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SOLVING BURGER-FISHER EQUATION (BFE) BY USING BANACH CONTRACTION METHOD (BCM)

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ABSTRACT

In this article, we propose a method for solving the Burger-Fisher equation. The proposed method is the Banach Contraction Method (BCM). The Burger-Fisher equation is a combination of the Burger equation and the Fisher equation. There have been many methods used to solve the problem associated with the Burger-Fisher Equation, such as the Adomian Decomposition Method (ADM) and the Local Discontinuous Galerkin (LDG) method. Therefore, this paper presents the results of the research findings with two main objectives: We used the BCM to solve the Burger-Fisher Equation, followed by validating the accuracy of the BCM with the exact solutions. Plus, calculations were mainly performed using the MAPLE 2018 software to facilitate the calculations and simplify the complicated results in this study. As for the scope of this study, we cover the partial differential equation specifically in terms of the Burger-Fisher Equation and the BCM. In summary, the method used in this study has introduced a straightforward mathematical method for solving various differential equations, which improves and expands our knowledge in the field of mathematics. Currently, all the articles related to Burger-Fisher Equation and BCM are applicable in this study. Also, a procedure for implementing the method is provided, along with an error analysis. As a result, we calculate the exact solution of Burger-Fisher Equation using BCM up to only. As for and continuously the solution was too long thus, we provide the error between the exact solution and BCM by using various value of α, β and σ as for x from 0 to 1 and for t = 1, 10, and 50. We prove that the new technique is more practical by comparing the outcomes of the exact solution with certain previously reported findings.

Keywords: *Banach Contraction Method, Burger-Fisher Equation, Numerical Method.*

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1. Introduction

Differential equations have many applications in technology and engineering, mainly in problems which have the shape of nonlinear differential equations. Different equations like fractional differential equations and partial differential equations can be solved by using numerical or analytical method (Daftardar-Gejji & Bhalekar, 2008) and a few applications for the Lane-Emden equations (Al-Jawary & Al-Qaissy, 2015) also an evolution equation

(Bhalekar & Daftardar-Gejji, 2010). This technique introduces a proper answer which merges with the exact answer “if solution exists” through progressive approximations.

The Burger-Fisher equation is useful in a variety of domains, including financial mathematics, gas dynamics, traffic flow, applied mathematics, and physics (Ismail et al, 2004). This equation depicts a standard model for understanding the interaction of the convection effect and diffusion transport. Ronald Aylmer Fisher (1890 -1962) and Johannes Martinus Burgers (1895-1981) are exposed via the Burger-Fisher equation (Kocacoban et.al, 2011).

According to a study done by Al-Jawary et al. (2018), to solve a nonlinear initial value issue, Daftardar-Gejji and Bhalekar suggested an effective iterative technique. The purpose of this research is to offer a different implementation of this method and the non-Newtonian fluid thin layer issue on a moving belt was considered. The suggested approach is founded on Banach's contraction principle.

This equation is basically used to solve the Burger-Fisher Equation by using the Banach Contraction Method (BCM) and to validate the accuracy of the BCM with the exact solutions.

2. Methodology and Implementation

In this section, it explains the research framework by displaying the flowchart of solving the Burger-Fisher Equation by using BCM, the methodology of the Burger-Fisher Equation and the Banach Contraction Method, and the implementation of BCM into Burger-Fisher Equation. Other than that, the Maple software were used to apply and calculate the Burger-Fisher Equation by using the method stated which BCM. Figure 1 below shows the flowchart of solving the Burger-Fisher Equation by using the BCM:

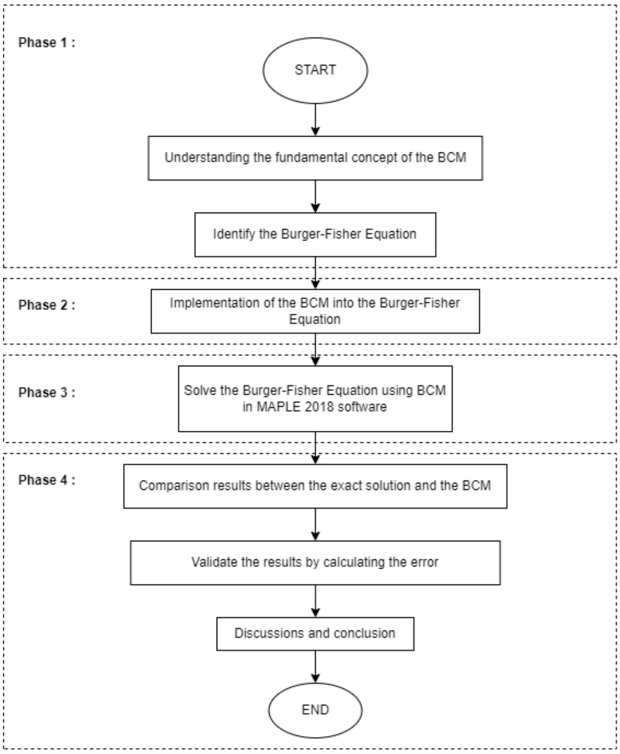

Figure 1:The flowchart of solving Burger-Fisher Equation by using BCM.

2.1 Implementation and Numerical examples

STEP 1: Understanding the fundamental concept of the BCM

Iterative technique is a mathematical step that will generate a series of progressed approximate answers for a category of problems. As a result of the iterative process, when the corresponding sequence converges given some initial approximations, an approximate solution that converges to the exact solution is generated. The following non-linear differential equation:

$$L\big(y(x)\big) + N\big(y(x)\big) + g(x) = 0 \tag{1}$$

With the boundary condition,

$$B\left(y, \frac{dy}{dx}\right) = 0, x \in D \tag{2}$$

Where x stands for the independent variable, y(x) is the unknown function, g(x) is a given known function, $L(.) = \frac{d^2}{dx^2}$ is the linear operator, N(.) is the non-linear operator, and B(.) is a boundary operator.

A non-linear functional equation is considered as

$$u(\bar{x}) = f(\bar{x}) + N\big(u(\bar{x})\big) \tag{3}$$

$f(\bar{x})$ presented as a known function while $u(\bar{x})$ presented as unknown function. Besides, N represented as the non-linear operator for the function equation (3). To apply the BCM, define consecutive approximations:

$$u_0 = f \tag{4}$$

$$u_1 = u_0 + N(u_0) \tag{5}$$

$$u_2 = u_0 + N(u_1) \tag{6}$$

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$$u_n = u_0 + N(u_{n-1}), n \in \mathbb{N} \tag{7}$$

The above equation is the general form of $u_n(x)$. $N(y)$ has unique fixed-point if N^k is a contraction for some positive integer k. So, the sequence generated by (7) is convergent. Hence, the solution for the above equation is,

$$u = \lim_{n\to\infty}(u_n) \tag{8}$$

In a word, by this study we aim to determine the BCM's ability to solve the Burger-Fisher equation. Thus, to know the accuracy of the BCM, it will be compared to the exact solution.

STEP 2: Identify the Burger-Fisher Equation

Burgers-Fisher Equation is a combination of both Burgers equation and Fisher equation. Burgers Equation is mostly used in several types of areas in mathematical physics, such as fluid mechanics, gas dynamics, traffic flow, quantum field, etc. It is because this equation is a fundamental non-linear partial differential equation (PDE). Therefore, the general equation for Burgers Equation is,

$$u_t + uu_x - \gamma u_{xx} = 0 \tag{9}$$

Besides that, the equation for Fisher Equation or known as the Fisher-Kolmogorov equation (KPP equation) is a reaction-diffusion equation first developed by Fisher in the context of population dynamics. It now arises in wide range of scientific applications and physical phenomena, and it can be used to describe a wide range of issues. The equation below is the Fisher equation:

$$u_t + \gamma uu_x = \beta u(1-u) \tag{10}$$

Thus, the combination of both Burgers equation and Fisher equation yields to the non-linear differential equation of Burger-Fisher equation which is,

$$u_t + \alpha u_{xx} - u_{xx} = \beta u(1-u) \tag{11}$$

However, it does have a general form which is the generalized Burgers-Fisher equation given,

$$u_t + \alpha u^n u_x - u_{xx} = \beta u(1-u^n), \alpha \neq 0, n > 0 \tag{12}$$

Where n, α, and β are constants.

With the initial condition

$$u(x,0) = f(x) \tag{13}$$

And the exact solution is,

$$u(x,t) = \left(\frac{1}{2} + \frac{1}{2}\tanh\left[-\frac{\alpha n}{2(n+1)}\left(x - \left(\frac{\alpha}{n+1} + \frac{\beta(n+1)}{\alpha}\right)t\right)\right]\right)^{\frac{1}{n}} \tag{14}$$

STEP 3: Implementing the BCM into the Burger-Fisher Equation

In this section, we apply the BCM to solve the Burger-Fisher Equation. This process begins with integration.

$$u_t + \alpha u^n u_x - u_{xx} = \beta u(1-u^n) \tag{15}$$

This equation will be rephrased as to simplify. Equation (1) was rephrased as:

$$u_t = -\alpha u^n u_x + u_{xx} + \beta u(1 - u^n) \tag{16}$$

It is when,

$$N\big(u(x,t)\big) = -\alpha u^n u_x + u_{xx} + \beta u(1 - u^n) \tag{17}$$

Therefore, the equation will be as shown :

$$u(x,t) = f + \int_0^t -\alpha u^n u_x + u_{xx} + \beta u(1 - u^n)\, dt \tag{18}$$

Then,

$$u_0 = f \tag{19}$$

$$u_1 = u_0 + \int_0^t N\big(u_0(x,t)\big)\, dt$$

$$u_2 = u_0 + \int_0^t N\big(u_1(x,t)\big)\, dt$$

$$\dots$$

Hence, in general the equation is:

$$u_{n+1} = u_0 + \int_0^t N\big(u_n(x,t)\big)\, dt \tag{20}$$

The solution in the form of power series is,

$$u_{n+1}(x,t) = \lim_{n\to\infty} u_n(x,t) \tag{21}$$

STEP 4: Solving the Burger-Fisher Equation using the BCM in Maple 2018 software

We started applying Burger-Fisher Equation and BCM in the software that was suggested in step 4. All the equations involved were run to obtain the results. This software simply makes the process less complicated because no manual calculations are required. The manual calculation is complicated because the Burger-Fisher Equation is quite tedious to handle.

STEP 5: Comparison results between the exact solution of Burger-Fisher Equation and the solution by BCM

Some comparison made between the exact solution of Burger-Fisher Equation and the solution by BCM. This comparison made by changing the parameters of alpha (α), beta (β) and sigma (σ). All the results collected by the software MAPLE were presented in a table with classification.

STEP 6: Validation by calculating the absolute error

The computation of the error is demonstrated in this section. In this study, we use the Burger-Fisher equation as the exact value whereas the BCM as the approximate value to determine the absolute error. The error of the results was calculated using the formula shown below.

$$\text{Absolute Error} = |V_A - V_E| \tag{22}$$

V_A = Approximated value

V_E = Exact value

STEP 7: Discussion and conclusions

Based on the comparison between the estimated value obtained by utilizing BCM and the exact value of Burger-Fisher Equation, we would finally draw a conclusion. In this research study, we also discussed whether all objectives were satisfied and achieved.

3. Results and discussion

We now have generalized Burger-Fisher equations' numerical solutions. The error for various values of α, β and σ are provided in order to confirm the effectiveness of the suggested method in contrast with the exact solution and the BCM. We will only display calculations for the BCM up to $u_1(x,t)$. This is because it takes too long to calculate $u_2(x,t)$ and other numbers. Consequently, the table will display the outcomes for our $u_2(x,t)$.

Example 1

Consider the generalized Burger-Fisher equation, Eq.(12), with the initial condition, Eq. (13), and the exact solution in Eq. (14). In Table 1, shown the error for σ = 1 and value of α = 0.01 and β = 0.01. The exact solution for the above α, β and σ is,

$$u(x,t) = \frac{1}{2} + \frac{1}{2}\tanh(-0.002500000000x + 0.002512500000t)$$

Therefore, the result by implementation of Banach contraction method are shown below:

$$u_0(x,t) = \frac{1}{2} - \frac{1}{2}\tanh(0.002500000000x)$$

$$\begin{aligned} u_1(x,t) = \frac{1}{2} - \frac{1}{2}\tanh(0.002500000000x) - (0.01000000000\,(0.5000000000 \\ - 0.5000000000\,tanh(0.002500000000x))\,(-0.001250000000 \\ + 0.001250000000\,tanh\,(0.002500000000x)^2) \\ - 0.002500000000\,tanh(0.002500000000x)\,(0.002500000000 \\ - 0.002500000000\,tanh(0.002500000000x)^2) \\ + 0.01000000000(0.5000000000 \\ - 0.5000000000\,tanh(0.002500000000x))\,(0.5000000000 \\ + 0.5000000000\,tanh(0.002500000000x)))t \end{aligned}$$

Example 2

Consider the generalized Burger-Fisher equation, Eq. (12), with the initial condition, Eq. (13), and the exact solution in Eq. (14). The result shown in Table 2 is the error for σ = 1 when value of α and β is 1.00. The exact solution for the above α, β, and σ is,

$$u(x,t) = \left(\frac{1}{2}+\frac{1}{2}\tanh(-0.2500000000x + 0.3750000000t)\right)^{1.000000000}$$

Hence, the result shown below is the result of implementation of BCM:

$$u_0(x,t) = \left(\frac{1}{2}-\frac{1}{2}tanh(0.2500000000x)\right)^{1.000000000}$$

$$\begin{aligned} u_1(x,t) = \left(\frac{1}{2}-\frac{1}{2}\tanh(0.2500000000x)\right)^{1.000000000} &- ((0.5000000000 \\ &- 0.5000000000\,tanh(0.2500000000x))\,(-0.1250000000 \\ &+ 0.1250000000\,tanh\,(0.2500000000x)^2) \\ &- 0.2500000000\,tanh\,(0.2500000000x)\,(0.2500000000 \\ &- 0.2500000000\,tanh(0.2500000000x)^2) + (0.5000000000 \\ &- 0.5000000000\;\,tanh(0.2500000000x))\,(0.5000000000 \\ &+ 0.5000000000\;\,tanh(0.2500000000x)))t \end{aligned}$$

Example 3

Using the generalized Burger-Fisher equation, Eq. (12), with the initial condition, Eq. (13), and the exact solution in Eq. (14). The result shown in Table 3 is the error for σ = 0.01 when the value of α = 1.00 and β = 0. The exact solution for the above α, β, and σ is,

$$u(x,t) = \left(\frac{1}{2}+\frac{1}{2}tanh(-0.004950495050x + 0.004901480248t)\right)^{100}$$

Consequently, the results of using the Banach contraction method are given below:

$$u_0(x,t) = \left(\frac{1}{2}-\frac{1}{2}tanh(0.004950495050x)\right)^{100}$$

$$u_1(x,t) = \left(\frac{1}{2} - \frac{1}{2}tanh(0.004950495050x)\right)^{100} - (100((0.5000000000 - 0.5000000000\,tanh(0.004950495050x))^{100})^{\frac{1}{100}}(0.5000000000 - 0.5000000000\,\tanh(0.004950495050\mathrm{x}))^{99}$$

Table 1: The error for σ = 1 shows the differences between exact solution and BCM.

x_i	t_i	α = β = 0.01		
		Exact solution	BCM	Error
0.0	1	0.5012562474	0.50122562474	3.0623E-05
0.1		0.5011312481	0.5011312481	0
0.2		0.5010062486	0.5010062486	0
0.3		0.50008812491	0.5008812491	-0.0007931
0.4		0.5007562494	0.5007562494	0
0.5		0.5006312497	0.5006312497	0
0.6		0.5005062498	0.5005062498	0
0.7		0.5003812499	0.5003812499	0
0.8		0.5002562500	0.5002562500	0
0.9		0.5001312500	0.5001312500	0
1.0		0.5000062500	0.5000062500	0
0.0	10	0.512559857	0.475062500	0.0374973572
0.1		0.512434935	0.474937502	0.0374974337
0.2		0.512310012	0.474812506	0.0374975057
0.3		0.512185087	0.474687514	0.0374975727
0.4		0.51206016	0.474562525	0.0374976353
0.5		0.511935232	0.474437539	0.0374976930
0.6		0.511810303	0.474312557	0.0374977462
0.7		0.511685372	0.474187577	0.0374977945
0.8		0.511560439	0.474062601	0.0374978383
0.9		0.511435506	0.473937628	0.0374978773
1.0		0.511305700	0.473812658	0.0374979117
0.0	50	0.562484145	0.375312500	0.1871716446
0.1		0.562361093	0.375187508	0.1871735850
0.2		0.562238033	0.375062531	0.1871755023
0.3		0.562114966	0.37493757	0.1871773962
0.4		0.561991892	0.374812625	0.1871792668
0.5		0.561868809	0.374687695	0.1871811143
0.6		0.561745720	0.374562781	0.1871829385
0.7		0.561622622	0.374437883	0.1871847392
0.8		0.561499517	0.37431300	0.1871865167
0.9		0.561376404	0.374188133	0.1871882711
1.0		0.561253284	0.374063282	0.1871900021

Table 2: The error for σ = 1 shows the differences between exact solution and BCM.

x_i	t_i	$\alpha = \beta = 1$		
		Exact solution	BCM	Error
0.0	1	0.679178699	0.312500000	0.3666786992
0.1		0.668187772	0.300119742	0.3680680301
0.2		0.657010463	0.287988782	0.3690216803
0.3		0.645656306	0.276120900	0.3695354065
0.4		0.634135591	0.264528573	0.3696070178
0.5		0.622459331	0.253222937	0.3692363941
0.6		0.610639234	0.242213749	0.3684254846
0.7		0.59868766	0.231509375	0.3671782853
0.8		0.586617579	0.221116781	0.3655007983
0.9		0.574442517	0.211041546	0.3634009707
1.0		0.562176501	0.201287885	0.3608886163
0.0	10	0.999447221	-1.375000000	2.374447221
0.1		0.999418896	-1.386326010	2.385744906
0.2		0.999389121	-1.395299489	2.394688610
0.3		0.999357820	-1.401922396	2.401280216
0.4		0.999324917	-1.406208292	2.405533209
0.5		0.999290330	-1.408182121	2.407472451
0.6		0.999253971	-1.407879855	2.407133826
0.7		0.999215751	-1.405348041	2.404563792
0.8		0.999175575	-1.400643253	2.399818828
0.9		0.999133343	-1.393831433	2.392964776
1.0		0.999088949	-1.384987173	2.384076122
0.0	50	1.0000000000	-8.875000000	9.8750000000
0.1		1.0000000000	-8.881640470	9.8816404700
0.2		1.0000000000	-8.876580694	9.8765806940
0.3		1.0000000000	-8.859892602	9.8598926020
0.4		1.0000000000	-8.831705478	9.8317054780
0.5		1.0000000000	-8.792204607	9.7922046070
0.6		1.0000000000	-8.741629202	9.7416292020
0.7		1.0000000000	-8.680269892	9.6802698920
0.8		1.0000000000	-8.608465618	9.6084656180
0.9		1.0000000000	-8.526600234	9.5266002340
1.0		1.0000000000	-8.435098538	9.4350985380

Table 3: The error for σ = 0.01 shows the differences between exact solution and BCM.

x_i	t_i	α = 1, β = 0		
		Exact solution	BCM	Error
0.0	1	1.286314996E-30	1.17551952E-30	1.10795476E-31
0.1		1.224468629E-30	1.11891092E-30	1.05557710E-31
0.2		1.165567281E-30	1.06500224E-30	1.00565040E-31
0.3		1.109472124E-30	1.01366598E-30	9.58061410E-32
0.4		1.056050780E-30	9.64780622E-31	9.12701582E-32
0.5		1.005177035E-30	9.18230295E-31	8.69467398E-32
0.6		9.567306119E-31	8.73904563E-31	8.28260486E-32
0.7		9.105968536E-31	8.31698158E-31	7.88986956E-32
0.8		8.666664268E-31	7.91510741E-31	7.51556858E-32
0.9		8.248351337E-31	7.53246708E-31	7.15884261E-32
1.0		7.850036855E-31	7.16814874E-31	6.81888117E-32
0.0	10	9.408893937E-29	4.65545E-30	8.943349232E-29
0.1		8.976075548E-29	4.43236E-30	8.532839703E-29
0.2		8.562957954E-29	4.21986E-30	8.140972345E-29
0.3		8.168653970E-29	4.01744E-30	7.766909706E-29
0.4		7.792316232E-29	3.82464E-30	7.409851834E-29
0.5		7.433134958E-29	3.64101E-30	7.069034160E-29
0.6		7.090336461E-29	3.4661E-30	6.743726110E-29
0.7		6.763181683E-29	3.29952E-30	6.433229728E-29
0.8		6.450964280E-29	3.14086E-30	6.136877857E-29
0.9		6.153009809E-29	2.98976E-30	5.854033405E-29
1.0		5.868673471E-29	2.84586E-30	5.584087203E-29
0.0	50	1.77446E-21	2.01218E-29	1.774457944E-21
0.1		1.70894E-21	1.91588E-29	1.708940662E-21
0.2		1.6458E-21	1.82414E-29	1.645804394E-21
0.3		1.58496E-21	1.73676E-29	1.584964081E-21
0.4		1.52634E-21	1.65351E-29	1.526337550E-21
0.5		1.46985E-21	1.57422E-29	1.469845619E-21
0.6		1.41541E-21	1.49870E-29	1.415411767E-21
0.7		1.36296E-21	1.42676E-29	1.362962310E-21
0.8		1.31243E-21	1.35824E-29	1.312426065E-21
0.9		1.26373E-21	1.29298E-29	1.263734384E-21
1.0		1.21682E-21	1.23083E-29	1.216821036E-21

The discussions on the results obtained will be elaborated further in this part. First and foremost, the results are validated by using σ = 1 and σ = 0.01. Each σ came with several examples of α and β which are α = β = 1, α = β = 0.01 and α =1 β = 0.

Based on the above tables, Table 1 and Table 2 shows σ = 1 while Table 3 shows σ = 0.01. Table 2 shows that highest error compared to the others as it is validated with σ =1 as t = 50. Additionally, based on the Table 1 and Table 2 where σ = 1, it shows high value of error compared to Table 3 where σ = 0.01. Hence, the value of σ give effects on the error as the increasing of value of σ, the higher the value of error. It is clearly proven when σ = 1 and σ = 0.01. It is the same as the value t. the increasing of t affects the error between exact solution and BCM where the value of t is 1, 10 and 50. Nevertheless, the increment of value x does not affect the value of error from each example.

As conclusion, as $t = 50$, the error of the exact solution and Banach Contraction Method increases. Thus, the increasing of t affects the error between exact solution and BCM but when x increases it does not affect the result. Therefore, the findings of the BCM and the exact solution showed a very high degree of agreement, which supports the BCM's validity.

4. Conclusion and Recommendations

4.1 Conclusions

In conclusion, mathematics is a field that involves many other scopes. Mathematics is not only about solving the issues of numbers, but it is also needed in physics, engineering, chemistry, and many other fields. For example, civil engineering is closely related to mathematics and uses almost every element of mathematics at some point to perform its tasks. Nowadays, it is crucial to have a good foundation in mathematics.

The Burger-Fisher equation was solved in this paper using the Banach Contraction Method (BCM), which requires extensive research on both the theory and computation of the Burger-Fisher Equation as well as BCM. Based on our research, this method has not yet been used to solve this equation. In order to solve the problem, it is essential to know the BCM's concepts. It has been established through the results and discussion that the Burger-Fisher equation used in this study is accurate. Different values for α, β, and σ are utilized in this study to test the approach's efficacy in comparison to the exact solution and the BCM.

Besides, this study was confined and limited to the observation and validation between the exact solution and the method that was used, which is BCM, therefore they are inadequate in producing outstanding results. Also, this study makes a valid and accurate contribution to the discipline. Using alternative techniques, such as the ADM, has made it more difficult to solve the Burger-Fisher Equation. The concept to solve this problem utilizing the Burger-Fisher Equation and the BCM comes from other approaches and components, such as Taylor's series. This research helps society better understand a person's ideas and discoveries by supporting society in this area. Unconsciously, our culture is establishing new standards for education, which is a great development. As time goes on, this research could become the basis for the development and growth of education in our society, even though it may currently reveal things that have not yet been discovered.

4.2 Recommendations

This research is important for assisting other mathematicians and researchers in solving challenging differential equations like the Burger-Fisher problem. While other techniques may be useful for solving other linear equations, methods that offer validity and accuracy are needed for solving non-linear equations. As our project benefits from a great deal of simplicity and an absence of complexity, we think that mathematicians could feel a little relieved. In addition, because this study was restricted to observing and validating the relationship between the precise solution and the technique used, BCM, they come up short of creating exceptional outcomes. In order to obtain adequate results, it is advised that future research incorporate improved comparison considering both the precise solution and additional numerical methods. It is easier and less complicated even though this method might not be the greatest in terms of accuracy. The Burger-Fisher equation employing BCM does not have a perfect precision, but its mathematical validity is significant and legitimate.

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