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E-BOOK OF EXTENDED ABSTRACTS

NUMERICAL SOLUTION OF TIME-DEPENDENT MICROPOLAR NANOFLUID FLOW OVER A LINEAR CURVED STRETCHING SURFACE USING BVP4C

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ABSTRACT

This study focused on unsteady micropolar nanofluid flow over a linear curved stretching surface, aiming to improve the numerical approach for solving heat transfer problems. Similarity transformations were applied to reduce the Partial Differential Equations (PDEs) into Ordinary Differential Equations (ODEs). The momentum and energy profile along with numerical results were obtained by using BVP4C solver in MATLAB. This study explores the effects of the curvature parameters on momentum profile and the stretching parameters on energy profile. The results indicate that momentum profile increased as curvature parameters increased. For energy profile, it shows that the energy profile decreased as stretching parameters increased. In addition, this study also explored the effects of curvature on wall shear stress, microrotation, heat transfer and mass transfer as reflected in the variation of skin friction, couple stress coefficient, Nusselt number and Sherwood number. These findings highlight the effectiveness of BVP4C in solving nonlinear systems in heat transfer problems and provide valuable insight into how different physical parameters influence the flow and temperature behavior. Overall, this study offers insight applicable to cooling system, biomedical field and material coating processes where curved and stretching surface interact with complex fluid like micropolar nanofluid.

Keywords: Micropolar, Nanofluid, Linear Curved Stretching Surface, BVP4C, Heat Transfer

INTRODUCTION

Heat transfer has caught much attention over the past few years and has widely been used in industry and engineering. Terms of micropolar were introduced by Eringen (1966). Choi and Eastman (1995) started to use the term nanofluid by adding nanoparticles in base fluid to an advanced heat transfer fluid. Combining nanoparticles with micropolar fluid enhances thermal properties and influences microrotation (Ahmad et al., 2021). In this study, micropolar nanofluid considered to be time-dependent and flow over a linear curved stretching surface which is more complex and widely applicable in real life problems.

Sakiadis (1961) was one of the earliest researchers who examine the boundary layer flow problem on continuously moving surfaces. Saleh et al. (2017) discovered that the boundary layer thickness increases for a curved sheet when compared to a flat sheet. In this study, BVP4C were used to solve micropolar nanofluid flow over a linear curved stretching surface.

BVP4C solver is a MATLAB function that is designed to solve ODEs. Ramzan et al. (2021) stated that a highly nonlinear system of equations such as fluid velocity and concentration in nanofluid can be solved by a BVP4C solver. In thermal engineering applications, this solver helps to provide an initial prediction for optimal parameter control (Wahid et al., 2021). To use BVP4C in MATLAB, PDEs need to be transformed into ODEs first. Hence, this study aims to transform PDEs into ODEs and solve the micropolar nanofluid flow over a linear curved stretching surface by using BVP4C solver.

METHODOLOGY

This study considered several PDEs which consist of the continuity, momentum, micropolar, energy and concentration equation (Abbas & Shatanawi, 2022). All these equations will be transformed into ODEs by using similarity variables. The ODEs were obtained as follows:

$$\begin{aligned}&\left(1+K_1\right)\left(F'''' + \frac{2}{\zeta+K_0}F''' - \frac{1}{\left(\zeta+K_0\right)^2}F'' + \frac{1}{\left(\zeta+K_0\right)^3}F'\right)\\&-\frac{K_0}{\zeta+K_0}\left(F''F' - FF'''\right) - \frac{K_0}{\left(\zeta+K_0\right)^2}\left(F'^{\,2} - FF''\right) - \frac{K_0}{\left(\zeta+K_0\right)^3}FF'\\&-K\left(G''+\frac{G'}{\zeta+K_0}\right) - \frac{\beta_0}{\zeta+K_0}\left(F'+\frac{\zeta}{2}F''\right) - \frac{\beta_0}{2}\left(3F''+\zeta F'''\right) = 0,\end{aligned} \tag{1}$$

$$\begin{aligned}&\left(1+\frac{K_1}{2}\right)\left(G''+\frac{G'}{\zeta+K_0}\right) + \frac{K_0}{\zeta+K_0}F'G - \frac{K_0}{\zeta+K_0}FG'\\&-K_1\left(2G+F''+\frac{1}{\zeta+K_0}F'\right) + \frac{\beta_0}{2}\left(\zeta G'+G\right),\end{aligned} \tag{2}$$

$$\frac{1}{Pr}\left(\theta'' + \frac{1}{K_0+\zeta}\theta'\right) + \frac{K_0}{K_0+\zeta}F\,\theta' - \frac{K_0}{K_0+\zeta}\theta + \left(N_B\,\phi'\,\theta' + N_T\,\theta'\,\theta'\right) - \frac{\beta_0}{2}\zeta\,\theta = 0, \tag{3}$$

$$\left(\phi''+\frac{1}{\zeta+K_0}\phi'\right) + \frac{K_0}{\zeta+K_0}F\phi' - \frac{K_0}{\zeta+K_0}\phi + \frac{N_T}{N_B}\left(\theta''+\frac{1}{\zeta+K_0}\theta'\right) - \frac{\beta_0}{2}\left(\zeta\,\theta'\right) = 0, \tag{4}$$

with boundary conditions of

$$\begin{aligned}&F'(0) = \lambda + \Pi\left(F''(0) - \frac{F'(0)}{K_0}\right),\ F(0) = \omega, \delta\theta'(0) + 1 = \theta(0),\ N_B\phi'(0) + N_T\theta'(0) = 0,\\&G(0) = -nF''(0),\ F'(\infty) = 1,\ F''(\infty) = 0,\ G(\infty) = 0,\ \theta(\infty) = 0,\ \phi(\infty).\end{aligned} \tag{5}$$

Next, this ODEs will be transformed into the first order form to obtain numerical results by using BVP4C solver in MATLAB software.

RESULTS AND DISCUSSION

Figure 1 shows effects of curvature (K_0) on momentum profile, $f(\zeta)$. The curvature parameter represents the effect of a curved surface on fluid flow. In real-world systems, many surfaces are not flat such as pipes or tubing in industrial application. To study the effects of curvature parameter, K_0 on momentum profile, $f(\zeta)$, K_0 were examined at values 0.3, 0.5, and 0.7. In contrast, the other parameters are considered fixed where β_0 = 0.1, $N_B = N_T = T_0 = K_1$ = 0.5, $\omega = \lambda = \Pi = \delta$ = 0.4, k = 10, Pr = 6.2 and n = 0. It indicates that as the value of curvature increases, momentum profile also increases. This means that the flow momentum at the surface becomes stronger with higher curvature. This result has not been reported in previous studies, highlighting its significance for future research in this topic.

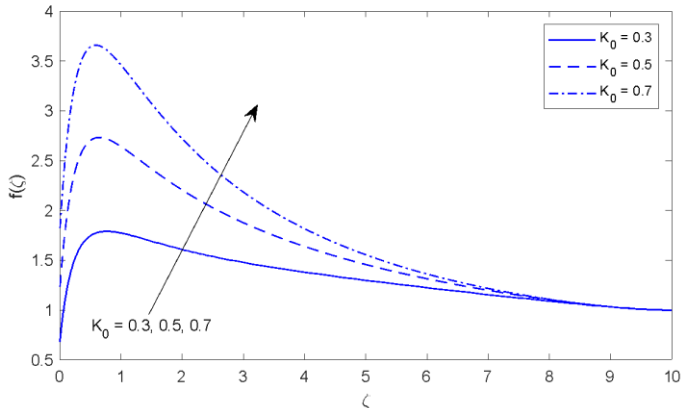

Figure 1: Momentum profile $f(\zeta)$ on different values of curvature parameter K_0

Figure 2 shows the effects of stretching parameter (ω) on energy profile, $\theta(\zeta)$. Stretching parameter plays a significant role in influencing fluids behavior and heat transfer rate. It represents how fast a surface is being stretched when interacting with a fluid such as micropolar nanofluid. By considering $\beta_0 = N_B = N_T = T_0 = K_1 = K_0 = 0.5$, $\delta = \lambda = \Pi = 0.4$, $k = 10, Pr\ Pr\ = 6.2$ and $n = 0$, the effects of stretching on energy profile were examined. The stretching parameter ω were analyzed at values 0.3, 0.5 and 0.7. It indicates that, as ω increased energy profile was reduced which means that the temperature of the fluids will drop since it absorbs less heat due to higher stretching. This result fills a gap in past studies and may be useful for future work.

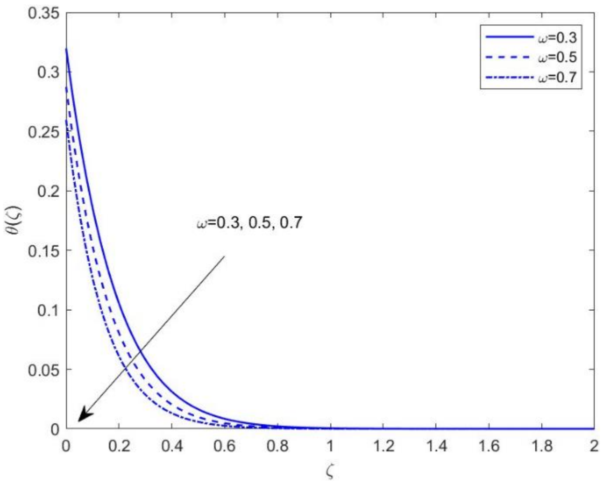

Figure 2: Energy profile $\theta(\zeta)$ on different values of stretching parameter ω

Table 1: Results of $C_f {Re_s}^{-1/2}, C_m Re_s, N_{u_s} {Re_s}^{-1/2}$ and $Sh_s {Re_s}^{-1/2}$ for different values of K_0 when $n = 0.0$.

K_0	$C_f {Re_s}^{-1/2}$	$C_m Re_s$	$N_{u_s} {Re_s}^{-1/2}$	$Sh_s {Re_s}^{-1/2}$
0.1	-858.9418461	-16.15112207	1.763189854	-1.763189854
0.3	-473.7042335	-10.14335702	1.696857490	-1.696857490
0.5	-498.9568583	-9.471270552	1.712402242	-1.712402242
0.7	-549.4206213	-9.019570392	1.736404595	-1.736404595

The numerical result was shown in Table 1 obtained by using BVP4C solver in MATLAB. The skin friction decreases sharply from K_0= 0.1 to 0.3, then slightly increases again at higher values. This means the wall shear stress is reduced at first but starts to rise as curvature gets stronger. The couple stress coefficient also decreases as K_0 increases, showing that higher curvature lowers the microrotation effect near the wall. For heat transfer, the Nusselt number slightly drops at first but then increases, suggesting that curvature helps improve temperature transfer after a certain point. For the Sherwood number, a small increase is seen from 0.1 to 0.3, followed by a decrease from 0.3 to 0.7, meaning mass transfer first weakens slightly and then becomes stronger again as curvature increases.

NOVELTY

This study provides new insights into unsteady flow of micropolar nanofluid over a linear curved stretching surface focusing on the effects of stretching and curvature, which are relevant for improving cooling systems and industrial processes involving micropolar nanofluid. Numerical results of skin friction, couple stress coefficient, Nusselt number and Sherwood number under the influence of curvature are also analyzed. Unlike previous studies, this work used BVP4C method to obtain the solution. To the best of our knowledge, these findings have not been previously reported, making this work original and contributing new understanding to the study of micropolar nanofluid flow.

CONCLUSION

This study solved the unsteady flow of micropolar nanofluid over linear curved stretching surfaces by using BVP4C. All the governing equations of the flow problems (PDEs) were reduced into ODEs by using similarity variables. Then, BVP4C was used to explore the numerical results, the effects of curvature parameters on momentum profile and the stretching parameters on energy profile. This study also helps improve the understanding of heat transfer in micropolar nanofluid, providing insight that can be applied to fields such as chemical processing, mechanical engineering and electronics cooling.

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