expert judgments.

Future work may also consider different methods for determining criteria weights, decision maker weights, and operator parameters. In this study, the decision makers are treated with equal importance, and the main criteria are assigned equal weights in the SYAI ranking stage. Although this is suitable for the present application, other decision making problems may require unequal weights to reflect different levels of expertise or criterion importance. Therefore, future studies may apply weighting methods such as entropy weight, AHP, best-worst method, or optimization-based techniques.

Another recommendation is to develop a computational tool for the SYAI-HTFGGHM method. The tool may allow users to upload decision matrices, define criteria types, assign weights, calculate rankings, and compare the results with other MCDM methods. This would make the proposed framework easier to apply, more transparent, and more useful for future decision making studies.

In conclusion, the SYAI-HTFGGHM framework can be further improved by extending it to other fuzzy sets, applying more advanced weighting procedures, and developing a user-friendly decision-support tool. These improvements would strengthen its application in more complex real-world decision-making problems

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APPENDICES

APPENDIX 1

SIMPLIFIED YIELDED AGGREGATION INDEX (SYAI) METHOD FOR ENHANCED MULTI CRITERIA DECISION MAKING

Dataset for SYAI Comparative Analysis

Table A.1

Criteria Description and Weights for Machine Selection.

Criteria	Criteria Names	Category	Opt.	Criteria Weights
C ₁	The size and the shape of the machine	Constructional parameters	max	0.070
C ₂	The volume or the capacity of the machine	Constructional parameters	min	0.070
C ₃	The construction of agitation and aeration system	Constructional parameters	max	0.070
C ₄	The number of the machines	Constructional parameters	max	0.140
C ₅	Investments	Economical parameters	min	0.200
C ₆	Terms of payment and maintenance	Economical parameters	max	0.080
C ₇	Operating costs	Economical parameters	min	0.120
C ₈	Warranty period	Technical parameters	max	0.125
C ₉	Delivery time	Technical parameters	min	0.050
C ₁₀	Maintenance conditions	Technical parameters	max	0.075

Table A.2

Ratings of The Five Alternatives According to The Defined Criteria.

Alternative	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈	C ₉	C ₁₀
A ₁	3	7	4	4	4	6	6	5	8	8
A ₂	4	6	5	5	5	5	8	8	9	9
A ₃	6	4	6	6	6	6	5	6	8	8
A ₄	5	6	6	6	4	5	7	7	9	9
A ₅	2	8	3	4	3	6	7	7	7	8

Source: (Štirbanović et al., 2019).

Coding for SYAI Software

```
# app.py import base64

from pathlib import Path import streamlit as st

import streamlit.components.v1 as components
st.set_page_config(page_title="SYAI-Rank", layout="wide")

APP_DIR = Path(_file_).resolve().parent

#----- Local images

def img_data_uri_try(candidates: list[str]) -> tuple[str, bool]:
    for name in candidates:

        p = Path(name)

        if not p.is_absolute():

            p = APP_DIR / name

        if p.exists() and p.is_file(): ext = p.suffix.lower()

        mime = "image/jpeg" if ext in (".jpg", ".jpeg") else
            "image/png" b64 =
            base64.b64encode(p.read_bytes()).decode("utf-8")

        return (f"data:{mime};base64,{b64}", True) return ("", False)

SCATTER_URI, SCATTER_FOUND = img_data_uri_try(
    ["scatter_matrix.png", "assets/scatter_matrix.png"])

CORR_URI, CORR_FOUND = img_data_uri_try(["corr_matrix.png",
    "assets/corr_matrix.png"])
```

```

# ----- Single source of truth for the sample CSV -----

def load_sample_csv_text() -> str:
    p = Path("/mnt/data/sample (1).csv")
    if p.exists():
        for enc in ("utf-8", "latin-1"):
            try:
                return p.read_text(encoding=enc)
            except Exception:
                pass
    # Fallback tiny demo
    return (
        "Alternative,Cost,Quality,Delivery\n"
        "A1,200,8,4\n"
        "A2,250,7,5\n"
        "A3,300,9,6\n"
        "A4,220,8,4\n"
        "A5,180,6,7\n"
    )

```

```

SAMPLE_CSV = load_sample_csv_text()

# ----- base page background (kept) -----

st.markdown("""

<style>

.stApp { background: linear-gradient(180deg, #0b0b0f 0%, #0b0b0f 35%, #ffe4e6
120%) !important; }

[data-testid="stSidebar"] { background: rgba(255, 228, 230, 0.08) !important;
backdrop-filter: blur(6px); }

</style>

""", unsafe_allow_html=True)

# ----- HTML APP -----

html = r"""

<!doctype html>

<html lang="en">

<head>

<meta charset="utf-8"/>

<meta name="viewport" content="width=device-width,initial-scale=1"/>

<title>SYAI-Rank</title>

<style>

... [HTML + JavaScript full code here, same as original app.py]

</style>

</head>

<body class="theme-dark">

... [full embedded HTML content with Streamlit component injection]

</body>

</html>

"""

# inject image URIs

html = html.replace("SCATTER_DATA_URI", SCATTER_URI or "")

html = html.replace("CORR_DATA_URI", CORR_URI or "")

```

```
components.html(html, height=4200, scrolling=True)
```

APPENDIX 2

HESITANT TRIANGULAR FUZZY GENERALIZED GEOMETRIC HERONIA OPERATOR

Table B.1
Hesitant Triangular Fuzzy Decision Matrix.

	G_1	G_2	G_3	G_4
A_1	$\{(0.3,0.4,0.5)\}$	$\{(0.6,0.7,0.8),(0.7,0.8,0.9)\}$	$\{(0.2,0.3,0.5)\}$	$\{(0.2,0.3,0.4),(0.3,0.4,0.5)\}$
A_2	$\{(0.6,0.7,0.8)\}$	$\{(0.2,0.4,0.5)\}$	$\{(0.3,0.4,0.5),(0.6,0.7,0.8)\}$	$\{(0.2,0.3,0.5),(0.4,0.5,0.6),(0.5,0.6,0.7)\}$
A_3	$\{(0.5,0.6,0.7)\}$	$\{(0.7,0.8,0.9),(0.8,0.9,1.0)\}$	$\{(0.2,0.3,0.4),(0.4,0.5,0.6)\}$	$\{(0.5,0.6,0.7)\}$
A_4	$\{(0.7,0.8,0.9)\}$	$\{(0.6,0.7,0.8),(0.7,0.8,0.9)\}$	$\{(0.1,0.2,0.3)\}$	$\{(0.3,0.4,0.5),(0.6,0.7,0.8)\}$
A_5	$\{(0.6,0.7,0.8)\}$	$\{(0.2,0.3,0.4),(0.5,0.6,0.7)\}$	$\{(0.4,0.5,0.6)\}$	$\{(0.2,0.3,0.4)\}$

Source: (Zhao et al., 2014).

APPENDIX 3

APPLICATION OF HESITANT TRIANGULAR FUZZY GENERALIZED GEOMETRIC HERONIAN MEAN AND SYAI METHOD IN EDUCATIONAL DECISION-MAKING

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17
DM1																	
A1	7	7	4	5	4,5	5	6	7	5	6	6	4	6	5	5	7	7
A2	6	7	4	6	5	4,5	6	7	4,5	6	4,5	4	6	5	5	7	7
A3	6	5	4,5	6	5	5	6	6	6	5	5	4	6	6	5	4	5
A4	5,6	5	4	6	5	5	5	6,7	4	5	5	4	6	5	4,5	5	5
DM2																	
A1	5,7	5,6	6	6	7	7	7	7	5	7	6	6,7	6	7	6,7	7	6,7
A2	6,7	7	6	6	6	7	6,7	6	5	7	6,7	6	4,5	6	6	7	7
A3	6	6	5	5	4,5	4	5	4,5	4	4	5	4,5	5	4	6	4,5	6,7
A4	7	7	7	6	7	7	7	7	5	7	6	6	6	4,5	6	5	6
DM3																	
A1	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
A2	7	7	7	6,7	7	7	7	7	7	7	7	7	7	7	7	6	7
A3	6	4,5	6	6	6	6,7	6	6	6	6,7	6	6	6	6	6	6	5
A4	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
DM4																	
A1	5,6	6	7	6	6	6	7	7	6	7	7	7	7	6	6	7	7
A2	6	6	6,7	6	6,7	6	7	6,7	6,7	7	7	7	7	6	6	7	7
A3	5,6	4	4	5	5	5	4,5	4	4	4,5	4	4	4	4	5	6,7	5
A4	5	6	6	6	6	6	6,7	6	6	6	6	6	6,7	6	6,7	7	7
DM5																	
A1	7	6,7	7	7	7	7	6	7	7	7	6	7	7	7	6	7	7
A2	4,5	6	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
A3	7	6	6	6	5	6	7	6	7	7	6	7	6	6	4	6	4,5
A4	7	6	7	7	7	7	7	7	7	7	7	6	7	6	6	7	7

Table C.2

Experts' Evaluations in Htfns by Decision Makers for Criteria C1–C3.

DM	Alt. C1: Student Engagement	C2: Language Comprehension	C3: Student Motivation
DM1	$A_1(0.7,0.9,1.0)$	$(0.5,0.7,0.9)$	$(0.7,0.9,1.0)$
	$A_2\{(0.5,0.7,0.9),(0.7,0.9,1.0)\}$	$\{(0.5,0.7,0.9),(0.7,0.9,1.0)\}$	$\{(0.7,0.9,1.0),(0.9,0.9,1.0)\}$
	$A_3\{(0.5,0.7,0.9),(0.7,0.9,1.0)\}$	$\{(0.5,0.7,0.9),(0.7,0.9,1.0)\}$	$(0.7,0.9,1.0)$
	$A_4\{(0.3,0.5,0.7),(0.5,0.7,0.9)\}$	$\{(0.5,0.7,0.9),(0.7,0.9,1.0)\}$	$\{(0.3,0.5,0.7),(0.5,0.7,0.9)\}$
DM2	$A_1(0.7,0.9,1.0)$	$\{(0.7,0.9,1.0),(0.9,0.9,1.0)\}$	$\{(0.7,0.9,1.0),(0.9,0.9,1.0)\}$
	$A_2\{(0.7,0.9,1.0),(0.9,0.9,1.0)\}$	$\{(0.7,0.9,1.0),(0.9,0.9,1.0)\}$	$\{(0.5,0.7,0.9),(0.7,0.9,1.0)\}$
	$A_3\{(0.5,0.7,0.9),(0.7,0.9,1.0)\}$	$\{(0.3,0.5,0.7),(0.5,0.7,0.9)\}$	$\{(0.3,0.5,0.7),(0.5,0.7,0.9)\}$
	$A_4(0.9,0.9,1.0)$	$\{(0.7,0.9,1.0),(0.9,0.9,1.0)\}$	$\{(0.7,0.9,1.0),(0.9,0.9,1.0)\}$
DM3	$A_1(0.9,0.9,1.0)$	$(0.9,0.9,1.0)$	$(0.9,0.9,1.0)$
	$A_2(0.9,0.9,1.0)$	$(0.9,0.9,1.0)$	$(0.9,0.9,1.0)$
	$A_3(0.7,0.9,1.0)$	$(0.7,0.9,1.0)$	$(0.7,0.9,1.0)$
	$A_4(0.9,0.9,1.0)$	$(0.9,0.9,1.0)$	$(0.9,0.9,1.0)$
DM4	$A_1(0.7,0.9,1.0)$	$(0.7,0.9,1.0)$	$\{(0.7,0.9,1.0),(0.9,0.9,1.0)\}$
	$A_2(0.7,0.9,1.0)$	$(0.7,0.9,1.0)$	$(0.9,0.9,1.0)$
	$A_3(0.3,0.5,0.7)$	$(0.5,0.7,0.9)$	$(0.3,0.5,0.7)$
	$A_4\{(0.5,0.7,0.9),(0.7,0.9,1.0)\}$	$\{(0.7,0.9,1.0),(0.9,0.9,1.0)\}$	$(0.7,0.9,1.0)$
DM5	$A_1(0.9,0.9,1.0)$	$(0.9,0.9,1.0)$	$\{(0.7,0.9,1.0),(0.9,0.9,1.0)\}$
	$A_2\{(0.7,0.9,1.0),(0.9,0.9,1.0)\}$	$(0.9,0.9,1.0)$	$(0.9,0.9,1.0)$
	$A_3\{(0.7,0.9,1.0),(0.9,0.9,1.0)\}$	$\{(0.5,0.7,0.9),(0.7,0.9,1.0)\}$	$(0.7,0.9,1.0)$
	$A_4\{(0.7,0.9,1.0),(0.9,0.9,1.0)\}$	$(0.9,0.9,1.0)$	$(0.9,0.9,1.0)$

Table C.3

Experts' Evaluations in Htfn by Decision Makers for Criteria C4–C6.

DM Alt.	C4: Teaching Effectiveness	C5: Cultural Appropriateness	C6: Readiness for Use
A ₁ {(0.5,0.7,0.9),(0.7,0.9,1.0)}	{(0.5,0.7,0.9),(0.7,0.9,1.0)}	(0.9,0.9,1.0)	
DM1(0.5,0.7,0.9) A ₂	{(0.5,0.7,0.9),(0.7,0.9,1.0)}	(0.9,0.9,1.0)	
A ₃ {(0.3,0.5,0.7),(0.5,0.7,0.9)}	{(0.5,0.7,0.9),(0.7,0.9,1.0)}	{(0.3,0.5,0.7),(0.5,0.7,0.9)}	
A ₄ {(0.3,0.5,0.7),(0.5,0.7,0.9)}	{(0.5,0.7,0.9),(0.7,0.9,1.0)}	(0.5,0.7,0.9)	
A ₁ {(0.7,0.9,1.0),(0.9,0.9,1.0)}	(0.7,0.9,1.0)	(0.9,0.9,1.0)	
DM2{(0.7,0.9,1.0),(0.9,0.9,1.0)} A ₂	(0.7,0.9,1.0)	(0.9,0.9,1.0)	
A ₃ {(0.3,0.5,0.7),(0.5,0.7,0.9)}	(0.5,0.7,0.9)	(0.5,0.7,0.9)	
A ₄ {(0.7,0.9,1.0),(0.9,0.9,1.0)}	(0.7,0.9,1.0)	{(0.5,0.7,0.9),(0.7,0.9,1.0)}	
A ₁ (0.9,0.9,1.0)	(0.9,0.9,1.0)	(0.9,0.9,1.0)	
DM3(0.9,0.9,1.0) A ₂	(0.9,0.9,1.0)	{(0.7,0.9,1.0),(0.9,0.9,1.0)}	
A ₃ (0.7,0.9,1.0)	(0.7,0.9,1.0)	{(0.5,0.7,0.9),(0.7,0.9,1.0)}	
A ₄ (0.9,0.9,1.0)	(0.7,0.9,1.0)	(0.9,0.9,1.0)	
A ₁ (0.9,0.9,1.0)	(0.7,0.9,1.0)	(0.9,0.9,1.0)	
DM4(0.9,0.9,1.0) A ₂	(0.7,0.9,1.0)	(0.9,0.9,1.0)	
A ₃ (0.3,0.5,0.7)	(0.3,0.5,0.7)	(0.5,0.7,0.9)	
A ₄ (0.9,0.9,1.0)	(0.7,0.9,1.0)	(0.9,0.9,1.0)	
A ₁ {(0.7,0.9,1.0),(0.9,0.9,1.0)}	{(0.7,0.9,1.0),(0.9,0.9,1.0)}	(0.9,0.9,1.0)	
DM5(0.9,0.9,1.0) A ₂	(0.9,0.9,1.0)	(0.9,0.9,1.0)	
A ₃ (0.7,0.9,1.0)	(0.5,0.7,0.9)	(0.7,0.9,1.0)	
A ₄ {(0.7,0.9,1.0),(0.9,0.9,1.0)}	(0.7,0.9,1.0)	(0.9,0.9,1.0)	

APPENDIX 4

Questionnaire Instrument

Use of Humor in Arabic Language Teaching and Learning

Use of Humor in Arabic Language Teaching and Learning

AMALAN HUMOR DALAM PENGAJARAN & PEMBELAJARAN BAHASA ARAB

Section 1: Respondent Information

Bahagian 1: Maklumat Responden

Please fill in the following information:

Sila isi maklumat berikut:

1. Age/Umur: _____
2. Gender/Jantina: ☐ Male/Lelaki ☐ Female/Perempuan
3. Education Level/Tahap Pengajian: ☐ Diploma/Diploma ☐ Bachelor's Degree/Ijazah
4. Master's Degree/Sarjana ☐ Doctor of Philosophy (PhD)
5. Teaching Experience in Arabic Language/Pengalaman Mengajar Bahasa Arab:
Less than 1 year/Kurang 1 tahun ☐ 1-3 years/1-3 tahun ☐ More than 3 years/Lebih 3 tahun

Section 2: Evaluation of the Use of Humor in Arabic Language Teaching

Bahagian 2: Penilaian Penggunaan Humor dalam Pengajaran Bahasa Arab

Likert Scale:

Skala likert: (Can choose more than 1)

1. Strongly Disagree / Sangat Tidak Setuju
2. Disagree / Tidak Setuju
3. Slightly Disagree / Sederhana Tidak Setuju
4. Neutral / Neutral
5. Slightly Agree / Sederhana Setuju
6. Agree / Setuju
7. Strongly Agree / Sangat Setuju

Criteria/Sub-criteria Kriteria/Sub-Kriteria	*Verbal Humor (verbal jokes, wordplay, language riddles) *Humor Verbal (Jenaka lisan, permainan kata, teka teki bahasa)	*Non-Verbal Humor (body language, facial expressions, funny gestures) *Humor Bukan Verbal (gaya badan, mimik muka, ekspresi lucu)	*Technology-/Audiovisual-Based Humor (funny videos, animations, language-related cartoons) *Humor Berasaskan Teknologi/Audiovisual (Video lucu, animasi, kartun berkaitan bahasa)	*Situational Humor (spontaneous acting, simulation of humorous situations in class) *Humor Situasi (lakonan spontan, simulasi situasi jenaka dalam kelas)
1. Student Engagement/ Keterlibatan Pelajar (Engagement)				
E1: Level of student interaction in class after humor is used/ Tahap interaksi pelajar dalam kelas selepas humor digunakan	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7
E2: Frequency of students responding to the humor used/ Kekurangan pelajar memberikan respons terhadap humor yang digunakan	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7
E3: Increase in attendance and participation in class/ Peningkatan kehadiran dan penyertaan dalam kelas	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7

Criteria/Sub-criteria Kriteria/Sub-Kriteria	*Verbal Humor (verbal jokes, wordplay, language riddles) *Humor Verbal (Jenaka lisan, permainan kata, teka teki bahasa)	*Non-Verbal Humor (body language, facial expressions, funny gestures) *Humor Bukan Verbal (gaya badan, mimik muka, ekspresi lucu)	*Technology-/Audiovisual-Based Humor (funny videos, animations, language-related cartoons) *Humor Berasaskan Teknologi/Audiovisual (Video lucu, animasi, kartun berkaitan bahasa)	*Situational Humor (spontaneous acting, simulation of humorous situations in class) *Humor Situasi (lakonan spontan, simulasi situasi jenaka dalam kelas)
2. Language Comprehension/ Pemahaman Bahasa				
LC1: Students' ability to understand Arabic language concepts through humor/ Keupayaan pelajar memahami konsep bahasa Arab melalui humor	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7
LC2: Improvement in test scores after the use of humor/ Peningkatan skor ujian selepas penggunaan humor	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7
LC3: Understanding of grammar and vocabulary through humor/ Kefahaman terhadap tata bahasa dan kosa kata melalui humor	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7

Criteria/Sub-criteria Kriteria/Sub-Kriteria	*Verbal Humor (verbal jokes, wordplay, language riddles) *Humor Verbal	*Non-Verbal Humor (body language, facial expressions, funny gestures)	*Technology-/Audiovisual-Based Humor (funny videos, animations, simulations)	*Situational Humor (spontaneous acting, simulation of humorous situations)
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	(Jenaka lisan, permainan kata, teka teki bahasa)	*Humor Bukan Verbal (gaya badan, mimik muka, ekspresi lucu)	language-related cartoons) *Humor Berasaskan Teknologi/ Audiovisual (Video lucu, animasi, kartun berkaitan bahasa)	situations in class) *Humor Situasi (lakonan spontan, simulasi situasi jenaka dalam kelas)
3. Student Motivation/ Motivasi Pelajar				
M1: Students' willingness to actively participate in classroom activities/ Kesiapan pelajar untuk terlibat secara aktif dalam aktiviti kelas	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7
M2: Level of students' interest and enjoyment in learning sessions/ Tahap minat dan keseronokan pelajar dalam sesi pembelajaran	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7
M3: Continuation of interest in learning Arabic outside the classroom/ Kesiambungan minat untuk belajar bahasa Arab di luar kelas	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7

Criteria/Sub-criteria Kriteria/Sub-Kriteria	*Verbal Humor (verbal jokes, wordplay, language riddles) *Humor Verbal (Jenaka lisan, permainan kata, teka teki bahasa)	*Non-Verbal Humor (body language, facial expressions, funny gestures) *Humor Bukan Verbal (gaya badan, mimik muka, ekspresi lucu)	*Technology-/Audiovisual-Based Humor (funny videos, animations, language-related cartoons) *Humor Berasaskan Teknologi/ Audiovisual (Video lucu, animasi, kartun berkaitan bahasa)	*Situational Humor (spontaneous acting, simulation of humorous situations in class) *Humor Situasi (lakonan spontan, simulasi situasi jenaka dalam kelas)
4. Teaching Effectiveness/ Keberkesanan Pengajaran				
T1: Teachers' perception of the effectiveness of humor in delivering lesson content / Persepsi tenaga pengajar terhadap keberkesanan humor dalam menyampaikan isi pelajaran	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7
T2: Ease for teachers in integrating humor into teaching/ Kemudahan tenaga pengajar mengintegrasikan humor dalam pengajaran	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7
T3: Effectiveness of humor in explaining complex concepts/ Keberkesanan humor dalam menjelaskan konsep yang kompleks	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7

Criteria/Sub-criteria Kriteria/Sub-Kriteria	*Verbal Humor (verbal jokes, wordplay, language riddles) *Humor Verbal (Jenaka lisan, pemainan kata, teka teki bahasa)	*Non-Verbal Humor (body language, facial expressions, funny gestures) *Humor Bukan Verbal (gaya badan, mimik muka, ekspresi lucu)	*Technology- /Audiovisual- Based Humor (funny videos, animations, language-related cartoons) *Humor Berasaskan Teknologi/ Audiovisual (Video lucu, animasi, kartun berkaitan bahasa)	*Situational Humor (spontaneous acting, simulation of humorous situations in class) *Humor Situasi (lakonan spontan, simulasi situasi jenaka dalam kelas)
5. Cultural Appropriateness / Kesesuaian Budaya				
C1: Students' acceptance of humor from diverse cultural backgrounds/ Penerimaan humor oleh pelajar dari pelbagai latar belakang budaya	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7
C2: Appropriateness of humor in line with religious and moral values/ Kesesuaian humor dengan nilai-nilai agama dan moral	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7
C3: Ability of humor to avoid sensitive or inappropriate elements/ Keupayaan humor untuk mengelakkan unsur sensitif atau tidak sopan	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7

Criteria/Sub-criteria Kriteria/Sub-Kriteria	*Verbal Humor (verbal jokes, wordplay, language riddles) *Humor Verbal (Jenaka lisan, pemainan kata, teka teki bahasa)	*Non-Verbal Humor (body language, facial expressions, funny gestures) *Humor Bukan Verbal (gaya badan, mimik muka, ekspresi lucu)	*Technology- /Audiovisual- Based Humor (funny videos, animations, language-related cartoons) *Humor Berasaskan Teknologi/ Audiovisual (Video lucu, animasi, kartun berkaitan bahasa)	*Situational Humor (spontaneous acting, simulation of humorous situations in class) *Humor Situasi (lakonan spontan, simulasi situasi jenaka dalam kelas)
6. Ease of Implementation/ Kesediaan Penggunaan				
I1: Ease for instructors in obtaining humor materials related to the Arabic language/ Kemudahan tenaga pengajar mendapatkan bahan humor berkaitan bahasa Arab	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7
I2: Compatibility of humor with existing teaching methods/ Keserasian humor dengan kaedah pengajaran sedia ada	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7	1-2-3-4-5-6-7

AUTHOR'S PROFILE



Wan Syaidatul Izzati binti Wan Abdul Rahman obtained her Bachelor of Science (Hons.) in Management Mathematics from Universiti Teknologi MARA (UiTM) Arau, Perlis in 2024, after completing her Diploma in Mathematical Sciences at UiTM Tapah in 2022. She is currently pursuing a Master of Science (Mathematics) by Research at the College of Computing, Informatics and Mathematics, Universiti Teknologi MARA (UiTM) Negeri Sembilan. Her research focuses on the development of advanced decision-making frameworks using fuzzy logic and mathematical modelling. She has experience in academic administration through her internship at Universiti Teknologi PETRONAS (UTP), where she developed strong communication and analytical skills. Her academic interests include applied mathematics, fuzzy systems, decision analysis, and computational intelligence.

LIST OF PUBLICATIONS

- Wan Syaidatul Izzati binti Wan Abdul Rahman, & Zahari Md Rodzi. (2025). Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean in MCDM. *Mathematical and Informatics Journal (MIJ)*, 6(2), 1–21. <https://doi.org/10.24191/mij.v6i2.4676>.
- Wan Syaidatul Izzati binti Wan Abdul Rahman, Zahari Md Rodzi, Zaifilla Farrina Zainuddin, & Faisal Al-Sharqi. (2026). Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean: An Advanced MCDM Approach. *Journal of*

Applied Mathematics and Informatics (JAMI), 42(2), 1–18. <https://doi.org/10.14317/jami.2026.083>.

Wan Syaidatul Izzati Wan Abdul Rahman, Zahari Md Rodzi, Nur Fathiah Fatin Mohamad Fauzi, & Muhammad Mukhlis Kamarul Zaman. (2025). *A Hybrid Decision Making Framework for Scoring Hesitant Triangular Fuzzy Information Using WASPAS in Data-Centric Multi-Criteria Analysis*. In *2025 6th International Conference on Artificial Intelligence and Data Sciences (AiDAS)*. IEEE. <https://doi.org/10.1109/AIDAS67696.2025.11213663>.

Wan Syaidatul Izzati binti Wan Abdul Rahman, Zahari Md Rodzi, & Zaifilla Farrina Zainuddin. (2025). *Hesitant Triangular Fuzzy Generalized Geometric Heronian Mean: An Advanced MCDM Approach*. *Electronic Journal of Applied Mathematics*, 44(1), 83–101. <https://www.ejpam.com/index.php/ejpam/article/download/6560/2443>

ACCEPTANCE:

Wan Syaidatul Izzati Wan Abdul Rahman, Zahari Md Rodzi, Syaiful Anam, Faisal Al-Sharqi, & Rogayah Mohd Zain. (2026). *Evaluation of Humor-Based Teaching Strategies Using an Intelligent Hesitant Triangular Fuzzy Decision Framework*. Accepted for publication and presentation in INFUS 2026.