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DETERMINING IMPORTANT PARAMETERS IN COVID-19 TRANSMISSION THROUGH THE SENSITIVITY ANALYSIS OF AN SIR MODEL

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ABSTRACT

COVID-19 is a novel coronavirus caused by the SARS-CoV-2 virus that affects humans worldwide. Mathematical models are widely used to understand and predict epidemic outbreaks. In this study, the susceptible-infected-recovered (SIR) model of COVID-19 transmission is formulated to identify significant parameters in controlling the number of infected cases. The constructed SIR model is modified to include quarantine compartment and vaccination parameter. The local sensitivity analysis is used to determine significant parameters in COVID-19 transmission. Since the initial disease transmission is highly related to the basic reproduction number, R_0, the sensitivity indices of the R_0 to every parameter are calculated. The results show that the most sensitive parameter that contributes to the reduction of COVID-19 infected cases is the quarantine rate, followed by the natural mortality rate. The finding revealed that quarantine measure is more effective in controlling the spread of COVID-19 than vaccination. These suggest that the health authorities should keep implementing the quarantine measure for infected cases despite already being vaccinated.

Keywords: *COVID-19, SIR Model, Sensitivity Analysis, Basic Reproduction Number*

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1. Introduction

The disease COVID-19 was identified in December 2019 in Wuhan, China. On 21st December 2021, more than 2.7 million confirmed cases have been reported globally. More than five million deaths and as of 19th December 2021, more than eight million vaccine doses have been confirmed [16]. During the outbreak of the disease, governments around the world have taken serious actions to slow down the spread of the disease. Some of the actions are social distancing, self-quarantine, and total lockdown. From March 2020 until May 2020, the Malaysian government has enforced the Movement Control Order (MCO) to minimize the number of infected people and the number of people going out [14].

Several mathematical models have been developed through many research and evaluate the rapid spread of infectious diseases to limit and prevent transmission through quarantine and other measures [5,11,13,14,17,18]. As the mathematical model, the Susceptible-Infected-Removed (SIR) model often used to model the spread of contagious disease [10]. The model has proven to be useful in analysing infectious diseases and suggesting the best control interventions. In most studies conducted [2,8,10,12], the crucial concept in sensitivity analysis is to analyze the sensitivity of the parameters to the basic reproduction number, as it indicates the initial disease transmission [7].

Despite the fact that COVID-19 has been around for more than a year, the epidemic still has a tremendous impact on our life. In order to understand better the dynamics of COVID-19 distribution, this study aims to construct a basic modified SIR COVID-19 transmission model with quarantine and vaccination measures. This research is vital to the general population since the contagious disease will keep spreading and evolving. Therefore, proper and adequate control measures are crucial to studying the growth trend of COVID-19 infection in Malaysia through the present mathematical model. These will be conducted through sensitivity analysis using the normalized forward sensitivity index [6,7].

2. Methodology and Implementation

2.1 Source of Data

The data used in this study is obtained via the open website *https://covid-19.moh.gov.my/terkini* by The Ministry of Health Malaysia. The COVID-19 data updated on this website includes the total number of active cases, deaths, and recovered patients. The number of hospitalized infected people who require breathing support is also available on the website.

2.2 Model Formulation

The proposed mathematical model of COVID-19 transmission is developed based on the well-known SIR model (1) introduced by Kermack and McKendrick in 1927 [3]. Table 1 represents the variable/parameter and its description involved in (1).

$$\begin{aligned}\frac{dS}{dt} &= -\beta(S)(I)\\ \frac{dI}{dt} &= \beta(S)(I)-\eta(I)\\ \frac{dR}{dt} &= \eta(I)\end{aligned} \tag{1}$$

Table 1: Variable/Parameter and its Description

Variable/Parameter	Description
$S(t)$	Susceptible population
$I(t)$	Infected population
$R(t)$	Recovered population
β	Transmission coefficient
η	Recovery rate

The SIR model in (1) is modified by including the quarantine measure compartment. Quarantine is one of the most efficient methods for preventing and controlling infectious diseases, particularly when medical treatment cannot be fully used. A new parameter, α , is introduced in the proposed model to represent the vaccination rate. Figure 1 depicts the schematic diagram of the proposed model.

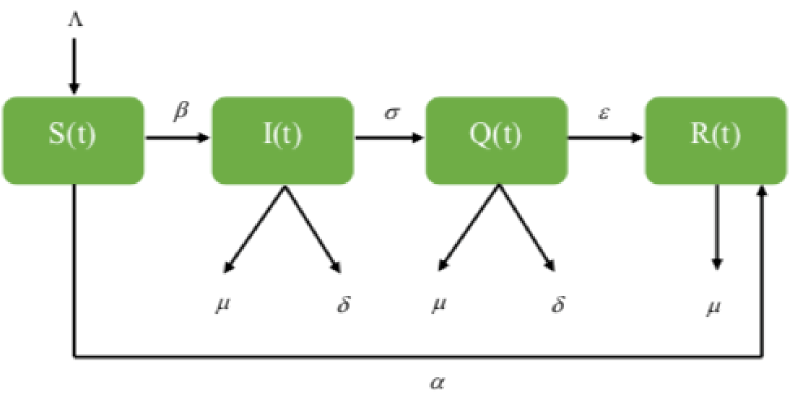

Figure 1: The schematic diagram of the proposed model (2).

The proposed model is given as follows:

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - \mu S - \beta SI - \alpha S \\
\frac{dI}{dt} &= \beta SI - \mu I - \sigma I - \delta I \\
\frac{dQ}{dt} &= \sigma I - \varepsilon Q - \mu Q - \delta Q \\
\frac{dR}{dt} &= \varepsilon Q - \mu R + \alpha S
\end{aligned} \tag{2}$$

Table 2 shows the descriptions of the variable and parameter used in model (2).

Table 2: Variable/Parameter and its Value

Variable/Parameter	Description	Value	Reference
$S(t)$	Susceptible population	-	-
$I(t)$	Infected population	-	-
$Q(t)$	Quarantine population	-	-
$R(t)$	Recovered population	-	-
Λ	Constant birth rate	0.016066	[4]
μ	Natural mortality death rate	0.0062	[9]
β	Transmission rate contact infection	0.036	[2]
α	Vaccination rate	0.0013	[13]
δ	Disease-induced death rate	0.02	[9]
ε	Quarantine healed	0.05	[5]
σ	Quarantine rate	0.28	[15]

2.3 Basic Reproduction Number, R_0

The total number of people who are directly infected by a single diseased person when introduced to a completely susceptible population is known as the basic reproduction number, R_0. Here, the next-generation matrix approach is used to compute the R_0. By utilizing the computed disease-free equilibrium point (DFE), $\bar{x} = \left(\frac{\Lambda}{\mu-\alpha}, 0, 0, \frac{\alpha\Lambda}{\mu(\mu-\alpha)}\right)$, the basic reproduction number obtained is as follows:

$$R_0 = \frac{\Lambda\beta}{(\mu+\sigma+\delta)(\mu+\alpha)} \tag{3}$$

2.4 Stability Analysis of Disease-Free Equilibrium Point (DFE)

Here, the local stability of the disease-free equilibrium point is presented using the following theorem:

Theorem 2.1 Suppose all the eigenvalues of $Df(x)$ have negative real parts. Then, the equilibrium points $x = \bar{x}$ of the autonomous system is locally asymptotically stable, and unstable if at least one of the eigenvalues has a positive real part [3].

The following theorem is the result of the local stability of the DFE of model (2).

Theorem 2.2 The disease-free equilibrium point $\bar{x}$ of model (2) is locally asymptotically stable when $R_0 < 1$, and unstable if $R_0 > 1$.

Proof. The Jacobian Matrix from the model (2) is

$$J = \begin{bmatrix} -\mu-\beta I-\alpha & \beta I & 0 & \alpha \\ \beta S & \beta S-\mu-\sigma-\delta & 0 & 0 \\ 0 & 0 & \varepsilon-\mu-\delta & \varepsilon \\ 0 & 0 & 0 & -\mu \end{bmatrix} \tag{4}$$

The eigenvalues obtained from the Jacobian Matrix (J) is

$$\begin{vmatrix} -\mu \\ -\varepsilon-\mu-\delta \\ -\mu-\alpha \\ -\dfrac{\alpha\delta+\alpha\mu+\alpha\sigma+\delta\mu+\mu^2+\mu\sigma-\beta\Lambda}{\mu+\alpha} \end{vmatrix}$$

$$\lambda_1 = -\mu, \lambda_2 = -\varepsilon-\mu-\delta, \lambda_3 = -\mu-\alpha, \lambda_4 = -\frac{\alpha\delta+\alpha\mu+\alpha\sigma+\delta\mu+\mu^2+\mu\sigma-\beta\Lambda}{\mu+\alpha} \tag{5}$$

The eigenvalues obtained can be transformed into a characteristic equation:

$$a_0\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0 \tag{6}$$

where the roots of the quartic equation are,

$$a_0 = (\mu+\sigma+\delta)(\alpha+\mu)$$
$$a_1 = (\mu+\sigma+\delta)[(\delta+\alpha+3\mu+\varepsilon)(\alpha+\mu)+(1-R_0)]$$
$$a_2 = (\mu+\sigma+\delta)[(\alpha\delta+2\alpha\mu+\alpha\varepsilon+2\delta\mu+3\mu^2+2\mu\varepsilon)(\alpha+\mu)+(\delta+\alpha+3\mu+\varepsilon)(1-R_0)]$$
$$a_3 = (\mu+\sigma+\delta)[(\delta+\alpha+3\mu+\varepsilon)(\alpha+\mu)+(\alpha\delta+2\alpha\mu+\alpha\varepsilon+2\delta\mu+3\mu^2+2\mu\varepsilon)(1-R_0)]$$
$$a_4 = (\mu+\sigma+\delta)(\delta+\alpha+3\mu+\varepsilon)(1-R_0)$$

It shows that all roots of quartic equation (34) are either negative or negative real parts if $R_0 < 1$. Therefore, by the Routh-Hurwitz criterion it is proved that disease-free equilibrium of model (2) is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

2.5 Sensitivity Analysis

Sensitivity analysis allows us to compute the relative change in a state variable when a parameter changes [6,7]. In this study, the sensitivity indices are calculated using the method introduced by Arriola and Hyman, commonly known as the normalized forward sensitivity index [6]. The definition of the method is given in Definition 1.

Definition 1 [6] The normalized forward sensitivity index of variable u, that depends differentiable on a parameter p, is defined by

$$\Upsilon_p^u = \frac{\partial u}{\partial p} \times \frac{p}{u} \tag{7}$$

Based on **Definition 1**, the sensitivity index for the basic reproduction number, R_0 is differentiable to the parameter, p can be defined as follows [5]:

$$\Upsilon_p^{R_0} = \frac{\partial R_0}{\partial p} \times \frac{p}{R_0} \tag{8}$$

3. RESULT AND DISCUSSION

As of January 2022, the total number of COVID-19 cases in Malaysia is recorded to be 98,318. Meanwhile, the number of recovered cases is 72,138. Figure 2 displays both daily cases and daily removals of COVID-19 for Malaysia beginning on January 1, 2022 and continuing until January 30, 2022.

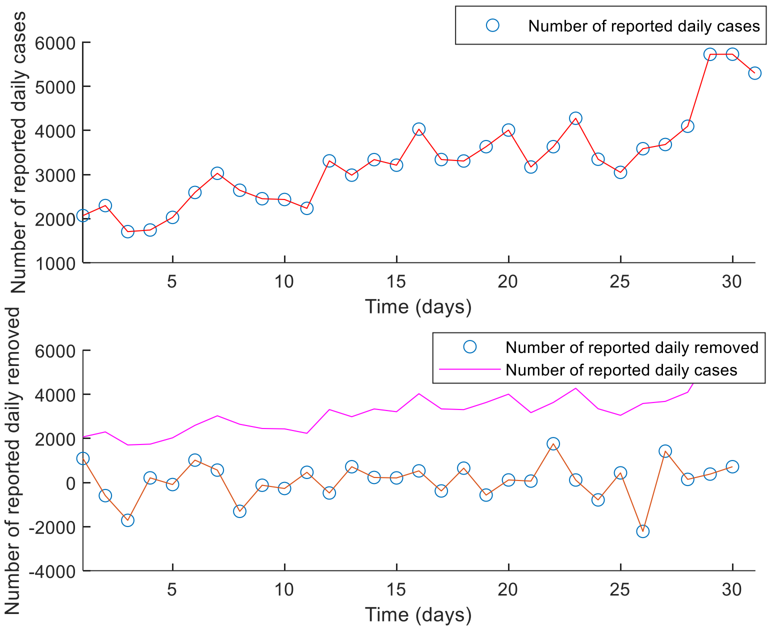

Figure 2: Daily Cases and Daily Removals of the COVID-19 Virus for Malaysia

The numerical results are simulated using MATLAB. In order to solve for model (2), the ODE45 solver is utilized, which implements the fourth Runge-Kutta (RK4) method. The parameter value used in the simulation is given in Table 3.

Table 3: Parameters and its Value

Parameter	Description	Value	Reference
Λ	Constant birth rate	0.016066	[4]
μ	Natural mortality death rate	0.0062	[9]
β	Transmission rate contact infection	0.036	[2]
α	Vaccination rate	0.0013	[13]

δ	Disease-induced death rate	0.02	[9]
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σ	Quarantine rate	0.28	[15]

Figure 3 and Figure 4 represent the figures of the SIR model and the proposed model (2) to ascertain whether the implementation of quarantine measures is successful in preventing the spread of COVID-19. It can be observed that for the SIR model, the quarantine compartment is vanished.

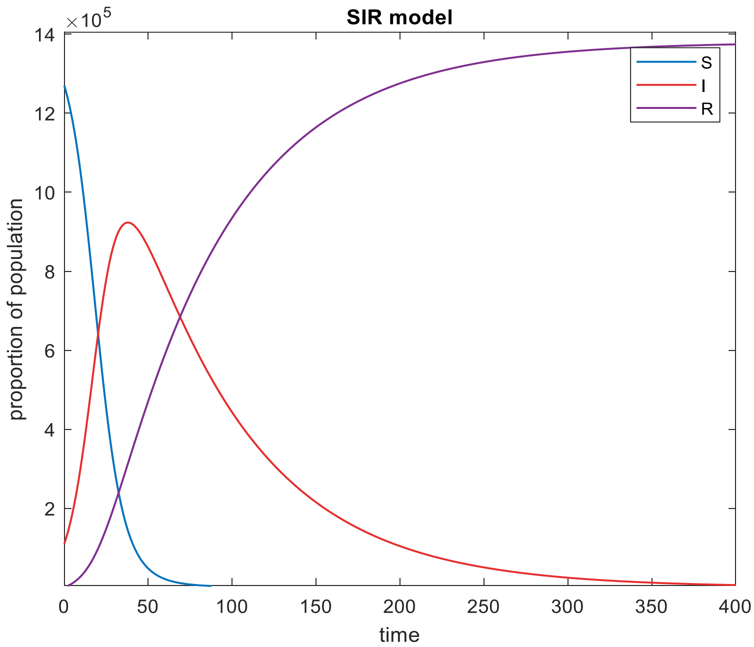

Figure 3: SIR Model

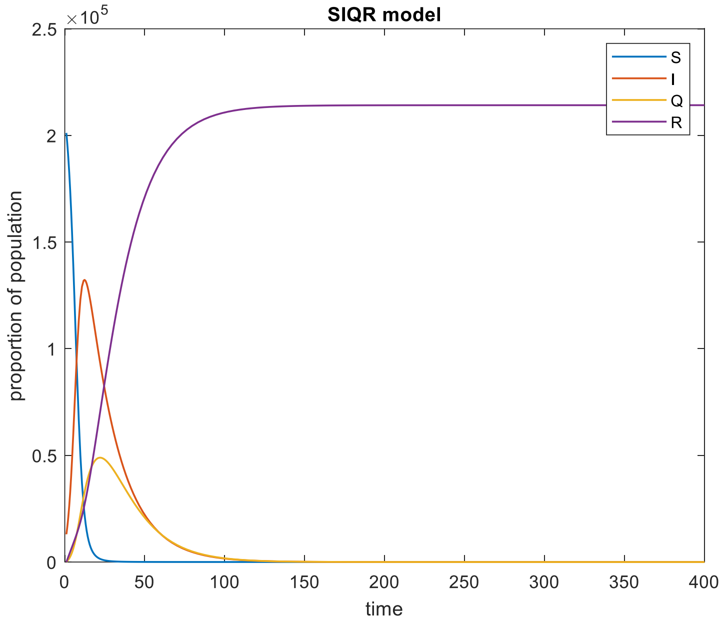

Figure 4: The numerical solution of the proposed model (2).

Since there are no preventative steps done to protect the susceptible population from becoming infected, the time it takes for the infected population to attain zero in Figure 3 is significantly longer than it should. Furthermore, Figure 3 suggests a high rate of transfers from susceptible to infected populations. Figure 4 shows that the time it takes for the infected population to reach zero is significantly lesser than without quarantine control. It is because quarantine procedures play an important role in preventing the spread of the COVID-19 virus. Figure 4 demonstrates that the number of susceptible individuals settled at a constant rate faster than the susceptible individuals in Figure 3. This concludes that it is possible to prevent the majority of the susceptible population from getting COVID-19 and eliminate the disease by enforcing quarantine regulations.

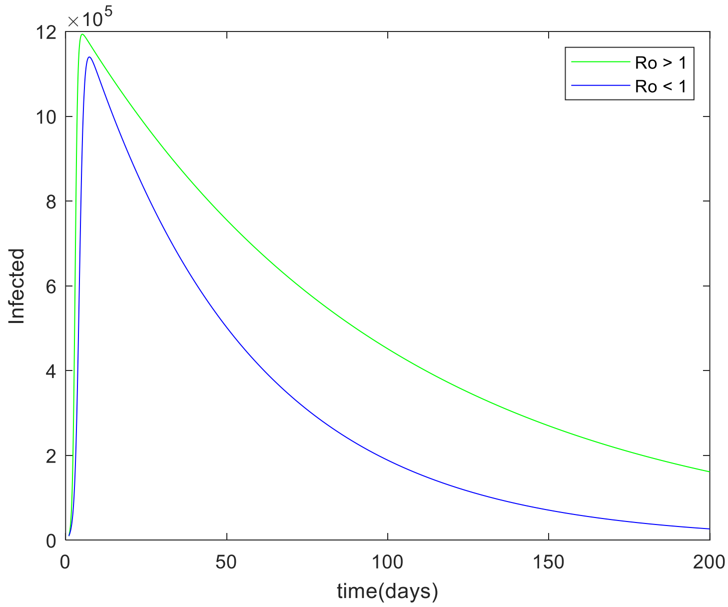

Figure 5: Infected Population from SIQR Model with different R_0 value

Figure 5 presents the graph of the infected population with two values of R_0. As a result, the plot for the infected population shows a more rapid decline when R_0 is less than one. Meanwhile, we can see that when R_0 is greater than one, the infected population stays at a high level for a considerable amount of time. Thus, the smaller the basic reproduction number, the faster the proportion of infected individuals decreases.

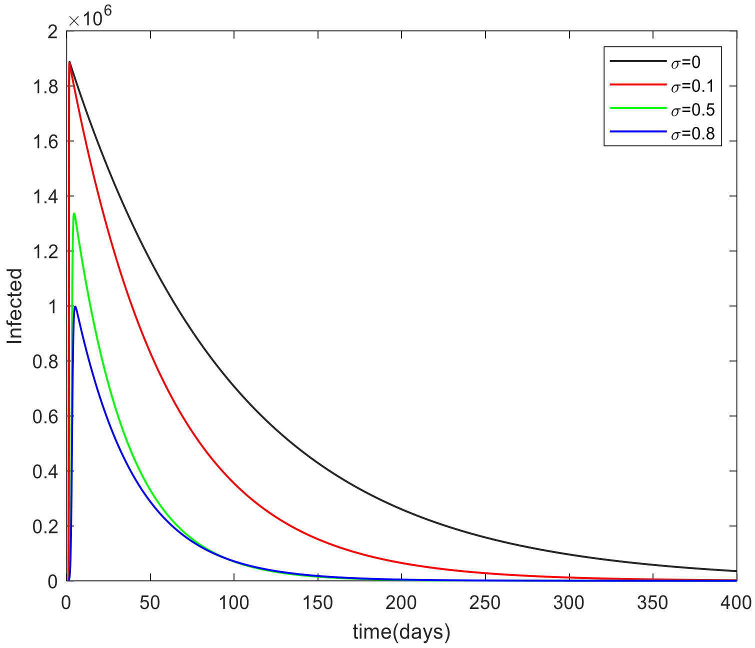

Figure 6: Infected Population with Different Values of σ

Figure 6 illustrates the graph of the infected population when the quarantine rate is chosen to be 0, 0.1, 0.5, and 0.8. This is where we determine whether or not a higher value parameter of quarantine rate will affect the infected population. Based on the information presented in Figure 6, we are able to draw the following conclusion: the number of infected people in the population decreases as the percentage of infected people who are quarantined increases. The higher the quarantine rate, the lesser the infected population. Quarantine measures have been shown to be effective in preventing the infected population from expanding at an exponential rate and also in reducing the number of susceptible people becoming infected.

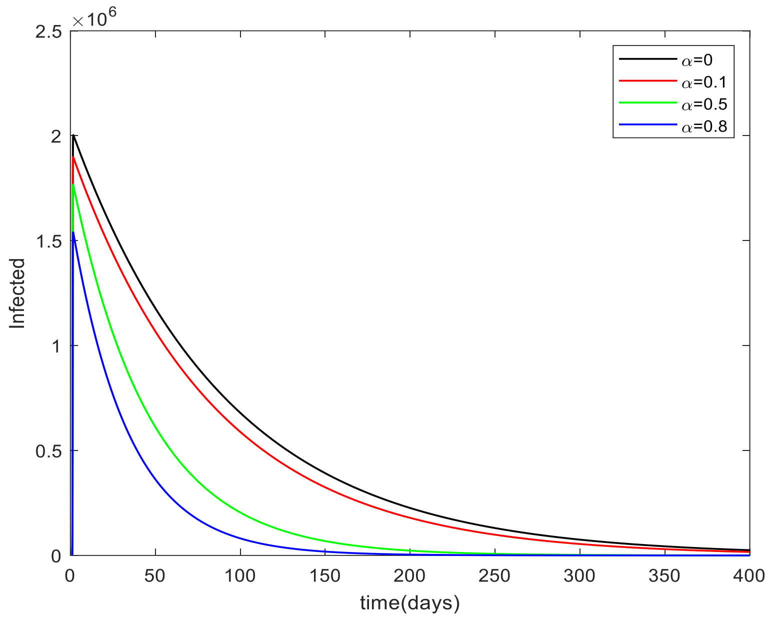

Figure 7: Infected population with different values of α

Figure 7 depicts the infected population with various vaccine rates set to be 0, 0.1, 0.5, and 0.8. From the figure, we can see that the higher the rate of vaccination, the lower the number of infected population cases. Based on the information presented in Figure 7, we came to the conclusion that the number of infected people in the population decreases as the number of vaccinated people increases. The vaccination program leads to effective control measures in preventing the spread of the COVID-19 virus. However, if we compare with the solution obtained in Figure 6, the infected population with a higher quarantine rate (0.8) is more efficient as the number of infected cases is significantly lower.

Table 4: Sensitivity Index

Parameter	Sensitivity Index
Λ	+1.0000
β	+1.0000
μ	-8.4690
α	-0.1733
δ	-0.6531
σ	-9.1444

Table 4 shows the sensitivity index of R0 to every parameter in the model (2). From the table, it is apparent that the most sensitive parameter is quarantine rate, σ and natural mortality rate, μ. Other significant parameters include the diseases induced death rate, δ, and vaccination rate α. The sensitivity indices can be interpreted as, for example, for $\sigma = -9.14438$, if we increase (or decrease) σ by 10%, then the basic reproduction number R_0 will be decreasing (or increasing) by approximately 91%. Similarly, for $\beta = 1.0000$, decreasing (or increase) β by 10% will decrease (or increase) R_0 by 10%. Figure 8, 9, 10, 11, 12 and 13 shows different result of infected population when the parameter increases by 10%. These figured verified the sensitivity index obtain in Table 4.

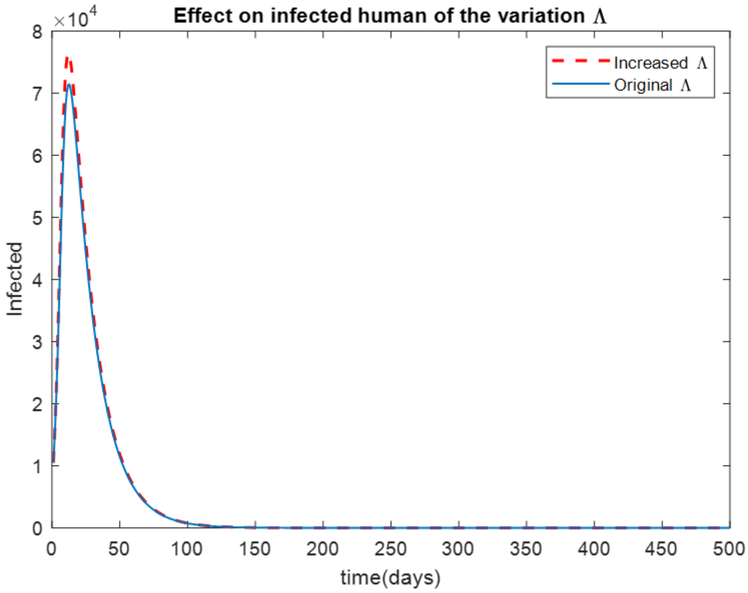

Figure 8: Infected human population with the original value of Λ and 10% increased

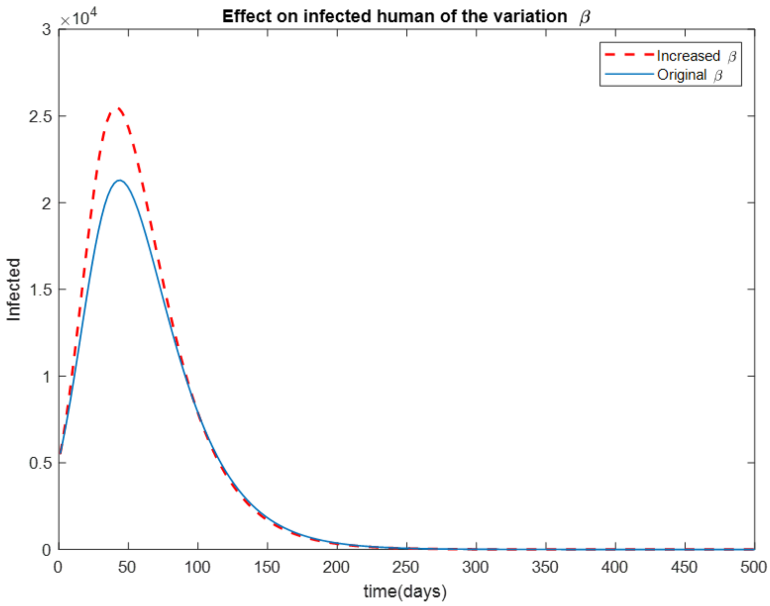

Figure 9: Infected human population with the original value of β and 10% increased

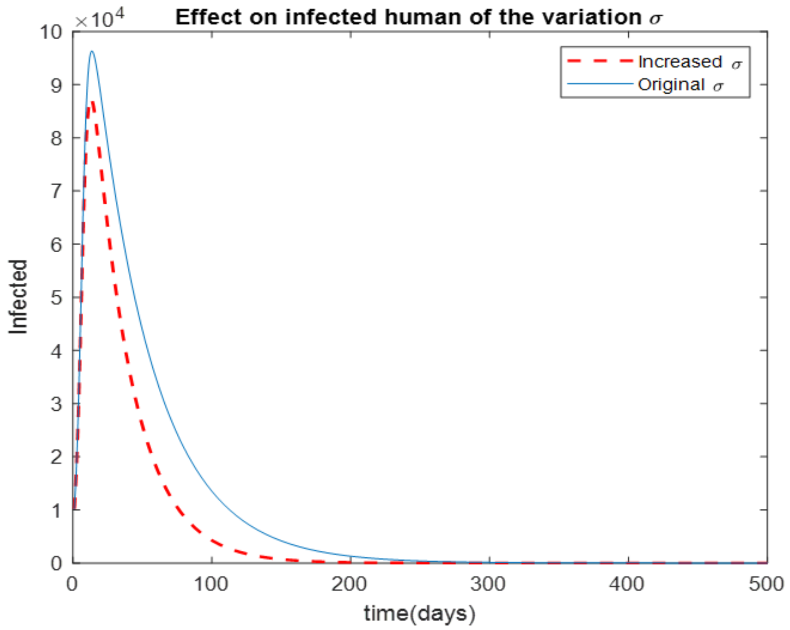

Figure 10: Infected human population with the original value of σ and 10% increased

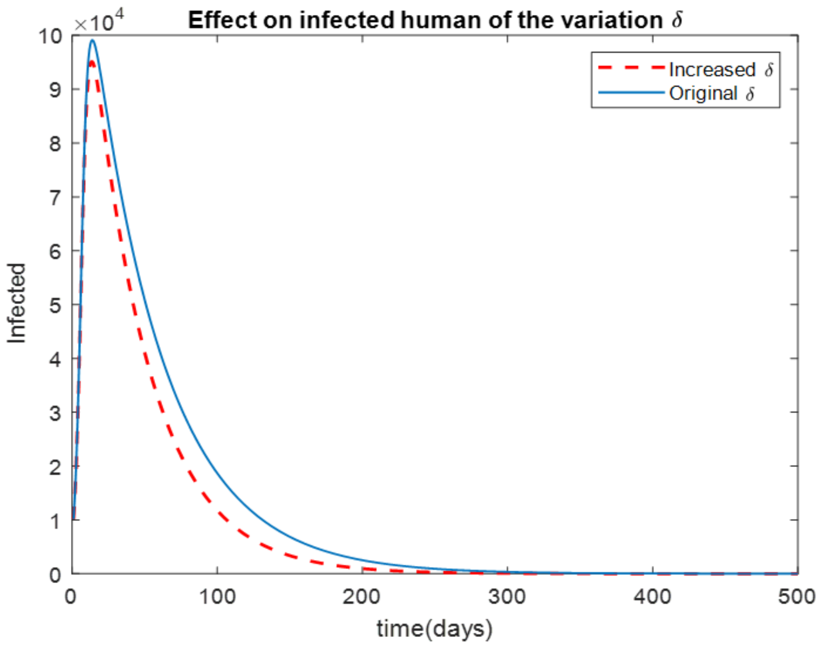

Figure 11: Infected human population with the original value of δ and 10% increased

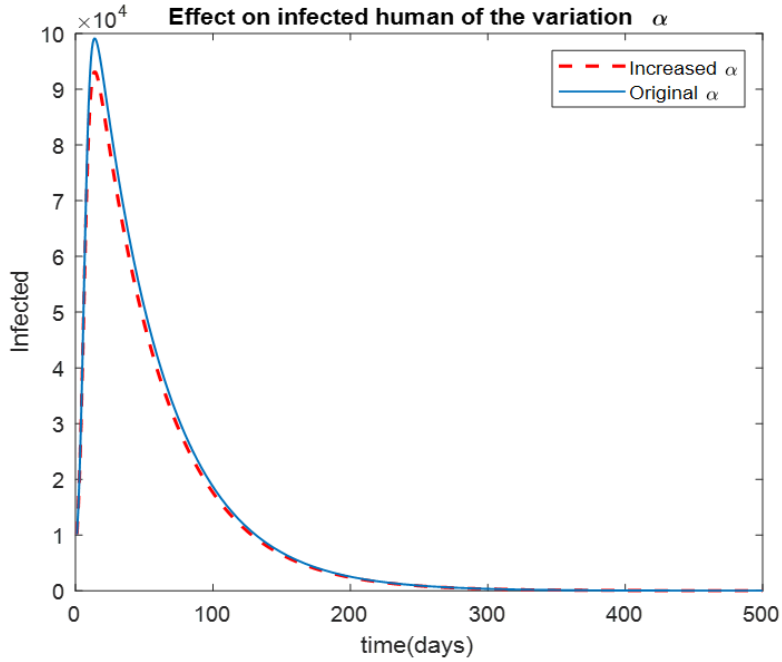

Figure 12: Infected human population with the original value of α and 10% increased

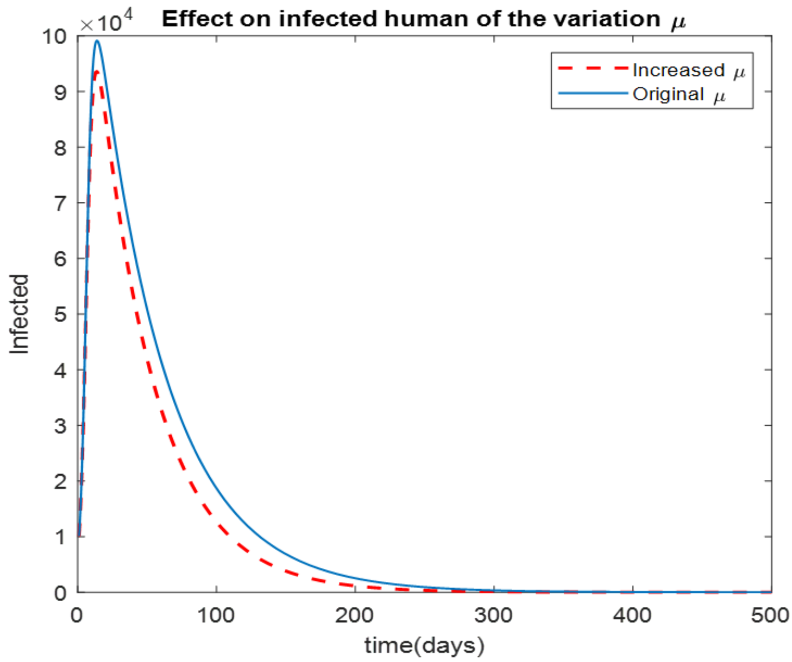

Figure 13: Infected human population with the original value of μ and 10% increased

From the results, we can interpret that the quarantine measure is significant in delaying the rise in the number of infected people. The results reveal that even with a high vaccination rate if the quarantine is not properly done, the number of infected populations is still significantly high. By carrying out the sensitivity analysis, we can demonstrate, in a quantitative manner, that the implementation of quarantine procedures contributes significantly to a reduction in the number of infectious populations. This study shows that quarantine and vaccination need to be carried out simultaneously in dealing with the spread of covid-19 in the community.

4. ACKNOWLEDGEMET

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