Mathematics Letters, 2 (2): 28-41, 2023
eISSN: 2948-3735

ANALYTICAL SOLUTION OF BLASIUS EQUATION USING LAPLACE TRANSFORM METHOD WITH ADOMIAN POLYNOMIALS: APPLICATION ON BOUNDARY LAYER FLUID FLOW PAST A FLAT PLATE

Zaileha Md Ali[1*] , Nur Dina Farhana Zu[2], Nuranis Suraya Mohd Atan[3], Nur Aziean Mohd Idris[4], Khadijah Abdul Hamid[5] and Hamidah Ayub[6]

School of Mathematical Sciences, College of Computing, Informatics and Mathematics, Universiti Teknologi MARA, 40450 Shah Alam, Selangor

[1*]zaile597@uitm.edu.my, [2]2019415344@istudent.uitm.edu.my, [3]2019207384@istudent.uitm.edu.my, [4]nuraziean@.uitm.edu.my, [5]khadijah435@uitm.edu.my, [6]hamidahayub@uitm.edu.my

(indicates the corresponding author)*

ABSTRACT

Blasius equation can be solved using analytical and numerical methods. However, it is known that numerical methods give an approximate solution. Therefore, this research uses an analytical approach to solve the Blasius equation, the governing equation of the boundary layer problem. This paper aims to solve the Blasius equation and its application on the boundary layer flow over a flat plate using Laplace Transform Method (LTM) with Adomian Polynomials, which helps to deal with the nonlinear terms. The governing partial differential equation is transformed into an ordinary differential equation. Then, the solutions are obtained using MAPLE software. The previous studies validate the results obtained. Next, the velocity profile of the boundary layer is determined. The velocity profile of the boundary layer showed that as the distance across the plate increase, the value of the velocity increases until it reaches 1.0 at the plate end.

Keywords: *Adomian Polynomials, Blasius equation, Boundary layer, Initial value problems, Laplace Transform Method.*

Received for review: 28-08-2023; Accepted: 22-10-2023; Published: 06-11-2023

1. Introduction

The Blasius problem can be solved analytically or numerically. An analytical method is a set of logical steps applied to solve a problem that is proven to find the exact answer to the problem (Brownlee, 2018) . The analytical method will give an exact answer. In contrast, the numerical method will provide an approximate solution and is not precise (Brownlee, 2018). The Homotopy Perturbation Method (HPM) was presented by Swaminathan et al. in 2021 to find an analytical solution to nonlinear problems in homogeneous reactions that occur in the mass-transfer boundary layer. Blasius equation is a third-order ordinary differential equation. The Blasius equation can be considered the first exact solution of the Navier-Stokes equation (Gerhart et al., 2021). Paul Richard Heinrich Blasius was the first person to successfully solve the Blasius equation analytically in 1908 (Gerhart et al., 2021). Blasius equation can describe the fluid's velocity profile in the boundary layer on a semi-infinite interval or flat plate (Makhfi & Bebbouchi, 2020). There was a research done by Selamat et al. (2019) on solving two forms of the Blasius equation and its application to the two-dimensional laminar viscous flow over a half-infinite domain.

The boundary layer problem is one of the fundamental problems in fluid mechanics (Rahmanzadeh et al., 2020). It was first introduced by an aerodynamicist, Prandtl, in 1904 (Schlichting & Gersten, 2017). A very thin layer close to the body, known as the boundary layer, is the layer in which the flow is affected by friction between the solid surface and the fluid (Schlichting & Gersten, 2017). The no-slip condition is one of the applications of the boundary layer problem where there is no slip between the fluid and the solid boundary layer. At this stage, the velocity of the fluid inside the boundary layer will increase from zero at the surface to the full value of the corresponding potential flow. Shah and Li (2018) research discussed the thermal and laminar boundary layer flow over prolate and oblate spheroids. Nandeppanavar et al. (2022) conducted research on laminar boundary flow over the layer of a wedge with an external magnetic field. One of the analytical methods that can be used to solve the boundary layer flow over a semi-infinite flat plate is the Laplace Transform method.

The Laplace Transform method (LTM) transforms a higher order differential equation into a polynomial form (Zill & Cullen, 2012). The LTM is an analytical method widely used in mathematics and its application (Deakin, 1981). The LTM first arose when Swiss mathematician Leonhard Euler wrote integrals that resemble the modern version and the first paper regarding Laplace transform in 1737 (Deakin, 1980). LTM is said to be used for the first time by Bateman in 1910. The method is named after the French mathematician and astronomer Pierre-Simon Marquis de Laplace. Aggarwal et al.(2018) applied Laplace Transform Method (LTM) to solve the population growth and decay problems. In 2019, Aggarwal et al., conducted research on applying LTM for solving improper integrals whose integrand consists of an error function with Adomian polynomials to deal with nonlinear terms.

Adomian Polynomials are integral to a nonlinear analysis by Adomian Decomposition Method (ADM), where ADM is a method to solve an ordinary and partial differential equation (Fatoorehchi & Abolghasemi, 2016). George Adomian first developed ADM from the 1970s to the 1990s (Hasan, 2012). Saelao and Yokchoo (2020) presented that the modified ADM is based on applying the conventional Adomian decomposition method, which only involves the calculation of the first Adomian polynomials. Another research conducted by Chang (2018) on deconstructing the nonlinear terms of the boundary value problem into a series of Adomian Polynomials, which helps to simplify the computations.

One of Blasius equation applications is on the boundary layer flow over a semi-infinite flat plate. Some researchers used a numerical approach to solve the Blasius equation, but the numerical method gives less accurate results. Therefore, this paper used an analytical approach to solve the Blasius equation on the boundary layer flow over a flat plate because the approach can give an exact solution. The Laplace transform method with Adomian polynomials is one of the practical analytical approaches for solving the governing equation that describes constant laminar flow over a semi-infinite flat plate. The Laplace transform method suits this problem since it does not need unnecessary linearization and the Adomian polynomials help simplify the nonlinear terms.

The research focuses to identify the governing equation of boundary layer flow over a semi-infinite flat plate, to solve the governing equation using Laplace transforms with Adomian polynomials and to obtain the results using the MAPLE software. Hence, the velocity effect of fluid flow over a flat plate is discussed.

2. Methodology

This study aims to solve the Blasius equation and its application on the boundary layer flow past a flat plate using the Laplace Transform Method (LTM) with Adomian Polynomials.

2.1 Transforming the Governing Equation to ODE using the Similarity Variable

The Navier-Stokes equation governs the governing equation for boundary layer flow over a semi-infinite flat plate as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}. \tag{2}$$

where

u : velocity component in x

v : velocity component in y

v : the fluid's viscosity

The equation (1) and (2) describes the continuity equation and momentum equation, respectively. The boundary conditions are given by

$$y=0,\, u=v=0, \tag{3}$$

$$y\to\infty,\, u\to U_{\infty}, \tag{4}$$

$$x=0,\, u=U_{\infty}. \tag{5}$$

Given that boundary layer equations from (1) and (2) with boundary conditions for the governing boundary layer equation are given by equation (3). The flow outside of the boundary layer is the uniform upstream flow $u = U$, which is

$$u\to U \qquad as \qquad y\to\infty \tag{6}$$

The equations (3) and (6), with the boundary conditions, are in nonlinear, partial differential equations for unknown velocity field u and v. The dimensionless similarity variable $\eta=y\sqrt{\frac{U}{vx}}$. Next, the stream function $\psi(x,y)$, can be written as

$$u=\frac{\partial\psi}{\partial y} \qquad\qquad v=-\frac{\partial\psi}{\partial x} \tag{7}$$

The dimensionless stream function $\psi-f(\eta)\sqrt{vxU}=0$ is defined as $f(n)=\frac{\psi}{\sqrt{vxU}}$, where $f=f(\eta)$ is an unknown function. From equation **(7)**,

$$u=\frac{\partial\psi}{\partial y}=\frac{\partial\psi}{\partial\eta}\frac{\partial\eta}{\partial y}=\sqrt{vxU}\frac{\mathrm{d}f}{\mathrm{d}\eta}\sqrt{\frac{U}{vx}}$$

$$U\frac{\mathrm{d}f}{\mathrm{d}\eta}=Uf'(\eta) \tag{8}$$

and

$$v=-\frac{\partial\psi}{\partial x}=-\left[\sqrt{vxU}\,\frac{\partial f}{\partial x}+\frac{1}{2}\sqrt{\frac{vU}{x}}f\right]$$

$$v=-\left[\sqrt{vxU}\,\frac{\partial f}{\partial x}\left(-\frac{1}{2}\eta\frac{1}{x}\right)+\frac{1}{2}\sqrt{\frac{vU}{x}}f\right]$$

$$v=\frac{1}{2}\sqrt{\frac{vU}{x}}\left[\eta\frac{\mathrm{d}f}{\mathrm{d}\eta}-f\right]$$

$$v=\frac{1}{2}\sqrt{\frac{vU}{x}}\,(\eta f'-f) \tag{9}$$

Differentiating the equation (8), it can be shown that

$$\frac{\partial u}{\partial x}=-\frac{U}{2x}\eta\frac{\partial^2 f}{\partial\eta^2} \tag{10}$$

$$\frac{\partial u}{\partial y}=U\sqrt{\frac{U}{vx}}\,\frac{\partial^2 f}{\partial\eta^2} \tag{11}$$

and

$$\frac{\partial^2 u}{\partial y^2}=\frac{U^2}{vx}\frac{\partial^3 f}{\partial\eta^3} \tag{12}$$

Substituting the equations (8) until (11) into equation (2), yield

$$U\frac{\mathrm{d}f}{\mathrm{d}\eta}\left(-\frac{U}{2x}\eta\frac{\partial^2 f}{\partial\eta^2}\right)+\frac{1}{2}\sqrt{\frac{vU}{x}}\left[\eta\frac{\mathrm{d}f}{\mathrm{d}\eta}-f\right]\left(U\sqrt{\frac{U}{vx}}\,\frac{\partial^2 f}{\partial\eta^2}\right)=v\left(\frac{U^2}{vx}\frac{\partial^3 f}{\partial\eta^3}\right) \tag{13}$$

$$Uf'\left(-\frac{U}{2x}\eta\frac{\partial^2 f}{\partial\eta^2}\right)+\frac{1}{2}\sqrt{\frac{vU}{x}}\,[\eta f'-f]\left(U\sqrt{\frac{U}{vx}}\,\frac{\partial^2 f}{\partial\eta^2}\right)=v\left(\frac{U^2}{vx}\frac{\partial^3 f}{\partial\eta^3}\right)$$

$$-\frac{U^2}{2x}\eta f'\left(\frac{\partial^2 f}{\partial\eta^2}\right)+\frac{U^2}{2x}\eta f'\left(\frac{\partial^2 f}{\partial\eta^2}\right)-\frac{U^2}{2x}f\left(\frac{\partial^2 f}{\partial\eta^2}\right)-\frac{vU^2}{vx}\left(\frac{\partial^3 f}{\partial\eta^3}\right)=0$$

$$-\frac{U^2}{2x}f\left(\frac{\partial^2 f}{\partial\eta^2}\right)-\frac{vU^2}{vx}\left(\frac{\partial^3 f}{\partial\eta^3}\right)=0$$

$$\frac{U^2}{2x}f\left(\frac{\partial^2 f}{\partial\eta^2}\right)+\frac{vU^2}{vx}\left(\frac{\partial^3 f}{\partial\eta^3}\right)=0$$

$$\frac{U^2}{x}f\left(\frac{\partial^2 f}{\partial\eta^2}\right)+\frac{2U^2}{x}\left(\frac{\partial^3 f}{\partial\eta^3}\right)=0$$

$$2\frac{\partial^3 f}{\partial \eta^3} + f\frac{d^2 f}{d\eta^2} = 0 \tag{14}$$

$$2f''' + ff'' = 0 \tag{15}$$

Hence, the governing equation (2) for the steady laminar flow over a semi-infinite flat plate known as Blasius Equation can be written as

$$f''' + \frac{1}{2}ff'' = 0. \tag{16}$$

For the boundary conditions, at $\eta = 0$ since $u = v = 0$ due to the no-slip boundary condition. Thus, from equation (9)

$$v(0) = 0 = -\frac{1}{2}f \Rightarrow f(0) = 0$$

From equation (8)

$$f'(0) = \frac{u(0)}{U} = 0 \Rightarrow f'(0) = 0$$

and as $\eta \to \infty$, the velocity u should approach the freestream velocity. $u \to U$ such that,

$$f'(\eta \to \infty) = \frac{U}{U} = 1 \Rightarrow f'(\infty) = 1$$

Therefore, the boundary conditions

$$y = 0; f(0) = f'(0) = 0, \tag{17}$$

$$y = \infty, f'(\infty) = 1 \tag{18}$$

2.2 Solve the ODE using the Laplace Transform Method with Adomian Polynomials

Laplace Transform Method (LTM) with Adomian Polynomials was used to solve the governing Blasius equation problem.

Theorem 2.2 The Laplace transform of nonlinearities (General) (Fatoorehchi & Abolghasemi, 2016)

Let N be a general nonlinear operator, which maps a Hilbert space H to H. Without loss of generality, and it is obvious that Ny can include differential or integral terms. For example, $Ny = y^2\sqrt{\frac{dy}{dt}}$ or $Ny = y\frac{dy}{dt} + e^{\frac{dy}{dt}} + \left(\frac{dy}{dt}\right)\sin(y)$. *Moreover, let* A_i *be the Adomian polynomials, which decompose the nonlinear operator N. Provided that* $y = \sum_{i=0}^{\infty} c_i t^i$ *it holds that,*

$$\mathcal{L}\{Ny\} = \sum_{i=0}^{\infty} \mathcal{L}\left\{A_i\left(c_0, \cdots, c_i t^i\right)\right\}$$

An analytical solution for this problem as follows

$$y(x) = \sum_{k=1}^{N} \left(-\frac{1}{2}\right)^k \frac{A^k \cdot 0.332057336^{k+1}}{(3k+2)!} x^{3k+2}! \tag{19}$$

where, $A_0 = A_1 = 0$ and

$$A_k = \sum_{r=0}^{k-1} \binom{3k-1}{3r} A_r A_{k-r-1},\ k \geq 2 \tag{20}$$

Consider the general function equation for ADM as follows

$$Lu + Nu + Ru = g, \tag{21}$$

where

L	:	invertible linear operator
N	:	nonlinear operator
R	:	remainder linear operator
g	:	bounded specified function
u	:	unknown function

However, Adomian Polynomials are applied to linearize the nonlinear term for the Blasius equation. Then, let L^{-1}as inverse operator of L and apply it to the equation (21) on both side

$$L^{-1}Lu + L^{-1}Nu + L^{-1}Ru = L^{-1}g \tag{22}$$

By choosing L as the nth-order derivatives and L^{-1} will become as an n-fold integration. Then $L^{-1}Lu = u - a$ where a come out from the integration. ADM decomposes the solution as $u = \sum_{n=0}^{\infty} u_n$. Let $u_n = L^{-1}g + a$, equation (22) become

$$u = u_0 - L^{-1}Nu - L^{-1}Ru \tag{23}$$

The ADM also composes the nonlinearity into a special infinite series of Adomian Polynomials as

$$Nu = \sum_{n=0}^{\infty} A_n, \tag{24}$$

where A_n is define as

$$A_n\left(u_0,...,u_n\right) = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} N \sum_{i=0}^{\infty} \lambda^i u_i\right]_{\lambda=0} \quad \text{b} \tag{25}$$

As a result, a recurrence relation can be built to calculate the remaining solution components as

$$u_{i+1} = -L^{-1}A_i - L^{-1}Ru_i \text{ ; } i \geq 0 \tag{26}$$

Laplace Transform Method will be applied to Equation (16) as follows

$$\mathcal{L}\{f'''\} + \frac{1}{2}\mathcal{L}\{ff''\} = 0 \tag{27}$$

In this case, the Laplace transform properties above, which is the Laplace transform for nth-order differentiation properties, will be applied to solve the ordinary differential equation system

$$s^3F(s) - s^2f(0) - sf'(0) - f''(0) + \frac{1}{2}\mathcal{L}\{ff''\} = 0 \tag{28}$$

By using theorem 2.2, the equation (28) can be written as

$$F(s) = \frac{c_2}{s^3} - \frac{1}{2}\left(\frac{1}{s^3}\right)\sum_{i=0}^{\infty}\mathcal{L}\left\{A_i\left(c_0,\ldots,c_i\eta^i\right)\right\} = 0 \tag{29}$$

Then, using the first two initial conditions, $c_0 = 0, c_1 = 0$ to calculate the Adomian Polynomials $Ny = ff''$ that decomposed where it led to

$$A_0\left(c_0\right) = c_0\frac{d^2}{d\eta^2}\left(c_0\right) = 0$$

$$A_1\left(c_0,c_1\eta\right) = c_1\eta\frac{d^2}{d\eta^2}\left(c_0\right) + c_0\frac{d^2}{d\eta^2}\left(c_1\eta\right) = 0$$

$$A_2\left(c_0,c_1\eta,c_2\eta^2\right) = c_2\eta^2\frac{d^2}{d\eta^2}\left(c_0\right) + c_1\eta\frac{d^2}{d\eta^2}\left(c_1\eta\right) + c_0\frac{d^2}{d\eta^2}\left(c_2\eta^2\right) = 0$$

$$A_3\left(c_0,c_1\eta,c_2\eta^2,c_3\eta^3\right)=c_3\eta^3\frac{d^2}{d\eta^2}\left(c_0\right)+c_2\eta^2\frac{d^2}{d\eta^2}\left(c_1\eta\right)+c_1\eta\frac{d^2}{d\eta^2}\left(c_2\eta^2\right)+c_0\frac{d^2}{d\eta^2}\left(c_3\eta 3\right)=0$$

$$A_4\left(c_0,c_1\eta,c_2\eta^2,c_3\eta^3,c_4\eta^4\right)=c_4\eta^4\frac{d^2}{d\eta^2}\left(c_0\right)+c_3\eta^3\frac{d^2}{d\eta^2}\left(c_1\eta\right)+c_2\eta^2\frac{d^2}{d\eta^2}\left(c_2\eta^2\right)+c_1\eta\frac{d^2}{d\eta^2}\left(c_3\eta 3\right)+c_0\frac{d^2}{d\eta^2}\left(c_4\eta^4\right)$$

$$=2c_2{}^2\eta^2$$

$$A_5\left(c_0,c_1\eta,c_2\eta^2,c_3\eta^3,c_4\eta^4,c_5\eta^5\right)=c_5\eta^5\frac{d^2}{d\eta^2}\left(c_0\right)+c_4\eta^4\frac{d^2}{d\eta^2}\left(c_1\eta\right)+c_3\eta^3\frac{d^2}{d\eta^2}\left(c_2\eta^2\right)+c_2\eta^2\frac{d^2}{d\eta^2}\left(c_3\eta 3\right)+c_1\eta\frac{d^2}{d\eta^2}\left(c_4\eta^4\right)+c_0\frac{d^2}{d\eta^2}\left(c_5\eta^5\right)$$

$$=8c_2c_3\eta^3$$

.

.

.

$$A_i\left(c_0,\ldots,c_i\eta^i\right)=\sum_{j=0}^{i}c_{i-j}\eta^{i-j}\frac{d^2}{d\eta^2}\left(c_i\eta^j\right)=\sum_{j=2}^{i}c_{i-j}c_jj(j-1)\eta^{i-2}\ ;\ i\geq 4 \qquad (30)$$

Apply Laplace transform in equation (30)

$$\mathcal{L}\left\{A_i\left(c_0,\ldots,c_i\eta^i\right)\right\}=\mathcal{L}\left\{\sum_{j=2}^{i}c_{i-j}c_jj(j-1)\eta^{i-2}\right\}$$

$$=\sum_{j=2}^{i}c_{i-j}c_jj(j-1)\mathcal{L}\{\eta^{i-2}\}$$

$$=\sum_{j=2}^{i}c_{i-j}c_jj(j-1)\frac{(i-2)!}{s^{i-1}}$$

$$=\frac{(i-2)!}{s^{i-1}}\sum_{j=2}^{i}c_{i-j}c_jj(j-1)\ ,\ i\geq 4 \qquad (31)$$

Substitute equation (31) into equation (29)

$$F(s)=\frac{c_2}{s^3}-\frac{1}{2}\sum_{i=4}^{\infty}\left[\frac{(i-2)!}{s^{i+2}}\sum_{j=2}^{i}c_{i-j}c_jj(j-1)\right] \qquad (32)$$

$$=\frac{c_2\eta^2}{2}-\frac{1}{2}\sum_{i=4}^{\infty}\left[\frac{(i-2)!}{(i+1)!}\eta^{i+1}\sum_{j=2}^{i}c_{i-j}c_jj(j-1)\right] \qquad (33)$$

Then, apply the basic properties of Laplace transform to equation (32) becomes (33)

In general form,

$$f = c_1\eta + c_2\eta^2 + c_3\eta^3 + c_4\eta^4 + c_5\eta^5 + \dots \tag{34}$$

Then, it is deduced to

$$c_1 = 0\,, \tag{35}$$

$$c_2 = \frac{c_2}{2}\,, \tag{36}$$

$$c_3 = 0\,, \tag{37}$$

$$c_4 = 0\,, \tag{38}$$

$$c_5 = \frac{c_2^{\,2}}{60} = \frac{\left(\frac{c_2}{2}\right)^2}{60}$$

$$c_5 = -\frac{c_2^{\,2}}{240} \tag{39}$$

$$c_6 = 0\,, \tag{40}$$

$$c_7 = 0\,, \tag{41}$$

$$c_8 = -\frac{11c_2c_5}{336} = -\frac{11}{336}\left(\frac{c_2}{2}\right)\left(-\frac{c_2^{\,2}}{240}\right)$$

$$c_8 = \frac{11c_2^{\,3}}{161{,}280} \tag{42}$$

$$c_9 = 0\,, \tag{43}$$

$$c_{10} = 0\,, \tag{44}$$

$$c_{11} = -\frac{1}{2}\left[\frac{1}{990}\left(58c_2c_8 + 20c_5^{\,2}\right)\right]$$

$$c_{11} = -\frac{1}{1980}\left[58\left(\frac{c_2}{2}\right)\left(\frac{11c_2^{\,3}}{161{,}280}\right) + 20\left(\frac{c_2^{\,2}}{240}\right)^2\right]$$

$$c_{11} = -\frac{1}{1980}\left[\frac{319c_2^{\,4}}{161{,}280} + \frac{c_2^{\,4}}{2880}\right]$$

$$c_{11} = -\frac{5c_2^{\,4}}{4{,}257{,}792} \tag{45}$$

The parameter c_2 will be determined by utilization of the last boundary condition, which is $y(\infty)=1$ using Pade Approximation of $f'(\eta)$ and get $c_2=0.3320573360$. Then, by substituting equations (35) until (45) into equation (34), yield

$$f(\eta)=\frac{c_2}{2}\eta^2-\frac{c_2}{240}\eta^5+\frac{11c_2{}^3}{161{,}280}\eta^8-\frac{5c_2{}^4}{4{,}257{,}792}\eta^{11}+\ldots \quad (46)$$

Hence,

$$f'(\eta)=c_2\eta-\frac{c_2}{48}\eta^4+\frac{11c_2{}^3}{20{,}160}\eta^7-\frac{5c_2{}^4}{387{,}072}\eta^{10}+\ldots \quad (47)$$

3. Results and Discussions

In this section, the validation of the results and velocity profile are discussed and analyzed. The findings are presented in a graph and tables.

3.1 Validation of the study

The analytical solutions of equation (46) are obtained using the MAPLE software. To verify the result of analytical solution of the Blasius equation using Laplace Transform Method (LTM), the value of $f(\eta)$ using LTM is validated with the numerical solution of the RCW method (Rahmanzadeh et al., 2020) and the analytical solution of Blasius (White, 2011). Table 1 shows the analytical and numerical solutions of the Blasius equation. From the table, the results obtained by LTM are in excellent agreement with the numerical solution of the RCW method and the analytical solution of Blasius. Hence, the result of this study is valid and reliable.

Table 1. Analytical and Numerical Solutions of Blasius Equation

	$f(\eta)$		
η	RCW Method (Rahmanzadeh, 2019)	Analytical Solution of Blasius (White, 2011)	Analytical Solution of Blasius using LTM (present study)
0.4	0.0266	0.0266	0.0266
0.8	0.1061	0.1061	0.1061
1.2	0.2379	0.2379	0.2379
1.6	0.4203	0.4203	0.4203
2	0.65	0.65	0.65
2.4	0.9223	0.9223	0.9223
2.8	1.231	1.231	1.231

3.2 The Blasius boundary layer solution for velocity over a flat plate

The results of equation (47) are obtained using the MAPLE software and are presented in Table 2.

Table 2. $f'(\eta)$ results of Blasius equation using LTM

η	$f'(\eta)$
0	0
0.4	0.1328

0.8	0.2648
1.2	0.3938
1.6	0.5168
2	0.6298
2.4	0.729
2.8	0.8112

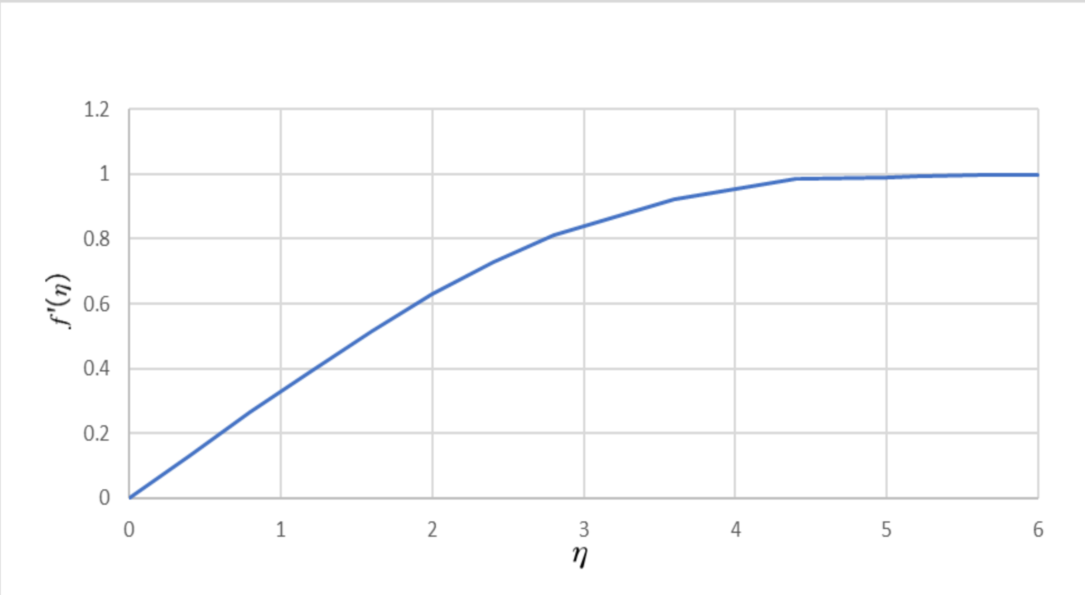

Figure 1. Blasius velocity profile $f'(\eta)$

The velocity profile, $f'(\eta)$ is shown in figure 1. From figure 1, the velocity profile starts at the value of zero and will increase until it reaches 1.0 at the end of the plate. At this point, it shows that the velocity of the fluid is similar to the upstream velocity.

4. Conclusion and Recommendations

In conclusion, this paper solves the Blasius equation and its application to the boundary layer flow over a flat plate analytically using Laplace Transform Method (LTM) with Adomian Polynomials. The findings of are listed as follows

- The results of $f(\eta)$ using LTM are validated with the numerical solution of RCW (Rahmanzadeh et al., 2020) and the analytical solution of Blasius (White, 2011). The results show that LTM is reliable and accurate to solve the Blasius equation.

- The velocity profile increases as the value of distance across the plate increase until it reaches 1.0 at the plate end. The result satisfies the boundary condition where $f'(\infty)=1$.

Finally, solving a similar ordinary differential equation using different boundary conditions is recommended for future research.

5. Acknowledgement

The authors would like to thank the School of Mathematical Sciences, College of Computing, Informatics and Mathematics, Universiti Teknologi MARA for supporting this project.

6. Funding

The author(s) received no specific funding for this work.

7. Conflict of Interest

The authors have no conflicts of interest to declare.

8. References

Aggarwal, S., Gupta, A., Singh, D., Asthana, N., & Kumar, N. (2018). Application of Laplace Transform for Solving Population Growth and Decay Problems. *International Journal of Latest Technology in Engineering, Management & Applied Science*, *7*, 141–145. https://www.researchgate.net/profile/Sudhanshu Aggarwal/publication/331064596_Application_of_Laplace_Transform_for_Solving_Population_Growth_and_Decay_Problems/links/5c6402bca6fdccb608beb5f9/Application-of Laplace-Transform-for-Solving-Population-Growth-an

Aggarwal, S., Singh, A., Kumar, A., & Kumar, N. (2019). Application of Laplace Transform for Solving Improper Integrals Whose Integrand Consisting Error Function. *Journal of Advanced Research in Applied Mathematics and Statistics*, *4*, 1–7. https://www.researchgate.net/profile/Sudhanshu Aggarwal/publication/335422727_Application_of_Laplace_Transform_for_Solving_Improper_Integrals_Whose_Integrand_Consisting_Error_Function/links/5d6510ee458515d6 1027c4d7/Application-of-Laplace-Transform-for-Solv

Brownlee, J. (2018). *Analytical vs Numerical Solutions in Machine Learning*. https://machinelearningmastery.com/analytical-vs-numerical-solutions-in-machine-learning/

Deakin, M. A. B. (1980). Euler's Version of the Laplace Transform. *The American Mathematical Monthly*, *87*(4), 264–269. https://doi.org/10.1080/00029890.1980.11995010

Fatoorehchi, H., & Abolghasemi, H. (2016). Series solution of nonlinear differential equations by a novel extension of the Laplace transform method. *International Journal of Computer Mathematics*, *93*(8), 1299–1319. https://doi.org/10.1080/00207160.2015.1045421

Gerhart, A. L., Hochstein, J. I., & Gerhart, P. M. (2021). *Munson, Young and Okiishi's Fundamentals of Fluid Mechanics* (9th ed.). Wiley.

Hasan, Y. (2012). *The Modified Adomian decomposition method*. Lap Lambert Academic Publishing.

Makhfi, A., & Bebbouchi, R. (2020). On the generalized Blasius equation. *Afrika Matematika*, *31*(5–6), 803–811. https://doi.org/10.1007/s13370-020-00762-9

Nandeppanavar, M. M., Madhusudhan, R., Nargund, A. L., Sathyanarayana, S. B., & Kemparaju, M. C. (2022). Application of HAM to laminar boundary layer flow over a wedge with an external magnetic field. *Heat Transfer*, *51*(1), 170–192. https://doi.org/10.1002/htj.22301

Rahmanzadeh, M., Asadi, T., & Atashafrooz, M. (2020). The Development and Application of the RCW Method for the Solution of the Blasius Problem. *Journal of Applied and Computational Mechanics*, *6*(1), 105–111. https://doi.org/DOI: 10.22055/JACM.2019.28250.1469

Saelao, J., & Yokchoo, N. (2020). The solution of Klein–Gordon equation by using modified Adomian decomposition method. *Mathematics and Computers in Simulation*, *171*, 94–102. https://doi.org/10.1016/j.matcom.2019.10.010

Schlichting, H., & Gersten, K. (2017). *Boundary-Layer Theory* (9th ed.). Springer Berlin Heidelberg. https://doi.org/10.1007/978-3-662-52919-5

Selamat, M. S., Halmi, N. A., & Ayob, N. A. (2019). A Semi Analytic Iterative Method for Solving Two Forms of Blasius Equation. *Journal of Academia*, *7*(2), 76–85. https://ir.uitm.edu.my/id/eprint/30591/1/30591.pdf

Shah, R., & Li, T. (2018). The thermal and laminar boundary layer flow over prolate and oblate spheroids. *International Journal of Heat and Mass Transfer*, *121*, 607–619. https://doi.org/10.1016/j.ijheatmasstransfer.2017.12.130

Swaminathan, R., Saravanakumar, R., Venugopal, K., & Rajendran, L. (2021). Analytical Solution of Non Linear Problems in Homogeneous Reactions Occur in the Mass-Transfer Boundary Layer: Homotopy Perturbation Method. *International Journal of Electrochemical Science*, *16*(6), 210644. https://doi.org/10.20964/2021.06.51

White, F. M. (2011). *Fluid Mechanics 7th Edition*. McGraw Hill. http://ftp.demec.ufpr.br/disciplinas/TM240/Marchi/Bibliografia/White_2011_7ed_Fluid-Mechanics.pdf

Zill, D. G., & Cullen, M. R. (2012). *Differential Equations with. Boundary-Value Problems* (7th ed.). Cengage Learning. http://www.uop.edu.pk/ocontents/DIFFERENTIAL%20EQUATIONS%20with%20Boundary-Value%20Problemsa%20Zill%20Cullen(1).pdf