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The Impact of Variable Seabed Bathymetry on Tsunami Wave Propagation: A One-Dimensional Shallow Water Equation Using Forward Time Central Space Scheme

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ABSTRACT

Tsunami wave dynamics are profoundly dictated by seafloor morphology, yet the specific influence of complex bathymetry on wave evolution remains a critical gap in predictive modeling. This research utilizes a finite difference approach to solve one-dimensional linearized shallow water equations (1D SWE), simulating tsunami propagation across three distinct seabed profiles: flat, sloping, and wavy. Our numerical results reveal that seafloor geometry is a primary driver of both wave amplification and velocity transformation. While flat beds maintain stable, low-hazard profiles characteristic of the deep ocean, sloping bathymetry triggers intense shoaling effects, maximizing wave height as the energy compresses in shallower coastal regions. Crucially, wavy seabed configurations induce complex energy dispersion and amplitude fluctuations, highlighting the non-linear risks posed by irregular underwater terrain. These findings underscore that precise bathymetric data is not merely a variable—it is a prerequisite for accurate early warning systems and robust coastal disaster mitigation strategies. This study provides a foundational framework for integrating high-resolution seafloor data into real-time risk assessments.

Keywords: Tsunami Wave Propagation, Seabed Bathymetry, Shallow Water Equation (SWE), Forward Time Central Space (FTCS), Shoaling Effect.

INTRODUCTION

Tsunamis are among the most destructive natural disasters, capable of causing catastrophic property damage, environmental devastation, and profound loss of life. Triggered by underwater landslides, volcanic eruptions, or seismic activity, these series of waves transform into hazardous flooding upon reaching the shore. Historical events, such as the 2004 Indian Ocean tsunami which claimed over 225,000 lives, underscore the urgent necessity for robust hazard assessment and effective disaster mitigation strategies.

While monitoring technologies have advanced, predicting the complex dynamics of tsunami wave propagation remains a significant challenge, particularly in coastal regions where the impact is most severe. Traditional physical modelling is often impractical due to high costs and logistical constraints, making mathematical modelling a critical alternative for developing efficient early warning systems. However, the accuracy of these simulations is highly dependent on how well they incorporate real-world variables, such as seabed variation, which are frequently oversimplified or neglected in standard models.

The primary motivation of this study is to bridge the gap between theoretical fluid dynamics and practical disaster preparedness by developing a simplified yet effective predictive framework for tsunami wave behavior. Driven by the need for alternatives to high-maintenance physical sensors and costly modeling, this research investigates how diverse underwater terrains—specifically flat, sloping, and wavy seabeds—directly influence wave amplitude and velocity. By applying a numerical finite difference approach to one-dimensional Shallow Water Equations (1D SWE), the study aims to improve forecasting reliability and optimize early warning systems through a deeper understanding of shoaling effects and energy dispersion. The modeling was specifically created to simulate tsunami propagation and visualize wave behavior, amplitude, and travel time across different coastal regions. Ultimately, these simulations provide a foundational framework for visualizing wave dynamics, ensuring more accurate coastal risk assessments and maximized evacuation windows for future tsunamigenic events.

SHALLOW WATER EQUATION

Linear and Non-Linear Shallow Water Equation

The Shallow Water Equations (SWE), also known as the de Saint-Venant equations, constitute a system of partial differential equations describing fluid flow where the vertical water depth is significantly smaller than the horizontal wavelength. First introduced in 1871 by French mathematician Adhémar Barré de Saint-Venant to model flood behavior, the SWE represents a simplified version of the Navier-Stokes equations. This simplification assumes a hydrostatic pressure distribution and uniform velocity, effectively reducing computational complexity by neglecting certain bathymetric conditions to facilitate analytical solutions. Today, the SWE remains the fundamental framework for solving ocean engineering problems, specifically tsunami wave propagation.

In the study by Eze et al. (2019), it covers two types of shallow water equations (SWEs), linear shallow water equation (LSWE) and nonlinear shallow water equation (NLSWE). The author mentioned that in calculating the tsunami wave propagation, LSWE is fit for region of open sea. However, it is not applicable in order to be implemented for tsunami wave propagation at shore. This led to the use of NLSWE that is more suited to match the condition of tsunamis when they approach and hit the shore area. Coriolis effect that happens due to rotation of the earth was discussed in the article which it gives impact to the energy distribution from a long-travel tsunami known as tele tsunami. Cho et al. (2007) also stated that LSWE is more suitable for long distance tsunami wave propagation.

In addition, in a study of coastal engineering reviewing on NLSWE in application of tsunami wave propagation (Brocchini & Dodd, 2008), the authors focus on the model in wave breaking and the movement approaching coastal region. The toughness and consistency of the equation in solving the tsunami wave propagation are the advantages of the NLSWE. However, it is lack with frequency dispersion which can lead to wrong predictions of wave propagation at open seabed. There are two types of equation form, primitive conservation form is for analytical solution while flux-conservative equation form is a mass and momentum in discrete form that fit in handling extreme changes.

Other than that, Liu et al. (2009) made a trial model tsunami for South and East of China Sea. The researcher selected a critical zone in depth between 400m and 500m. Since the mean depth for China Sea in south area is more than 1000m, the LSWE is suitable to employ while in east area with mean depth of 300m, the use of NLSWE is fit. The wave amplitude is influenced by the frictional effect.

Therefore, the nonlinear term in NLSWE become useful in forecasting the tsunami at coastal area. Additionally, the author used open boundary in indicated the real-world scenario which the fluid can flow in and out. In conclusion, the author believed that LSWE is adequate for the bathymetry of South China Sea (open sea) and NLSWE is fit when the wave approach to coast area since the ocean depth become shallow.

Method In Solving Shallow Water Equation

The mathematical complexity of tsunami modeling has led to a diverse array of numerical and analytical techniques designed to solve the Shallow Water Equations (SWE) across varying marine environments. Semi-analytical approaches, such as the Elzaki Adomian Decomposition Method (EADM), have been successfully used to simulate mid-ocean wave amplification and run-up without the need for discretized variables, providing a continuous representation of velocity and height (Varsoliwala and Singh, 2021). Recent advancements done by Izadi et al. (2024) include the application of the Variational Iteration Method (VIM) and the Residual Power Series Method (RPSM) to nonlinear fractional SWE, which streamline computation by eliminating the need for complex linearization or discretization. Furthermore, Karunakar and Chakraverty (2017, 2018) research into Homotopy Perturbation Methods (HPM) and Sunil Kumar (2020) research on Modified Homotopy Analysis Transform Methods (MHATM) has demonstrated that these techniques can effectively handle uncertainty and fractional coupled systems, often reaching exact solutions faster than traditional iterative processes. Time-fractional models, such as work by Tandel et al. (20??) used the Fractional Reduced Differential Transform Method (FRDTM), further enhance our understanding of tsunami behavior across irregular shore gradients and complex bathymetry.

Complementing these analytical trends are robust grid-based numerical methods that prioritize stability and spatial accuracy. The Method of Lines (MOL) has proven effective in transforming partial differential equations into systems of ordinary differential equations, which are then resolved using high-order schemes like the 4th-order Runge-Kutta method to match real-world observations (Ismail et al., 2007; Mousa, 2018). For coastal regions, Chen et al. (2024) and Supian & Rusli (2018) used the Finite Difference Method (FDM), while Hashentuya et al. (2013) used Finite Element Method (FEM) that are widely utilized, often employing nested or staggered grids to transition from coarse ocean resolutions to the finer details required near the shore. Specialized schemes, such as the Leap-frog method, incorporate dispersion-correction factors to account for the physical frequency dispersion often neglected by the Linearized SWE (LSWE) (Cho et al., 2007). In this study, the Forward Time Central Space (FTCS) scheme is applied due to its directness and verified ability to approximate wave height and velocity. While the FTCS scheme can be susceptible to instability if the Courant-Friedrichs-Lewy (CFL) condition is not strictly managed, it remains a highly accurate tool for simulating the initial stages of tsunami propagation when optimized parameters are utilized.

Seabed Topography

Seabed morphology is a critical determinant of tsunami wave dynamics, as variations in seafloor geometry fundamentally alter wave profiles during propagation. In deep-ocean regions characterized by relatively constant depths, wave height and velocity typically remain stable, though they may undergo gradual increases over time because of dispersion (Varsoliwala & Singh, 2021). Conversely, sloping seabeds modeled by linear depth functions trigger significant shoaling effects as the tsunami approaches coastal areas. During this transition, wave velocity decreases while amplitude increases sharply, leading to a much greater amplification of coastal run-up.

For irregular seafloor conditions, such as wavy seabeds utilizing sinusoidal profiles, the interaction between the wave and the bathymetry becomes significantly more complex. Research by Belibassakis et al. (2019) investigate these irregular terrains reveals that features like Bragg scattering and resonant reflection against opposing currents can induce unpredictable amplitude fluctuations. These distinct behaviors across flat, sloping, and wavy profiles highlight the essential need for high-resolution bathymetric data in numerical models. Accurately representing these seafloor variations is paramount for effectively predicting wave behavior and assessing the potential impact on coastal infrastructure.

METHODOLOGY

Overview of the Proposed Framework

The core of this study is in modelling tsunami wave propagation using Shallow Water Equations (SWE) which is normally applied to explain the dynamics of long surface waves in shallow water where the horizontal scale is much larger than the vertical depth. The one-dimensional Linear Shallow Water Equation (1D LSWE) is used in this study because this model is simple to compute and capable of describing the tsunami propagation in deep and moderately shallow waters. These equations are derived based on the assumptions of incompressible, inviscid, and hydrostatic flow. The 1D LSWE consists of a coupled system of partial differential equations representing the conservation of mass and the conservation of momentum (Brocchini & Dodd, 2008). The governing classical wave equations based on Karunakar and Chakraverty (2021) are given by:

$$\frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} = 0$$
$$\frac{\partial M}{\partial t} + gD\frac{\partial \eta}{\partial x} = 0 \qquad (1)$$

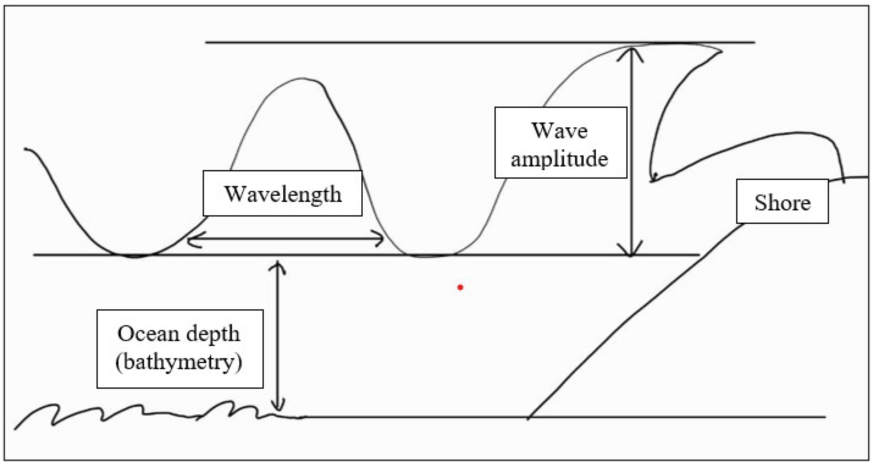
Here, $\eta(x,t)$ the surface elevation of the water, $M(x,t)$ is the depth average velocity and water velocity in x-direction, g is the acceleration due to gravity. $D = \eta + h$ is the total water depth, where $h(x)$ is the seabed profile under undisturbed water (see Figure 1). The model is built under several assumptions where the fluid is incompressible and homogeneous where the density is constant, the vertical acceleration is negligible, and the pressure distribution is hydrostatic. These assumptions enable to reduce the complex three-dimensional fluid motion into a one-dimensional system suitable for numerical modelling.

The 1D SWE can be used to investigate the behavior of the wave as it propagates along a straight coast or channel and its crossing over the underlying bathymetry to give an idea about the dynamic in the wave as it passes through. The model is also effective when used in reproducing the shoaling effect in which the amplitude of the waves grows. The SWE (Equation 1) been solved by discretize both space and time using Forward Time Centre Space (FTCS). In this process, i and j are referred to the Cartesian coordinates of the direction, x at time, t as shown in Equation 2 below:

$$\eta_{i,j+1} = \eta_{i,j} - \frac{1}{2}c\,(M_{i+1,j} - M_{i-1,j})$$
$$M_{i,j+1} = M_{i,j} - \frac{1}{2}cg\,(\eta_{i,j} + h)(\eta_{i+1,j} - \eta_{i-1,j}) \tag{2}$$

Figure 1

Illustration of Tsunami Wave Structure



The coupled system of one-dimensional SWE that been used for the tsunami wave propagation simulation. The variable of h is showing the ocean depth and correspond to the seabed variations. To analyse how different seabed topographies influence tsunami wave propagation, three distinct seabed profiles are used into the simulation model. Each profile represents a different physical condition commonly found in oceanic and coastal environments. These variations are applied by modifying the water depth function of $h(x)$ across the spatial domain. The seabed profiles considered are as follows:

i. **Sloping Seabed using Hyperbolic Tangent Profile**

This profile simulated a gradual change from deep to shallow water which typical in coastal areas. It is model using a hyperbolic tangent function (Park, 2007; Supian & Rusli, 2018) with some modification been made

$$h(x) = 25 - 25\tanh\left(\frac{x-70}{8}\right). \tag{3}$$

This function creates a smooth slope surface that becomes shallower as the wave propagates toward the shore. This allows the study of wave shoaling and amplification near coastal regions.

ii. **Flat Seabed using Constant Depth Profile**

As for trial and error in this study, the flat seabed represents condition in deep ocean where the bathymetry remains uniform. It is defined as

$$h(x) = 50. \tag{4}$$

This scenario allows for the investigation of tsunami behaviour in open sea where minimal interaction with the seafloor.

iii. **Wavy Seabed using Sinusoidal Profile**

To simulate complex underwater which refers to irregular surface in a sinusoidal depth variation is used with trial and error for theoretical purpose in the study done by making modification from tangent hyperbolic function in sloping seabed (Park, 2007; Supian & Rusli, 2018) to wavy seabed using the sine function of

$$h(x) = 35 + 15\sin\left(\frac{2\pi x}{L}\right). \tag{5}$$

This profile introduces periodic fluctuations of the bottom topography which allowing the study of wave dispersion, reflection, and local amplification due to irregular topography.

These three configurations of seabed present a basis of comparison that helps in evaluating the behavior of the tsunami sea waves under various bottom conditions. The changes in the wave's amplitude and their velocity during the time are discussed to understand the physical processes of the wave transformation, energy concentration, and the possible coastal impacts.

ANALYSIS AND RESULTS

The primary objective of the simulation is to provide sufficient temporal and spatial range for a tsunami wave, initiated by a Gaussian pulse, to develop fully and propagate across the entire computational domain. This extended duration is essential for observing critical wave characteristics, including amplitude amplification, coastal shoaling, and energy dispersion under flat, sloping, and wavy seabed scenarios.

To solve the governing equations (Equation (2) – (5)), the numerical simulation employs specific boundary condition to ensure stability and physical accuracy. Following the approach described by Park (2007) fixed boundary conditions implemented at $x = 0$ and $x = L = 100$. Specifically, the parameters are set such that the surface elevation and velocity at the boundaries are defined by $\eta(0,t) = 0, \eta(100,t) = 0,$ $M(0,t) = 0$ and $M(100,t) = 0$.

i. **Sloping Seabed using Hyperbolic Tangent Profile**

Figure 2

Simulation of tsunami over a slopping seabed at t = 0.0s.

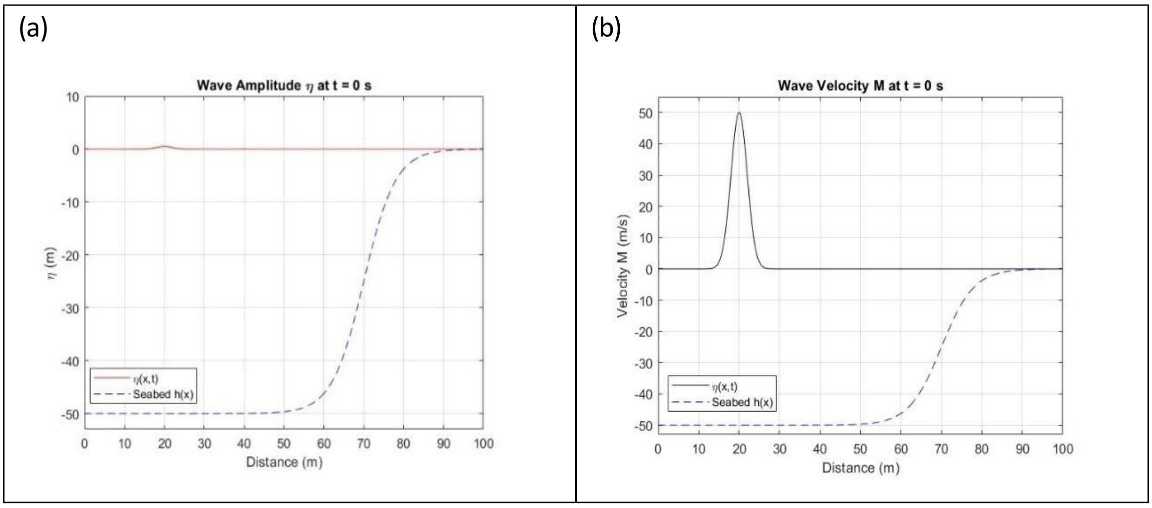


Figure 3

Simulation of tsunami over a slopping seabed at t = 0.5s.

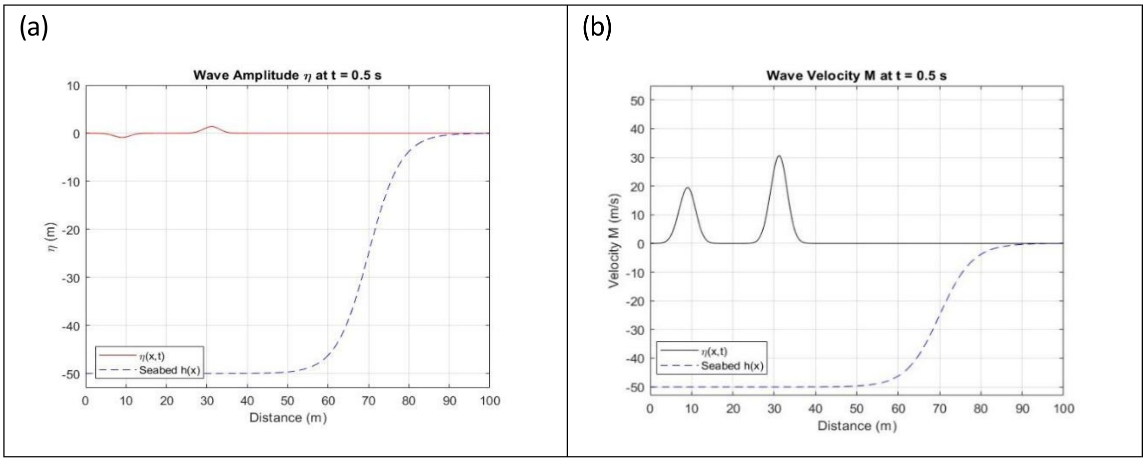


Figure 4

Simulation of tsunami over a slopping seabed at t = 1.0s.

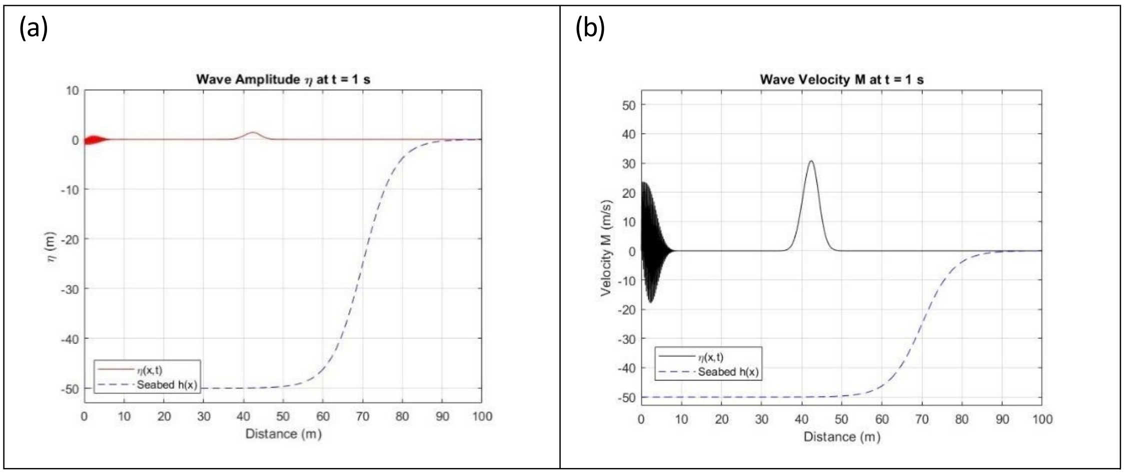


Figure 5

Simulation of tsunami over a slopping seabed at t = 2.0s.

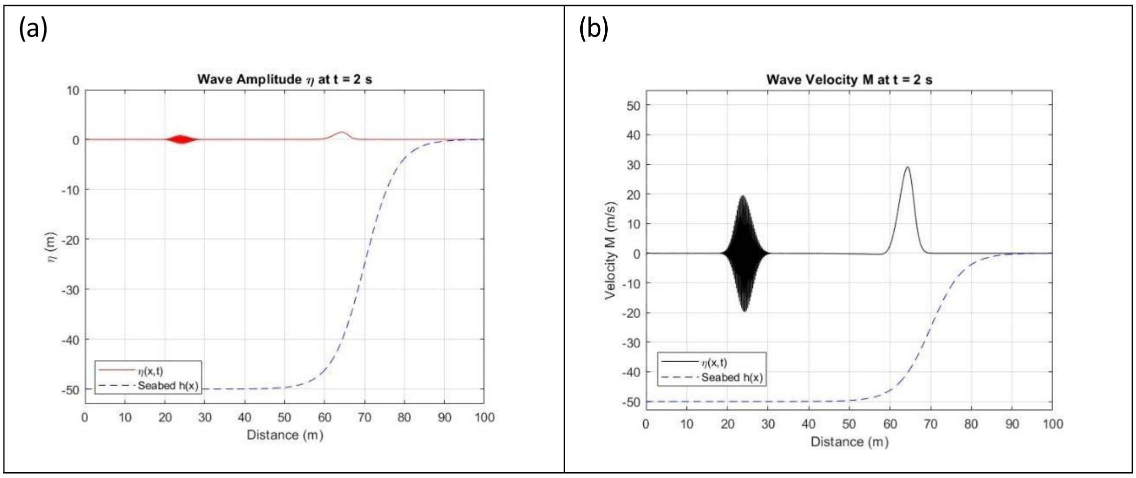


Figure 6

Simulation of tsunami over a slopping seabed at t =3.0s.

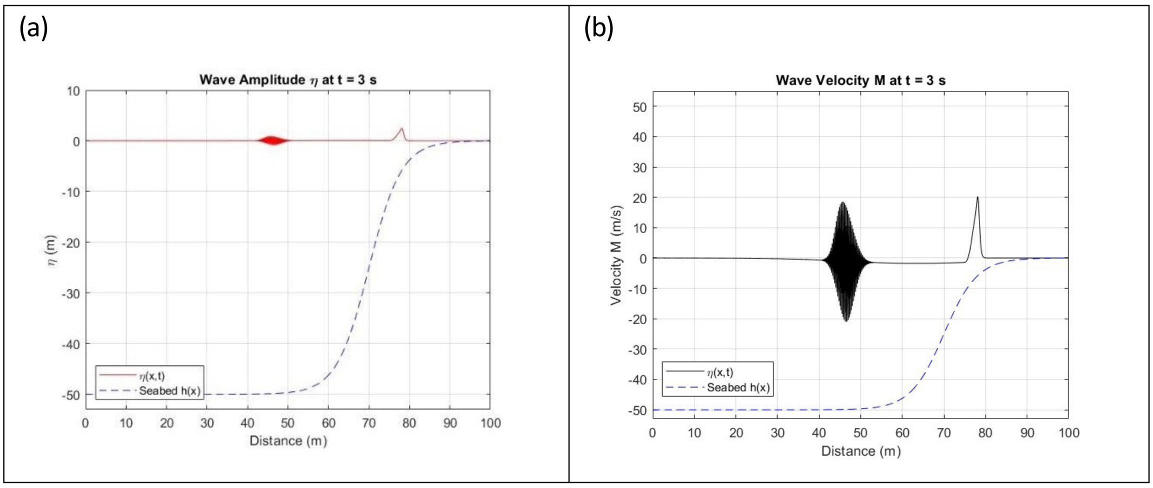


Figure 7

Simulation of tsunami over a slopping seabed at t =4.0s.

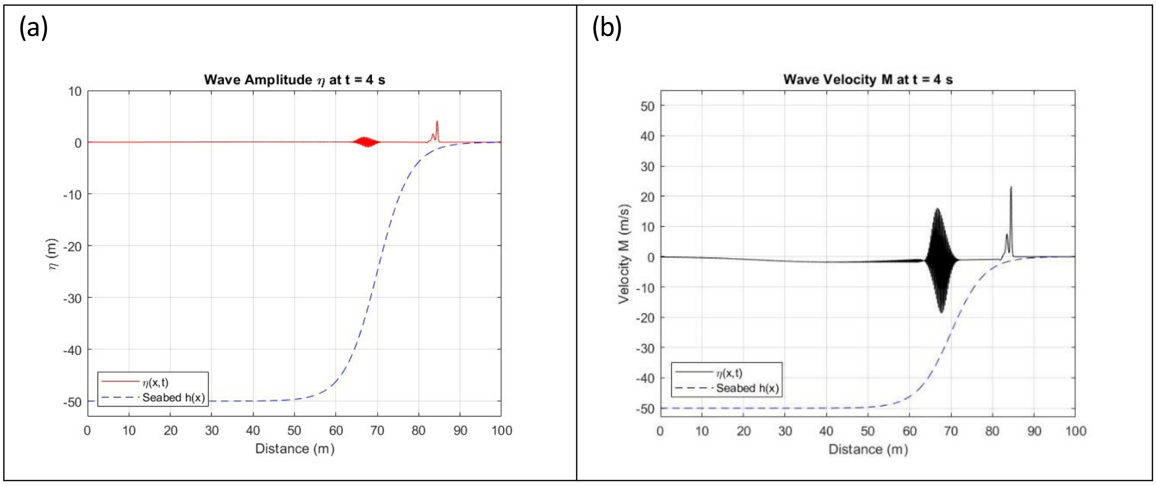


Figure 8

Simulation of tsunami over a slopping seabed at t =5.0s.

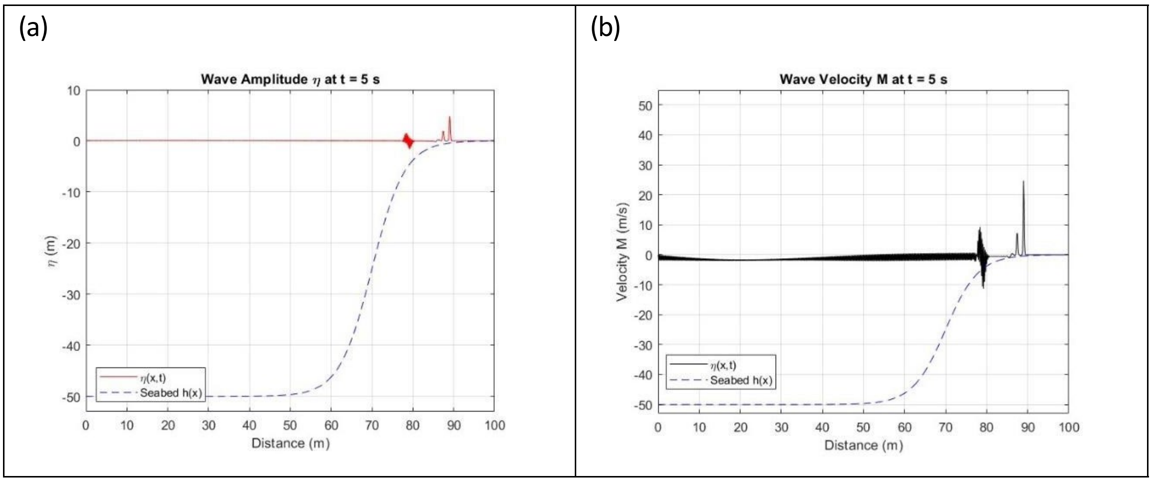
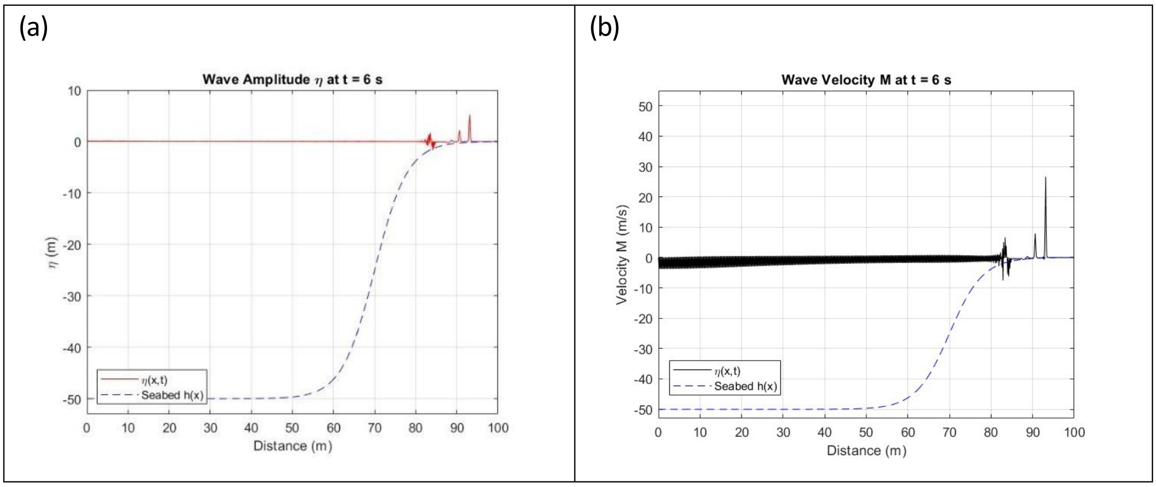


Figure 9

Simulation of tsunami over a slopping seabed at t =6.0s.



This profile in Figures 2 – 9, allows to explore the shoaling effects, where the wave amplitude increases as the water becomes shallower. From the results of simulation, the wave amplitude and velocity at $t = 0s$ and $t = 0.5s$ closely match those reported in the referenced journal**.** However, when $t = 1s$ until $t = 6s$, the simulation results begin to diverge. Based on article written by Park (2007), the result is not matching due to some modification made by the authors in the simulation.

Initially, at $t = 0.0$ s, the amplitude of this wave is 0.5m at $t = 0.0$s and it increases significantly over time, to a peak of 5.14m at t = 6.0s. In contrast, the wave velocity begins at 50.00 m/s and decreases as the wave moving. The fluctuating occurs slightly before settling on 26.61m/s. This behavior is consistent with the shoaling effect, where in which the wave energy is compressed in shallower waters, resulting in increased amplitude and decreased velocity.

The stable increase in amplitude of the wave when the tsunami the wave travel from deeper to shallower water. The water depth decreasing and the wave speed become slow down, thus the energy of the wave is conserve which it transforms the loses of kinetic energy become increases of potential energy. Therefore, it shows the wave height increasing. The waves become dangerous against the shorelines. The steep bathymetry enables the gradual build-up causing high heights of waves with considerable destructive capabilities.

ii. **Flat Seabed using Constant Depth Profile**

Figure 10

Simulation of tsunami over a flat seabed at t = 0.0s.

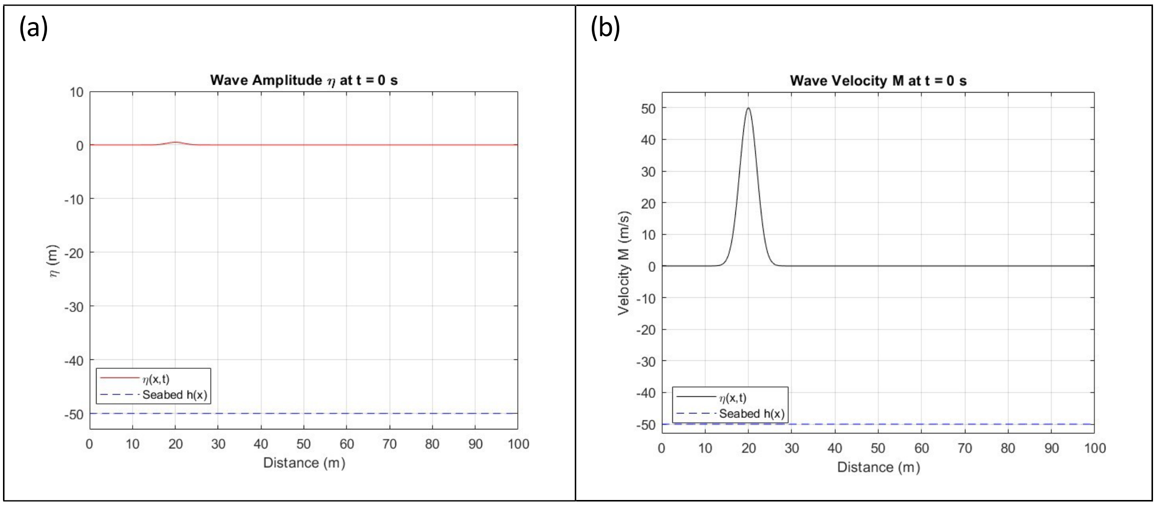


Figure 11

Simulation of tsunami over a flat seabed at t = 0.5s.

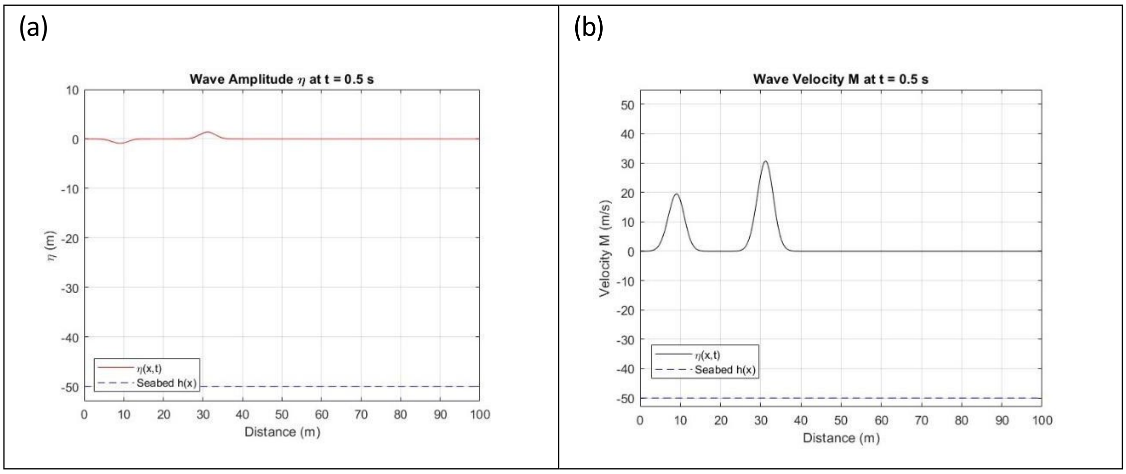


Figure 12

Simulation of tsunami over a flat seabed at t = 1.0s.

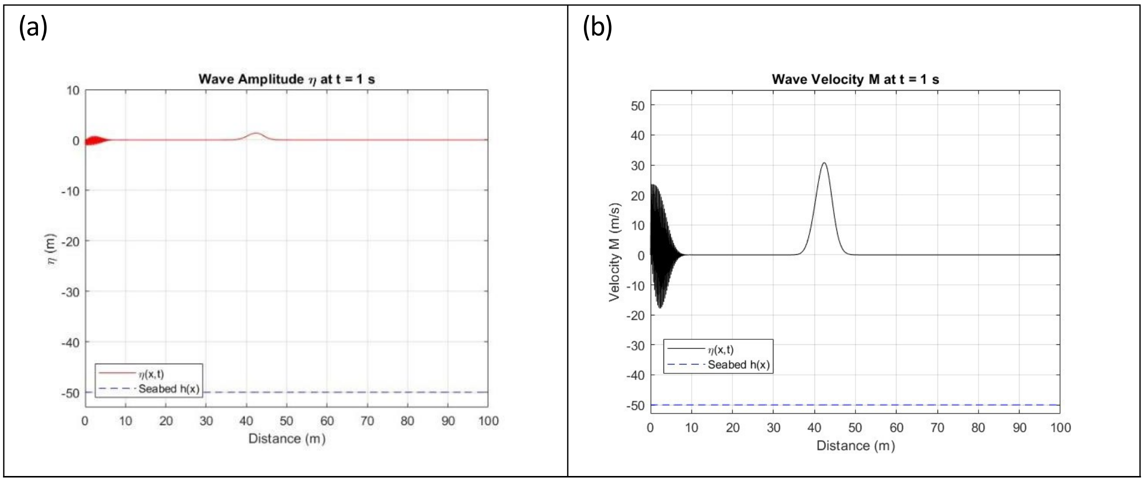


Figure 13

Simulation of tsunami over a flat seabed at t = 2.0s.

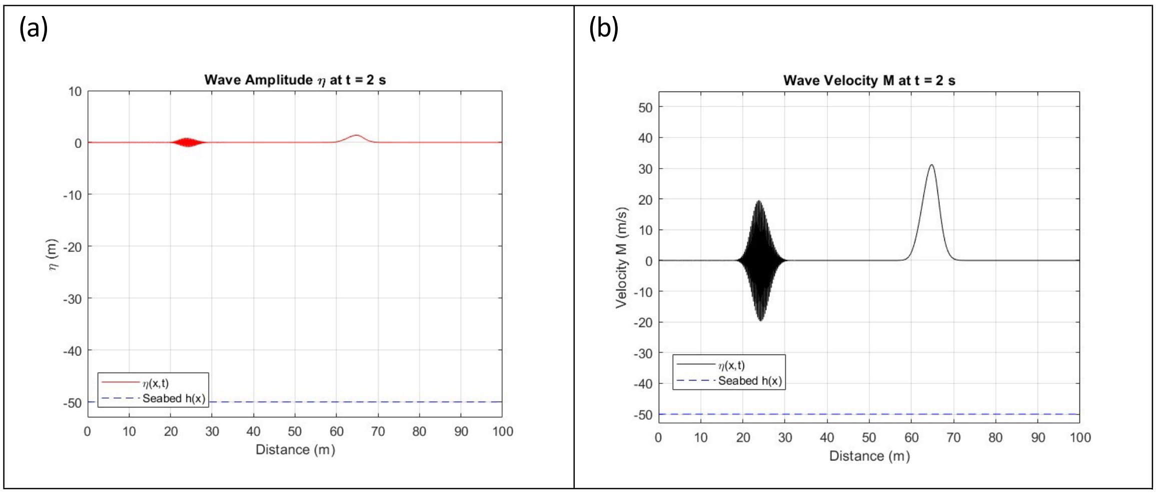


Figure 14

Simulation of tsunami over a flat seabed at t = 3.0s.

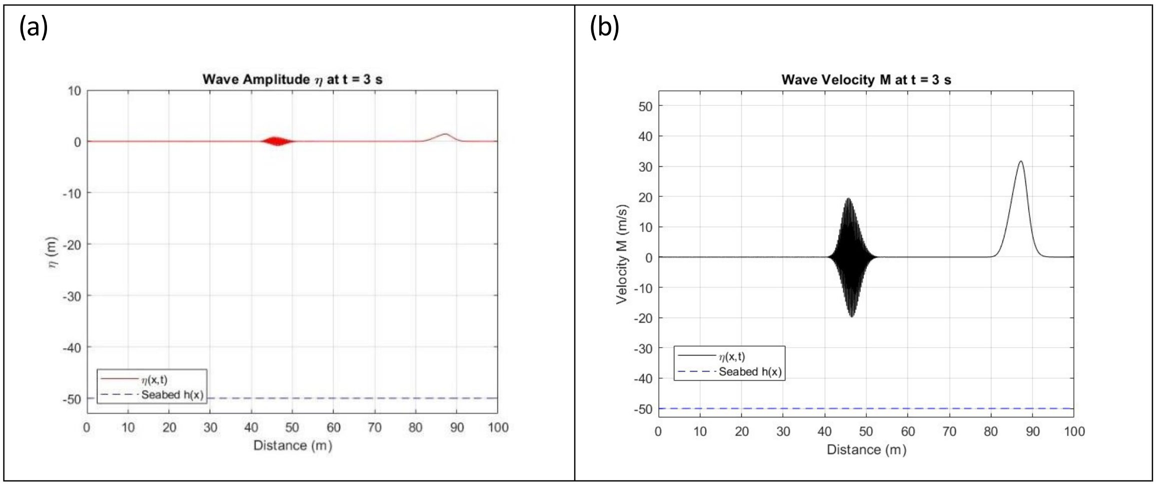


Figure 15

Simulation of tsunami over a flat seabed at t = 4.0s.

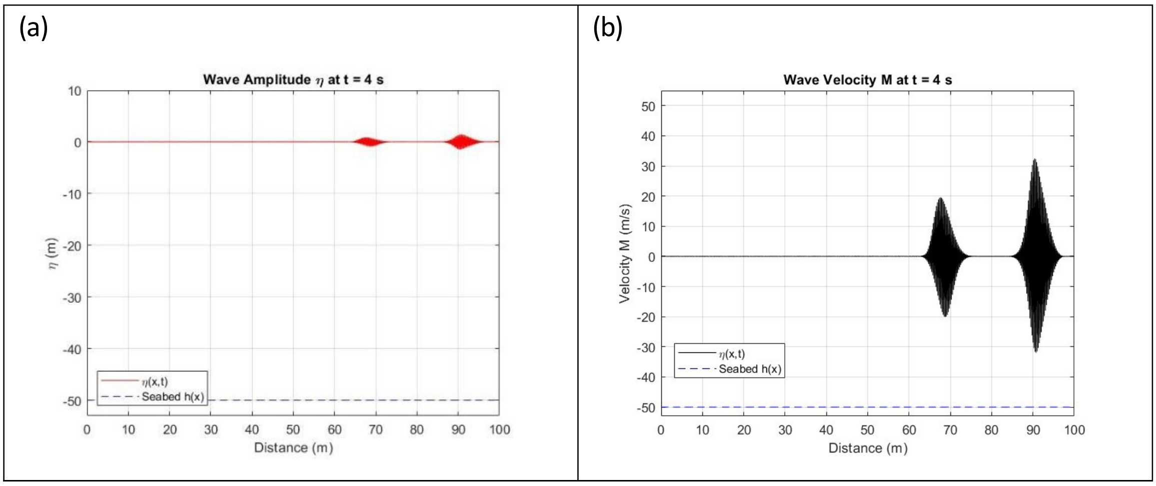


Figure 16

Simulation of tsunami over a flat seabed at t = 5.0s.

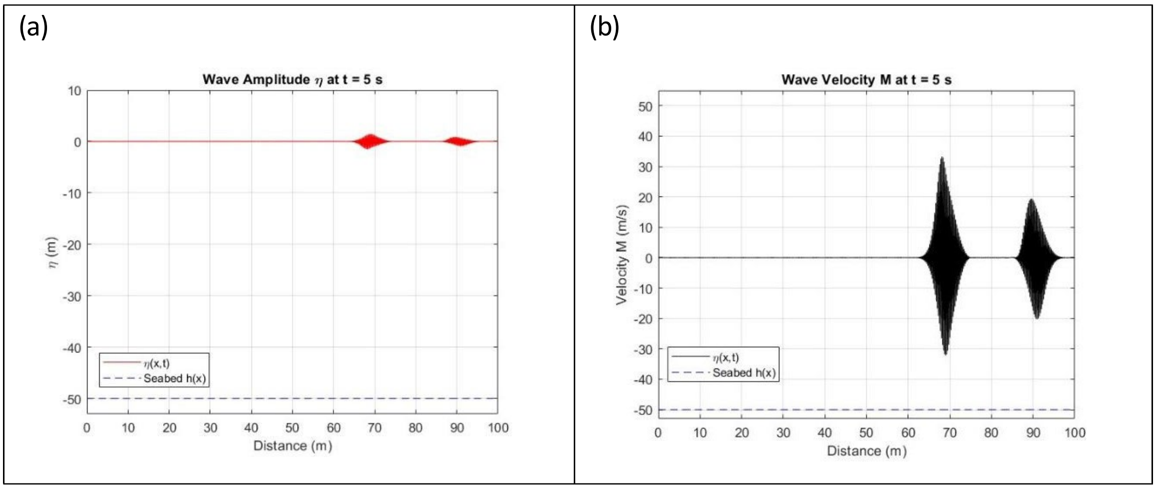
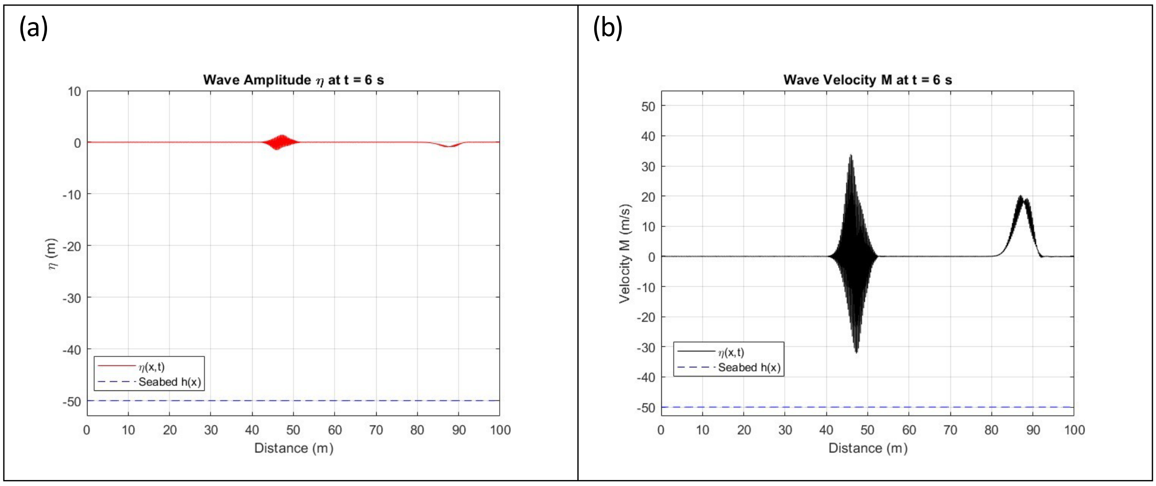


Figure 17

Simulation of tsunami over a flat seabed at t = 6.0s.



The simulations in Figures 10 – 17, examines how the tsunami wave evolves in terms of amplitude and velocity across time. This simulation results point out that the tsunami wave is relatively stable when it comes to its dynamics until its initial phase. The amplitude grows slightly over time starting from 0.5m to about 1.45m at t = 4.0s, and the velocity also shows a rise to go between slightly above 30.64m/s to 32.22m/s. However, the wave amplitude sharply reduces with a negative value of -0.8043m at t = 5.0s and the value of velocity also decreases to 17.99m/s.

The negative value of wave amplitude showing the temporary drop happen in the water surface below the undisturbed water level. This phenomenon is valid in actual tsunami. The negative value appeared due to hydrodynamic principles which are wave reflection and the drawdown phase. It turns out that when the tsunami wave reaches the shore area, it reflects to the origin, and it interact with the next incoming wave. The overlapping happens between crest of wave and trough then causing the wave height decreased or called as destructive interference. Next the draw-down phase is a situation as it retreats from the land after its maximum run-up (Carrier et al., 2003). When the draw-down happened, the water surface falls below its still water level, resulting in negative water surface amplitude. Under constant-depth conditions, the energy of the wave is least disrupted or converted so early in the propagation normally may be smooth, but later there is the possibility of interference and reverse.

In addition, there is no shoaling or energy that makes the wave not to increase very much in power. Instead, crest and trough components may cause phase cancellation, particularly during latter parts of the simulation. This indicates how an even seabed profile helps in dissipating the energy of the tsunami. Therefore, the tsunami is less dangerous than the sloping and uneven seabed. In total, the flat seabed scenario presents the characteristic wave appearance that is more stable and less amplification with respect to wave traveling in deep ocean regions where early detection systems may be designed to offer maximum protection because the risk of instantaneous impact is less in the deep ocean areas.

iii. **Wavy Seabed using Sinusoidal Profile**

Figure 18

Simulation of tsunami over a wavy seabed at t = 0.0s.

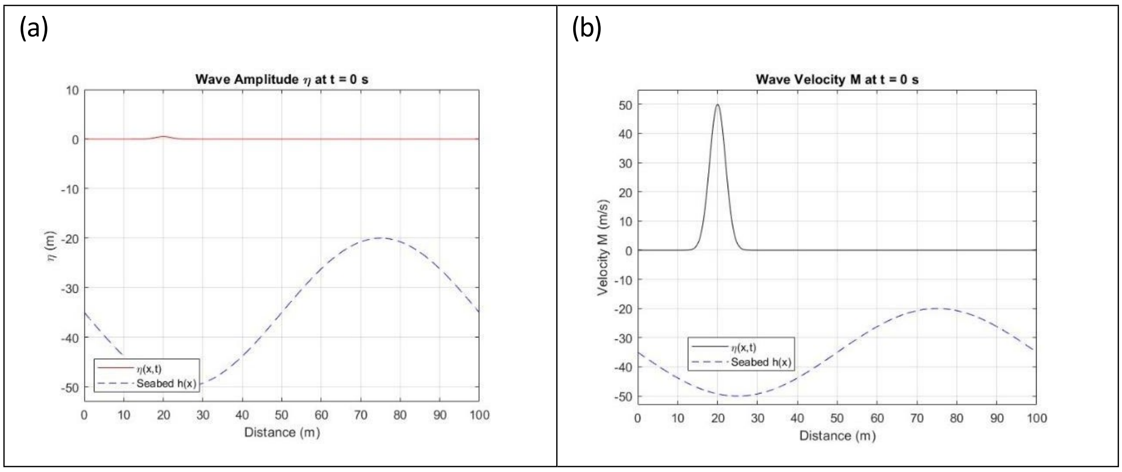


Figure 19

Simulation of tsunami over a wavy seabed at t = 0.5s.

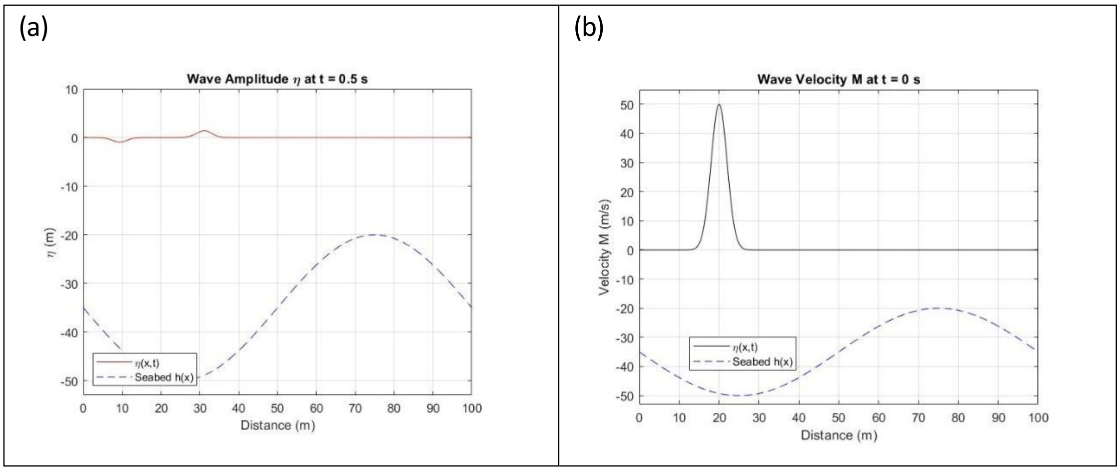


Figure 20

Simulation of tsunami over a wavy seabed at t = 1.0s.

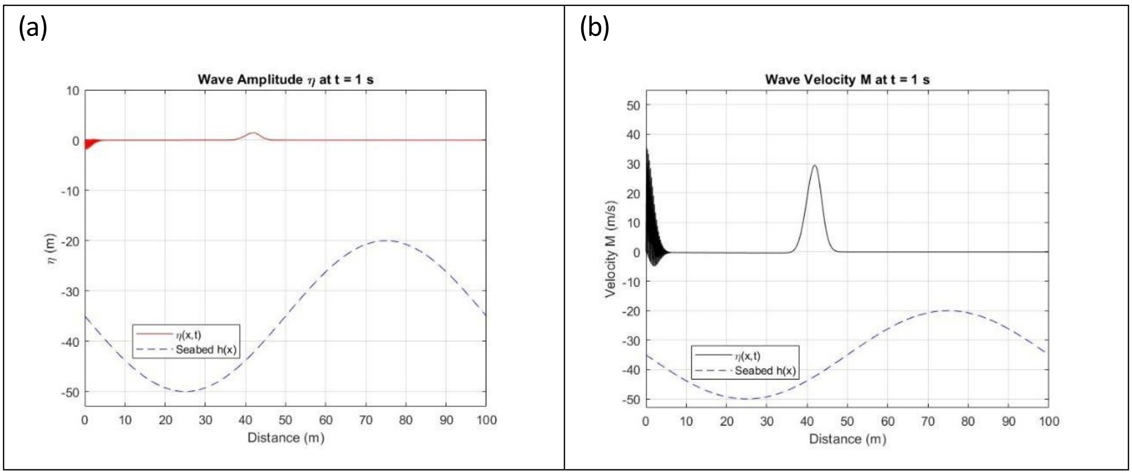


Figure 21

Simulation of tsunami over a wavy seabed at t = 2.0s.

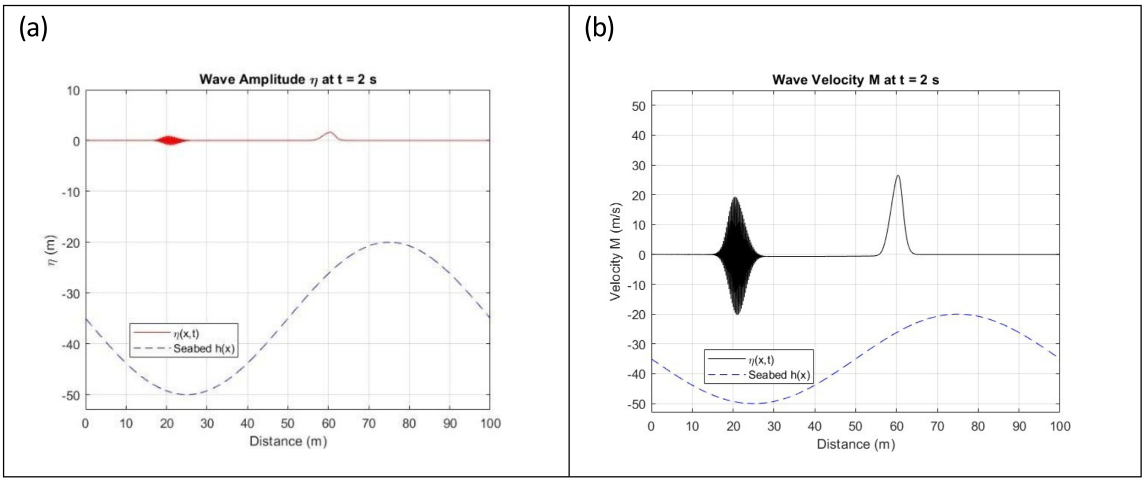


Figure 22

Simulation of tsunami over a wavy seabed at t = 3.0s.

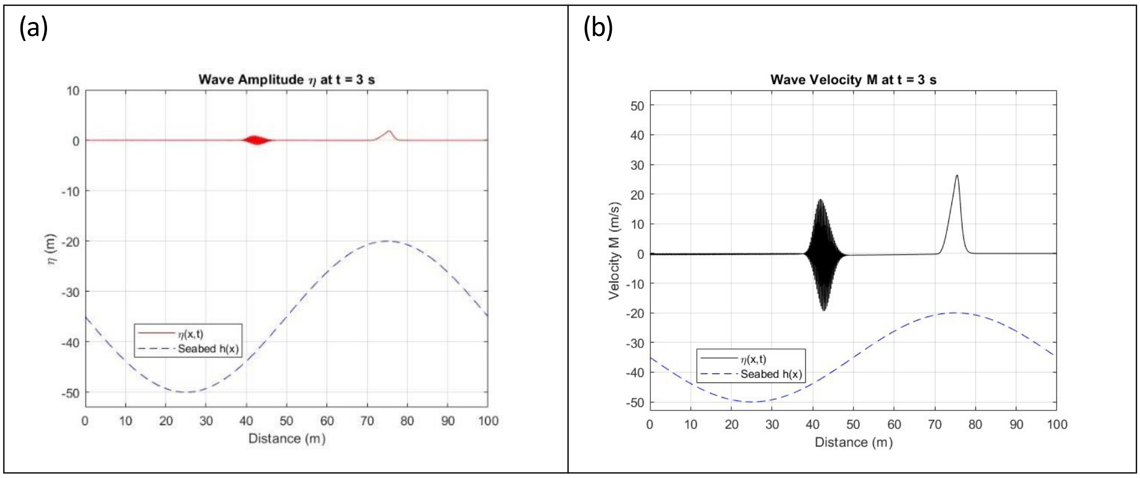


Figure 23

Simulation of tsunami over a wavy seabed at t = 4.0s.

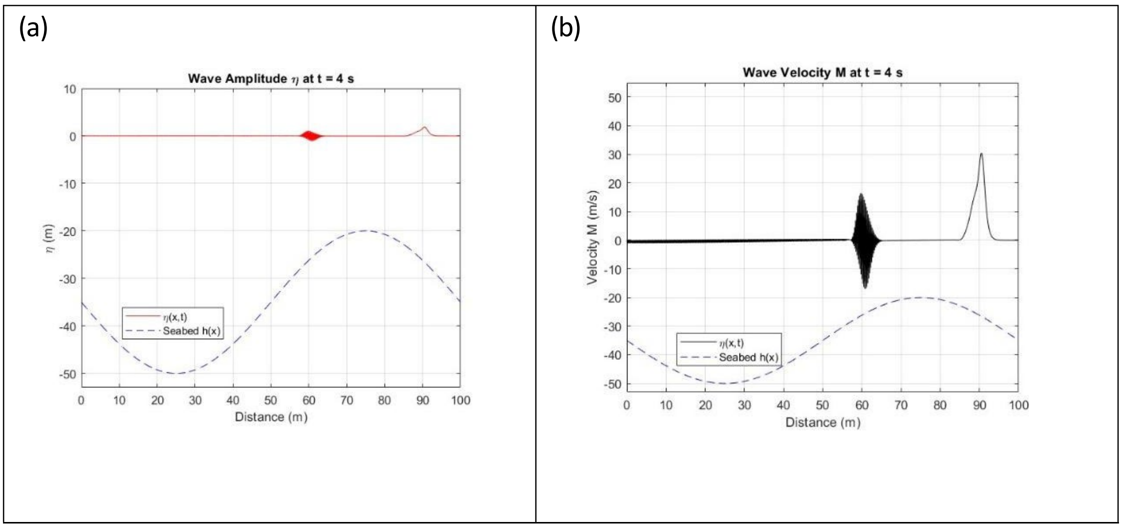


Figure 24

Simulation of tsunami over a wavy seabed at t = 5.0s.

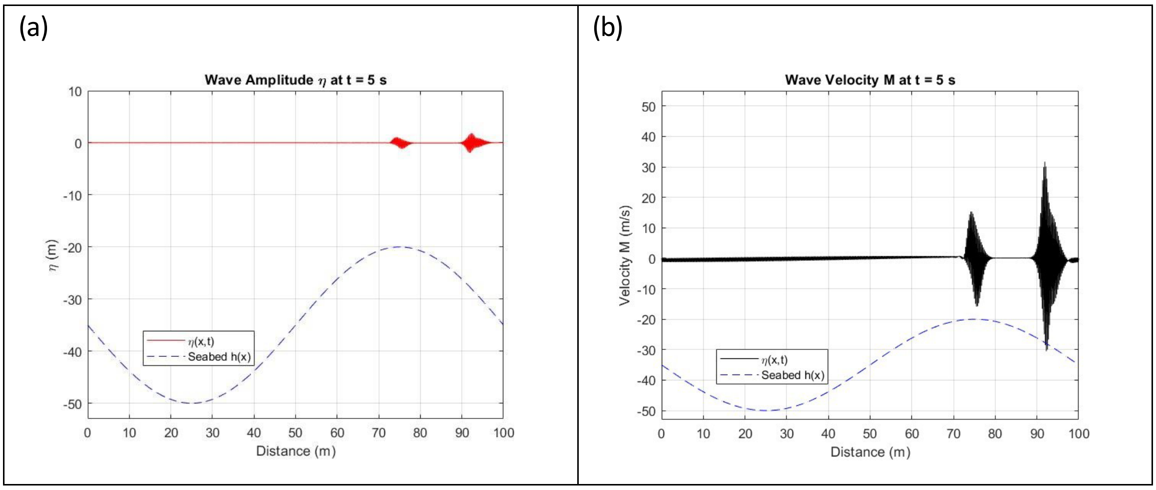
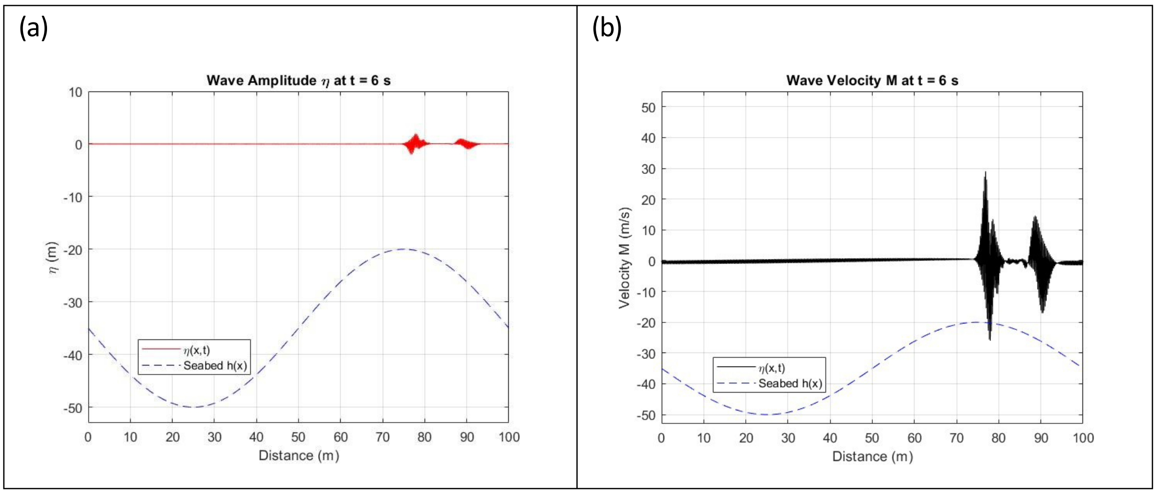


Figure 25

Simulation of tsunami over a wavy seabed at t = 6.0s.



This topography in Figures 18-25, creates periodic variations in seabed depth that allow dealing with wave dynamics in varying bathymetry. The amplitude and velocity of the waves in this simulation is not monotonic because of the dispersion of energy and refraction of the waves caused by the change in the depth of the seabed. The wave is increase steadily with the value of amplitude being 0.5m to the highest point of about 1.85m at t = 3.0s. However, there is a downward trend to 1.02m at t = 6.0s. It is also the case with the velocity that fluctuate over time and at the last time step its value reaches minimum 14.65m/s.

These oscillation shows that the irregular or uneven seabed channels the energy of the waving in a non-uniform way making certain areas amplify and others decline in amplitude locally. The wavy profile also does not result in a monotonic amplification of the waves unlike the case with the sloped or the flat seabed profiles. Instead, it contains some new dynamic interactions of partial reflections, interferences, and local energy concentration.

The findings indicate that abnormal geometries of seabed may cause challenges in the tsunami propagation as its speed and height may change in an unexpectable manner. These settings can caused complexity in tsunami prediction and need bathymetry data with a high resolution in order to model it accurately. Tsunami waves that traveling over irregular seabed will have varied of amplification which depends on the shape and position of seabed. At this point, the wave is not dangerous since the wave height is not above 2m. The situation demonstrates the relevance to take into consideration the complexity of the real seabed in the prediction of tsunami hazard, especially on the coasts bordered with reefs, islands, or the elements of the underwater topography.

CONCLUSION

This study demonstrates that seafloor morphology is a fundamental determinant of tsunami wave behaviour, directly influencing risk levels in coastal environments. By applying a Forward Time

Central Space (FTCS) scheme to 1D Linearized Shallow Water Equations, we quantified how distinct seabed profiles—flat, sloping, and wavy—transform wave amplitude and velocity. Our simulations confirm that while flat beds maintain stable profiles, sloping bathymetry triggers dangerous shoaling effects, maximizing wave height as energy compresses near the shore. Furthermore, wavy configurations introduce complex energy dispersion and unpredictable fluctuations, highlighting the hidden risks posed by irregular underwater terrain.

Ultimately, these findings prove that precise, high-resolution bathymetric data is not merely a secondary variable but a prerequisite for the accuracy of early warning systems. By integrating these seafloor variations into predictive frameworks, coastal managers can better visualize wave dynamics, optimize evacuation windows, and strengthen disaster mitigation strategies for future tsunamigenic events.

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Conflict of Interest
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REFERENCES

Belibassakis, K., Touboul, J., Laffitte, E., & Rey, V. (2019). A mild-slope system for Bragg scattering of water waves by sinusoidal bathymetry in the presence of vertically sheared currents. *Journal of Marine Science and Engineering*, *7*(1). https://doi.org/10.3390/jmse7010009

Brocchini, M., & Dodd, N. (2008). Nonlinear Shallow Water Equation Modeling for Coastal Engineering. *Journal of Waterway, Port, Coastal, and Ocean Engineering*, *134*(2), 104–120. https://doi.org/10.1061/(asce)0733-950x(2008)134:2(104)

Carrier, G. F., Wu, T. T., & Yeh, H. (2003). Tsunami run-up and draw-down on a plane beach. *Journal of Fluid Mechanics*, *475*, 79–99. https://doi.org/10.1017/S0022112002002653

Chen, P., Oppong, B., Lee, E.-J., & Huang, X. (2024). *Mathematical Models of Tsunami Propagation*. https://doi.org/doi:10.20944/preprints202408.0659.v1

Cho, Y. S., Sohn, D. H., & Lee, S. O. (2007). Practical modified scheme of linear shallow- water equations for distant propagation of tsunamis. *Ocean Engineering*, *34*(11–12), 1769–1777. https://doi.org/10.1016/j.oceaneng.2006.08.014

Deleanu, D., & Dumitrache, C. L. (2019). Explicit finite difference schemes for solving the one – dimensional tsunami wave propagation equations. *Journal of Physics Conference Series*, *1297*(1), 012005. https://doi.org/10.1088/1742- 6596/1297/1/012005

Department of the Malaysian Meteorological (2010). *Seismic and Tsunami Observations of Malaysia Seismology Course) Malaysian Meteorological Department*. Malaysian Government. https://iisee.kenken.go.jp/net/shiva/update/Malaysia.pdf

Dubey, R. K. (2015). On the stability of Forward in Time and Centred in Space (FTCS) scheme for scalar hyperbolic equation. *arXiv (Cornell University)*. https://doi.org/10.48550/arxiv.1503.02629

Eze, C. L., Uko, D. E., Gobo, A. E., Sigalo, F. B., & Israel-Cookey, C. (2009). Mathematical Modelling of Tsunami Propagation. *J. Appl. Sci. Environ. Manage*, *13*(3), 9–12.

Hager, W. H., Castro-Orgaz, O., & Hutter, K. (2019). Correspondence between de Saint- Venant and Boussinesq. 1: Birth of the Shallow–Water Equations. *Comptes Rendus - Mecanique*, *347*(9), 632–662. https://doi.org/10.1016/j.crme.2019.08.004

Ismail, A.I.N., Karim, F., Roy, G.D. and Meah, M.A. (2007) Numerical Modeling of Tsunami via Method of Lines. World Academy of Science, Engineering and Technology, 1, 341-349.

Izadi, M., Yadav, S. K., & Methi, G. (2024). Two efficient numerical techniques for solutions of fractional shallow water equation. *Partial Differential Equations in Applied Mathematics*, *9*, 100619. https://doi.org/10.1016/j.padiff.2024.100619

Kamal, A. (2023, December 26). Boxing Day Tsunami: Malaysia ditches Tsunami buoys for enhanced tide gauges [NSTTV]. *NST Online*. https://www.nst.com.my/news/nation/2023/12/994137/boxing-day-tsunami-malaysia-ditches-tsunami-buoys-enhanced-tide-gauges#google_vignette

Karunakar, P., & Chakraverty, S. (2017). Comparison of solutions of linear and non-linear shallow water wave equations using homotopy perturbation method. *International Journal of Numerical Methods for Heat and Fluid Flow*, *27*(9), 2015–2029. https://doi.org/10.1108/HFF-09-2016-0329

Karunakar, P., & Chakraverty, S. (2018). Solving shallow water equations with crisp and uncertain initial conditions. *International Journal of Numerical Methods for Heat and Fluid Flow*, *28*(12), 2801–2815. https://doi.org/10.1108/HFF-09-2017-0351

Karunakar, P., & Chakraverty, S. (2021). Homotopy perturbation method for predicting tsunami wave propagation with crisp and uncertain parameters. *International Journal of Numerical Methods for Heat and Fluid Flow*, *31*(1), 92–105. https://doi.org/10.1108/HFF-11-2019-0861

Kawahara, M., Kodama, T., & Kinoshita, M. (1988). Finite element method for tsunami wave propagation analysis considering the open boundary condition. *Computers & Mathematics With Applications*, *16*(1–2), 139–152. https://doi.org/10.1016/0898-1221(88)90028-4

Kumar, S., Odibat, Z., Aldhaifallah, M., & Nisar, K. S. (2020). A comparison study of two modified analytical approach for the solution of nonlinear fractional shallow water equations in fluid flow. *AIMS Mathematics*, *5*(4), 3035–3055. https://doi.org/10.3934/m ath.2020197

Laman web rasmi Jabatan Meteorologi Malaysia. (n.d.). https://www.met.gov.my/en/pendidikan/tsunami/

Liu, Y., Shi, Y., Yuen, D. A., Sevre, E. O. D., Yuan, X., & Xing, H. L. (2009). Comparison of linear and nonlinear shallow wave water equations applied to tsunami waves over the China Sea. *Acta Geotechnica*, *4*(2), 129–137. https://doi.org/10.1007/s11440-008-0073-0

Moon, W. C., Sidek, L. M., Lau, T. L., Puay, H. T., Majid, T. A., Wahab, A. K. A., & Teo, F. Y. (2022). A shared vision on the 2004 Indian Ocean tsunami in Malaysia: hazard Assessments, Post-Disaster Measures and research. *Journal of Marine Science and Engineering*, *10*(8), 1088. https://doi.org/10.3390/jmse10081088

Mousa, M. M. (2018). Efficient numerical scheme based on the method of lines for the shallow water equations. *Journal of Ocean Engineering and Science*, *3*(4), 303–309. https://doi.org/10.1016/j.joes.2018.10.006

Park, J. (2007). *Numerical Simulation of Wave Propagation Using the Shallow Water Equations One dimensional Shallow Water Equations*. *Harvey Mudd College*.

Supian, N. M., & Rusli, N. (2018). Approximating one-dimensional coupled shallow- water equations for predicting tsunami wave propagation using finite difference method. *AIP Conference Proceedings*, *2013*, 020024. https://doi.org/10.1063/1.5054223

Tandel, P., Patel, H., & Patel, T. (2021). Tsunami wave propagation model: A fractional approach. *Journal of Ocean Engineering and Science*, *7*(6), 509–520. https://doi.org/10.1016/j.joes.2021.10.004

The complete Sabah, Malaysia earthquake report (2025). Earthquakelist.org. https://earthquakelist.org/malaysia/sabah/#all-latest-earthquakes

Varsoliwala, A. C., & Singh, T. R. (2021). Mathematical modeling of tsunami wave propagation at mid ocean and its amplification and run-up on shore. *Journal of Ocean Engineering and Science*, *6*(4), 367–375. https://doi.org/10.1016/j.joes.2021.03.003

Yamamoto, K., Hashentuya, N., & Watanabe, M. (2013). Finite element analysis of tsunami propagation. *Advances in Intelligent Systems Research/Advances in Intelligent Systems Research*. https://doi.org/10.2991/wiet-13.2013.4