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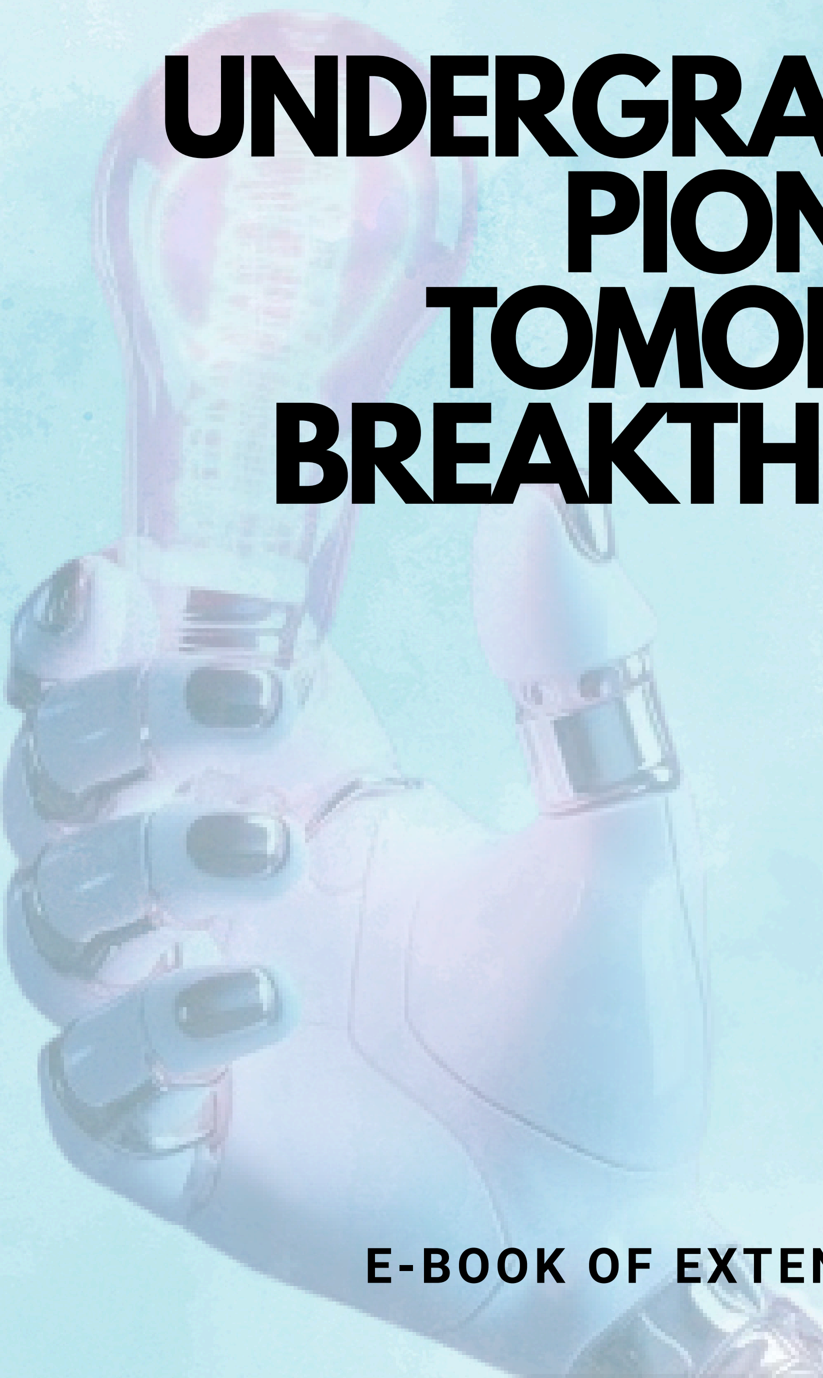

E-BOOK OF EXTENDED ABSTRACTS

NUMERICAL SOLUTION OF UNSTEADY MICROPOLAR NANOFLUID MODEL OVER EXPONENTIALLY CURVED SURFACE WITH CHEMICAL REACTION USING BVP4C

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ABSTRACT

This study examines the time-dependent behavior of micropolar nanofluids over exponentially curved surfaces in the presence of chemical reaction. Micropolar nanofluids are fluid systems that combine the beneficial microstructure effects of micropolar fluids and the improved thermal properties of nanofluids due to presence of nanoparticles in suspension. Micropolar nanofluids have several applications for heat transfer in industry. The partial differential equations (PDEs) for the governing equations of motion, micropolar motion, energy, and concentration are developed and reduced into a system of nonlinear ordinary differential equations (ODEs) using a form of similarity transformations. The nonlinear ODE system of equations is solved numerically using the bvp4c solver, a collocation method, from MATLAB. The solutions obtained from the bvp4c solver are compared to existing studies using the shooting method, and the results verify the accuracy of the bvp4c method of numerical solution. The study considers the effect of several important parameters including curvature *k,* unsteadiness *ß*, micropolar *K*, Prandtl *Pr*, and Schmidt *Sc* numbers, on physical profiles of velocity, micropolar motion, temperature, and concentration. The study finds that velocity of the micropolar fluid increases as the *k* and *K* numbers increase, and that the micropolar profiles increase with increasing *ß* and decrease as *K* increases. The temperature and concentration profiles are found to increase with curvature while decreasing with *Pr* and *Sc* numbers respectively.

Keywords: Micropolar Nanofluid, Exponentially Curved Surface, Chemical Reaction, Bvp4c, Numerical Solution

INTRODUCTION

Fluid dynamics is foundational to engineering, medicine, and environmental uses, especially for the estimation of boundary boundary layer flow properties. Nanofluids are nano-particle suspensions with a minimum particle size of 100 nm or below, which show enhanced thermal conductivity and heat transfer, and therefore, can be advantageous in applications involving cooling and heating systems. Micropolar fluids (Eringen, 1966) are an extension of non-Newtonian fluids, which incorporate effects caused by micro-rotation, and are relevant for things like suspension flows, liquid crystals, and biological fluids.

This work provides a model for unsteady micropolar nanofluids over exponentially curved surfaces taking into consideration chemical reactions. Curved geometries have a significant impact to velocity distribution, pressure gradients, and heat transfer quality, and exponentially curved surfaces show distinctive flow characteristics as was examined during uniform curvature surfaces. Chemical reactions which are endothermic and exothermic can continue to alter the boundary layer behaviour in opposing ways.

The problem statement indicates the need for better heat transfer fluids in applications on cooling systems, electronics, and energy systems. Currently available research lacks extensive studies related to micropolar nanofluids over exponentially curved surfaces with chemical reactions, particularly using robust solvers like bvp4c.

The objectives are:

i. To reduce nonlinear PDEs governing micropolar nanofluid flow into ODEs using similarity transformations.
ii. To solve the reduced equations using MATLAB's bvp4c solver.
iii. To validate results against the shooting method and analyze the impact of physical parameters.

METHODOLOGY

Based on the conservation laws, continuity, momentum, micropolar, energy, and concentration governing equations are presented and derived according to Fuzhang et al. (2022). Using similarity transformations enables the conversion of the PDEs into the following system of ODEs:

$$\begin{aligned}&(1+K)\left(f^{iv}+\frac{2}{\eta+k}f'''-\frac{1}{(\eta+k)^2}f''+\frac{1}{(\eta+k)^3}f'\right)\\&+\frac{k}{\eta+k}\left(ff'''-3f'f''+ff''+f'^2\right)-\frac{4k}{(\eta+k)^2}f'^2\\&-K\left(h''+\frac{1}{\eta+k}h'\right)-\beta\left(\eta f'''+\frac{\eta}{\eta+k}f''+\frac{2}{\eta+k}f'+3f''\right)=0,\end{aligned} \tag{1}$$

$$\begin{aligned}&\left(1+\frac{K}{2}\right)\left(h''+\frac{1}{\eta+k}h'\right)-K\left(2h+f''+\frac{1}{\eta+k}f'\right)\\&+\frac{k}{\eta+k}\left(f'h-fh'\right)-\beta\left(3h+\eta h'\right)=0,\end{aligned} \tag{2}$$

$$\begin{aligned}&\frac{1}{Pr}\left(\theta''+\frac{1}{\eta+k}\theta'\right)+\frac{k}{\eta+k}\left(f\theta'-Af'\theta\right)+Nb\theta'\varphi'\\&+Nt\left(\theta'\right)^2+\delta\theta-\beta\left(2\theta+\eta\theta'\right)=0,\end{aligned} \tag{3}$$

$$\begin{aligned}&\frac{1}{Sc}\left(\phi''(\eta)+\frac{1}{\eta+k}\phi'(\eta)\right)+\frac{k}{\eta+k}\left(f(\eta)\phi'(\eta)-Bf'(\eta)\phi(\eta)\right)\\&+\frac{Nt}{Nb}\frac{1}{Sc}\left(\theta''(\eta)+\frac{1}{\eta+k}\theta'(\eta)\right)-\beta\left(4\phi(\eta)+\eta\phi'(\eta)\right)=0,\end{aligned} \tag{4}$$

with boundary conditions as follows:

$$\begin{aligned}&f(\eta)=0,\ h(\eta)=-\eta f''(\eta),\ f'(\eta)=1,\\&\beta t\theta'(\eta)+1=\theta(\eta),\ \beta c\phi'(\eta)+1=\phi(\eta),\ \ at\ \ \eta=0,\ f'(\eta)\to 0,\ f''(\eta)\to 0,\\&h(\eta)\to 0,\ \theta(\eta)\to 0,\ \phi(\eta)\to 0\ \ at\ \ \eta\to\infty\end{aligned} \tag{5}$$

To solve the equations using bvp4c in MATLAB software, the ODEs will be reduced to a system of first-order equations.

RESULTS AND DISCUSSION

The results achieved using bvp4c are validated with shooting method in previous studies; Sajid et al. (2010) and Sajid et al. (2011) which shows close agreement and affirming the solver's accuracy. The key findings include the velocity profile $f'(\eta)$ increased with higher curvature parameter k and micropolar parameter K, indicating better momentum transfer near the boundary layer. The micropolar profile $h(\eta)$ showed the trade-off between microrotation and micropolar resistance, with larger values of K decreasing microrotation and the unsteadiness parameter β encouraging it. In terms of temperature distribution $\theta(\eta)$, both curvature k and the Prandtl number Pr enhanced heat transport; in particular, Pr reduced the thickness of the thermal boundary layer. On the other hand, the concentration profile $\phi(\eta)$ increased with increasing curvature but declined with increasing Schmidt number Sc because higher Sc values inhibit mass diffusivity and provide thinner concentration boundary layers. Overall, the research demonstrates that curvature and micropolar effects are critical in fluid flow behaviour, while diffusivity and chemical reaction parameters significantly affect heat and mass transfer performance. In addition, graphical plots are employed to depict the flow behaviour and heat transfer performance, providing a complete visualisation of the system response for changing conditions. The results in Figure 1 and Figure 2 present some of the main trends and physical observations from the numerical simulations, paving the way for further discussion and interpretation.

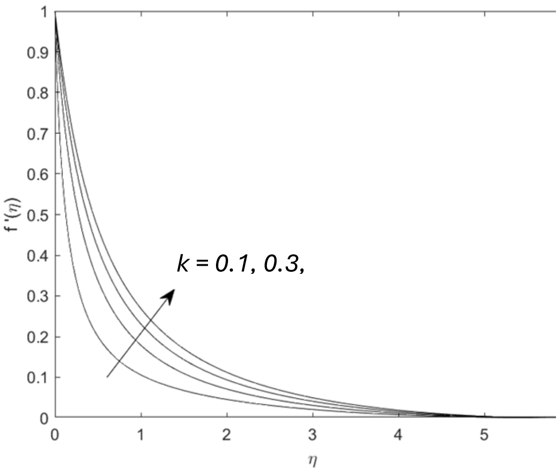

Figure 1: Velocity profile $f'(\eta)$ for different values of k

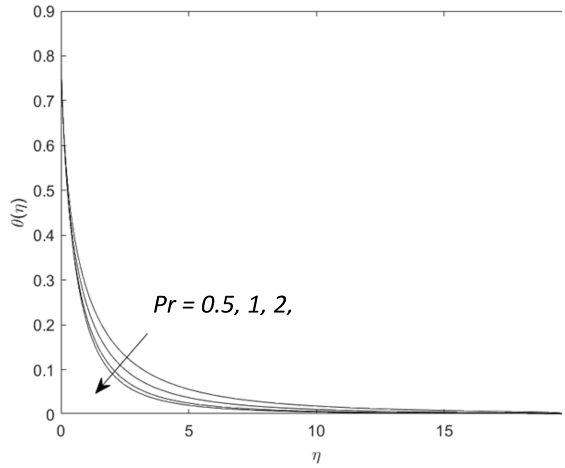

Figure 2: Temperature profile $\theta(\eta)$ for different values of Pr

Table 1: Skin friction and coupling stress numbers when all other parameters are set as: $\beta t = \beta c = 0.1$, $Sc = 1$, $n = Nb = 0.5$, $A = B = 0.3$, $Nt = \delta = 0.4$, $Pr = 4$

k	K	β	$C_f Re_s^{1/2}$	$C_m Re_s$
0.5	0.1	0.1	-0.48391	1.072053
1.0	-	-	-0.63957	0.329143
0.5	0.1	0.2	-0.55508	0.830222
-	0.2	-	-0.69022	0.587362
0.5	0.2	0.0	-0.54072	1.019638
-	-	0.1	-0.61705	0.783008

Table 2: Nusselt and Sherwood numbers were determined by establishing the parameters βt = 0.1, βc = 0.2, Nb = 0.2, Nt = 0.2, δ = 0.1, A = 0.1 and B = 0.1

k	K	β	Sc	Pr	$-\theta'(0)$	$-\phi'(0)$
0.1	0.1	0.1	1	0.4	1.74618	1.97429
0.3	-	-	-	-	1.03409	1.25814
1	0.1	0.1	1	0.4	0.64459	0.84836
-	0.3	-	-	-	0.64737	0.85137
0.1	0.1	0	1	0.1	1.59993	1.71370
-	-	0.2	-	-	1.72954	2.12720
0.7	0.1	0.1	0.5	0.1	0.61230	0.81874
-	-	-	1.5	-	0.61181	1.13100

The numerical results in Table 1 and Table 2 indicate that the increase in curvature (high k) increases skin friction (more drag on the fluid) but results in lower couple stress (diminishes the microrotational effects). For heat and mass transfer, higher curvature will reduce both Nusselt and Sherwood numbers, impeding thermal and concentration performance slightly. In addition, higher unsteady flow (higher β) increases both Nusselt and Sherwood numbers which improves the heat and mass transfer rates. The effect of microrolar parameter K increases skin friction noticeably, however this has low significance in heat and mass transfer. Finally, higher Schmidt number Sc increases the Sherwood number significantly improving the mass transfer.

NOVELTY

The present research offers fresh knowledge about the unsteady flow of the micropolar nanofluids over exponentially curved surfaces with chemical reactions, the numerical solutions of which have been conducted using the bvp4c solver. The impact of curvature, unsteadiness and chemical reactions on heat and mass transfer are discussed in finer details and are original contribution to the topic.

CONCLUSION

The problem of unstable micropolar nanofluid flow over exponentially curved surfaces with chemical reactions was successfully solved numerically in this paper using MATLAB's bvp4c solver. The solver showed accuracy and reliability in comparison to the shooting approach. Crucial physical characteristics such as curvature, unsteadiness, micropolar effects, Prandtl, and Schmidt numbers have a significant impact on the profiles of temperature, concentration, velocity, and microrotation. Future work could explore additional physical effects including magnetic fields or variable viscosity to enhance the model's applicability further. Applying the knowledge gained to the construction of sophisticated cooling systems, microreactors, and thermal management technologies can demonstrate the usefulness of micropolar nanofluids in engineering and industrial applications.

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