Research Article

Finding The Perimeter of Kitchen Knife by Newton's Divided-Difference Interpolation Method

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Abstract: *This study focuses on the application of Newton's Divided Difference Interpolation method for measuring the perimeter of objects. Consequently, employing Newton's Divided-Difference Interpolation method proves to be an effective approach for determining the perimeter of the object, specifically the knife. The object such as the knife is chosen as this object can fit on the graph paper for further calculation. Knives also have unique shape with various types of curves along the circumference. This interpolation method enables us to approximate the curve of the object's boundary accurately, allowing for precise measurements of its perimeter. The study demonstrates that Newton's Divided Difference Interpolation method can be effectively used to calculate the perimeter of objects, as evidenced by the small percentage relative error obtained from the findings.*

Keywords: Newton's divided difference interpolation, object perimeter

1. INTRODUCTION

Newton's Divided Difference Interpolation is a way of finding an interpolation polynomial that fits a particular set of points or data (Das & Chakrabarty, 2016). Newton's interpolation is a classical polynomial interpolation approach and plays a significant role in numerical analysis and image processing. The interpolation function of most classical approaches is unique to the given data. In this paper, univariate and bivariate parameterized Newton-type polynomial interpolation methods are introduced (Zou et al., 2020). This study plans to estimate the perimeter of a knife using Newton's divided difference method. The kitchen knife has a complex geometric shape, a finite number of data points, and non-linear variations in its perimeter. Kitchen knife have curved, irregular and intricate blade designs. These geometries pose a challenge for direct measurements of perimeter using conventional methods. This study is about proving the effectiveness of Newton's Divided Difference Interpolation method of measuring perimeter of various geometries.

2. METHOD & MATERIAL

This project involves the following steps:

Step 1: Data Collection

The object of interest is sketched on the graph paper as in Figure 1. Next, the shape of the object is divided into several parts or sections based on the curve line of the object. All sections will be calculated as separate equations. In Figure 1, the perimeter of the knife is divided into 11 equations. Lastly, the set of coordinate data for each section will be identified.

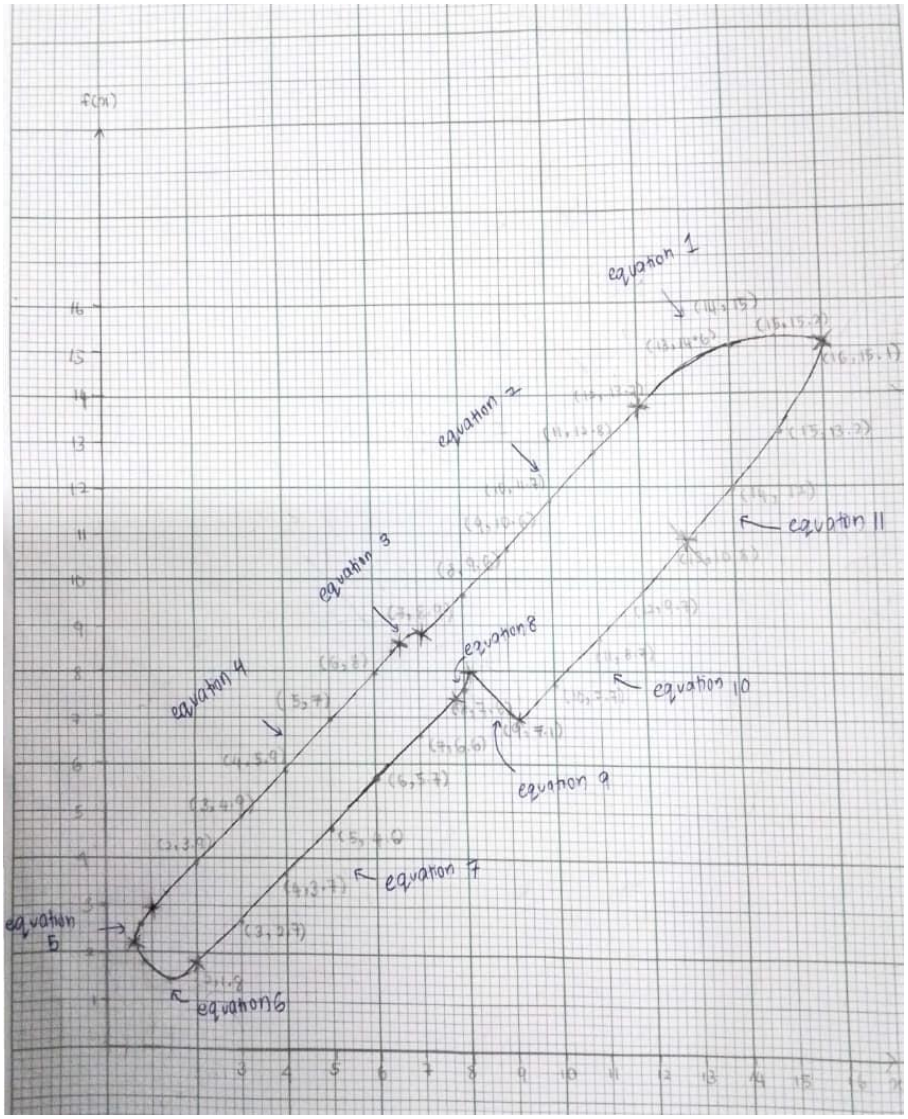

Figure 1: Knife plotted on graph paper.

Step 2: Equations

To find the equations using Newton's Divided Difference Interpolation method, the value of $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$ needs to be obtained from the set of data collection. The variable xi represents the distinct values and yi represents the corresponding function values. All the data points need to be arranged in ascending value to follow order of x-values.

Step 2a: Construct a finite divided difference table.

The finite divided difference can be calculated using equation (1), (2), and (3):

$$f[x_i] = y_i \tag{1}$$

$$f[x_i, x_{i+1}] = \frac{f[x_{i+1}] - f[x_i]}{(x_{i+1}) - x_i} \tag{2}$$

$$f[x_i, x_{i+1}] = \frac{f[x_i, x_{i+1}, \ldots, x_{i+k}] - f[x_i, x_{i+1}, \ldots, x_{i+k}]}{(x_{i+1}) - x_i} \tag{3}$$

Continue this process until all finite divided difference are calculated.

Step 2b: Find the interpolating polynomial equation.

Construct the interpolating polynomial equation using the finite divided difference table. The polynomial can be written as equation (4) or (5).

The Newton's Forward Divided Difference Interpolation Formula

$$P(x) = f[x_0] + (x - x_0) f[x_0, x_1] + (x - x_0)(x - x_1) f[x_0, x_1, x_2] + \ldots + (x - x_0)(x - x_1)(x - x_2)\ldots(x - x_n) f[x_0, x_1, x_2, \ldots, x_n] \tag{4}$$

or the Newton's Backward Divided Difference Interpolation Formula,

$$P(x) = f[x_n] + (x - x_n) f[x_{n-1}, x_n] + (x - x_n)(x - x_{n-1}) f[x_n, x_{n-1}, x_{n-2}] + \ldots + (x - x_n)(x - x_{n-1})(x - x_{n-2})\ldots(x - x_x) f[x_0, x_1, x_2, \ldots, x_n] \tag{5}$$

Step 2c: Repeat step 2a and 2b for all the parts or sections of the knife to find equation for each part.

Step 3: Approximate perimeter

The arc length of each part of the object will be calculated using equation (6).

$$\text{Arc Length, L} = \int_a^b \sqrt{1 + (P'(x))^2}\ dx \tag{6}$$

Hence, the approximate perimeter of the object can be obtained by totalling up all 11-arc length.

Step 4: Validation and accuracy

The exact perimeter is obtained using traditional techniques where the exact perimeter can be calculated by using a thread and a ruler. Thread is used to trace the entire perimeter, then the length of the thread is measured using a ruler. To compare the approximation perimeter obtained through Newton's Divided Difference Interpolation, the accuracy and reliability need to be evaluated by calculating the percentage relative error using equation (7).

$$\text{Percentage relative error, } \varepsilon\% = \left|\frac{error}{exact\ value}\right| \times 100\% \tag{7}$$

3. FINDINGS

This study discusses the use of Newton's Divided Difference Interpolation method to find the equations of an object. Hence, the application of this method shows how the perimeter of the knife is obtained.

3.1 Solution

There are 11 equations that are obtained using Newton's Divided Difference Interpolation method. The 11 equations represent all 11 parts of the knife as shown in Figure 1. Microsoft Excel is used as an app to fasten the calculations for Step 2. Some of the equations are quadratic and some are linear equations. The list of polynomial equations can be obtained in equation (8) – (18).

$$P(x) = -0.175x^2 + 5.2x - 23.5 \quad (8)$$

$$P(x) = x + 1.6 \quad (9)$$

$$P(x) = -2.5x^2 + 34.5x - 112.2 \quad (10)$$

$$P(x) = 1.02x + 1.88 \quad (11)$$

$$P(x) = 1.25x^2 + 1.25x + 1.6 \quad (12)$$

$$P(x) = 1.1111x^2 - 3.1111x + 3.5778 \quad (13)$$

$$P(x) = 0.96x - 0.12 \quad (14)$$

$$P(x) = 2.5x^2 - 38.5x + 155.6 \quad (15)$$

$$P(x) = -x + 16.2 \quad (16)$$

$$P(x) = x - 2.3 \quad (17)$$

$$P(x) = 0.35x^2 - 11.35x + 68.7 \quad (18)$$

3.2.2 Result

After constructing the 11 polynomial equations which are shown in equation (8) until (18), the arc length is calculated using equation (6). The arc length is calculated one by one for all 11 polynomial equations. Then, the perimeter of the kitchen knife is obtained by totalling up all the arc length. The total arc length obtained is 43.0052cm which is the approximate perimeter of the kitchen knife. Besides, the exact perimeter of the kitchen knife when measured by thread and ruler is 43.9 cm.

$$error = exact\ value - approximate\ value = 43.9 - 43.0052 = 0.8948$$

The difference between exact and the approximate perimeter is small, that is less than 1cm.

$$\text{Percentage relative error, } \varepsilon\% = \left|\frac{error}{exact\ value}\right| \times 100\%$$

$$= \left|\frac{0.8948}{43.9}\right| \times 100\%$$

$$= 2.0383\%$$

4. DISCUSSION

The results obtained from the findings show a very good result where the error found is equal to 0.8948cm and the percentage relative error is 2.0383%. Since the error is small, it has proven that this method is effective. The relative error also indicates that the results is very close to the exact value (Percentage error, n.d). Therefore, in the future study, there is a need to develop a software application that can run the three-step method that has been presented in Topic 2. The software that will be developed in the future will benefit everyone from different field, and not only mathematicians who can get the benefits from its development. Overall, using the kitchen knife is an appropriate object to evaluate the effectiveness of Newton's Divided Difference Interpolation method to find the perimeter object of interest because the knife size can fit on the graph paper to be drawn in finding all the points needed when using Newton's Divided Difference Interpolation method. Besides, the exact value of the knife also can be measured easily using thread and ruler.

5. CONCLUSION

Given the small percentage of relative error, the approximate perimeter calculated using the arc length equation is remarkably close to the exact value. Consequently, employing Newton's Divided-Difference Interpolation Method proves to be an effective approach for determining the perimeter of the object, specifically the knife. This interpolation method enables the researchers to approximate the curve of the object's boundary accurately, allowing for precise measurements of its perimeter. By dividing the object into smaller intervals and constructing a polynomial that passes through the given data points, Newton's Divided Difference Interpolation method provides a reliable estimation of the knife's perimeter. This mathematical technique accounts for variations in the shape of the object effectively handles irregularities, ultimately delivering reasonably accurate value for its perimeter. Therefore, utilising Newton's Divided Difference Interpolation method is a beneficial strategy in the approximately measured the knife's perimeter, ensuring a high level of precision.

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