SIMILARITY-BASED FUZZY INFERIOR RATIO (FIR) VARIANTS FOR SOLVING SUPPLIER SELECTION PROBLEM

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ABSTRACT

The Fuzzy Inferior Ratio (FIR) method has been acknowledged as one of the most beneficial approaches for the Multi-Criteria Decision Making (MCDM) in a fuzzy environment. It simultaneously uses a compromise solution that considers the distance of the alternatives with both Fuzzy Positive Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution (FNIS). In this research, a method that integrates three different similarity measures (SMs) namely Similarity Measure 1, Similarity Measure 2 and Similarity Measure 3, into the Fuzzy Inferior Ratio (FIR) was proposed. The first SM using graded mean integration representation distance. Second SM using the center of gravity (COG) points and geometric distance. The third SM uses the geometric distance, the perimeter and the area of two trapezoidal fuzzy numbers. SM has the advantage since it uses different perspectives of fuzzy numbers other than geometric distance. It can be calculated by using other aspects such as area, perimeter and center of gravity of the fuzzy numbers. With the integration of SMs in FIR, the loss of information can be minimized. In this study, the proposed method that is FIR with different SMs is implemented to rank suppliers in the supply chain system. Then, the result is analyzed by comparing the ranking of the suppliers by using three different similarity measures. It is found that the results are consistent for all the variants of similarity-based FIR. The result of the three different SMs shows the same ranking and the best supplier to choose is Supplier 2.

Keywords: *Fuzzy Inferior Ratio, Similarity Measure, Supplier Selection.*

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1. Introduction

Decisions are made every day in daily life, even the simplest ones like what to eat for breakfast or what subject to study today. This is why decision-making is so crucial. Fuzzy set theory is being utilized to solve a variety of issues in the engineering, medical, business, health, and natural sciences. Due to the vagueness and fuzziness of the data, Multi-Criteria Decision Making (MCDM) is generally known as the decision-making topic with the fastest growing problem. American statesman Benjamin Franklin invented MCDM and used it to make crucial decisions using a simple paper system. In 384–322 BC, a famous Greek philosopher and polymath Aristotle described "preferences" as "rational desires", which was discovered by Köksalan et. al., (2013). This shows that the MCDM method has been there since long before someone connected rational decision-making to human desires. The basic goal of MCDM is to form a basis for more logical and efficient decision making, especially when multiple criteria need to be considered.

The study of MCDM is strongly related to other topics, such as Multi-Attribute Decision Making (MADM), Multi-Attribute Utility Theory (MAUT), Multi-Objective Decision Making (MODM), and Public Choice Theory (PCT), according to Massam (1988). The use of the MCDM method in real-world decision-making scenarios may result in highly serious limited information and the criteria containing imprecision information. For example, because the characteristics of complex engineering problems require the kind of information suggested by Kahraman (2008), decision-makers encounter various problems when dealing with incomplete and vague information in MCDM problems.

Multi-Attribute Decision Making (MADM) and Multi-Objective Decision Making (MODM) are two basic approaches to MCDM problems. MADM refers to choosing between various courses of action in the presence of multiple, frequently conflicting attributes. Vinogradova (2019) claims that MADM methods are focused on identifying the most suitable comparative suppliers or ranking options based on their relevance to the evaluated objective. According to Zhang et. al., (2021), because it can be successfully used to make decisions in multiple homogenous options, it has become a topic of attention for scholars. According to Mohanty et. al., (2018), MADM can be used in complex problems, for example, in the selection of an office chair with prominent features satisfying ergonomic needs (both physical and psychological needs) and focusing on the requirements of the user in terms of conflicting criteria with the aim of solving the task of selection of n ergonomically designed product.

The number of suppliers is effectively infinite for MODM problems, and the tradeoffs between the design criteria are often expressed as continuous functions. A systematic decision-making process is the MODM method. A decision-maker(DM) can use it to define his decision, develop his own set of values, frame the situation, and assess the conclusion in a systematic manner. By establishing the decision suppliers (what options are available) and the decision criteria (what is relevant to the decision-maker), the analyst can break down a complex decision into smaller and simpler components. Once the project context and structure have been established, the suppliers are then ranked against the selection criteria. Following a final score allocation to the decision options, these criteria are then weighted according to the decision maker's value structure.

Moreover, MODM problems can often be classified into four types. In order to find an efficient solution, DM is not required to obtain any information from the first group of MODM problems. All of the assumptions underlying these algorithms relate to DM's preferences. The reduction of deviations of the objective functions from an ideal solution known as the "utopian point" is the goal of L-P Metric methods, which are among the most used algorithms. The process of minimizing deviations cannot start until all objectives have been normalized because they all have different natures.

Fuzzy MCDM methods, however, enable more practical outcomes in decision-making problems. Fuzzy sets are typically used in Analytical Hierarchical Processes (AHP), Analytical Net-work Processes (ANP), and Technique for Order Preference by

Similarity to Ideal Solution (TOPSIS) methods. According to Kaya et. al., (2019), it has been observed that the solutions are more responsive when using the Fuzzy Multicriteria Optimization and Compromise Solution (VIKOR), Fuzzy Elimination and Choice Expressing the Reality (ELECTRE), Fuzzy Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE), Fuzzy Axiomatic Design (AD), and Fuzzy Decision Making Trial and Evaluation Laboratory (DEMATEL) methods.

The Fuzzy Inferior Ratio (FIR) method was proposed by Hadi-Vencheh & Mirjaberi (2014) to overcome the weaknesses of a number of compensatory decision-making methods, the most of which only evaluate one of the ideal solutions in order to determine the rating index. It was highlighted that the single remotest value does not automatically make the supplier the best choice. The result was the simultaneous proposal of a compromise solution between the positive and negative ideal solutions. By integrating the similarity measure in the decision-making process, the FIR method is improved. This reduces the amount of information that is lost as a result of simplification. A decision-making procedure was established by introducing a new similarity-based compromise solution and inferior ratio. An application was illustrated to validate its effectiveness. Other fuzzy decision-making approaches that compute using the distance method in the computation may employ the similarity-based approach.

A similarity measure is an algorithmic method for calculating the degree of similarity between two items in some aspects. The degree of similarity between two data objects. In the context of data mining, it is a distance with dimensions that indicates object features. This method calculates the degree of similarity between mapping sources. A numerical representation of how similar two feature vectors are based on a comparison. A similarity measure, also known as a similarity function, is a real-valued function that determines how similar two items are. It categorizes the images according to their similarity to the query image. A tool for calculating the degree of resemblance between two things. A comparison of how similar any two objects are represented in multidimensional space. A number between "0" and "1" is frequently used to describe the degree of a relationship between two variables, with "0" indicating no similarity or exclusion and "1" indicating perfect identity similarity. The general approach is to describe data features as multidimensional points and then calculate distances between them. A number expressing the strength of links between concept map elements or between two idea maps.

Additionally, similarity measures (distance measure) is a significant means for measuring uncertain information. The degree of similarity between fuzzy sets is represented by the fuzzy similarity measure. Pythagorean fuzzy similarity measures were proposed by Zhang (2016) for multi-attribute decision-making situations. For pattern recognition, medical diagnosis, and clustering analysis, Peng & Dai (2017) lists many new distance and similarity measures and discusses their transformation relations. Wei & Wei (2018) discovered that certain Pythagorean fuzzy cosine functions are used to deal with decision-making difficulties. The third and fourth axioms, as well as the ability to differentiate between positive and negative differences and deal with division by zero problems, are not met by a number of existing similarity measures. Due to the above counterintuitive phenomena of the current PFS similarity measurements, it could be difficult for decision-makers to select optimal or convincing suppliers.

The selection of suppliers is crucial for any company since it affects how the company is perceived. In order to minimize costs and increase value, every company will concentrate on the advantages and disadvantages of potential purchases. To find a supplier, however, we must consider all factors, not only the cost of purchases, as this will have an impact on the company's image. Additionally, it is important for a company to have a positive relationship with the supplier in order to obtain the best possible pricing and the supplier's guarantee that the buyer will receive the best quality as a result of their trust. This could have a positive effect on the company that makes a genuine effort to select the best supplier. According to Yücenur et. al., (2011), selecting good suppliers is important for the suppliers to perform well, and this difficulty involves both tangible and intangible factors.

Many MCDM methods only consider either positive or negative solutions when determining how to rank the suppliers. However, the best choice need not be the one that is closest distance to Fuzzy Positive Ideal Solution (FPIS) and farthest distance from Fuzzy Negative Ideal Solution (FNIS). A balance between the distance from FPIS and FNIS is required. The distance between the suppliers is considered by FIR when using both FPIS and FNIS simultaneously. A very important tool is the use of SMs to assess how similar two fuzzy numbers are to one another. In addition to geometric distance, it also considers other perspectives of fuzzy numbers, such as area, perimeter, the center of gravity, etc. With the integration of three different SMs in FIR, information loss can be avoided.

There are three (3) objectives for this study. The first objective is to propose a similarity measure based Fuzzy Inferior Ratio (FIR) procedure. The second objective is to implement the proposed method in supplier selection using secondary data. Lastly, the ranking of the suppliers are compared based on three different similarity measures.

2. Preliminaries

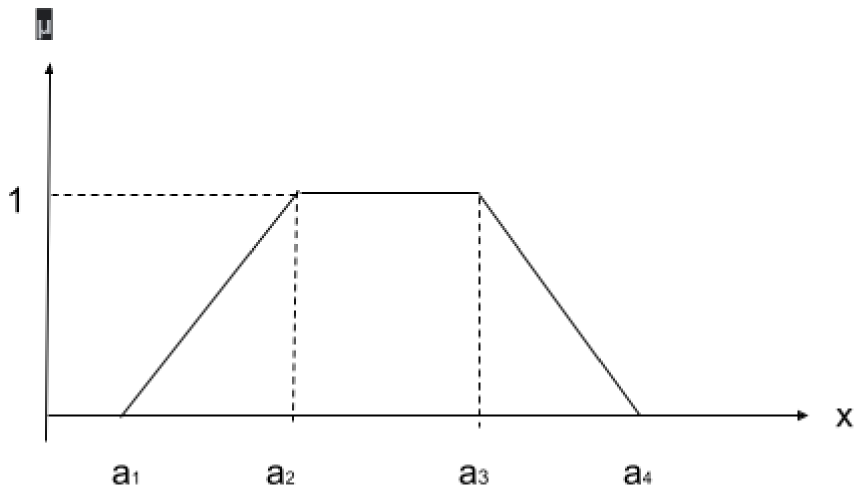
In this section, some definitions related trapezoidal fuzzy numbers are presented as follows:

Definition 1: Definition of trapezoidal fuzzy number (Kaufmann & Gupta, 1987).
A trapezoidal fuzzy number (TFN) $\tilde{A} = (a_1, a_2, a_3, a_4)$ is a fuzzy set defined by a membership function $\mu_{\tilde{A}}(x) : \mathbb{R} \Rightarrow [0,1]$ such that

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a_1)}{(a_2-a_1)}, & a_1 \le x \le a_2 \\ 1 & , a_2 \le x \le a_3 \\ \frac{(x-a_4)}{(a_3-a_4)}, & a_3 \le x \le a_4 \\ 0 & , otherwise \end{cases}$$

where

$a_1, a_2, a_3, a_4 \in$ R, and $a_1 < a_2 < a_3 < a_4$



Trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$

Figure 1: Trapezoidal Fuzzy Number

Definition 2: Definition of Arithmetic operations of fuzzy numbers (Gani & Assarudeen, 2012).

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ be two TFNs. The arithmetic operations of fuzzy numbers are as follow:

i) Addition: $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

ii) Subtraction: $\tilde{A} - \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$

iii) Multiplication: $\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$

iv) Scalar Multiplication, $\lambda\tilde{A} = (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4)$, $\lambda > 0$

Definition 3: Definition of the defuzzification process (A., Lotfi, Malkhalifeh & T., 2016)

The defuzzification process of transforming a fuzzy number A into a crisp value is given by

$$A = \frac{(a_1 + a_2 + a_3 + a_4)}{4}$$

Definition 4: First Similarity Measure (Hsieh and Chen, 2009)

The similarity measure using graded mean integration representation distance, where the degree of similarity *S(A,B)* between the FNs *A* and *B* is calculated as follows:

$$S_1(A, B) = \frac{1}{1 + d(A,B)} \tag{2}$$

where d(A,B) = |P₁(A) – P₁(B)], and P₁(A) and P₁(B) are the mean graded integration representation of A and B. For trapezoidal FNs, they are defined as follows:

$$P_1(A) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$
$$P_1(B) = \frac{b_1 + 2b_2 + 2b_3 + b_4}{6}$$

Definition 5: Second Similarity Measure (Chen & Chen, 2003)
Let $A = (a_1, a_2, a_3, a_4)$ and $B = (b_1, b_2, b_3, b_4)$ be two trapezoidal FNs. The center of gravity (COG) points (X_a, Y_a) and geometric distance (X_b, Y_b) of trapezoidal FNs *A* and *B* are used. The COG point (X_a, Y_a) for FN *A* is calculated as follows:

$$Y_a = \begin{cases} \frac{1}{6}\left(\frac{a_3 - a_2}{a_4 - a_1} + 2\right) & \text{, if } a_1 \neq a_4 \\ \frac{1}{2} & \text{, if } a_1 = a_4 \end{cases}$$

$$X_a = \frac{Y_a(a_3 + a_2) + (a_4 + a_1)(1 - Y_a)}{2}.$$

The degree of similarity $S_2(A, B)$ is calculated as follows:

$$S_2(A, B) = \left(1 - \frac{\sum_{i=1}^{4} |a_i - b_i|}{4}\right) \times (1 - |X_a - X_b|)^{\frac{S_a - S_b}{2}} \times \left(\frac{min(Y_a, Y_b)}{max(Y_a, Y_b)}\right) \tag{3}$$

where $S_2(A, B) \in [0, 1]$.

Definition 6: Third Similarity Measure (Hejazi , 2011)
This similarity measure takes into account the geometric distance, the perimeter of the two trapezoidal FNs, and the area of the two trapezoidal FNs. This similarity measure is the latest of the four and is defined as

$$S_3(A, B) = \left(1 - \frac{\sum_{i=1}^{4} |a_i - b_i|}{4}\right) \times \left(\frac{min(P_2(A), P_2(B))}{max(P_2(A), P_2(B))}\right) \left(\frac{min(Area(A), Area(B)) + 1}{max(Area(A), Area(B)) + 1}\right) \tag{5}$$

where P_2 *(A)* and P_2 *(B)* are the perimeters of trapezoidal GFN *A* and *B*, which are defined as

$$P_2(A) = \sqrt{(a_1 - a_2)^2 + 1} + \sqrt{(a_3 - a_4)^2 + 1} + (a_3 - a_2) + (a_4 - a_1)$$
$$P_2(B) = \sqrt{(b_1 - b_2)^2 + 1} + \sqrt{(b_3 - b_4)^2 + 1} + (b_3 - b_2) + (b_4 - b_1)$$

and areas of two trapezoidal GFNs are calculated as follows:

$$\text{Area }(A) = \frac{1}{2}(a_3 - a_2 + a_4 - a_1)$$
$$\text{Area }(B) = \frac{1}{2}(b_3 - b_2 + b_4 - b_1)$$

3. Methodology

This chapter includes a brief discussion of each step taken in the Fuzzy Inferior Ratio.

3.1 FUZZY INFERIOR RATIO

The Fuzzy Inferior Ratio approach was utilized in this project, and it is explained in the following section. Step 1 until step 11 are involved:

Step 1: Determine the Experts, Criteria, and Options.

The first step is to form k decision makers, Dk, k=1,2…r, to evaluate the importance of the criteria, Cj, j =1,2…n and the ratings of the suppliers, A_i, i=1,2…m based on the given criteria. The evaluation is represented as a trapezoidal fuzzy number.

Step 2: Determine the Linguistic Variable for Criteria and Suppliers.

The second step is to choose appropriate linguistic variables for evaluating and ratings of suppliers. The scale and the linguistic terms used for the importance of weight and ratings of suppliers are trapezoidal fuzzy numbers in the interval [0,1] and [0,10] in all seven linguistic terms respectively. It is shown in Table 1 and Table 2:

Table 1: Linguistic terms of the Importance Weight of the criteria.

Linguistic term for the Importance Weight	Fuzzy number
Very Low (VL)	(0.0,0.0,0.1,0.2)
Low (L)	(0.1,0.2,0.2,0.3)
Medium Low (ML)	(0.2,0.3,0.4,0.5)
Medium (M)	(0.4,0.5,0.5,0.6)
Medium High (MH)	(0.5,0.6,0.7,0.8)
High (H)	(0.7,0.8,0.8,0.9)
Very High (VH)	(0.8,0.9,1.0,1.0)

Table 2: Linguistic terms of the ratings for the suppliers

Linguistic term of Rating for the Suppliers	Fuzzy number
Very Poor (VP)	(0,0,1,2)

Poor (P)	(1,2,2,3)
Medium Poor (MP)	(2,3,4,5)
Fair (F)	(4,5,5,6)
Medium Good (MG)	(5,6,7,8)
Good (G)	(7,8,8,9)
Very Good (VG)	(8,9,10,10)

Step 3: Determine the Importance Weight of Criteria.
The experts will evaluate the importance weight of each criterion in this step. The linguistic term variable will be a guidance from the experts to make the evaluation. W_j denotes the weight of criterion C_j based on the linguistic preference determined by decision makers.

Step 4: The weight of criteria is aggregated.
The aggregated fuzzy rating can be determined as $(\tilde{w}_j^k)=(\tilde{w}_{j1}^k, \tilde{w}_{j2}^k, \tilde{w}_{j3}^k, \tilde{w}_{j4}^k)$ where j=1, 2, …,q; k=1, 2, …, r.

Use weighted arithmetic mean for weight.

$$\tilde{w}_j = \frac{1}{k}(\tilde{w}_j^1 + \tilde{w}_j^2 + \cdots + \tilde{w}_j^k) \tag{1}$$

Find the aggregated fuzzy weight $\tilde{x}_{ij}$ of the suppliers A_i under criterion C_j given by $\tilde{x}_{ij}=(a_{ij}, b_{ij}, C_{ij}, d_{ij})$, where; $i = 1, 2, \cdots, p;\ j = 1, 2, \cdots, q;\ k = 1, 2, \cdots, r.$

$$x_{ij} = \frac{1}{k}(\tilde{x}_{ij}^1 + \tilde{x}_{ij}^2 + \cdots + \tilde{x}_{ij}^k) \tag{2}$$

Step 5: Construct the Decision Matrix.
Given *m* suppliers and *n* criteria, a multi-criteria decision-making problem can be presented in a matrix form as below

$$D = \begin{array}{c c} & \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_m \end{array} & \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{pmatrix} \end{array} = [x_{ij}]_{m\times n} \tag{3}$$

$$\tilde{w}_j = [\tilde{w}_1, \tilde{w}_2, \ldots \tilde{w}_n],\ i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$$

where $\tilde{w}_j$ and $\tilde{x}_{ij}$ are linguistic values that can be described by trapezoidal fuzzy numbers.

Step 6: Normalized the fuzzy decision matrix.

For benefit criteria where $x_j^{(4)} = max_{1 \le i \le n} x_{ij}^{(4)}$

$$v_{ij} := \left(\frac{x_{ij}^{(1)}}{x_j^{(4)}}, \frac{x_{ij}^{(2)}}{x_j^{(4)}}, \frac{x_{ij}^{(3)}}{x_j^{(4)}}, \frac{x_{ij}^{(4)}}{x_j^{(4)}} \right), \ i = 1, 2, ..., n \tag{4}$$

For cost criteria where $x_j^{(1)} = min_{1 \le i \le n} x_{ij}^{(1)}$

$$v_{ij} := \left(\frac{x_j^{(1)}}{x_{ij}^{(4)}}, \frac{x_j^{(1)}}{x_{ij}^{(3)}}, \frac{x_j^{(1)}}{x_{ij}^{(2)}}, \frac{x_j^{(1)}}{x_{ij}^{(1)}} \right), \ i = 1, 2, ..., n \tag{5}$$

Step 7: Transform the normalized fuzzy decision matrix via linear scale transformation by letting $r_{min} = 0$ and $r_{max} = 1$. So, the fuzzy normalized decision matrix is:

$$N = \begin{array}{c c} & \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_m \end{array} & \begin{pmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & v_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ v_{m1} & v_{m2} & \cdots & v_{mn} \end{pmatrix} \end{array}$$

Then, construct the weighted transformed normalized fuzzy decision matrix ṽ by multiplying the weight ($\tilde{w}_j$) of evaluation criteria with the transformed normalized fuzzy decision matrix N as:

$$\tilde{v} = [\tilde{v}_{ij}]_{m \times n}, \ i = 1,2,..., m \ ; \ j = 1,2,...n \text{ where } \tilde{v}_{ij} = N \times \tilde{w}_j \tag{6}$$

Step 8: Determine the Fuzzy Positive Ideal Solution (FPIS), A^+ and Fuzzy Negative Ideal Solution (FNIS), $\bar{A}$.

$$A^+ := \left(max_{1 \le i \le n} v_{ij}^{(1)}, max_{1 \le i \le n} v_{ij}^{(2)}, max_{1 \le i \le n} v_{ij}^{(3)}, max_{1 \le i \le n} v_{ij}^{(4)} \right) \tag{7}$$

$$A^- := \left(min_{1 \le i \le n} v_{ij}^{(1)}, min_{1 \le i \le n} v_{ij}^{(2)}, min_{1 \le i \le n} v_{ij}^{(3)}, min_{1 \le i \le n} v_{ij}^{(4)} \right) \tag{8}$$

Step 9: Calculate the degree of similarity between fuzzy number, v_{ij} and FPIS and degree of similarity between fuzzy number, v_{ij} and FNIS by using Fuzzy Similarity Measure Formula as in definition 4 to 6.

$$S\,(A_i, A^+) \text{ and } S\,(A_i, A^-)$$

Step 10: Calculate the similarity-based compromise solution for each supplier.

$$\zeta\,(A_i) := \frac{\Sigma S\,(A_i, A^+)}{max_{1 \le i \le n} \Sigma S\,(A_i, A^+)} - \frac{\Sigma S\,(A_i, A^-)}{min_{1 \le i \le n} \Sigma S\,(A_i, A^-)}, \quad i = 1, 2, ..., n \tag{9}$$

Step 11: Calculate the ratio for the suppliers.

$$IR_p(i) := \frac{\zeta(A_i)}{\min_{1\le i\le n} \zeta(A_i)} \tag{10}$$

The ranking order of all applicants is determined where the lowest value will be placed in the first ranking position.

4. Implementation

The proposed method, FIR with similarity measure is illustrated here using the data from Gani, & Assarudeen, (2012). A high-tech manufacturing company ran a survey in order to find a suitable material supplier for purchasing essential components for new products. Experts assessed the significance of criteria and ranked the options.

A committee of three (3) Decision Makers namely D1, D2, and D3 had been set up to decide the best supplier. Five (5) benefits criteria which were Profitability of Supplier (C1), Relationship Closeness (C2), Technological Capability (C3), Conformance Quality (C4), and Conflict Resolution (C5) were considered. Five (5) suppliers were chosen and named as A1, A2, A3, A4, and A5.

The linguistic terms for the importance of criteria are shown in Table 1:

The linguistic terms for the ratings of the suppliers are shown in Table 2:

Table 3 shows evaluation of the importance of criteria by three decision makers.

Table 3: Evaluation of the Importance of Criteria by Decision Makers.

Criteria	**D1**	**D2**	**D3**
C1	H	H	H
C2	VH	VH	VH
C3	VH	VH	H
C4	H	H	H
C5	H	H	H

Table 4 shows the conversion of evaluation of the importance of five criteria by three decision makers and the aggregated weights values of the criterion Ci under decision makers were obtained using weighted arithmetic mean for weight from equation (1).

Table 4: The Aggregated Weights of Five Criteria and its Defuzzified Value.

Criteria	D1	D2	D3	Aggregated Value	Defuzzification Value
C1	(0.7,0.8,0.8,0.9)	(0.7,0.8,0.8,0.9)	(0.7,0.8,0.8,0.9)	(0.70,0.80,0.80,0.90)	0.80
C2	(0.8,0.9,1.0,1.0)	(0.8,0.9,1.0,1.0)	(0.8,0.9,1.0,1.0)	(0.80,0.90,1.00,1.00)	0.93
C3	(0.8,0.9,1.0,1.0)	(0.8,0.9,1.0,1.0)	(0.7,0.8,0.8,0.9)	(0.77,0.87,0.93,0.97)	0.89
C4	(0.7,0.8,0.8,0.9)	(0.7,0.8,0.8,0.9)	(0.7,0.8,0.8,0.9)	(0.70,0.80,0.80,0.90)	0.80
C5	(0.7,0.8,0.8,0.9)	(0.7,0.8,0.8,0.9)	(0.7,0.8,0.8,0.9)	(0.70,0.80,0.80,0.90)	0.80

Table 5 shows the Three Decision Makers' evaluations of the Suppliers based on each of the Criteria.

Table 5: The Ratings of Five Suppliers under Five Criteria by Three Decision-Makers.

Criteria	Supplier	Decision-makers		
		D1	D2	D3
$C1$	**A1**	MG	MG	MG
	A2	G	G	G
	A3	VG	VG	G
	A4	G	G	G
	A5	MG	MG	MG
$C2$	**A1**	MG	MG	VG
	A2	VG	VG	VG
	A3	VG	G	G
	A4	G	G	MG
	A5	MG	G	G
$C3$	**A1**	G	G	G
	A2	VG	VG	VG
	A3	VG	VG	G
	A4	MG	MG	G

	A5	MG	MG	MG
$C4$	**A1**	G	G	G
	A2	G	VG	VG
	A3	VG	VG	VG
	A4	G	G	G
	A5	MG	MG	G
$C5$	**A1**	G	G	G
	A2	VG	VG	VG
	A3	G	VG	G
	A4	G	G	VG
	A5	MG	MG	MG

The aggregated fuzzy weight $\tilde{x}_{ij}$ of the suppliers A_i under criterion C_j is obtained in equation (2) as below and the results are shown in Table 6 below.

Table 6: Aggregated Fuzzy Rating for Five Suppliers regarding Five Criteria.

	C1	**C2**	**C3**	**C4**	**C5**
A1	(5.00,6.00,7.00,8.00)	(6.00,7.00,8.00,8.67)	(7.00,8.00,8.00,9.00)	(7.00,8.00,8.00,9.00)	(7.00,8.00,8.00,9.00)
A2	(7.00,8.00,8.00,9.00)	(8.00,9.00,10.0,10.0)	(8.00,9.00,10.0,10.0)	(7.67,8.67,9.33,9.67)	(8.00,9.00,10.0,10.0)
A3	(7.67,8.67,9.33,9.67)	(7.33,8.33,8.67,9.33)	(7.67,8.67,9.33,9.67)	(8.00,9.00,10.0,10.0)	(7.33,8.33,8.67,9.33)
A4	(7.00,8.00,8.00,9.00)	(6.33,7.33,7.67,8.67)	(5.67,6.67,7.33,8.33)	(7.00,8.00,8.00,9.00)	(7.33,8.33,8.67,9.33)
A5	(5.00,6.00,7.00,8.00)	(6.33,7.33,7.67,8.67)	(5.00,6.00,7.00,8.00)	(5.67,6.67,7.33,8.33)	(5.00,6.00,7.00,8.00)

The normalized aggregated fuzzy decision matrix for the suppliers with regard to the benefit criteria are shown in Table 7.

Table 7: Normalized Aggregated Fuzzy Decision Matrix of Each Supplier with relation to the Criteria

	C1	**C2**	**C3**	**C4**	**C5**
A1	(0.52,0.62,0.72,0.83)	(0.60,0.70,0.80,0.87)	(0.70,0.80,0.80,0.90)	(0.70,0.80,0.80,0.90)	(0.70,0.80,0.80,0.90)
A2	(0.72,0.83,0.83,0.93)	(0.80,0.90,1.00,1.00)	(0.80,0.90,1.00,1.00)	(0.77,0.87,0.93,0.97)	(0.80,0.90,1.00,1.00)

A3	(0.79,0.90,0.97,1.00)	(0.73,0.83,0.87,0.93)	(0.77,0.87,0.93,0.97)	(0.80,0.90,1.00,1.00)	(0.73,0.83,0.87,0.93)
A4	(0.72,0.83,0.83,0.93)	(0.63,0.73,0.77,0.87)	(0.57,0.67,0.73,0.83)	(0.70,0.80,0.80,0.90)	(0.73,0.83,0.87,0.93)
A5	(0.52,0.62,0.72,0.83)	(0.63,0.73,0.77,0.87)	(0.50,0.60,0.70,0.80)	(0.57,0.67,0.73,0.83)	(0.50,0.60,0.70,0.80)

The weighted transformed normalized fuzzy decision matrix with respect to all criteria is calculated using equation (6) and the values were presented in Table 10.

Table 8: Weighted Transformed Normalized Fuzzy Decision Matrix

Supplier	$C1$	$C\mathbf{2}$	$C3$	$C4$	$C5$
A1	(0.00,0.17,0.34,0.58)	(0.00,0.23,0.50,0.67)	(0.31,0.52,0.56,0.78)	(0.22,0.43,0.43,0.69)	(0.28,0.48,0.48,0.72)
A2	(0.30,0.51,0.51,0.77)	(0.40,0.68,1.00,1.00)	(0.46,0.70,0.93,0.97)	(0.32,0.55,0.68,0.83)	(0.42,0.64,0.80,0.90)
A3	(0.40,0.63,0.74,0.90)	(0.27,0.53,0.67,0.83)	(0.41,0.64,0.81,0.91)	(0.38,0.62,0.80,0.90)	(0.33,0.53,0.59,0.78)
A4	(0.30,0.51,0.51,0.77)	(0.07,0.30,0.42,0.67)	(0.10,0.29,0.43,0.65)	(0.22,0.43,0.43,0.69)	(0.33,0.53,0.59,0.78)
A5	(0.00,0.17,0.34,0.58)	(0.07,0.30,0.42,0.67)	(0.00,0.17,0.37,0.58)	(0.00,0.19,0.31,0.56)	(0.00,0.16,0.32,0.54)
FPIS, A^+	(0.40,0.63,0.74,0.90)	(0.40,0.68,1.00,1.00)	(0.46,0.70,0.93,0.97)	(0.38,0.62,0.80,0.90)	(0.42,0.64,0.80,0.90)
FNIS, A^-	(0.00,0.17,0.34,0.58)	(0.00,0.23,0.42,0.67)	(0.00,0.17,0.37,0.58)	(0.00,0.19,0.31,0.55)	(0.00,0.16,0.32,0.54)

The values for FPIS and FNIS were obtained using equations (7) and (8) as shown in Table 8.

The first similarity measure is calculated using equation (2) in Definition 4 as below:

$$S_1(A_i, A^+) = \frac{1}{1 + d(A_i, A^+)}, \quad S_1(A_i, A^-) = \frac{1}{1 + d(A_i, A^-)}$$

where $d(A_i, A^+)$ and $d(A_i, A^-)$ are the distance of all suppliers from both FPIS and FNIS respectively. The distances were calculated using equations in Definition 4.

This process is repeated using Similarity Measure 2 (Definition 5) and Similarity Measure 3 (Definition 6). The results are shown in section Results and Discussion.

The compromise solution of five suppliers is shown in Table 9. The value of $\zeta(A_i)$ for the suppliers is calculated using equation (9).

$$\zeta(A_i) := \frac{\Sigma S(A_i, A^+)}{\max_{1 \le i \le n} \Sigma S(A_i, A^+)} - \frac{\Sigma S(A_i, A^-)}{\min_{1 \le i \le n} \Sigma S(A_i, A^-)}, \quad i = 1, 2, ..., n$$

Table 9: Compromise Solution for Each Supplier.

Supplier	**Criteria**	$S(Ai, A^{+})$	$\Sigma S(Ai, A^{+})$	$S(Ai, A^{-})$	$\Sigma S(Ai, A^{-})$	**ζAi**
$A1$	$C1$	0.711	3.839	1.000	4.420	-0.434
	$C2$	0.695		0.973		
	$C3$	0.806		0.793		
	$C4$	0.802		0.846		
	$C5$	0.824		0.809		
$A2$	$C1$	0.868	4.792	0.798	3.578	0.000
	$C2$	1.000		0.682		
	$C3$	1.000		0.666		
	$C4$	0.924		0.743		
	$C5$	1.000		0.690		
$A3$	$C1$	1.000	4.627	0.711	3.676	-0.062
	$C2$	0.826		0.796		
	$C3$	0.926		0.703		
	$C4$	1.000		0.700		
	$C5$	0.875		0.765		
$A4$	$C1$	0.868	3.952	0.798	4.293	-0.375
	$C2$	0.699		0.965		
	$C3$	0.707		0.920		
	$C4$	0.802		0.846		
	$C5$	0.875		0.765		
$A5$	$C1$	0.711	3.466	1.000	4.965	-0.664
	$C2$	0.699		0.965		
	$C3$	0.666		1.000		
	$C4$	0.700		1.000		

$C5$	0.690	1.000

The Inferior Ratio for each supplier is calculated using the formula below:

$$IR_p(i) := \frac{\zeta(A_i)}{\min_{1 \le i \le n} \zeta(A_i)}$$

According to *IR(A,i)*, the ranking order of all suppliers is determined. The lowest value is placed in the first ranking position. The values of inferior ratio and ranking for all suppliers are shown in Table 10.

Table 10: Inferior Ratio and the Ranking using Similarity Measures 1.

	SM1	
Supplier	**IR(A,i)**	**Rank**
A1	0.654	4
A2	0.000	1
A3	0.093	2
A4	0.565	3
A5	1.000	5

The ranking of suppliers is A2 $\prec$ A3 $\prec$ A4 $\prec$ A1 $\prec$ A5. Based on the Fuzzy Inferior Ratio method, the most preferred supplier among the experts is A2 compared to other suppliers. The least preferred supplier is A5. Next, calculation of ranking is based on FIR with different SM.

5. Results and Discussion

Table 11 shows the ranking for the defuzzified values of the weight of each criterion.

Table 11: The ranking of the importance of the Criteria

Criteria	**Weight**	**Ranking**
C1: Profitability of Supplier	0.80	3
C2: Relationship Closeness	0.93	1
C3: Technological Capability	0.89	2
C4: Conformance Quality	0.80	3
C5: Conflict Resolution	0.80	3

It shows that the most important criteria is C2 (Relationship Closeness) with a weightage of 0.93. Secondly, C3 is Technological Capability with a weightage of 0.89. The other criteria which are C1, C4, and C5 namely Profitability of Supplier, Conformance Quality, and Conflict Resolution share the same weightage which is 0.80. The highest rank of the criterion for the suppliers based on the experts is C2 which is Relationship Closeness. The second highest rank is C3 which is Technology Capability. The last three Criteria that share the same rank are C1, C4, and C5 which are Profitability of Supplier, Conformance Quality, and Conflict Resolution.

Table 12 shows the results of the ranking of the suppliers by applying 3 different SMs in FIR. The first SM, S1 is based on the graded mean integration representation distance. Next, the calculation of S2 is based on the center of gravity (COG) and geometric distance. Lastly, S3 is based on geometric separation, the area, and perimeter of the two trapezoidal FNs.

Table 12: The Ranking of the Suppliers using FIR with Different Similarity Measures.

	SM1		**SM2**		**SM3**	
Supplier	**IR(A,i)**	**Rank**	**IR(A,i)**	**Rank**	**IR(A,i)**	**Rank**
A1	0.654	4	0.609	4	0.633	4
A2	0.000	1	0.000	1	0.000	1
A3	0.093	2	0.092	2	0.132	2
A4	0.565	3	0.504	3	0.559	3
A5	1.000	5	1.000	5	1.000	5

Table 12 shows the ranking of suppliers after using the 3 different types of similarity measures in FIR are consistent with the result $A2 \prec A3 \prec A4 \prec A1 \prec A5$. The evaluations made by three decision-makers on the ratings of suppliers based on 5 benefits criteria which are Profitability of Supplier, Relationship Closeness, Technological Capability, Conformance Quality, and Conflict Resolutions.

6. Conclusion and Recommendations

The Fuzzy Inferior Ratio (FIR) approach has been acknowledged as one of the most beneficial approaches for multi-criteria decision-making in a fuzzy environment since it simultaneously considers a compromise solution between the positive and negative aspects of the evaluation (Ahmad et. al., 2020). The similarity measure is the numerical value that expresses the intensity of links between two or more items or variables (Ifenthaler, 2012).

A strategy that incorporates similarity measure (SM) into fuzzy inferior ratio (FIR) was proposed, since SM has an advantage in employing fuzzy numbers from other angles other than geometric distance. For instance, it can take into account the fuzzy numbers' area, perimeter, and center of gravity. The loss of information can be reduced by integrating SM with FIR.

A suggested strategy was used to incorporate the secondary data from the prior study in order to rank the providers in the supply chain system. The ranking of the suppliers was then compared using FIR with three different similarity measures. It is discovered that the outcomes are consistent for all similarity-based FIR variations.

To remain competitive and adapt to rapidly changing markets in the contemporary global marketplace, businesses must strengthen their agility. Selecting a supplier is one of the most crucial decisions a business must make in order to produce the greatest products for its clients. In addition, a supplier is more than just a source of high-quality products, we must also consider how they operate and whether they get along with the business, as the latter must ultimately place its full faith in them. The corporation can cut costs and obtain the newest materials with advanced technologies if a wise decision was made in choosing the supplier.

Based on the outcome obtained, it can be stated that the Fuzzy Inferior Ratio (FIR) Method is a methodical strategy that can be utilized to rank each supplier when choosing the best supplier. The results of the three similarity measures show that the ranking is the same and the best supplier is Supplier 2. Criteria 2 which is Relationship Closeness has the highest weight in the importance of the criteria compared to the other criteria. Criteria 3 which is the Technological Capability is the second-most important criteria while Criteria 1, 4, and 5 namely Profitability of Supplier, Conformance Quality, and Conflict Resolution are considered as least significant.

The Fuzzy Inferior Ratio Method can also be used in larger groups of people and for other types of decision-making, such as choosing the best products with similar functions but different aesthetics (such as color, shape, and material used for the product) and the target markets will be those who will purchase the products in line with their preferences. In addition, this method can be used to choose suitable office chairs or to help sports shoe manufacturers choose which materials and types of sports shoes will provide the highest-quality products on the market. Lastly, the Fuzzy TOPSIS, Fuzzy VIKOR, Fuzzy Analytic Hierarchy Process (FAHP), and Fuzzy ELECTRE are some more techniques that can be utilized to assess the preference of choosing suppliers.

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