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RELATION BETWEEN PI, PHI, THE GREAT PYRAMID OF GIZA AND SQUARE BASED PYRAMID

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ABSTRACT

The mathematical constant Pi (π) appears to have been programmed into The Great Pyramid of Giza with a value of 3.1419. It took around 2000 years for this value to be rediscovered with such precision. To build the Giza Pyramid, Pi and the Golden Ratio Phi were employed to determine its proportions. Since the Great Pyramid is used to find the Pi and Phi values, the other four similar base square pyramids with different dimensions are chosen to identify the criteria needed in investigating the value of Pi and Phi. The Red Pyramid, Pyramid of Cestius, Louvre Pyramid and Luxor Las Vegas Pyramid are used to find and to compare the values of Pi and Phi with the original values. This study identifies whether square base pyramids can be used to find Pi and Phi values. This study investigates the approximate values of Pi and Phi by using any square based pyramid with the Great Pyramid of Giza as a point of reference. The features and the relationship of Pi and Phi of the Great Pyramid of Giza and other square based pyramids are explored. From the dimension of the pyramid, one of the circle formulas is utilized to calculate Pi's value and the value of Phi is calculated by using the Pythagorean Theorem. Then, the ratio is calculated to be a reference on the measurement of the pyramid to get a value of Pi and Phi with the lesser error compared to the error that is calculated based on the Great Pyramid of Giza. The result demonstrates that the dimensions of Great Giza are the most suitable to have stable and sturdy pyramid. The stability or the relative error of a pyramid can be improved by modifying the dimensions and the values of Pi and Phi of the pyramid. According to our findings, not all square based pyramids can be used to obtain the value of Pi and Phi as the main dimensions of the pyramids are not suitable such as base length, height, and the angle of slanted faces. Further studies in physics and architecture are required to build a stable pyramid, in the real world, that is able to stand on specified geographical structure and also able to withstand the weather of the region.

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Keywords *Great Pyramid of Giza, Pi, Phi, Square Based Pyramids.*

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1. Introduction

The Giza Pyramids are the world's largest and most famous pyramid formations. It was built on a rocky plateau on the west bank of the Nile in Giza to honor several of Egypt's Fourth Dynasty pharaohs during the Old Kingdom period. The Old Kingdom, which lasted from 2686 to 2181 BC, was Egypt's first great era of civilization. The Great Pyramid of Giza (also known as Cheops Pyramid or Pyramid of Khufu) in Cairo, Egypt, was completed in 2560 BC, more than 4500 years ago. It was the world's highest man-made building for over 3800 years, is one of the Seven Wonders of the Ancient World. The Pyramids of Giza, Saqqara, Dahshuor, and Abu Rawash, as well as the ancient structures of the Memphis region, were declared as a UNESCO World Heritage Site in 1979. The largest pyramid stands 481 feet (147 meters) above the plateau and its stone masses, believed to number around 2.3 million, weigh between 2.5 and 15 tons on average. For the different tiers, The Great Pyramid builders used stones of various sizes and heights. For many layers, stones of various sizes and heights were used. The weight of an average pyramid stone block is 2.5 tons, according to most Egyptologists. 8000 tons of granite were shipped in from Aswan, which is over 800 kilometers away. The pyramid's greatest granite stones, lying above the "King's" chamber, weigh between 25 and 80 tons each. The great pyramid was built with around 500,000 tons of mortar. Many of the Great Pyramid's casing stones and inner chamber blocks were fitted together with great accuracy.

It has caught the attention of archaeologists as well as numerologists who may make erroneous claims throughout history due to the vastness of the ancient building, its architecture, and historical significance of Great Pyramid. The mathematical constant Pi (represented by the Greek symbol) appears to have been programmed into the Great Pyramid with a value of 3.1419. It took around 2000 years for this value to be rediscovered with such precision. So, how did the Egyptians come up with is still questionable. In fact, the Egyptians may have mistakenly, but not accidently, incorporated into their Great Pyramid with "great" precision without even realizing it! Scientific logic and high school arithmetic are used to explain the phenomenon, rather than numerology or dubious pseudo-scientific Pyramidology.

After the Great Pyramids of Khufu and Khafre at Giza, the Red Pyramid is Egypt's third largest pyramid. Pharaoh Sneferu (2575-2551 BC), Khufu's father, constructed the Red Pyramid. The construction took ten years and seven months, according to ancient graffiti. The Red Pyramid is Sneferu's second pyramid (the first one is Bent Pyramid), and it is the first real smooth-sided pyramid to be built. It stands 345 feet tall with a 43° angle. This is the corrected angle that was employed at the older Bent Pyramid when the original angle was found to be too steep.

The dimensions of the Louvre Pyramid in Paris, France are consciously identical to those of the Great Pyramid, although there is a significant scale difference. The late I.M. Pei, a Chinese-born architect and founder of Pei Cobb Freed & Partners, built the glass-and-metal structure that lies atop the Louvre's underground, yet light-filled, the foyer that connects the museum's three pavilions which are Denon, Richelieu, and Sully and its measurements constitute a tiny Great Pyramid of Giza with a square base and apex of 71 feet (Whitney, C., Ph.D., 2019). They built the Great Pyramid of Giza in a pyramid shape because they wanted a towering, monumental, and a large edifice, and the simplest way to achieve these attributes was to build it in a pyramid shape. The Louvre Pyramid, on the other hand, expresses itself in symbolic, historical, and metaphorical ways by making several connections to the old Giza Pyramid. It is understandable given the museum's extensive collection of ancient Egyptian artefacts.

There are other pyramids that are less well-known including the Pyramid of Cestius which can be found in the Italian city of Rome. The Pyramid of Rome is a historic structure modelled after an Egyptian pyramid that was built to serve as a tomb. Although dwarfed by the Egyptian pyramids in terms of size, the monument is nonetheless quite large with a square base of 29.5 meters on all sides and a height of 36.4 meters. A rectangular hole with a barrel vault serves as a burial chamber on the interior. It measures around 20 feet long, 13 feet broad, and nearly 16 feet tall. When compared to its Egyptian rivals, the Pyramid of Cestius is noticeably thinner and more pointed in terms of structural bearing. Although it is unclear why there is a difference, there are some theories. One theory is that the use of cement allowed for a taller structure, and that the design was chosen to give the pyramid as much height as feasible. Others claim this is due to the pyramid's inspiration coming from those in Jebel Barkal, presently in Sudan, which are significantly steeper (Marta, 2020).

Another pyramid that resembles The Great Pyramid of Giza, Luxor Las Vegas is a black pyramid hotel and casino complex on the Las Vegas Strip's southern end. It has been regarded as a great example of 1990's postmodernist architecture since its opening in October 1993. Outside the pyramid, a 110-foot-tall sculpture of the Great Sphinx of Giza completes the resort's Ancient Egyptian theme. Circus Enterprises designed and built Luxor, which cost $375 million and took only 18 months to finish. It was built with reinforced concrete and topped with a steel skeleton to form the pyramid. The Luxor has the world's largest atrium, with 30-storeys and 120000 square feet of floor space with 2,000 slot machines and 87 table games. Luxor Sky Beam, with a brightness of 42.3 billion candela, is the world's brightest light beam. MGM Resorts International now owns the hotel, which is the third largest in Las Vegas and the sixth largest in the world.

There has been a lot of speculations regarding how the Great Pyramid was built and what rule was employed to determine its proportions. For this last point, two theories have been proposed: one claims that Pi was employed to determine the proportion base-height, while the other claims that the Golden Ratio Phi was used. The intriguing thing is that both ideas fit the Great Pyramid proportions to within two or three decimal places, which is excellent enough for building. This fact is used to demonstrate that many algebraic relationships between Pi and the Golden Ratio can be set with the same precision in this study. The Great Pyramid geometry can also be used to obtain exact transcendental Pi-Phi relationships. It is further demonstrated that these correlations between Pi and Phi are unique to the geometry of the square pyramid.

1.1 Giza Pyramid Dimensions

The Great Pyramid's cross-section yields a right-angled triangle, with the vertical from the peak, a half base, and the slant side (Bartlett, 2014). Furthermore, using the Great Pyramid's measurements, we can see if this triangle follows golden ratio proportions. The exact dimensions of the Great Pyramid are crucial to theories concerning its proportions. There are two main hypotheses that have been advocated. One claims that Pi was employed to determine the proportion base-height, while the other claims that the Golden Ratio Phi was used (Miguel, Ramirez & Galena, 2018). It is further demonstrated that these correlations between Pi and Phi are unique to the geometry of the square pyramid.

In addition, the 70-70-99 Pythagorean Isosceles Triplets design appears to have some validity since it explains how Pi was precisely integrated into the Great Pyramid without the Egyptians having to grasp the value of Pi or the Pythagorean Theorem (Yochim, B., 2015). It offers a possible explanation for why the Great Pyramid has indented sides.

1.2 Pi Value Theorem

Many approximate values of Pi may be found in the early history of mathematics. The most frequent way is to build a many-sided polygon and use the perimeter and diameter to get Pi's estimate (Allain, 2016). Pi (π) is a mathematical constant which value is the circumference to diameter the ratio of any circle. This is the same as the ratio of a circle's area to its radius squared (Iyer & Srivastava, 2010). According to Verner (2003), the value of Pi is included in the proportions of the Great Pyramid of Giza in the form of 3+1/7 = 22/7. The ancient Babylonians were the first to estimate the area of a circle by multiplying three times the radius's square, yielding Pi = 3. Egyptians calculated the value of Pi by matching the area of an 8-sided square with the area of a circle with a diameter of 9. This provided the approximate figure of 3.1605, which was a more accurate representation of Pi (Allen & Donald, 2017). Some researchers believe this is the product of a purposeful design proportion, while others believe the ancient Egyptians had no notion of Pi and that the observed pyramid design was solely dependent on the slope choice.

Archimedes' approach was one of the most prominent attempts to approximate the value of Pi (Dhar, 2020). Archimedes, who is a Greek mathematician, deduced that the value of Pi must fall between the two extremes. He calculated the perimeters of two polygons which one encircled within the circle and the other within which the circle was circumscribed. He established that the value of Pi was between 22/7 and 223/71 by using this method (Groleau & Rick, 2003).

1.3 The Golden Ratio (Phi)

The Great Pyramid's exterior measurements revealed a mastery of the Golden Ratio (Phi). We will see now that this value was also integrated into the internal geometry. Professor Petrie (1883) established that the horizontal length of the Grand Gallery was 1688.9 B" at the floor level and 1648.45 B" at the roof level, a difference of 40.45 B". In Egypt, the "Sacred Cubit" was 25 Pyramid Inches. For example, each base side of the Pyramid had a side length of 365.243 Sacred Cubits, which when multiplied by four equals 1460.972 Sacred Cubits. 36,524.3 Pyramid Inches is calculated by multiplying this value by 25. The fascinating phenomenon is that the measurement of 40.45/25 = 1.618 is an accurate approximation of the Golden Ratio's value.

It is also clear that the total amount of inward slope was developed to learn about the Golden Ratio. The measurements recorded by Professor Petrie were taken in British Inches, which are 0.11 percent smaller than the Pyramid Inch. Further research into the Golden Ratio's application leads us to the Entrance Passage, precisely the place where the Ascending Passage begins. Professor Flinders Petrie (1883) measured the commencement of the entry at the face of the Great Pyramid's northern face, and the Ascending Passage roof starts vertically 1110.9 British Inches on the sloping floor of the Entrance Passage. The external measurement of the Great Pyramid's height appears to be in a Phi relationship with the intersection of the Ascending Passage and the Entrance Passage.

The mathematical relationship that explains the golden ratio is represented by the three sides of a golden ratio pyramid. The base of this triangle, known as a Kepler triangle, is 1 and the hypotenuse is Phi. The square root of Phi is the height of this right triangle. This triangle demonstrates one of the golden ratio's distinctive properties ($1+Phi = Phi^2$) using the Pythagorean theorem ($a^2+b^2=c^2$). According to the Pythagoras theorem, the hypotenuse of a triangle with a base of 220c and a height of 280c is equal to the square root of the sum of 48400 and 78400, which is 356.0898763. As the base equals 55 times 4 and the hypotenuse is 89, Flinders Petrie discovered that the triangle's dimensions were multiples of 4 cubits. It is possible the fact that the numbers 55 and 89 are the Fibonacci sequence's 10th and 11th terms, respectively, with Phi as the limit of its subsequent quotients, may be purely coincidental. As a result, the secant of the inclination angle of the pyramid face on its base is 89/55 = 1.61818..., which is significantly more precise. Pi = 22/7 and Phi = 89/55 are now possible Egyptian constants.

The Phi findings are created automatically if any right-angle triangle with dimensions a, b, and c that fulfil c/b = b/a. During the three millennia of Egyptian civilization, any engineer must have considered the conditions in which the proportionality requirement is satisfied. The Pythagoras theorem dictates that the sides must fulfill a:b:c = 1:root(Phi):Phi.

2.0 Methodology

This project aims to discover the relationship between Pi, Phi, The Great Pyramid of Giza, and square based pyramid. Based on research done by Miguel. R (2018), exact transcendental Pi-Phi relationships can be also obtained from the Great Pyramid geometry. The procedure of finding the relationship is described as follows.

Step 1: Calculate the dimension of the pyramid.

This section presents a calculation of pyramid dimension for The Great Pyramid of Giza, Red Pyramid, Pyramid of Cestius, Louvre Pyramid and Luxor Las Vegas Pyramid. All pyramids use the same formula as we are searching for the same values for Pi and Phi.

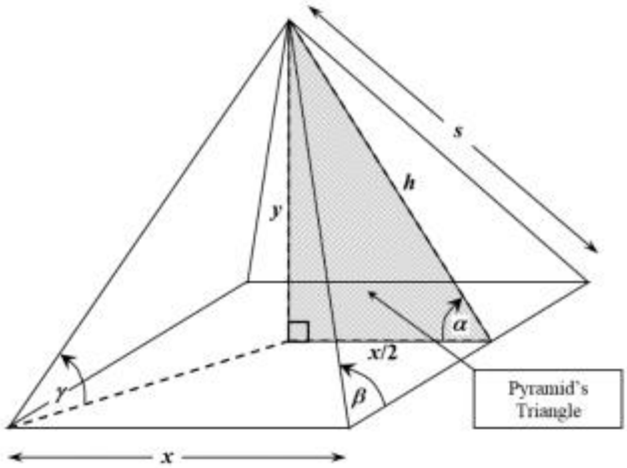

Figure 1: Great Pyramid variables

To retrieve the value of Pi and Phi, these are the items that we need:

a) Original length of the side of the square base, x in meter

b) Average length of the side of the square base, $\bar{x}$ in meter, where n is the number of sides of the pyramid

$$\bar{x} = \frac{\sum x}{n}$$

c) Original height of the pyramid, y in meter

d) Angle of pyramid,

$$a = \tan^{-1}\left(\frac{y}{\left(\frac{\bar{x}}{2}\right)}\right)$$

e) Average height of the triangular faces, h in meter

$$h = \sqrt{y^2 + \left(\frac{\bar{x}}{2}\right)^2}$$

f) Estimated average length of the pyramid's edges, s in meter

$$s = \sqrt{\left(\frac{\sqrt{2\bar{x}^2}}{2}\right)^2 + y^2}$$

g) Estimated angle of the triangular faces, β

$$\beta = \tan^{-1}\left(\frac{h}{\left(\frac{\bar{x}}{2}\right)}\right)$$

h) Estimated angle of the edges, γ

$$\gamma = \tan^{-1}\left(\frac{y}{\left(\frac{\sqrt{2\bar{x}^2}}{2}\right)}\right)$$

Step 2: Calculate Pi value based on the dimension of the pyramid.

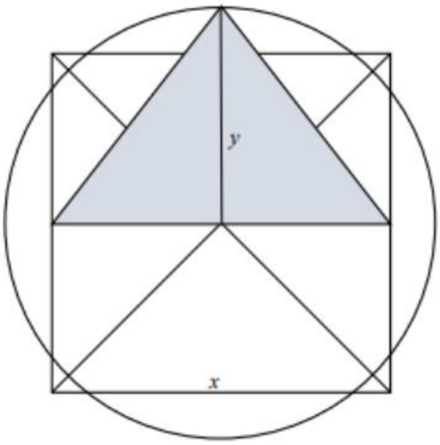

Figure 2: The perimeter of the Pyramid's base is equal to the length of a circumference generated with the Pyramid's height

We utilized one of the circle formulas to calculate Pi's value, which is the circumference of the circle as the perimeter of the pyramid of the base equals the circumference with radius equal to the pyramid's height.

The formula of the circumference, C:

$$C = 2\pi r$$

since $r = y$

$$C = 2\pi y$$

Then, the value of Pi calculated.

$$4x \approx 2\pi y$$

$$\pi \approx \frac{2x}{y}$$

The error of estimated Pi value is calculated to check whether the result of calculation agrees with the theoretical prediction or results from other calculations. The error is calculated to find the accuracy of the calculation.

$$E_\pi = \frac{estimated\ Pi\ value - \pi}{\pi} \times 100\%$$

Step 3: Calculate Phi value based on the dimension of the pyramid.

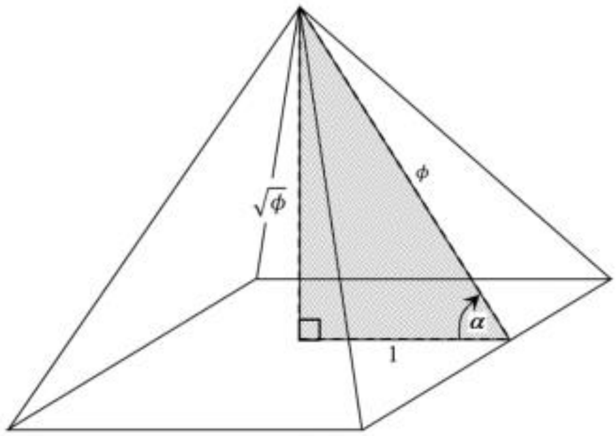

Figure 3: The square of the ratio of the Great Pyramid's height and half of its base is equal to the Golden Ratio

In addition to the relationships of the pyramid's geometry to Phi and Pi, it is also possible that the pyramid was constructed using a completely different approach that simply produced the phi relationship. The writings of Herodotus in Gnomon, From Pharos to Factals (Gazale, 1999) make a vague and debated reference to a relationship between the area of the surface of the face of the pyramid to that of the area of a square formed by its height.

$$y^2 = \frac{xh}{2}$$

Then, the value of Phi calculated by using the Pythagorean Theorem, $c = \sqrt{a^2 + b^2}$.

$$y^2 = \frac{xh}{2}$$

$$y^2 = \frac{x}{2}\sqrt{\left(\frac{x}{2}\right)^2 + y^2}$$

$$\frac{y^4}{\left(\frac{x}{2}\right)^2} = \left(\frac{x}{2}\right)^2 + y^2$$

$$\frac{y^4}{\left(\frac{x}{2}\right)^4} = 1 + \frac{y^2}{\left(\frac{x}{2}\right)^2}$$

$$\left(\frac{2y}{x}\right)^4 = \left(\frac{2y}{x}\right)^2 + 1$$

Let $\varphi = \left(\frac{2y}{x}\right)^2$,

$$\varphi^2 = \varphi + 1$$

Solving the equation,

$$\varphi = \left(\frac{2y}{x}\right)^2$$

The error of estimated Phi value is calculated, the same step in finding the error for estimated Pi value,

$$E_\varphi = \frac{estimated\ Phi\ value - \ \varphi}{\varphi} \times 100\%$$

Step 4: Calculating the ratio of the pyramid dimension, ratio average length to average height.

Ratio is calculated to be a reference on the measurement of the pyramid to get a value of Pi and Phi with the lesser error compared to the error that is calculated based on the Great Pyramid of Giza. Two ratios were calculated by first assigning the average length as 1 and the second ratio is assigning the average height by 1. We need to calculate two ratios because we want to discover the shape of a pyramid either it is short and wide or tall and narrow.

a) First ratio:

$$\frac{average\ length}{average\ length} : \frac{average\ height}{average\ length}$$

$$1 : \frac{average\ height}{average\ length}$$

b) Second ratio:

$$\frac{average\ length}{average\ height} : \frac{average\ height}{average\ height}$$

$$\frac{average\ length}{average\ height} : 1$$

3.0 Implementation

Step 1: Calculation of the dimensions of the pyramids.

The average length and height of all pyramids are found in articles and journals. Then, the value is calculated in Microsoft Excel

Table 1: Dimensions of square-based pyramid

Items	Pyramids				
	The Great Pyramid of Giza, Egypt	Luxor Pyramid, Las Vegas	Red Pyramid, Egypt	Cestius Pyramid, Italy	Louvre Pyramid, France
Length, x	230.36 m	196.9 m	219.28 m	29.6 m	34 m
Average length, $\bar{x}$	230.36 m	196.9 m	219.28 m	29.6 m	34 m
Height, y	146.72 m	106.68 m	104.42 m	37 m	21.6 m
Angle of pyramid, α	51.8669	47.2975	43.6031	68.1986	51.7959
Average	186.5293 m	145.1655 m	151.4083 m	39.85022 m	27.48745 m

height of triangular faces, h					
Estimated average length of the pyramid's edges, s	219.2251 m	175.4008 m	186.9369 m	42.50976 m	32.31965 m
Estimated angle of the triangular faces, β	58.30508	55.85522	54.09039	69.62547	58.2647
Estimated angle of the edges, γ	42.01048	37.45998	33.95795	60.50379	41.93784

Step 2: Calculation of Pi value based on the dimension of the pyramid.

Table 2: Pi value based on pyramid dimensions

Items	Pyramids				
	The Great Pyramid of Giza, Egypt	Luxor Pyramid, Las Vegas	Red Pyramid, Egypt	Cestius Pyramid, Italy	Louvre Pyramid, France
Estimated Pi value	3.140131	3.691414	4.199962	1.6	3.148148

Error, E_π	0.04653%	17.50134%	33.68893%	49.07042%	0.208668%

Step 3: Calculation of Phi value based on the dimension of the pyramid.

Table 3: Phi value based on pyramid dimensions

Items	Pyramids				
	The Great Pyramid of Giza, Egypt	Luxor Pyramid, Las Vegas	Red Pyramid, Egypt	Cestius Pyramid, Italy	Louvre Pyramid, France
Estimated Phi value	1.622649	1.17418	0.907046	6.25	1.614394
Error, E_φ	0.285448%	27.43152%	43.94133%	286.2722%	0.224689%

Step 4: Calculation of the ratio of the pyramid dimension, ratio average length to average height.

a) First ratio $\left(1 : \frac{average\ height}{average\ length}\right)$

Table 4: First ratio of pyramid dimension

Items	Ratio	Dimension of Pyramid of Giza by using first ratio
Length, x	1	230.36

Average length, $\bar{x}$	1	230.36
Height, y	0.63661	146.64
Angle of pyramid, α	51.8535	51.8535
Average height of triangular faces, h	0.809489	186.4739
Estimated average length of the pyramid's edges, s	0.951458	219.1779
Estimated angle of the triangular faces, β	58.29747	58.29747
Estimated angle of the edges, γ	41.99679	41.99679
Estimated Pi value	3.141614	
Error, E_π	0.001535%	
Estimated Phi value	1.621089	
Error, E_φ	0.189067%	

b) Second ratio $\left(\frac{average\ length}{average\ height} : 1\right)$

Table 5: Second ratio of pyramid dimension

Items	Ratio	Dimension of Pyramid of Giza by using second ratio
Length, x	1.5708	230.4678
Average length, $\bar{x}$	1.5708	230.4678
Height, y	1	146.72
Angle of pyramid, α	51.8539	51.8539
Average height of triangular faces, h	1.271555	186.5626
Estimated average length of the pyramid's edges, s	1.494559	219.2817
Estimated angle of the triangular faces, β	58.29767	58.29767
Estimated angle of the edges, γ	41.99716	41.99715
Estimated Pi value	3.1416	
Error, E_π	0.000234%	
Estimated Phi value	1.621131	
Error, E_φ	0.191675%	

4.0 Result and discussion

This work demonstrated that, of all the possible regular right pyramids, only the geometry of the square based pyramid algebraically relates two of the most amazing mathematical constants with two-three decimal precision. To find the exact values of Pi and Phi, the pyramid must be a right-angled triangle with a square-based pyramid where all the faces of the triangle are isosceles. Isosceles triangle is a triangle that has exactly two sides of equal length. Particularly, all the faces would be slanted at the same angle. The other characteristics that must be considered to find the exact values of Pi and Phi are the pyramid's base length and height. Pyramid's base length, height, and angle of slanted faces are all related to determine the centre of gravity of a pyramid. When the centre of gravity of a pyramid is lower, the pyramid is more stable. These criteria are significant in determining the stability and strength of a pyramid.

Through these findings, we can observe that the dimensions of Great Giza are the most suitable to have stable and sturdy pyramids. The estimated Pi value for Great Giza is 3.140131 which is the closest approximation to the exact Pi value of 3.1416 compared to other square-based pyramids. Great Giza has the least error of Pi value of 0.04653% while Cestius Pyramid in Italy has Pi value of 1.6 with the biggest error of 49.07042%. This indicates that the durability of the Cestius Pyramid will not be as strong as the Great Giza. The second and third pyramids that have strong durability are the Louvre Pyramid and Luxor Pyramid which have Pi values of 3.148148 and 3.691414 with the relative errors of 0.208668% and 17.50134% respectively. On the other hand, the Phi value found in Great Giza is 1.622649 with the relative error of 0.285448% which is the second nearest to the exact Phi of 1.61818. The Louvre Pyramid has a more accurate Phi value of 1.614394 with lower relative error of 0.224689% compared to Great Giza while the pyramid that has the most distant Phi value is Cestius Pyramid with 6.25. The stability or the relative error of a pyramid can be improved by modifying the dimensions and the values of Pi and Phi of the pyramid.

Two ratios are calculated as a reference on the measurement of the pyramid to obtain a value of Pi and Phi with less error than the error calculated based on the Great Pyramid of Giza.

Based on the first ratio of $1 : \frac{average\ height}{average\ length}$, the shape of the pyramid will look short but wide as the average length is 1 and the average height is 0.63661. The relative error of Pi and Phi is lower than the relative error that is calculated from the Great Pyramid of Giza. The comparison of the errors is as below:

Table 6: Comparison of relative error of Pi and Phi by first ratio

Error	By using first ratio	By using the Great Pyramid of Giza
Error, E_{π}	0.001535%	0.04653%
Error, E_{φ}	0.189067%	0.285448%

Based on the second ratio of $\frac{average\ length}{average\ height}:1$, the shape of the pyramid will look narrow but high as the average length is 1.5708 and the average height is 1. The relative error of Pi and Phi is lower than the relative error that is calculated from the Great Pyramid of Giza. The comparison of the errors is as below:

Table 7: Comparison of relative error of Pi and Phi by second ratio

Error	By using second ratio	By using the Great Pyramid of Giza
Error, E_{π}	0.000234%	0.04653%
Error, E_{φ}	0.191675%	0.285448%

5.0 Conclusion and Recommendation

The Great Pyramid of Giza built in 2490 before the century (B.C.) is well known as the founder of the Pi and Phi value. The pyramid dimension and characteristics of the pyramid is important as it will affect the shape, size, and centre of gravity of the pyramid as well as the estimated value of Pi and Phi. The comparison of five square based pyramids has been done to discover the value of estimated Pi and Phi. This study used simple mathematical equations to find the precise Pi, Phi value and the right pyramid dimensions to achieve Pi and Phi value with the smallest error. The result of this study can hopefully help the academicians, especially teachers, to reflect on their skills on how to relate mathematics with nature to make students understand and love mathematics.

According to our findings, not all square based pyramids can be used to obtain the value of Pi and Phi as the main dimensions of the pyramids are not suitable such as base length, height, and the angle of slanted faces. Some of the pyramids' slanted faces angle is way too steep,

68° and some are sloping, 43°, which may lead to unstable pyramids and inaccurate Pi and Phi values. It can effectively assist teachers, as the eye opener, to build the replica of the pyramid to expose to the students about the root of Pi and Phi value. In other way, the architects might also be interested in building another pyramid, slightly the same as the Great Pyramid of Giza but with the result of the most accurate Pi and Phi value. This research suggests the second ratio of the pyramid dimension is much suitable to use as it gives the better result of Pi and Phi value with the smallest error compared to the Great Pyramid of Giza. Further studies in physics and architecture are required to build a stable pyramid, in the real world, that is able to stand on specified geographical structure and also able to withstand the weather of the region.

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