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SYSTEMATIC REVIEW OF LITERATURE ON VARIOUS CLASSES OF DIMENSIONAL ALGEBRA

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ABSTRACT

This systematic review of literature provides a comprehensive overview of the various classes of Dimensional Algebra (DA) and their applications in solving complex mathematical problems in different domains. DA represents a significant extension of conventional algebraic systems, offering a powerful framework for addressing mathematical problems in physics, computer science, engineering, and mathematics itself. The selection of literature for this review was based on predefined inclusion and exclusion criteria, including articles published in the English language, peer-reviewed scholarly works, and publications related to DA and its various classes. The search was restricted to articles published in English in a peer-reviewed journal between 2002 and 2022. Nine hundred and thirty-eight articles were initially identified through searches, and an additional 47 articles were screened through other resources. Each article was screened for adherence to inclusion criteria, and studies were excluded in accordance with exclusion criteria, which consisted of non-academic sources, duplicate publications, and works that do not contribute significantly to the understanding of DA. The screening was done by examining the title and abstract, and if needed, the entire article. This review provides a valuable resource for researchers and practitioners seeking to effectively harness the potential applications of DA in various domains.

Keywords: *Classification of Algebra, Dimensional Algebra, Leibniz Algebra, Two-Dimensional Algebra,*

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1. Introduction

In recent years, the field of algebra has witnessed remarkable advancements, particularly in Dimensional Algebra (DA). DA represents a significant extension of conventional algebraic systems, offering a powerful framework for addressing complex mathematical problems in various domains, including physics, computer science, engineering, and mathematics itself. As this area continues to evolve, a comprehensive understanding of the diverse classes of DA is essential for researchers and practitioners to effectively harness its potential applications.

At its core, Dimensional Algebra incorporates diverse mathematical constructs such as geometric products, versos, conformal transformations, and homogeneous coordinates, forming a versatile toolbox that facilitates the manipulation of higher-dimensional entities (Hestenes, 1966; Dorst et al., 2007). These fundamental concepts provide a rigorous mathematical framework to represent complex structures and phenomena within an intuitive geometric context.

The classification of Dimensional Algebra provides a framework for understanding their properties and relationships. Such classification is crucial in algebraic research, as they allow researchers to explore the similarities and differences between various classes of algebras. By studying the structures that emerge within each class, researchers gain valuable insights into the nature of these algebras and uncover connections to other mathematical fields.

The adoption of Dimensional Algebra transcends traditional disciplinary boundaries, finding applications in various scientific domains. Notably, in physics, DA has played a pivotal role in understanding relativistic phenomena, electromagnetism, quantum mechanics, and spacetime representations (Lasenby et al., 1998; Sobczyk, 2011). Similarly, in computer graphics, DA has proven instrumental in achieving real-time rendering, collision detection, and animation control through the utilization of geometric algebra (Hildenbrand, 2011; Hestenes & Sobczyk, 1984). Furthermore, in robotics and control theory, DA has enabled elegant solutions to kinematics, dynamics, and path planning problems (Sommer, 2011; Faugeras, 1993).

This systematic literature review has a primary objective of extensively examining the diverse categories of Dimensional Algebra. The review explores the prevailing research trends and highlights the similarities in the purpose of the studies, mathematical techniques employed, and research methods. These aspects are presented in detail in Table 1.

Table 1: Themes of this study.

No.	Themes of This Study
1.	Purpose of the studies
2.	Mathematical techniques
3.	Methods

By delving into an extensive array of scholarly works, this review endeavours to present a holistic perspective on the development, applications, and advancements within the field, thereby offering invaluable insights to researchers, practitioners, and scholars interested in exploring the intricacies of higher-dimensional mathematics. As Dimensional Algebra continues to gain prominence as an indispensable mathematical tool, this systematic literature review endeavours to provide a comprehensive and well-structured overview of the various classes of DA. Ultimately, this exploration will foster a deeper understanding of higher-dimensional spaces and inspire further breakthroughs in mathematics and its manifold applications.

2. Literature Review

2.1 Classification of Algebras

Algebras play a fundamental role in various branches of mathematics and have been extensively studied for their rich mathematical structures. The classification of algebras is an important area of research that aims to categorize algebras into distinct classes based on their properties and characteristics. The classification of algebras is a challenging task due to the vast diversity of algebraic structures and their intricate interrelationships. It involves identifying common features among algebras and devising systematic methods to differentiate and group them. Over the years, various approaches have been developed to tackle this problem, such as the use of homological and cohomological methods, representation theory, and categorical techniques.

Homological and cohomological techniques have been extensively employed in the classification of algebras. In particular, the use of Hochschild and cyclic homology, as well as derived categories, has proven to be fruitful in understanding algebraic structures. Notable contributions include the work of Gerstenhaber (1964), who introduced the notion of deformations of algebras, and Deligne (1998), who studied the cohomology of associative algebras. These methods provide deep insights into the structure and properties of algebras and have led to significant advancements in their classification.

Representation theory is another powerful tool used in the classification of algebras. It involves studying the different ways in which algebras can be realized as linear transformations on vector spaces. By investigating the representations of algebras, one can uncover hidden symmetries and connections between seemingly unrelated structures. The work of Artin and Procesi (1975) on classifying simple algebras using quivers is a notable example of the successful application of representation theory in algebra classification. This approach provides a concrete and visual framework for understanding algebraic structures and has been influential in various branches of mathematics.

Categorical methods have gained prominence in recent years for the classification of algebras. Category theory provides a unifying language to study algebraic structures and their relationships. By considering the morphisms and functors between different categories, one can analyse the behaviour and properties of algebras in a broader context. Notable contributions include the theory of Hopf algebras, where the categorical viewpoint has played a crucial role in their classification. Categorical techniques offer a systematic and abstract framework for studying algebras, opening new avenues for understanding their classification.

2.2 Two-Dimensional Algebra

Two-dimensional algebras represent a class of mathematical structures that have attracted considerable interest over the years. Originating from complex analysis, they have found applications in various areas such as geometry, theoretical physics, computer graphics, and quantum mechanics. Two-dimensional algebra presents a captivating realm of mathematical exploration, encompassing diverse algebraic structures and their intricate properties.

The study of two-dimensional algebras dates to the late 19th century when mathematicians such as William Kingdon Clifford and Hermann Grassmann made pioneering contributions. Clifford algebras, named after William Clifford, provide a framework for extending the real numbers to include geometric properties and transformations (Clifford, 1878). Grassmann algebras, on the other hand, introduced by Hermann Grassmann, paved the way for the development of multilinear algebra and exterior calculus (Grassmann, 1844). These early works laid the foundation for further exploration of two-dimensional algebras. Two-dimensional algebras possess distinct characteristics that differentiate them from higher-dimensional algebras. One notable feature is their non-commutative multiplication, where the order of operations affects the result, unlike the commutative nature of real numbers. Two-dimensional algebras can also exhibit non-associativity, where the grouping of operations influences the outcome. Studying these properties enhances our understanding of the mathematical structures within two-dimensional algebras.

Ahmed et al. (2017) developed a groundbreaking classification framework for two-dimensional algebras, considering both commutative and non-commutative cases. This systematic approach facilitates a comprehensive categorization of two-dimensional algebras, including their classes, subclasses, and subclasses. By establishing this structured classification, researchers gain a solid foundation for exploring the inherent relationships and dependencies among different types of two-dimensional algebras. In the study on two-dimensional algebraic identities, Ahmed et al. (2020) shed light on the fundamental properties and constraints governing these algebraic systems. They investigate the relationships between addition and multiplication, unravelling the flow of identities within two-dimensional algebras. This exploration deepens our understanding of the underlying structure and behaviour of these algebraic systems, paving the way for further analysis and discovery.

Understanding the symmetries and transformations applicable to two-dimensional algebras is crucial for comprehending their flow. Ahmed et al. (2018) provides valuable insights into the symmetries, transformations, and mappings inherent within two-dimensional algebras through their research on automorphism groups and derivation algebras. Automorphism groups focus on preserving the algebraic structure through transformations, while derivation algebras examine derivations and their associated properties. This investigation reveals the symmetries, mappings, and transformations that preserve the essential characteristics of two-dimensional algebras, highlighting their flow and interconnections.

2.3 Leibniz Algebras

Leibniz algebras, a class of algebraic structures, have been the subject of extensive study in recent years. Researchers have made significant contributions to the classification, description, derivations, completeness, representations, and understanding of nilpotency in Leibniz algebras. These investigations have deepened our comprehension of the diverse characteristics and properties of this mathematical structure.

In the area of classification and description of Leibniz algebras, Rakhimov et al. (2012) and La Rosa and Mancini (2022) have made noteworthy contributions. Rakhimov et al. provide a detailed analysis of specific classes of Leibniz algebras, unveiling their structural properties and establishing connections with other algebraic structures. Their work lays the foundation for a deeper understanding of the diversity and characteristics of Leibniz algebras. Similarly, La Rosa and Mancini focus on two-step nilpotent Leibniz algebras, exploring their properties and investigating their relation to other algebraic objects. By classifying and describing various classes of Leibniz algebras, these studies contribute to the overall knowledge and comprehension of this mathematical structure.

The study of derivations of Leibniz algebras has also been a subject of interest. Rakhimov and Al-Nashri (2012) delve into the properties of derivations in Leibniz algebras, with a specific focus on certain classes. Their research investigates the properties of these derivations and establishes connections between Leibniz algebras and other algebraic structures. Understanding derivations is crucial for comprehending the transformational properties of Leibniz algebras and their relationship with other mathematical objects. The findings of Rakhimov and Al-Nashri shed light on the behaviour and characteristics of derivations in the context of Leibniz algebras.

Another aspect of Leibniz algebras that has been explored is their completeness. Boyle et al. (2020) contribute to the study of complete Leibniz algebras, which possess specific closure properties. Their research focuses on characterizing the conditions for the existence of complete Leibniz algebras and explores the associated properties. Investigating complete Leibniz algebras provides insights into the structural aspects and closure properties of these algebraic structures. The findings of this study contribute to the understanding of complete Leibniz algebras and their role in various mathematical contexts.

The representation theory of Leibniz algebras has also been a topic of investigation. Fialowski and Mihálka (2014) explore the relationship between Leibniz algebras and their representations, shedding light on the connection between these algebraic structures and the broader field of representation theory. By analysing the representations of Leibniz algebras, this research provides a framework for understanding the interplay between Leibniz algebras and other mathematical objects, enhancing our understanding of the structural properties of Leibniz algebras.

Nilpotent Leibniz algebras, which extend to an infinite-dimensional setting, have also received attention. Lavau and Palmkvist (2020) examine the concept of infinity-enhancing in Leibniz algebras, exploring their properties in this expanded framework and enhancing our understanding of the algebraic structure. In a related vein, Hosseini et al. (2021) focus on characterizing nilpotent Leibniz algebras by their multiplier, a measure of the failure of the Leibniz identity. This research investigates the relationship between nilpotency and the multiplier of Leibniz algebras, providing valuable insights into the classification and characterization of these algebras.

3. Methodology/Implementation

This study employed the Systematic Literature Review (SLR) method, which aims to gather, identify, analyze, and synthesize multiple studies on various classes of Dimensional Algebra. SLR is a literature review that adheres strictly to scientific methods to avoid systematic or biased errors (Kwon & Lee, 2019). To ensure a structured process, this study followed the stages recommended by Peter (2019) in the Preferred Reporting Items for Systematic Reviews and Meta-Analysis (PRISMA) guidelines. These stages include identification, screening, eligibility, and inclusion. The research question was addressed through a methodical approach, as depicted in Figure 1. Following the data search, 15 relevant articles were selected for further analysis, and their details can be found in Table 2.

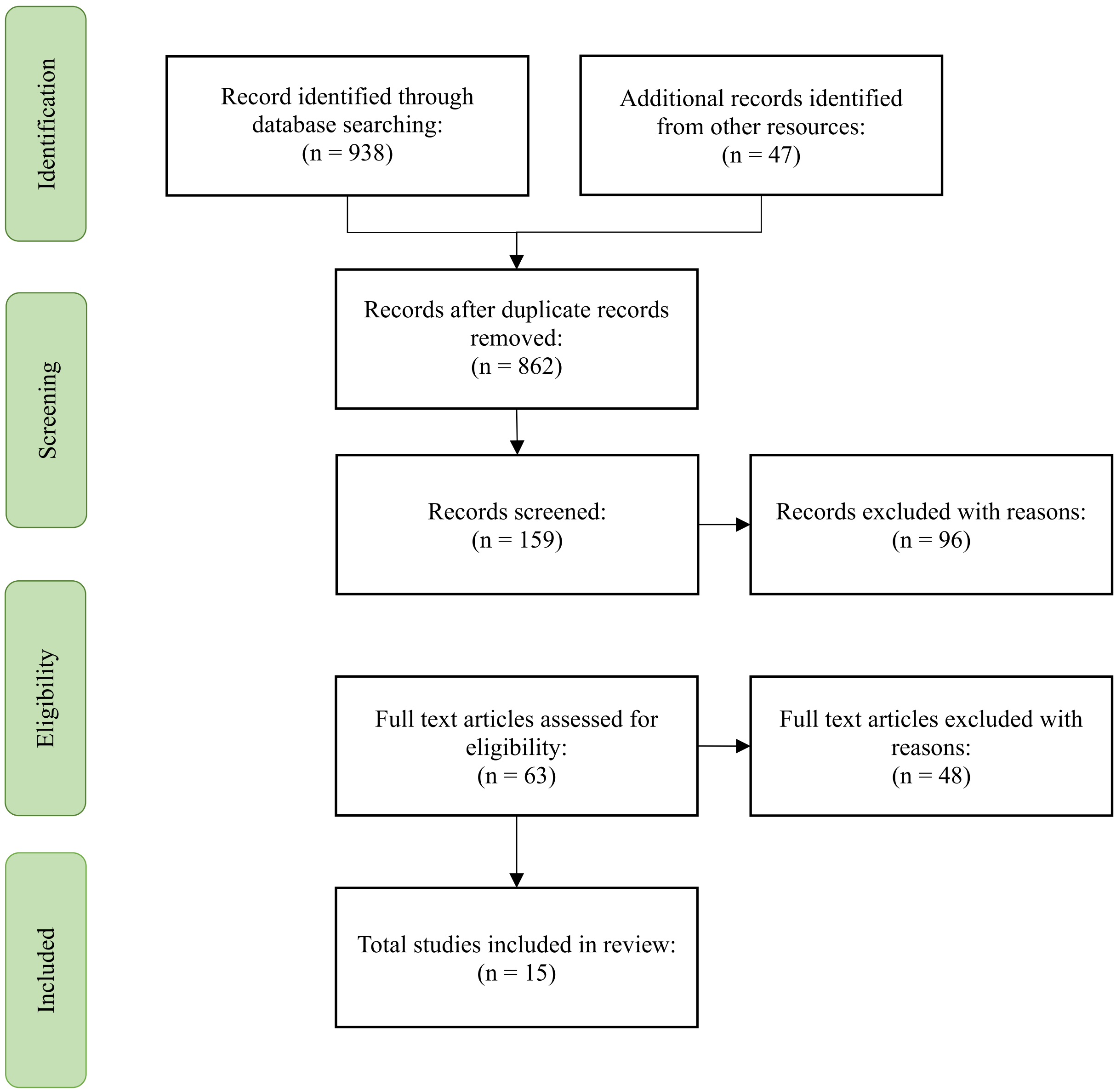

Figure 1: Research Data Search & Selection Flow

Table 2: Identity of the articles to be analysed further.

No	Article Title	Year	Author's Name	Journal Name
1.	Complete Leibniz algebras	2020	Kristen Boyle, Kailash C. Misra, Ernest Stitzinger	Journal of Algebra
2.	Representations of Leibniz Algebras	2014	Fialowski, A., & Mihálka, É. Zs.	Algebras and Representation Theory
3.	On Derivations of Some Classes of Leibniz Algebras	2012	Rakhimov, I. S., & Al-Nashri	Journal of Generalized Lie Theory and Applications
4.	Description of some classes of Leibniz algebras	2012	Rakhimov, I.S., Rikhsiboev, I. M., Khudoyberdiyev, A. Kh., & Karimjanov, I. A.	Linear Algebra and Its Applications
5.	Infinity-enhancing of Leibniz algebras	2020	Lavau, S., & Palmkvist, J.	Letters in Mathematical Physics
6.	Two-step nilpotent Leibniz algebras	2022	La Rosa, G., & Mancini, M.	Linear Algebra and Its Applications
7.	Characterizing nilpotent Leibniz algebras by their multiplier	2021	Hosseini, S. N., Edalatzadeh, B., & Salemkar, A. R.	Journal of Algebra
8.	Complete lists of low dimensional complex associative algebras	2009	I.S. Rakhimov, I.M. Rikhsiboev, W. Basri	-
9.	Contractions of low dimensional complex associative algebras	2017	Mohammed, N. F., Rakhimov, I. S., & Husain, S. K.	AIP Conference Proceedings
10.	Cohomology spaces of low dimensional complex associative algebras	2017	Mohammed, N. F., Rakhimov, I. S., & Husain, S. K.	AIP Conference Proceedings
11.	Centroids and derivations of low-dimensional Leibniz algebra	2017	Husain, S. K., Rakhimov, I. S., & Basri, W.	AIP Conference Proceedings
12.	Complete Classification of Two-Dimensional Algebras	2017	Ahmed, H., Bekbaev, U., & Rakhimov, I. S.	AIP Conference Proceedings
13.	Identities on Two-Dimensional Algebras	2020	Ahmed, H., Bekbaev, U., & Rakhimov, I. S.	Lobachevskii Journal of Mathematics
14.	The Automorphism Groups and Derivation Algebras of Two-Dimensional Algebras	2018	Ahmed, H., Bekbaev, U., & Rakhimov, I. S.	Journal of Generalized Lie Theory and Applications
15.	Local and 2-Local Derivations and Automorphisms on Simple Leibniz Algebras	2019	Ayupov, S., Kudaybergenov, K., & Omirov, B.	Bulletin of the Malaysian Mathematical Sciences Society

To ensure the comprehensiveness of this systematic literature review, a rigorous and systematic search strategy will be employed to identify relevant academic articles, conference papers, books, and other authoritative sources. The search will encompass electronic databases such as Google Scholar, IEEE Xplore, ScienceDirect, Google Scholar, and other specialized repositories related to mathematics and computer science. The search terms will include variations of "Dimensional Algebra," "Two-Dimensional Algebra," "Classification of Algebra," "Leibniz Algebra," and other relevant synonyms.

The selection of literature for this review will be based on predefined inclusion and exclusion criteria. Inclusion criteria will include articles published in the English language, peer-reviewed scholarly works, and publications related to Dimensional Algebra and its various classes. Exclusion criteria will consist of non-academic sources, duplicate publications, and works that do not contribute significantly to the understanding of Dimensional Algebra.

The search was restricted to articles published in English in a peer-reviewed journal between 2002 and 2022. The authors limited articles to the last twenty years to ensure practices being evaluated were still relevant to current educational standards (Sahlberg, 2016). Nine hundred and thirty-eight articles were initially identified through searches; an additional 47 articles were screened through the other resources. Each was screened for adherence to inclusion criteria. Inclusion criteria will include articles published in the English language, peer-reviewed scholarly works, and publications related to Dimensional Algebra and its various classes. Studies are excluded in accordance with exclusion criteria, which consist of non-academic sources, duplicate publications, and works that do not contribute significantly to the understanding of Dimensional Algebra. The screening was done by examining the title and abstract, and if needed the entire article.

After applying the screening procedures, 15 articles met all inclusion criteria. Once the articles were identified, the extracted data will be subjected to a thematic analysis to identify common themes, key concepts, and emerging patterns in the literature. Study themes included the purpose of the studies, mathematical techniques, and methods of the research, as seen in Table 3(a), 3(b), and 3(c).

Table 3(a): Data collected from the articles.

No.	Article Title	Purpose of the Studies	Mathematical Techniques	Methods
1.	Complete Leibniz algebras	To investigate the properties and structure of complete Leibniz algebras, which are a generalization of Lie algebras. The authors aim to establish basic results about complete Leibniz algebras and their relationship with Lie algebras.	Group theory, Lie theory, and homological algebra.	Theoretical and computational methods, algebraic and geometric methods
2.	Representations of Leibniz Algebras	The purpose of the study in this article is to investigate the representations of Leibniz algebras. There is connection built between the structure of the Leibniz algebra and its representation theory, which is important in the study of Lie algebras and their applications. The article explores the relationship between the representation of a Leibniz algebra and the subalgebras and quotient algebras associated with it.	Lie algebras, universal enveloping algebras, and homological algebra	Algebraic and geometric methods
3.	On Derivations of Some Classes of Leibniz Algebras	To investigate the properties of derivations in specific classes of Leibniz algebras, namely filiform and homogeneous algebras. The authors aim to determine the structure of the derivations and their relation to the	Leibniz algebra theory and their derivations, and linear algebra	Algebraic and analytical methods

		algebras' structure. The study also explores the automorphisms and cohomology of these algebras.		

Table 4(b): Data collected from the articles.

No.	Article Title	Purpose of the Studies	Mathematical Techniques	Methods
4.	Description of some classes of Leibniz algebras	To investigate the properties and structure of these algebras and to provide a classification of them. Several important subclasses of Leibniz algebras are examined, including solvable, nilpotent, and semi simple algebras, and explore their properties. The study also contributes to the development of the theory of Leibniz algebras.	Leibniz algebra theory, linear algebra and abstract algebra	Algebraic and geometric methods
5.	Infinity-enhancing of Leibniz algebras	The purpose of the study is to investigate the concept of "infinity-enhancing" of Leibniz algebras, which involves introducing higher order operations and studying their properties. A framework is established for studying infinite-dimensional Leibniz algebras that have previously been difficult to analyse using conventional methods.	Leibniz algebra theory, theory of homotopy Lie algebras and Lie algebra cohomology	Algebraic and geometric methods
6.	Two-step nilpotent Leibniz algebras	To investigate the properties and structure of two-step nilpotent Leibniz algebras. Specifically, the authors aim to classify these algebras up to isomorphism and describe their derivations and automorphisms. The study is primarily focused on exploring the algebraic properties of these structures and their applications in mathematical physics and geometry.	Leibniz algebra theory, group theory, and homological algebra	Theoretical analysis and computational methods
7.	Characterizing nilpotent Leibniz algebras by their multiplier	The objective of this study is to characterise nilpotent Leibniz algebras by their multiplier, a numerical invariant related to the algebra. The authors are interested in investigating the link between the multiplier and other Leibniz algebraic properties, as well as the necessary and sufficient conditions under which two nilpotent Leibniz algebras might share the same multiplier.	Leibniz algebra theory, multiplier, cochain complex theory and homological algebra	Algebraic and geometric methods
8.	Complete lists of low dimensional complex associative algebras	The purpose of the study is to provide complete lists of low dimensional complex associative algebras, where all complex associative algebras up to dimension six are classified and provided a systematic approach to the classification problem. The article presents a comprehensive list of all such algebras, along with their structural properties.	Linear algebra, representation theory, and the theory of commutative and non-commutative rings	Theoretical analysis and computer-aided calculations
9.	Contractions of low dimensional complex associative algebras	The purpose of the study is to investigate the contractions of low dimensional complex associative algebras, specifically to study the properties and structure of these contractions, as well as their relationship with certain classes of algebras such as Lie algebras and Poisson algebras.	Linear algebra, commutative algebra, and homological algebra	Algebraic and geometric methods

Table 5(c): Data collected from the articles.

No.	Article Title	Purpose of the Studies	Mathematical Techniques	Methods
10.	Cohomology spaces of low dimensional complex associative algebras	This research aims to explore low-dimensional complex associative algebras' cohomology spaces. The authors seek to find out the structure of these cohomology spaces and how they relate to the algebras' algebraic properties.	Homological algebra, complex analysis, and algebraic topology	Algebraic topology and computer software
11.	Centroids and derivations of low-dimensional Leibniz algebra	To investigate the properties of low-dimensional Leibniz algebras, specifically their centroids and derivations. The study aims to determine the structure of the algebra and the properties of its elements, such as the ideal and subalgebra generated by the centroids, and the automorphisms and derivations of the algebra. These properties are explored and applied to specific examples of low-dimensional Leibniz algebras.	Linear algebra, group theory, Lie algebra theory, and Leibniz algebra theory	Theoretical analysis and examples
12.	Complete Classification of Two-Dimensional Algebras	To provide a complete classification of two-dimensional algebras by identifying all possible algebraic structures that can exist in two dimensions. The study uses mathematical techniques to systematically analyze and categorize the different types of two-dimensional algebras, providing a comprehensive framework for future research in this area.	Lie algebras, differential equations, and linear algebra	Theoretical analysis and computational techniques
13.	Identities on Two-Dimensional Algebras	The purpose of the study is to investigate certain algebraic structures known as two-dimensional algebras and their properties. The authors focus specifically on the study of identities, which are equations that hold true for all elements of an algebraic structure. They explore the implications of these identities and use them to classify different types of two-dimensional algebras.	Lie algebras, group theory, and linear algebra	Algebraic and geometric methods
14.	The Automorphism Groups and Derivation Algebras of Two-Dimensional Algebras	To investigate the automorphism groups and derivation algebras of two-dimensional algebras. The authors provide a comprehensive understanding of the structures of these algebras and explore their properties and relationships with other mathematical structures. The authors employ a range of mathematical techniques and tools to analyze the automorphism groups and derivation algebras of these algebras, including group theory, representation theory, and algebraic geometry.	Group theory, Lie theory, and algebraic geometry	Algebraic, geometric, and analytic methods
15.	Local and 2-Local Derivations and Automorphisms on Simple Leibniz Algebras	To investigate the structure of simple Leibniz algebras under the action of local and 2-local derivations and automorphisms. The authors examine the commutators and Lie brackets of various algebraic structures in order to analyse the features of these structures. The results of the study may also have implications for the study of other types of algebras and their automorphisms.	Leibniz algebra theory, Lie algebras, derivations, and automorphisms	Theoretical analysis and computational methods

4. Results And Discussion

4.1 Purpose of the Studies

The purpose of the Classification of Dimensional Algebras is to study and categorize a broad class of mathematical structures known as dimensional algebras. Dimensional algebras are algebraic systems that exhibit properties related to the dimensionality of vector spaces. They provide a powerful framework for understanding and manipulating geometric and algebraic structures in various mathematical disciplines.

By classifying dimensional algebras, researchers aim to uncover their underlying properties, relationships, and symmetries. This classification enables a systematic analysis of their structural features and provides insights into their applications across different areas of mathematics and physics. Moreover, it helps to identify new connections between seemingly disparate mathematical theories, leading to the development of novel mathematical tools and techniques.

The study of dimensional algebras encompasses a wide range of topics, including Lie algebras, associative algebras, and Jordan algebras. Techniques from representation theory, algebraic geometry, and algebraic topology are often employed in the classification process. Ultimately, the classification of dimensional algebras contributes to a deeper understanding of fundamental mathematical structures and paves the way for advancements in fields such as quantum mechanics, differential geometry, and mathematical physics.

4.2 Mathematical Techniques

Table 6: Mathematical techniques in the studies

Mathematical Techniques in the Studies	
Group Theory	n=5
Lie algebra	n=5
Homological algebra	n=5
Leibniz algebra theory	n=6
Linear algebra	n=7

Table 4 above shows the different mathematical techniques of the studies. The techniques used in each study differ according to the purpose of the study. From the 15 selected articles, there are more mathematical techniques used, however, this table shows the most used techniques among the articles. As seen there, group theory, lie algebra and homological algebra are all being used in 5 articles. Whereas Leibniz algebra theory and linear algebra theory are used in 6 and 7 articles respectively. Certain techniques may complement each other in terms of their limitations and strengths.

By integrating multiple techniques, mathematicians can address a wider range of questions and explore different aspects of the Dimensional Algebras. This interdisciplinary approach or combining two or more fields of study can lead to a more comprehensive understanding and classification of these algebras, uncovering connections and patterns that might have been missed by using a single technique alone. Furthermore, the combination of techniques can foster innovation and inspire the development of new mathematical methods specific to the classification of Dimensional Algebras. As researchers encounter challenges or gaps in their analysis, they may discover new ways to bridge these gaps by leveraging the strengths of different techniques.

4.3 Methods

Table 7: Methods in the studies

Methods in the Studies	
Algebraic Method	n=9
Geometric Method	n=7
Theoretical Method	n=6
Computational Method	n=6

Table 5 above provides an overview of the research methods used in the articles reviewed. There are four research methods used, namely Algebraic method, Geometric method, Theoretical method, and Computational methods. 9 studies used Algebraic methods, 7 studies with Geometric methods, 6 studies with theoretical methods and 6 studies used a Computational method.

When employing multiple methods for the classification of dimensional algebras, there are several advantages and benefits. The use of multiple methods allows for a more comprehensive and in-depth analysis of the algebras. Each method brings its own unique perspective and insights into the structures and properties of dimensional algebras. By combining algebraic, geometric, theoretical, and computational approaches, researchers can gain a more complete understanding of the algebras' characteristics from different angles. This multiple viewpoint helps validate the results and ensures a more accurate classification.

Secondly, employing multiple methods helps cross-validate the findings and lessen potential limitations or biases inherent in a single method. Different methods often have their own strengths and weaknesses. By leveraging multiple approaches, researchers can compensate for the limitations of one method with the strengths of another. For instance, if there are some difficulties in algebraic methods handling certain complexities, geometric or computational methods can provide alternative means to overcome those challenges. This interdisciplinary approach promotes a more reliable classification process, reducing the chances of errors or oversights and increasing confidence in the obtained results.

5. Conclusion And Recommendations

As the field of algebra continues to evolve, researchers and practitioners must strive to gain a comprehensive understanding of the diverse classes of Dimensional Algebra (DA). This study has highlighted the importance of employing multiple approaches, including algebraic, geometric, theoretical, and computational methods, to gain a more complete understanding of the characteristics of DA from different angles. By leveraging multiple approaches, researchers can compensate for the limitations of one method with the strengths of another, promoting a more reliable classification process and increasing confidence in the obtained results.

Thematic analysis of the fifteen articles identified in this study revealed common themes, key concepts, and emerging patterns in the literature. The purpose of the studies varied, with some investigating the properties and structure of complete Leibniz algebras, while others focused on the representations of Leibniz algebras. Mathematical techniques employed in the studies included group theory, Lie theory, homological algebra, and universal enveloping. Methods of research included theoretical and computational methods, as well as algebraic and geometric methods.

Based on the findings of this study, it is suggested that future research should prioritize the development of new classes of Dimensional Algebra. Although the review identified some existing classes, there is a clear need for new ones that can specifically address various mathematical problems in different domains. Achieving this requires the application of innovative mathematical techniques and the integration of Dimensional Algebra with other mathematical systems.

Additionally, it is recommended that future research should explore the potential applications of Dimensional Algebra in various domains. Although the review identified some areas where DA can be applied, further investigation is needed to fully understand its potential applications. This exploration can be facilitated through the creation of case studies and fostering collaboration between researchers and practitioners from different domains.

Furthermore, future research should focus on developing effective teaching and learning strategies for Dimensional Algebra. Despite some existing studies on the subject, there is a clear need for better strategies that can enhance students' understanding and application of Dimensional Algebra. This can be achieved through the smart integration of technology and the use of innovative teaching methods.

Finally, practitioners in various domains should be encouraged to explore the potential applications of Dimensional Algebra in their respective fields. Although the review highlighted some applicable domains, there is still untapped potential waiting to be discovered. To facilitate this, information should be widely disseminated, and adequate training and support should be provided to practitioners.

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8. Conflict of Interest

The authors have no conflicts of interest to declare.

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