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MINIMIZING MAKESPAN OF JOB SHOP SCHEDULING PROBLEM USING MIXED INTEGER LINEAR PROGRAMMING

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ABSTRACT

Scheduling is the practice of allocating resources to projects in order to guarantee that they are finished in a timely manner. Making decisions on a regular basis is a common practice in many manufacturing and service businesses. It means managing and improving how resources are distributed among tasks over time. In addition to being a key instrument for increasing productivity and lowering costs in the product and service system, production scheduling is also crucial to the manufacturing stage of the product life cycle. The Job Shop Scheduling Problem (JSSP), which is the most often used in schedule theory, provides a solid representation of the overall domain but is notoriously challenging to solve. To accommodate inconsistent demand, manufacturers frequently buy more production machinery. The older machines are frequently utilized to balance production when the newer ones are overworked. When a manufacturer adds multiple production capabilities, scheduling job orders becomes a vital factor. The actual earliness and tardiness of a timetable would fluctuate randomly with the makespan delay since random machine breakdown cannot be predicted in advance. A faulty schedule could result in increased production expenses or possibly customer loss. Generally, the purpose of this study is to minimize the makespan of job shop scheduling problem. The JSSP will then be modelled as a mixed integer linear programming model and solved by using Excel Solver. This study includes three phases, which starts with data acquisition, construction of mixed integer linear programming model, solution of the model and performance review. There are three sets of data (3 × 3 JSSP, 4 × 4 JSSP, and 6 × 6 JSSP) involved in this study.

Keywords: *Excel Solver, Job Shop Scheduling Problem, Makespan, Mixed Linear Programming, Scheduling*

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1. Introduction

The definition of scheduling is a process of assigning resources to tasks to ensure that those tasks are completed in a reasonable amount of time. The general problem is to find a schedule, known as a feasible schedule, which is also optimal with respect to some performance criterion, where tasks (e.g., basic tasks) are distributed across resources (e.g., machines). For scheduling problems, it proposed a functional classification approach. This scheme classifies problems using the following dimensions: requirement generation, processing complexity, scheduling criteria, parameter variability, and scheduling environment (Jones et al., 1999). Most problems encountered in fields such as scheduling, assignment, and vehicle routing are NP-hard. These problems require efficient solutions (Mallikarjuna et al., 2014).

According to Wu and Sun (2019), many manufacturing and service industries use scheduling to make decisions on a routine basis. It involves the control and optimization of the allocation of resources to tasks over time. Production scheduling plays a vital role in the manufacturing phase of the product life cycle, and it is also an important tool for improving productivity and reducing costs in the product and service system. For example, it is possible that the new machinery runs at higher cutting speeds and revolutions per minute, reduces setup times, expands tooling capacity, has updated manufacturing software, has an automatic inspection device, and/or can handle more capacity. As a result, upgrading the entire shop is typically done in phases, with management relying on a mix of new and old equipment. In general, the planning department usually allocates and schedules resources in favour of new equipment; however, older resources are still normally used, though not as often, to meet required demand and to handle breakdowns. Consequently, managers are faced with a novel production environment. Scheduling errors can lead to high manufacturing costs or even customer losses (Vital-Soto et al., 2020).

Schedule theory's most popular model is the job-shop, which is a good representation of the general domain but notoriously difficult to solve. It has been studied and developed the most in deterministic scheduling theory, serving as a testbed for different solution techniques, old and new, and being highly motivated by practical requirements, it is of particular significance (Mallikarjuna et al., 2014). Given a set of machines and a set of jobs, the job-shop scheduling problem (JSSP) seeks to find schedules that minimize certain objectives (e.g., makespan). In order to solve this problem, most research assumes that the manufacturing environment is static, which means that production will follow the predetermined schedule precisely (Wu & Sun, 2019).

1.1 Background of Study

In a job shop system, job shop scheduling is a vital part of production management. Reasonable job shop scheduling is required for the effective execution of production operations (Yu, 2021). JSSP is a standard and essential combinatorial optimization problem in operational research and management science. JSSP has been explored extensively by academics in engineering and academic sectors due to its extraordinarily broad technical and social applicability backgrounds (Xiong et al., 2022). JSSP includes job activities with varying machine sequences and processing durations. Machine operations can be scheduled using a variety of parameters, such as processing time, lead time, makespan, free float, or due dates and many others. JSSP is an NP-hard optimization problem in which different machines are allocated to different workloads while minimising any applicable specified criteria. Therefore, several algorithms and problem-solving strategies have been created and applied over time to solve JSSP (Liaqait et al., 2021).

A variety of mathematical models of the problem have been addressed by several researchers. There is a need for an efficient, universal technique in solving JSSP. One of the methods in solving JSSP is using Mixed Integer Linear Programming method (MILP). There have been several studies that applied MILP to solve the scheduling problem through the years with various objectives. In this study, we are going to use MILP in order to minimize the overall makespan of JSSP.

1.2 Problem Statement

Increased globalization, population growth, the development of new products, high levels of production, and extreme consumption have all contributed to the development of an economy (Ambrogio et al., 2020). Manufacturers often acquire more manufacturing equipment to meet unpredictable demand. This equipment is often equipped with new technologies and enhanced capabilities (Vital-Soto et al., 2020). The fast-changing market requirement has led to make-to-order production being one of the main production methods of the enterprise. In the case of some complex and large products like heavy machine tools, they are usually customized with a single order or a relatively small order, and production is done in the form of discrete manufacturing (Wang & Lu, 2021). The planning department usually prioritizes the new equipment when assigning and scheduling resources. When newer machines are overloaded, the older ones are often used to balance production. In a company, scheduling job orders becomes a crucial factor when multiple manufacturing resources are added. A wrong schedule could result in increased manufacturing costs or even customer loss (Vital-Soto et al., 2020) and the actual earliness of a schedule will vary randomly with the makespan delay since random machine breakdown cannot be predicted in advance (Wu & Sun, 2019). Therefore, this study will employ a Mixed Integer Linear Programming (MILP) model to minimize makespan of JSSP.

1.3 Objectives

The objectives of this study are listed as follows:

1. To apply the job shop scheduling problem as a mixed integer linear programming model.
2. To minimize the makespan of job shop scheduling problem by using mixed integer linear programming implemented by Excel Solver.

1.4 Significant and Benefit of Study

The goal of this study is to develop a schedule to handle the problem of job shop scheduling. Large-scale problems can be solved with minimal computational effort using the proposed technique. The contribution of this study is to consider the challenge of developing efficient solutions with the goal of concurrently reducing the makespan of tasks to increase manufacturing system flexibility. As an outcome, a complex, automated schedule optimization system capable of generating optimal production plans was created. This study focused on minimizing makespan on JSSP. This study also can be useful to future research since there are several fields of jobs, for example, operation research manager. In addition, in modern industries, JSSP allows the achievement of several benefits, such as cost reduction and time reduction, as well as customer satisfaction. Research to date has mostly been geared towards reducing time-consuming objectives, such as making the makespan as short as possible and speeding up the flow rate.

1.5 Scope and Limitation of Study

This study focuses on minimizing makespan of job shop scheduling problem. A secondary data collection method is applied in this study, where it only involves 3×3, 4×4, and 6×6 JSSP data. In order to solve the problem, the MILP model will then be solved by using Excel Solver. This study will only be covered on JSSP as general.

2. Methodology and Implementation

This section presents three phases of the methodology and implementation of JSSP using MILP. Phase one begins with data acquisition by selecting 3×3 JSSP which is modified from an example by Pinedo (2016), 4×4 JSSP which is taken from Asano and Ohta (2002) and 6×6 JSSP which is formulated by Fisher and Thompson (Chaudhuri & De, 2010). Table 1, Table 2, and Table 3 represents the 3×3, 4×4, and 6×6 JSSP, respectively.

Table 1. 3 × 3 JSSP

Job	**Machine (Processing Time)**		
1	1(5)	2(10)	3(4)
2	3(4)	1(5)	2(6)
3	3(5)	2(3)	1(7)

Table 2. 4 × 4 JSSP

Job	**Machine (Processing Time)**			
1	3(5)	2(10)	4(8)	1(13)
2	1(5)	3(13)	2(10)	4(6)
3	2(7)	1(9)	4(8)	3(8)
4	1(7)	2(6)	3(7)	4(8)

Table 3. 6 × 6 JSSP

Job	**Machine (Processing Time)**					
1	3(1)	1(3)	2(6)	4(7)	6(3)	5(6)
2	2(8)	3(5)	5(10)	6(10)	1(10)	4(4)
3	3(5)	4(4)	6(8)	1(9)	2(1)	5(7)
4	2(5)	1(5)	3(5)	4(3)	5(8)	6(9)
5	3(9)	2(3)	5(5)	6(4)	1(3)	4(1)
6	2(3)	4(3)	6(9)	1(10)	5(4)	3(1)

Phase two is to apply MILP model proposed by Fattahi et al. (2006) to solve the model of JSSP. Lastly is phase three to shows the solution of the model with minimizing makespan as objective by represent the data using disjunctive graph and visualize the solution using solution schedule. All 3 × 3, 4 × 4 and 6 × 6 JSSP will be solved using Excel Solver.

3. Result and Discussion

To analyse the compatibility and ability of the developed model in the previous part, the model was further tested on three JSSP data which are 3 × 3, 4 × 4 and 6 × 6 in Excel Solver. Based on Figure 1, there are three jobs and three machines with 12 conjunctive arcs (solid line) and nine disjunctive arcs (dotted line). Each operation represented by O_{ij} where the small number at the front represent machine (i) while the small number at the back represent job (j). Each operation will pair with the same job or machine. For example, O_{11} can go to the same job (O_{21}) or to the same machine which have two options (O_{12}, O_{13}). Then, we enter all operation with its processing times in excel solver. Excel solver executed the solution of route for finding minimum makespan.

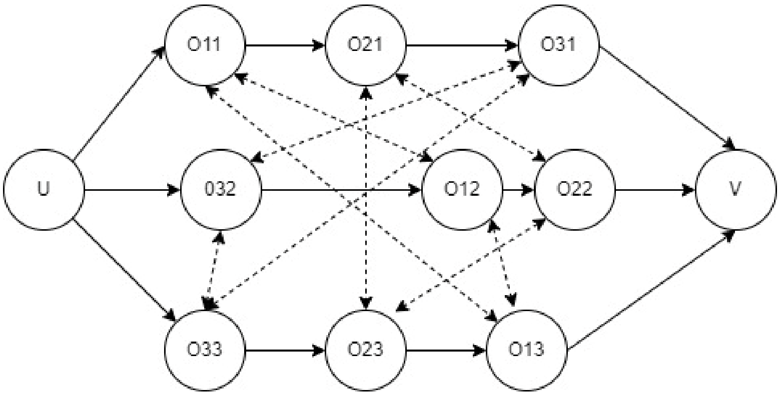

Figure 1. Initial Disjunctive Graph of 3 × 3 JSSP

In the excel, constraints of exit and entry path for each node set in column G to I and L to N respectively. Exit and entry path for each node must exactly once, hence the constraints set manually '1' in column I and N. Next, all nodes of the operations inserted in column C, together with their possible paths in column D. The numbers inserted in column B represents the processing times of each operation. The objective cell added in column B, which represents the objective to minimize the makespan of the operations. Next, column A was filled by using Excel Solver, an add-in program by Microsoft. This column determines the path for each node in column C, where the chosen node will appear number '1'. The solution of route obtained from the Excel Solver executed into the final disjunctive graph of 3 × 3 JSSP. The solution satisfied the conditions of the model which are each job follow a specific sequence of operations and each machine perform one operation at a time.

The solution schedule can be visualized to find the minimum makespan as shown in Figure 2. To visualize the minimum makespan of 3 × 3 JSSP data, it needs to prioritize the operation chosen by excel solver and must follow the sequence of operations where minimum processing times must run first. For example, in order for O_{13} to run, O_{11}, O_{23} and O_{33} must run first.

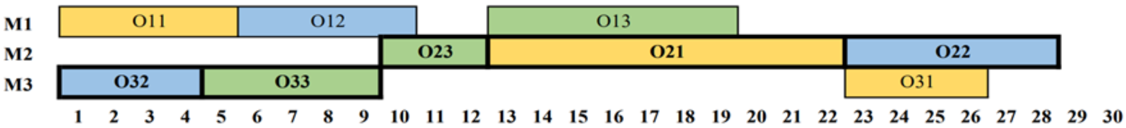

Figure 2. Solution schedule for 3 × 3 JSSP

From Figure 2, the yellow and blue box represents for Job 1 and Job 2, respectively, while the green box represents for Job 3. The bold box represents the makespan of the operation, where the path of the makespan starts from $O_{32} \rightarrow O_{33} \rightarrow O_{23} \rightarrow O_{21} \rightarrow O_{22}$. The total makespan for 3 × 3 JSSP based on Figure 13 is 28.

Thus, the model continues tested for other two JSSP which are 4 × 4 and 6 × 6 JSSP according to the method above. The data of 4 × 4 and 6 × 6 JSSP were taken from Asano and Ohta (2002) and Fisher and Thompson (Chaudhuri & De, 2010), respectively. Hence, the minimum makespan obtain for those two data are 47 and 60 as shown as in Figure 3 and Figure 4, respectively.

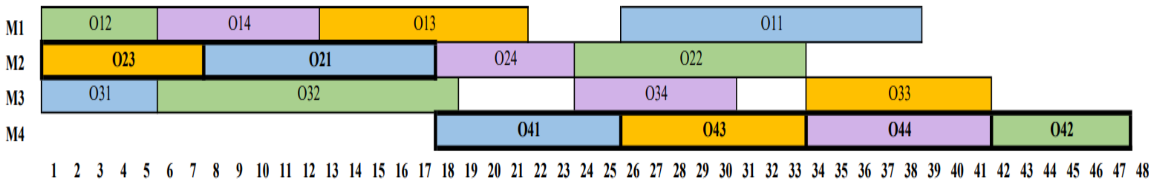

Figure 3. Solution schedule for 4 × 4 JSSP

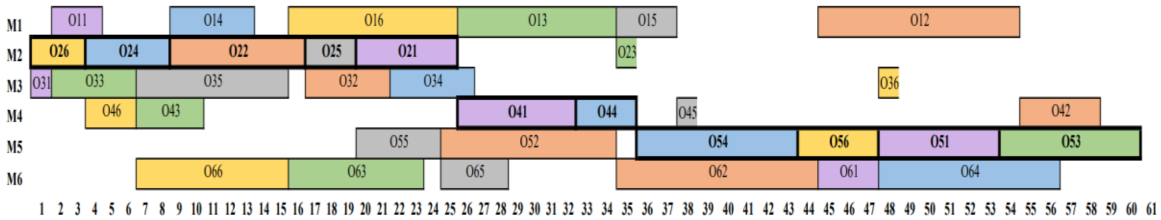

Figure 4. Solution schedule for 6 × 6 JSSP

4. Conclusion and Recommendations

In this study, we have dealt with one performance criteria of JSSP. An effective mixed integer linear programming is presented. The objective function considered is the minimization of makespan. Three problems, which are 3 × 3 JSSP, 4 × 4 JSSP and 6 × 6 JSSP have been selected to evaluate the performance of the proposed model. The first objective of the study is to model the job shop scheduling problem as a mixed integer linear programming model. The obtained results showed the efficiency of the proposed model. Next, our objective is to solve the mixed integer linear programming model by using Excel Solver. Excel Solver calculated the best route with minimum makespan for each JSSP. Then, solution schedule is constructed. Therefore, from the solution schedule, we can calculate the minimum makespan.

Since this study only focused on the JSSP, it is recommended that further study should be carried out on other types of scheduling problems such as flow shop and open shop problems. Hence, this study only concerned about makespan. So, others can also investigate other objective functions such as tardiness. Besides that, it is suggested to add more criteria analysis such as maximum lateness, maximum earliness and maximum cost. Other than that, the investigation on JSSP might be too simple compared to the real-world problems that have more complicated constraints, more flexible objective functions and more dynamic features. As a result, it is highly anticipated that these approaches will be extended to include more realistic settings, as generating an efficient and fine-tuned schedule in a reasonable time is becoming increasingly important.

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