Mathematics Letters, 2 (2): 197-209, 2023
eISSN: 2948-3735

DISTANCE CALCULATION OF TWO POINTS BY USING PARAMETRIC EQUATIONS FOR TEACHING AND LEARNING PROCESSES

Nabila Humayra Sheik Ghazali[1], Mohd Agos Salim Nasir[2*], Nurzalina Harun[3]

College of Computing, Informatics and Mathematics, Universiti Teknologi MARA,40450 Shah Alam, Malaysia

[1]nabilahumayra03@gmail.com, [2*]mohdagos066@uitm.edu.my, [3]nurzalina1587@uitm.edu.my

(indicates the corresponding author)*

ABSTRACT

Time and distance data were collected to obtain a polynomial graph by plotting the distances of x and y versus time, t and to determine a parametric equation form for two of the quadratic equations. To begin the experiment, data on time in minutes from the initial point, Subang Jaya to the end point, Taman Melati was collected and marked in Table 1. The first data of time, t was added to the second one to achieve the second data of time, t for all 27 data of time, t. The distances of x and y were likewise measured and recorded. This process was repeated for 26 times to achieve additional sets of results. From the plotting of x versus t and y versus t, two polynomial graphs were obtained. From here, two quadratic equations were obtained with the highest power of 2. Since one of the equations has a negative determinant value, which is the x(t) equation, this implies a maximum value graph whereas the y(t) equation has a positive determinant value, which infers a minimum value graph of a polynomial. Moreover, an elimination method was conducted on both the x(t) and y(t) equations. When the equation of x(t) was rewritten in the form $ax^2 + bx + c = 0$, t in terms of x equation was obtained. From there, t(x) was inserted into y(t) to achieve a y(x) equation.

Keywords: *Actual Value, Calculated Value, Calculate Distance, Parametric Equations.*

Received for review: 14-09-2023; Accepted: 22-10-2023; Published: 06-11-2023

1. Introduction

Naturally, the two most fundamental components of human behaviour and existence are time and distance. When any particle moves from one position to another, time and distance are the two vital elements that are continuous and can be recorded for further investigation, resulting in various outcomes. It is a process that all particles go through in order to function in certain scenarios. For instance, in the human body itself, the smallest unit that can sustain life on its own and makes up all living things as well as the body's tissues, which are cells that need to travel from one cell body to an adjacent cell body to properly function as a carrier and fulfill its related mechanism. In the physical nature of time, time discusses the theory of general relativity in relation to spacetime events. However, the numerical findings vary depending on the observer. In general relativity, the matter of the minute of this moment in time is only significant in relation to a certain observer.

Time, t can be defined as a continuous sequence of existence and occurrences that takes place in what seems to be an unstoppable order from the past, through the present, and into the future. A practical definition of time is very helpful in the conduct of both advanced experiments and daily activities. A quantitative component that is a part of several measures that are used to compare the duration of events or the time frames between the two events, to measure the rate of change of quantities in the actual world. In geometry, a three-dimensional space, often known as 3D-space, is a mathematical domain where a point's location is determined by three values, which are the coordinates, and time is frequently referred to as the fourth dimension. Furthermore, in both the International System of Units (SI) and International System of Quantities, time counts as one of the seven basic physical quantities, and the SI unit of time is the second. However, every occurrence in the house, the neighbourhood, the nation, and the entire world may still be documented using measurement units such as minutes (m), hours (hrs), days, weeks, seasons, years, centuries, and eras. It states that observing a specific number of repetitions of one or more standard circular events, case in point, a free-swinging pendulum is considered one standard unit of time, such as the second. Time is also used to define other values, which include velocity, so it would be redundant to describe time in terms of other numbers.

Distance is a measurement of how far away two things or locations are, either numerically or occasionally qualitatively. In physics or common language, distance can be referred to either as a physical length or as a measurement based on other factors. For example, "two counties over". This term is frequently used metaphorically to refer to a measurement of the difference between two identical objects due to the fact that the concept of spatial cognition provides a rich source of conceptual analogies in human understanding. To point out, given that distance only depends on magnitude and not direction, distance is a scalar number, and can only have positive or zero values. A scalar number is a real number that can be fully explained by a magnitude or numerical quantity alone. The meter (m) is the commonly used unit of measurement for distance, but greater distances can also be expressed in kilometres (km), and smaller distances can be expressed in centimetres (cm) or millimetres (mm). However, the distance traversed in this study is denoted by the letters x and y.

When a particle travels from 0 milliseconds and from 0 millimetre to t time and x distance, a mechanism is carried out to determine an equation called a parametric equation of motion. The coordinates of every point that makes up a geometrical object, such as a curve or surface, can often be expressed using parametric equations; these objects are referred to as parametric curves and parametric surfaces, respectively. Parametric equation is a type of equation that uses an independent variable known as a parameter, commonly represented by t, and dependent variables that are specified as continuous functions of the parameter and independent of other variables. When necessary, more than one parameter can be used. In this experiment, time, t represents the independent variable, whereas distance of x and y as dependent variables. Therefore, this experiment is conducted with one independent variable and two independent variables. To give an example, the equation $y=x^2$ can be expressed as a pair of equations in the parametric form $x=t$, and $y=t^2$, rather than the Cartesian form of $y=x^2$. This conversion of equations to its parametric form, called parameterization, greatly improves the efficiency of differentiating and integrating curves, which is a parametric curve or the graph of the equation. Throughout the experiment, it is predicted that a polynomial graph of x versus t and y versus t with its own quadratic equation will be obtained. From there, a parametric equation of y versus x can be determined as the second prediction using the parameter elimination method.

When both x and y are expressed in terms of a third variable t, referred to as a parameter, such curves as the cycloid may be handled most effectively: [$x=f(t)$, $y=g(t)$]. This project is aimed at identifying the parametric equation from two different equations of $x(t)$ and $y(t)$, and the graphs of $x(t)$ and $y(t)$ are both considered as quadratic equations.

2. Methodology/Implementation

Initially, the time taken to travel from Subang Jaya to Taman Melati was taken and recorded in minutes using a mobile phone timer. To ensure maximum accuracy, time, t was set to 0.00 minutes before the recording began. The distance traveled shown in Figure 1 using Google Maps was also measured and recorded to put into Table 1. To specify, each point and location were precisely measured using their longitudes and latitudes, starting from the initial point (0,0) located at UiTM Shah Alam. All the steps were repeated to achieve an additional 26 sets of findings for all 27 locations. The data recorded then was transferred into the Microsoft Office Excel Spreadsheet to generate three different scatter charts. From there, in order to derive quadratic equations of x in terms of t and y in terms of t, a polynomial trendline graph was selected for both graphs. The $x(t)$ equation was then rewritten into $ax^2 +bx+c=0$ form. The a, b and c values were inserted into this formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{1}$$

In such a way, t was calculated in terms of x. The $t(x)$ equation was then computed into y of the x equation to find a parametric equation in order to obtain the $y(x)$ equation. The x against y graph was plotted between the longitudinal distance, x, and the latitudinal distance, y. A graphing calculator was then used to create two separate graphs, where one graph indicates the positive value while the other graph indicates the negative value, with their own quadratic equations. However, only one of these two equations will be selected as the final parametric equation based on its similarities. Finally, the actual value of the equation and the experimental value can be compared to one another in order to assess the accuracy of the earlier computations by substituting one of the distance, x values in each of these equations.

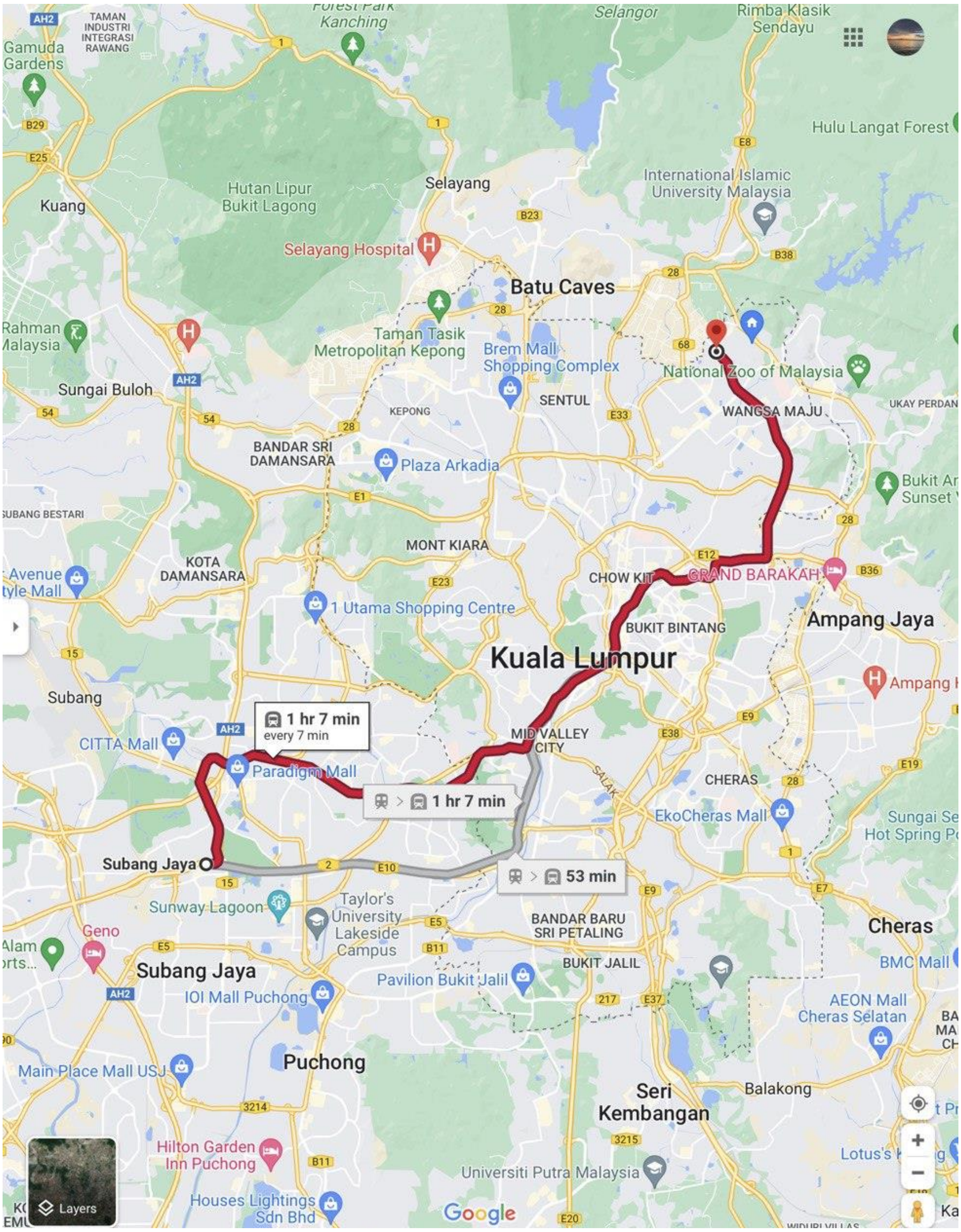

Figure 1. The map of the journey from Subang Jaya to Taman Melati

3. Results and Discussion

3.1 Results

Distance, x and y, and time, t was measured and recorded into Table 1.

Table 1. Time, distance of x and y travelled from Subang Jaya to Taman Melati

Place	Distance, km		Time, minutes
	x	y	t
Subang Jaya	7.49	2.02	0.00
CGC - Glenmarie	7.79	3.27	2.08
Ara Damansara	7.48	4.66	4.27
Lembah Subang	8.05	5.16	5.51
Kelana Jaya (KJ24)	9.54	5.21	7.75
Taman Bahagia	10.43	5.01	9.02
Taman Paramount	11.56	4.35	10.41
Asia Jaya	13.35	4.26	11.89
Taman Jaya	14.04	4.24	12.97
KL Gateway - Universiti	15.86	5.38	16.21
Kerinchi (KJ18)	16.6	5.49	17.62
Abdullah Hukum	17.57	5.83	19.02
Bangsar	18.26	6.87	20.55
KL Sentral	19.03	7.54	21.88
Pasar Seni	20.08	8.44	24.26
Masjid Jamek	20.21	9.26	25.62
Dang Wangi	20.89	9.98	27.68
Kampung Baru	21.3	10.55	29.06
KLCC	22.13	10.27	30.33
Ampang Park	22.68	10.38	31.39
Damai	23.27	10.88	32.64
Dato' Keramat	24.06	10.97	34.19
Jelatek	24.48	11.21	34.76
Setiawangsa	24.56	12.13	35.86
Sri Rampai	24.97	14.65	39.06
Wangsa Maju	24.11	15.45	40.21
Taman Melati	23.01	16.99	41.5

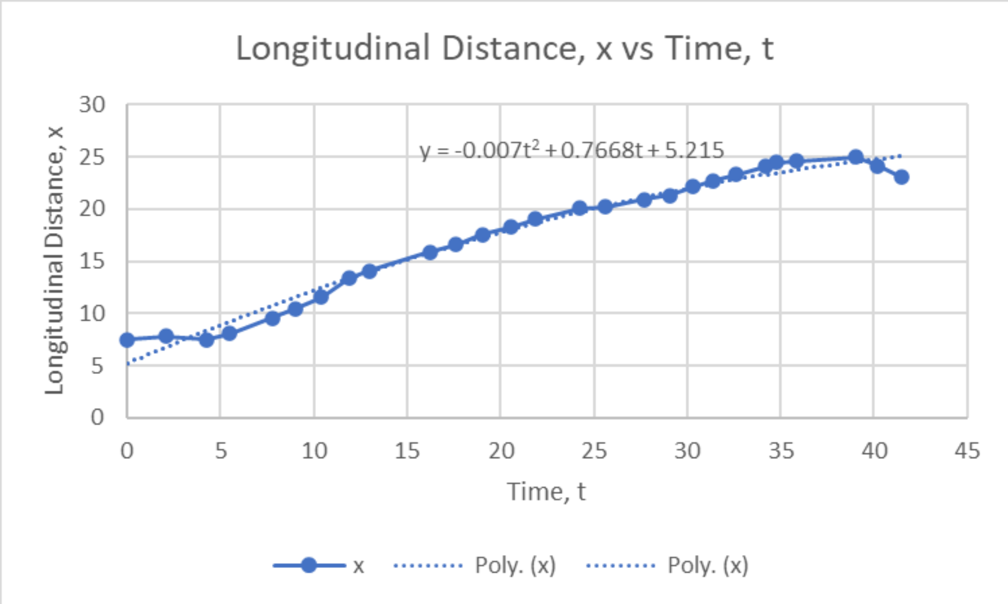

Figure 2. The graph of Longitudinal Distance, x vs Time, t

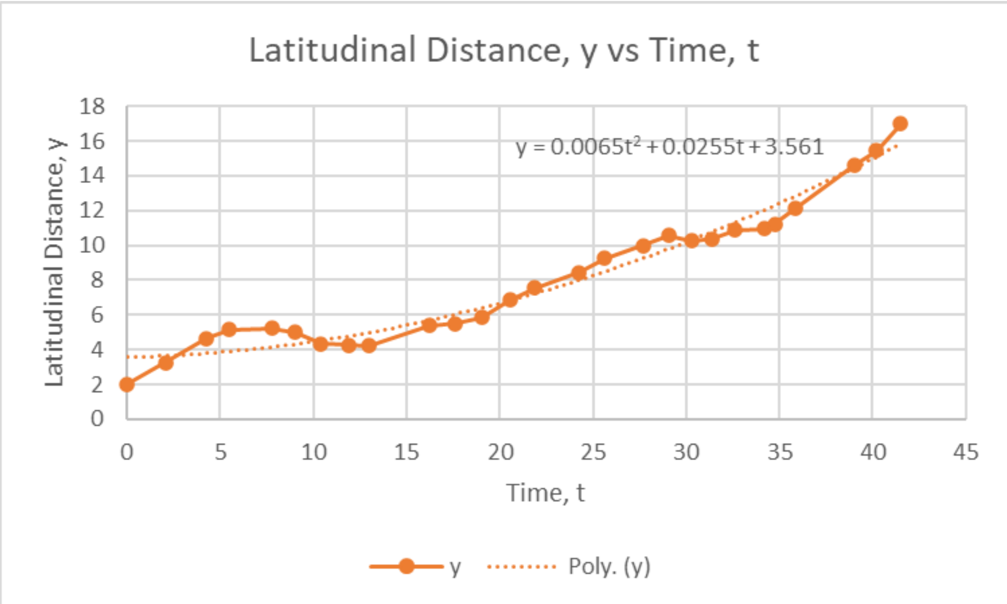

Figure 3. The graph of Latitudinal Distance, y vs Time, t

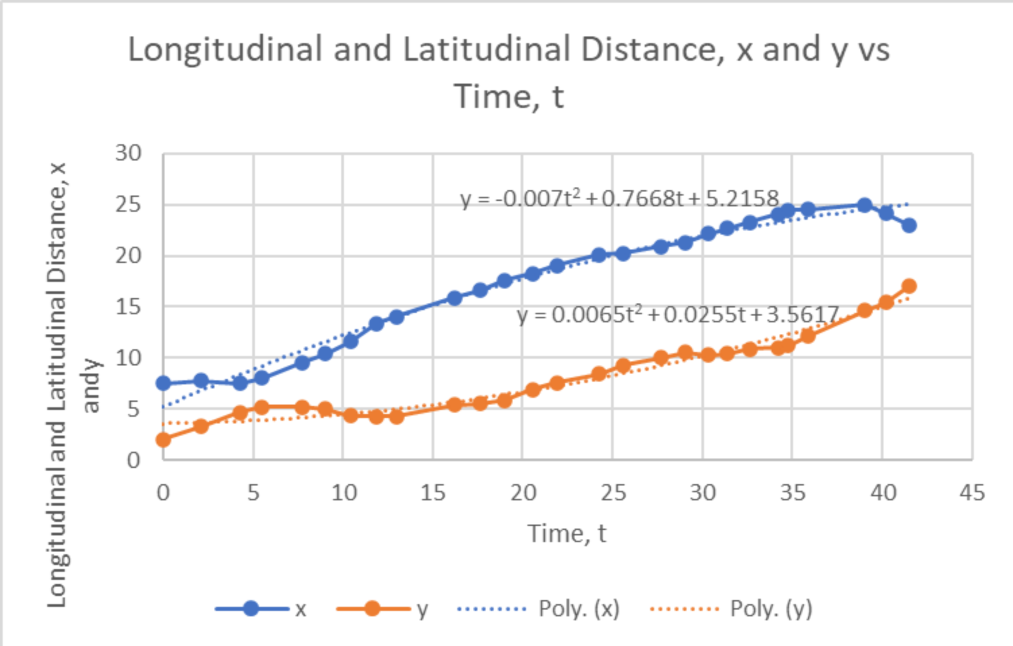

Figure 4. The graph of Longitudinal and Latitudinal Distance, *x* and *y* vs Time, *t*

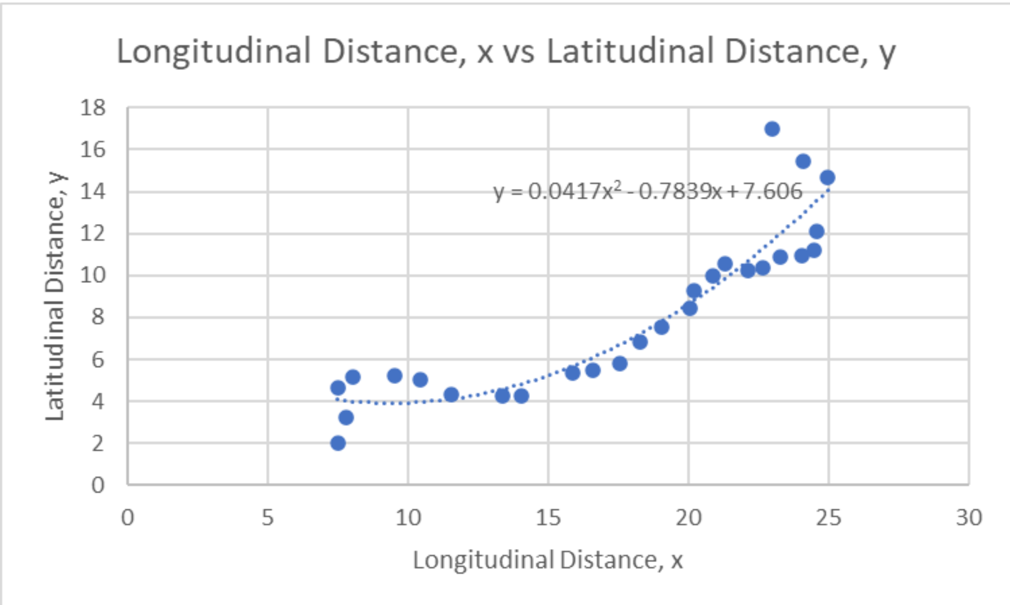

Figure 5. The graph of Longitudinal Distance, *x* vs Latitudinal Distance, *y*

$$x(t) = -0.007t^2 + 0.7668t + 5.2158 \tag{2}$$

$$y(t) = 0.0065t^2 + 0.0255t + 3.5617 \tag{3}$$

$$-0.007t^2 + 0.7668t + 5.2158 - x = 0 \tag{4}$$

$$y(x) = 0.0147x^2 - 0.7839x + 7.606 \tag{5}$$

To obtain the parametric equation, the equations of *x(t)* and *y(t)* can be solved simultaneously. From Equation (2), *t* in terms of *x* can be solved by using the following formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{where } a = -0.007,\ b = 0.7668 \text{ and } c = 5.2158 - x$$

$$t = \frac{-0.7668 \pm \sqrt{0.7668^2 - 4(-0.007)(5.2158 - x)}}{2(-0.007)}$$

$$t = \frac{-0.7668 \pm \sqrt{0.734 - 0.028x}}{-0.014} \tag{6}$$

Next, substitute *t* into Equation (3) to achieve the parametric *y(x)* equation.

$$y(x) = 0.0065\left(\frac{-0.7668 \pm \sqrt{0.734 - 0.028x}}{-0.014}\right) + 0.0255\left(\frac{-0.7668 \pm \sqrt{0.734 - 0.028x}}{-0.014}\right) + 3.5617 \tag{7}$$

The graph of final equation *y(x)* can be obtained by using Desmos Graph, graphing calculator.

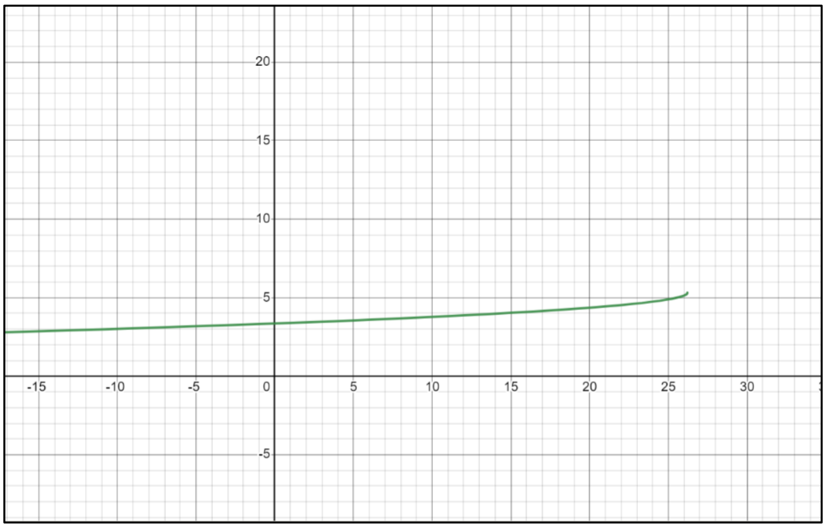

Figure 6. Graph of *y(x)* equation

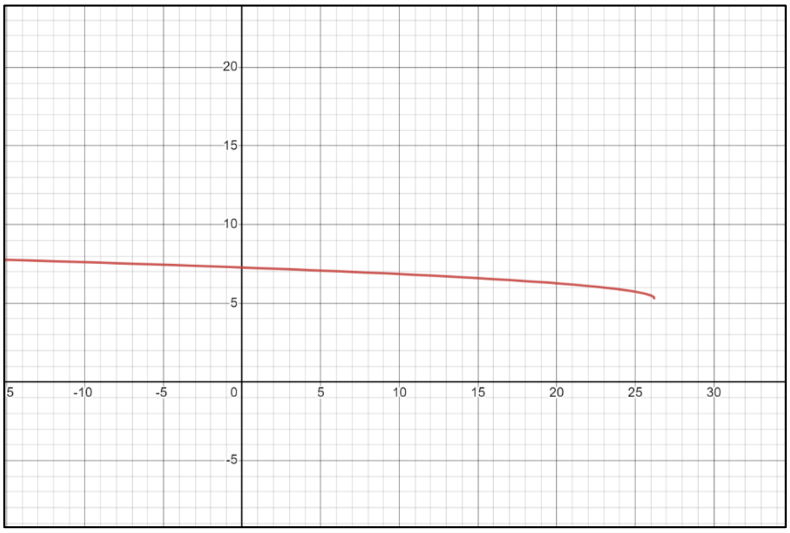

Figure 7. Graph of *y(x)* equation

From Figure 6 and Figure 7, two distinct equations can be derived from the *y(x)* parametric Equation (7) which are:

$$y(x)=0.0065\left(\frac{-0.7668+\sqrt{0.734-0.028x}}{-0.014}\right)+0.0255\left(\frac{-0.7668+\sqrt{0.734-0.028x}}{-0.014}\right)+3.5617 \tag{8}$$

and

$$y(x)=0.0065\left(\frac{-0.7668-\sqrt{0.734-0.028x}}{-0.014}\right)+0.0255\left(\frac{-0.7668-\sqrt{0.734-0.028x}}{-0.014}\right)+3.5617 \tag{9}$$

By observation, the graph in Figure 6 that was initially obtained from Equation (8) is most identical to the parametric equation graph in Figure 5. Therefore, Equation (8) which represents Figure 6 was chosen as the final parametric equation.

To compare the accuracy of the calculations, Equation (5) and Equation (8) can be compared by substituting one of the distances, *x* value:

$$\begin{aligned} y(7.79) &= 0.0147(7.79)^2-0.7839(7.79)+7.606 \\ &= 2.3915 \end{aligned}$$

$$\begin{aligned} y(7.79) &= 0.0065\left(\frac{-0.7668+\sqrt{0.734-0.028(7.79)}}{-0.014}\right)+0.0255\left(\frac{-0.7668+\sqrt{0.734-0.028(7.79)}}{-0.014}\right)+3.5617 \\ &= 3.5929 \end{aligned}$$

3.2 Discussion

From the results above, it can be concluded that the results obtained are consistent with the prediction stated. All the results obtained also managed to meet the objective of the study, which is to identify the parametric equation from two different equations of $x(t)$ and $y(t)$.

Based on Table 1, the distances x and y in kilometres and the time t in minutes were measured and marked. The distance of x was the longitude of each of the locations starting from Subang Jaya to Taman Melati from the origin (0,0), located at UiTM Shah Alam marked on the graph. In contrast, the distance y was the latitude of the locations from the origin (0, 0). 27 values of x, y and t were recorded and marked into the table.

In Figure 2, the graph entitled "Time vs. Distance" illustrates the correlation between the amount of time elapsed, t and the corresponding distance traveled, x. The x-axis represents time, measured in minutes, while the y-axis represents distance, measured in kilometres. By analyzing the graph, a generally increasing trend where the line rises as time progresses was observed, starting from 7.49 kilometres from the initial point (0,0) at time 00.0 minutes to 23.01 kilometres at time 41.5 minutes. This suggests a positive relationship between distance x and time, indicating that as more time passes, the distance traveled also increases. Throughout the graph, there are a few downward sloping graphs, which indicate a decreasing value of distance, x as time, t increases. Therefore, there might be some fluctuations or variations in the slope of the line. These variations could be due to factors like changes in speed, breaks, or different stages of the journey. It is essential to look for notable features, such as peaks or valleys in the line of the graph. Peaks represent moments when the distance traveled is at its maximum within a specific time period, while valleys indicate instances where the distance is at a minimum. The peak of this specific graph is when the maximum distance traveled was 24.97 kilometres in 39.06 minutes in total. In contrast, the minimum distance traveled was only 7.48 kilometres in 4.27 minutes, indicating the graph's valley.

The relationship between the amount of time elapsed, t and the associated distance traveled, y, was then illustrated in the next figure, Figure 3 with the title "Time vs. Distance." The x-axis represents time in minutes, whereas the y-axis represents distance in kilometres. By evaluating the graph, a second increasing trend where the line rises as time progresses was observed, starting from 2.02 kilometres from the initial point (0,0) at time 0.00 minutes to 16.99 kilometres at time 41.5 minutes. This implies a strong correlation between distance of y and time, t, showing that as time goes on, the distance y traveled also rises. The slope of the line may change or fluctuate over the course of the graph. Throughout the graph, there are a few downward-sloping graphs scattered, indicating a decreasing value of distance, y as time, t increases. These variances may be the result of things like changes in speed, train stops, or various sections of the trip. It is crucial to search for distinguishing characteristics, such as peaks or troughs in the graph's line. Peaks reflect periods of time when the distance traveled is at its greatest, and valleys represent periods of time when it is at its smallest. For this y versus t graph, the maximum distance traveled was the same as the end point of the journey, which is 16.99 kilometres for a total of 41.5 minutes, where the peak of the graph occurred. The graph's valley is shown by the smallest distance traveled, which was 2.02 kilometres for 0.00 minutes.

As for the third Time vs. Distance graph, both x of t graph and y of t graph were combined into one whole graph. The blue line graph represents the x in terms of t graph, whereas the orange line graph represents the y in terms of t graph. As both of these graphs were combined, we saw two initially increasing patterns that stayed identical, where the line got higher with time. This implies that time and distance have a positive connection, showing that as more time passes, the distance traveled also grows. Additionally, intersections or crossings between multiple lines on the graph can provide insights into the relationship between different variables. For example, if there are multiple lines representing different modes of transportation, intersections might indicate when one mode overtakes another in terms of distance covered over time. However, this time versus distance graph does not show any intersections or crossings between the two multiple lines. To further explain, there is no significant linear relationship between the distance of *x* and the distance of *y*. By examining specific data points or markers on the graph, one can make observations and draw conclusions. For instance, significant spikes in the line could signify moments of high speed or acceleration, while sudden drops might indicate deceleration or stops along the journey.

A graph of *x(t)* and *y(t)* generates two distinct quadratic equations with different coefficients. The first quadratic equation formed by distance, *x* in terms of time, *t* graph was $x(t) = -0.007t^2 + 0.7668t + 5.2158$. In this equation, the coefficients are negative for the quadratic term, $-0.007t^2$, but positive for the linear term, $0.7668t$ and the constant term, 5.2158. The second quadratic equation formed was $y(t) = 0.0065t^2 + 0.0255t + 3.5617$. The positive coefficient of the quadratic term, $0.0065t^2$ indicates an upwardly opened parabola, while the negative coefficient of the quadratic term, $0.007t^2$ indicates that the parabola opens downward. The positive value coefficient will generate a minimum point graph. In contrast, the negative value coefficient will have a maximum point rather than a minimum point. To find the roots or solutions of the equation, we can use methods like factoring, completing the square, or the quadratic formula. However, since the equation is not in its factored form and the quadratic term coefficient is not easily factorable, we can utilize the quadratic formula. The quadratic formula states that the roots can be found using the Equation (1); $t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Plugging in the values from the quadratic equation, we can calculate the *t* value.

In this case, Equation (2) was selected. This step requires isolating *t* on one side of the equation. Once the expression for t in terms of *x* is obtained, the equation is then substituted into the second parametric equation, Equation (3). Variable *t* was replaced with the expression obtained in the previous step. This will yield an equation solely in terms of *x* and *y*. The equation was simplified to obtain the final equation:

$$y(x) = 0.0065\left(\frac{-0.7668 \pm \sqrt{0.734 - 0.028x}}{-0.014}\right) + 0.0255\left(\frac{-0.7668 \pm \sqrt{0.734 - 0.028x}}{-0.014}\right) + 3.5617$$

which represents the relationship between *x* and *y* without the parameter *t*. Eliminating the parameter method may not always result in a simple or explicit equation between *x* and *y*. Sometimes, the resulting equation might still be implicit or have higher-order terms. As *t* varies, this parametric equation generates different points on the curve formed by the two quadratic equations.

The relationship between the longitudinal distance, *x*, and the latitudinal distance, *y*, was demonstrated using Figure 6, or the *x* versus *y* graph. A graph with an upward slope that represents a positive coefficient quadratic equation value can be seen clearly. From there, two distinct graphs were obtained using a graphing calculator, one of which showed an ascending parabola and the other, by contrast, a falling parabola. Since the upward-sloping parabola has more similarities with the parametric *y(x)* equation, graph in Figure 7 and its equation obtained from the derivation of the graph, Equation (8):

$$y(x) = 0.0065\left(\frac{-0.7668 + \sqrt{0.734 - 0.028x}}{-0.014}\right) + 0.0255\left(\frac{-0.7668 + \sqrt{0.734 - 0.028x}}{-0.014}\right) + 3.5617$$

was chosen to be the last and final *y(x)* equation. Lastly, to compare the accuracy of the calculations made previously, Equation (5) and Equation (8) can be compared with each other by substituting one of the distance values, *x*, into both of these equations. The first value obtained was 2.3915, while the second value was 3.5929. Given that there was only a 1.2014 difference between the two readings, neither of these results indicated a significant difference. However, since there are some differences between the experimental value and the actual value, there are still some mistakes and errors that reduce the accuracy of these estimates.

4. Conclusion and Recommendations

This study was a success since the objective was met. A parametric equation from two distinct *x(t)* and *y(t)* quadratic equations using the parameter elimination method was obtained. The data collected and analysed in Table 1 provided insights into the relationship between distance, time, and latitude and longitude coordinates. The elimination of the parameter t allowed us to express the relationship between x and y solely in terms of *x*. Furthermore, the x versus y graph in Figure 6 clearly demonstrates the relationship between the longitudinal distance *(x)* and the latitudinal distance *(y)*. Two distinct graphs were obtained, one showing an ascending parabola and the other a descending parabola. The upward-sloping parabola was found to have more similarities with the parametric equation *y(x)* derived from Figure 7. Thus, Equation (8) was chosen as the final representation of the relationship between *x* and *y*.

Furthermore, to assess the accuracy of the calculations, Equation (5) and Equation (8) were compared by substituting a distance value, *x*, into both equations. Although there was a slight discrepancy between the experimental and actual values, the difference was not significant. Overall, it is important to acknowledge that there may have been mistakes and errors in the calculations, leading to a reduction in accuracy. These findings contribute to the understanding of the variables involved and provide insights into the patterns and variations observed in the data. Further research and exploration can be conducted to delve deeper into the implications and applications of these parametric equations in relevant fields.

5. Acknowledgement

The authors would like to thank the College of Computing, Informatics and Mathematics, Universiti Teknologi MARA for supporting this project.

6. Funding

The author(s) received no specific funding for this work.

7. Conflict of Interest

The authors have no conflicts of interest to declare.

8. References

Stewart, J. (2016). *Calculus*. Cengage Learning.

Time (2012). Definition for time - Oxford Dictionaries Online (World English). Retrieved June 10, 2023, from https://web.archive.org/web/20120704084938/http://oxforddictionaries.com/definition/time.

Dowden, B. (n.d.) *Time | Internet Encyclopedia of Philosophy*. Retrieved June 10, 2023, from http://www.iep.utm.edu/time/.

Anon (2019) *Scalars and Vectors*. http://www.physicsclassroom.com. Retrieved June 10, 2023.

Dawkins, P. (2022) *Calculus II - Parametric Equations and Curves*. (n.d.). Tutorial.math.lamar.edu. https://tutorial.math.lamar.edu/classes/calcii/parametriceqn.aspx. Retrieved June 12, 2023.

Hartman, G. Libretexts. (2020). *9.2: Parametric equations*. Mathematics LibreTexts. https://math.libretexts.org/Bookshelves/Calculus/Calculus_3e_(Apex)/09%3A_Curves_in_the_Plane/9.02%3A_Parametric_Equations. Retrieved June 12, 2023. Retrieved June 12, 2023.