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α- CUT BASED HIERARCHICAL FUZZY TOPSIS FOR TABUNG HAJI PILGRIMAGE GUIDE SELECTION

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ABSTRACT

Tabung Haji (TH) is the Malaysian hajj pilgrims fund board that manages pilgrimage activities and provides investment services for Malaysian Muslim communities. To ensure effective service, TH has a stringent selection of her staff. In the selection process of PIHTAS (Hajj Facilitators in the Holy Land), uncertainty and subjective perception-based evaluation are inevitable. In this paper, a fuzzy approach is introduced to cater to this problem. Integration of decision-making methods between fuzzy AHP (FAHP) and fuzzy TOPSIS (FTOPSIS), known as Hierarchical Fuzzy TOPSIS is utilized for this purpose. An α-cut approach is used to perform the calculation in the method where it can give a better representation of the evaluation result as it captures a more accurate shape of the operation. Using the proposed method, the criteria weight is determined using FAHP and the ranking of alternatives is obtained using FTOPSIS. The selection procedure involves two stages which are the evaluation of the application form and the interview process. It is found that the fuzzy approach gives a consistent result as compared to the common practice of the TH selection process and the fuzzy approach has the advantage of accommodating ambiguity and subjective matters.

Keywords: *Criteria weight determination, Hierarchical fuzzy TOPSIS, Ranking of alternatives, Tabung Haji.*

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1. Introduction

Tabung Haji or Lembaga Tabung Haji is the Malaysian hajj pilgrims fund board and was established in 1962 to enable Muslims to perform the Hajj, one of the tenets of their faith and help the Malayan rural economy. The fundamental concept behind their corporation was to enable Muslims to save for pilgrimage without fear of the money being tainted by riba (usury). *TH* has two strategies which are to improve the social-economic problems of the Muslims and allows them to invest and effectively participate in the nation's economic development (Salleh, 2013). It also provides a variety of services such as investment, savings, hajj/umrah service, plantations and many more.

To serve better, *TH* has a stringent process of selecting her staff with effective development programs. Two courses of PEKTA (Hajj course facilitators) to guide and teach about hajj process in the homeland and PIHTAS (Hajj facilitators in the Holy Land) to manage the hajj process and aid the pilgrims in the Holy Land are conducted as a tool for selecting suitable candidates as pilgrimage guides. Hussin et. al. (2014) observed about the opportunities and challenges for the Guidance Division which is in the Hajj Department. Staff groups of Tabung Haji which are PIHTAS (Hajj facilitators in the Holy Land) and PEKTA (Hajj course facilitators) are focused on the supervision needs of pilgrims either in Malaysia or in the Holy Land. The requirement to be a PIHTAS must be fulfilled by first being appointed as a PEKTA. PEKTA candidature is open to the public who are interested, committed and willing to provide the best service on duty. In the staff selection process of PIHTAS, two stages of evaluation will be made. The first stage is the evaluation of the application form. Criteria that are involved in this evaluation process are academic qualification, language proficiency, working experience and evidence related to an application for this position. The minimum requirement for academic qualification is having a background of religious education with a diploma in Islamic Studies and language proficiency in Arabic and Malay is a must. The applicants will be evaluated according to the following criteria (Nuradli, personal communication, December 28, 2016): The candidate

- Must be a PEKTA who holds a valid teaching credential that is validated by the State Department of Islamic Development.
- Has already performed Hajj.
- Has attended Muzakarah Haji and Intensive Classes on Tafaqquh Hajj.
- Not involved in any business related to bringing pilgrims for Hajj and Umrah.
- Capable of working more than 60 days

The PIHTAS has the responsibility of handling and providing an integrated guidance module in four courses which are the basic hajj course, intensive hajj course, special hajj course and prime hajj course for the pilgrims (Portal Tabung Haji, August 16, 2016). The lack of PIHTAS for TH becomes one of the crucial problems faced by pilgrims in the Holy Land. On average there is only one PIHTAS to guide 1000 pilgrim at Mecca which is far from adequate.

Since the selection process, PEKTA and PIHTAS is a complex process that is multi-dimension and subjective, a fuzzy approach to making the decision is suitable to be used. Fuzzy set theory was introduced Zadeh (1965) to represent a vagueness environment effectively. It is useful in the area of decision-making that involves human perception in the evaluation. The decision-making methods can be categorized into three main types which are the multiple criteria decision-making methods (MCDM), multiple attribute decision-making methods (MADM) and Multiple objective decision-making methods (MODM). Multiple criteria decision making (MCDM) is the most recognized method in operations research by many researchers and is considered both a computational and mathematical tool to solve the subjective evaluation of criteria and obtain a rating of alternatives (Mardani, Jusoh, & Kazimieras, 2015).

The analytic hierarchy process (AHP) method presented by Wind & Saaty (1980) used a hierarchical system of elements via pairwise comparisons to solve multi-criteria decision-making (MCDM) problems. The fuzzy AHP method was introduced to deal with complexity and uncertainty in decision-making problems (Saaty, 1988). Some variants for determining the criteria weights using FAHP were proposed which are Logarithmic Least Square Method (Wang et al., 2016)), the Extent Analysis Method (Zhu et al., 1999) and the Lambda-Max method (Csutora & Buckley, 2001). Mohd Saadon, Dom, & Mohamad (2010) stated that the Lambda-Max method has the advantage of being able to control any type of fuzzy number in pairwise comparisons and can be computed easily. FAHP has been used in many applications in decision making and a comprehensive review of its application was discussed by Russo & Camanho (2015).

Hwang and Yoon (1981) introduced the technique for order performance by similarity to ideal solution TOPSIS to solve MCDM problems based on the distance measurement from the positive and the negative ideal solutions. Chen (2000) extended the TOPSIS method into a fuzzy environment and applied it to a selection problem. Fuzzy numbers were utilized to represent the evaluation that caters to vagueness and subjectivity matters (Qin et al., 2008). Dymova et al. (2013) proposed a new distance formula of the alternatives from the ideal solutions to overcome the drawbacks of hamming distance and Euclidean distance used in previous FTOPSIS variants. Fuzzy TOPSIS is very viable to be used in solving decision-making problems because it is easy to implement and can prevent the risk of inconsistencies in ranking the alternatives (Md Saad, Ahmad, Abu, & Jusoh, 2013). FTOPSIS has been used in many applications that involve ranking, ordering, prioritizing and categorization. A survey of its applications was discussed in Behzadian et al. (2012). FTOPSIS has also been extended and integrated with other methods. A combination of the pairwise comparison method of AHP and the compensatory approach of TOPSIS under a fuzzy environment had been introduced by Wang, Cheng and Kun-cheng, (2009). They stated that the fuzzy TOPSIS by Chen has some problems that mislead the basic concept of TOPSIS and sometimes affect the results. Therefore, they came out with fuzzy hierarchical TOPSIS to overcome the problem. This method is suitable to deal with fuzziness and provide a more precise weight of criteria. The fuzzy hierarchical TOPSIS has also been used effectively by researchers such as Awasthi and Chauha (2012), Bao et al. (2012), Tyagi et al. (2014) and Dagdeviren et al. (2009) to solve diverse types of decision-making problems.

Zadeh (1965) also introduced the concept of α-cut or α-level set of a fuzzy set. It deals with the values at the vertical axis which consequently give a corresponding interval at the horizontal axis of the fuzzy set representation. The α-cut is known as a fuzzy algebra to get a crisp range from a fuzzy range (Noor-E-Alam, Hasin, Ullah, & Lipi, 2008). The α-cut of a fuzzy number is always bounded, closed and in the form of an interval (Siler & Buckley, 2005) and any fuzzy set can be described by specifying its α-cuts. Guang-Yuan and Wen-Quan (1985) applied the α-cut method to a structural design with various design levels α where the nonlinear problem was solved and obtained a sequence of solutions while optimum design problems with fuzzy resources showed the α-cuts strategy has been recognized as the standard method. In the arithmetic operation of the three tuples fuzzy numbers (a,b,c), in particular the multiplication and the division of triangular fuzzy numbers, the shape does not give a true representation since only three endpoints are used in the computation. Applying the α-cut method will give an interval that covers the whole values under consideration. This will ensure all information represented by the fuzzy number is preserved. The shape of the fuzzy number after undergoing the operation may consist of either a linear, a quadratic or irrational form of a curve. Dutta, Boruah, & Ali (2011) pointed out that the α-cut method can perform a variety of fuzzy operations such as obtaining exponential values, extracting the root and taking a logarithm with a better representation of the output. The α-cut method has been implemented in the TOPSIS method (Wang & Elhag, 2006; Chu & Lin, 2009; Zou et al., 2012) to solve several decision-making problems.

In this paper, the α-cut based integrated method of FAHP and FTOPSIS is used to determine the weight of the criteria in the selection of PIHTAS and the obtaining the ordering of the applicants to be appointed to the post. This is to allow a fair and reasonable process of selection to be made. The paper is organized as follows. The first section is the introduction followed by the methodology that will be developed and used in the selection of PIHTAS in section 2. Section 3 is on the implementation together with the result and discussion. The conclusion will be in the final section of the paper.

2. Methodology and Implementation

Some elementary concepts will first be given in this section before the presentation of the proposed method.

A fuzzy set A of a universe X is defined as:

$$A = \{(x, \mu_A(x)) | x \in X\}$$

where $\mu_A : X \to [0,1]$ is a membership function and $\mu_A(x)$ is a degree of belongingness of $x \in X$. A fuzzy set is normal if at least one of its elements attains the height of 1 else the set is non-normal. It is also to be said convex if and only if

$$\mu_A(\lambda x_1 + (1-\lambda)x_2 \geq \min(\mu_A(x_1), \mu_A(x_2))$$

for all x_1, x_2 in X and all $\lambda \in [0,1]$, where min denotes the minimum operator. A fuzzy number is a convex normalized fuzzy set of the real line R whose membership function is piecewise continuous. A triangular fuzzy number A can be defined as triplets (a_1, a_2, a_3) where the membership $\mu_A(x)$ is determined as follows:

$$\mu_A(x) = \begin{cases} 0, & x \epsilon[-\infty, a_1] \\ \dfrac{x - a_1}{a_2 - a_1}, & x \epsilon[a_1, a_2] \\ \dfrac{a_3 - x}{a_3 - a_2}, & x \epsilon[a_2, a_3] \\ 0, & x \epsilon[a_3, +\infty] \end{cases}$$

The α-cut of fuzzy number A can be defined as $A^\alpha = \{x | f_A(x) \geq \alpha\}$, where $x \in R, \alpha \in [0,1]$. A^α is a non-empty bounded closed interval contained in R and can be denoted as an interval $A^\alpha = [A_l^\alpha, A_u^\alpha]$, where A_l^α and A_u^α are its lower and upper bounds, respectively. For example, if triangular fuzzy number A = (a, b, c), then the α-cut of A can be expressed as:

$$A^\alpha = [(b-a)\alpha + a, (b-c)\alpha + c]$$

A fuzzy hierarchical TOPSIS method with α-cut operations is proposed in this paper. The proposed procedure is the integration between FAHP and FTOPSIS, which will be applied in selecting PIHTAS candidates for TH. The evaluation of the applicants will be based on the information obtained from the application form. FAHP method will be used to determine the criteria weights when the pair-wise comparison of each criterion is evaluated in the form of a linguistic scale and represented in triangular fuzzy numbers and interval representation. FTOPSIS with α-cut operations will determine the order of preference among them. The successful applicants will be called for an interview. There are seven (7) applicants who have applied for the post. The application forms of the candidates based on fuzzy inputs were evaluated by three panels from TH. Besides, the weights of criteria involved in the evaluation are first determined so that they will be used in the evaluation process of obtaining the order of preference of the applicants.

2.1 Phase 1: Determination of Criteria Weight using λ-max FAHP

The determination of criteria weight is carried out by the decision-makers using the following linguistic scales and corresponding fuzzy numbers as in Table 1.

Table 1. Linguistic scales for criteria weight evaluation

Linguistic scale	Triangular fuzzy scale	Triangular fuzzy reciprocal scale
Just equal	(1,1,1)	(1,1,1)
Equally important	(1/2,1,3/2)	(2/3,1,2)
Weakly important	(1,3/2,2)	(1/2,2/3,1)
Strongly more important	(3/2,2,5/2)	(2/5,1/2,2/3)
Very strongly more important	(2,5/2,3)	(1/3,2/5,1/2)
Absolutely more important	(5/2,3,7/2)	(2/7,1/3,2/5)

The steps in the method of the λ-max FAHP (Csutora &Buckley, 2001) is given below:

Step 1: Identify the criteria and alternatives of the decision-making problem. This step is to decide the criteria and alternatives that can be used in this research.

Step 2: Structuring the hierarchical structure based on fuzzy AHP Comparisons.

Step 3: Determine the pairwise comparison matrices.

Step 4: Calculate the fuzzy weight value of each criterion.

Step 4.1: Apply the α-cuts to obtain the positive decision matrix from the decision-makers, DMs.

i. Let α = 1 to obtain the positive reciprocal matrix in the middle bound of decision-maker k, $T_m^k = \left[t_{ijm}^k\right]_{nxn}$. The middle bound w_{im}^k will be calculated by using the geometric mean formula as shown below:

$$A^k = \left[\prod_{j=1}^{n} a_{ij}\right]^k = \left[a_{11}^1 \otimes a_{12}^2 \otimes \ldots \otimes a_{1n}^k\right]^{\frac{1}{k}} \tag{1}$$

$$w_m^k = (w_{im}^k)_{nx1} = \left(\frac{A^k}{\sum A^k}\right), i = 1,2,\ldots,n \tag{2}$$

where,

w_m^k = the middle weight vector.

ii. Let α = 0 to obtain the positive reciprocal matrix in the lower bound and upper bound of decision-maker k, $T_l^k = \left[t_{ijl}^k\right]_{nxn}$ and $T_u^k = \left[t_{iju}^k\right]_{nxn}$. The lower bound and upper bound will be calculated by using the geometric mean formula:

$$w_l^k = (w_{il}^k)_{nx1} = \left(\frac{A^k}{\sum A^k}\right), i = 1,2,...,n \tag{3}$$

$$w_u^k = (w_{iu}^k)_{nx1} = \left(\frac{A^k}{\sum A^k}\right), i = 1,2,...,n \tag{4}$$

where,

w_l^k = The lower weight vector.

w_u^k = The upper weight vector.

Step 4.2: Use the smallest possible Q_l^k and the largest possible Q_u^k to minimize the fuzziness of the weights. These two constants are calculated as follows:

$$Q_l^k = \min\left\{\frac{w_{im}^k}{w_{il}^k} \middle| 1 \le i \le n\right\} \tag{5}$$

$$Q_u^k = \max\left\{\frac{w_{im}^k}{w_{iu}^k} \middle| 1 \le i \le n\right\} \tag{6}$$

The new lower bound and upper bound is defined as

$$w_l^{k*} = \left(w_{il}^{k*}\right)_{nx1}, w_{il}^{k*} = Q_l^k w_{il}^k, i = 1,2,...,n \tag{7}$$

$$w_u^{k*} = \left(w_{iu}^{k*}\right)_{nx1}, w_{iu}^{k*} = Q_u^k w_{iu}^k, i = 1,2,...,n \tag{8}$$

Step 4.3: The fuzzy weight determined by the decision-makers can be obtained by aggregating $\left(w_l^{k*}, w_m^k, w_u^{k*}\right)$ and it is defined as follows:

$$\tilde{w}_i^k = \left(\frac{1}{k}\right) \otimes \left(w_l^{k*}, w_m^k, w_u^{k*}\right), i = 1,2,...,n.$$

2.2 Phase 2: Ranking of Candidates using α-cut based FTOPSIS

In this phase, the candidate will be evaluated using the α-cut based FTOPSIS and ranked accordingly. The steps involved are as follows:

Step 1: Form a committee of decision-makers, then identify the evaluation criteria. Transform fuzzy numbers of the appropriate linguistic variables of the ratings for to criteria into α-cut representation.

Step 2: Aggregate the performance ratings and perform normalization using the formula below.

$$x_{ij} = \frac{1}{k} \otimes (x_{ij1} \oplus x_{ij2} \oplus \ldots \oplus x_{ijk}) \tag{9}$$

$$\bar{x}_{ij} = \frac{x_{ij}}{x_{ij}^{+}}, x_{ij}^{+} = \max_{i} x_{ij} \tag{10}$$

where $\bar{x}_{ij}$ is a normalized rating of the j^{th} criterion.

Step 3: Perform multiplication of normalized rating and normalized weight as below:

$$\bar{x}_{ij} \otimes \tilde{w}_i^{\alpha} = \left[a_{ij}\alpha + c_{ij}, b_{ij}\alpha + d_{ij}\right] \otimes \left[p_{ij}\alpha + q_{ij}, r_{ij}\alpha + s_{ij}\right] \tag{11}$$

$\tilde{w}_i^{\alpha}$ is the normalized weight of j^{th} criterion.

Step 4: Normalize the multiplication values between ratings and the weight of criteria.

Step 5: Substitute the α-cut value with 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1 for each criteria and aggregate the values.

Step 6: Determine the fuzzy positive-ideal solution (FPIS, A^*) and fuzzy negative-ideal solution (FNIS, A^-).

$$A^* = \{\tilde{v}^*{}_1, \tilde{v}^*{}_2, \ldots, \tilde{v}^*{}_n\},$$

$$A^- = \{\tilde{v}^-{}_1, \tilde{v}^-{}_2, \ldots, \tilde{v}^-{}_n\},$$

where A^- is associated with benefit criteria and A^* is associated with cost criteria.

$$A^* = [(1,1,1), (1,1,1), (1,1,1), (1,1,1), (1,1,1)(1,1,1)]$$

$$A^- = [(0,0,0), (0,0,0), (0,0,0), (0,0,0), (0,0,0)(0,0,0)]$$

where $j = 1,2,\ldots,n$.

In terms of α-cuts, (FPIS, A^*) and (FNIS, A^-) will be in the form

$$A^* = [1,1]$$

$$A^- = [0,0]$$

Step 7: Calculate the distance for each alternative for (FPIS, A^*) and (FNIS, A^-) with each α-levels using the operation of interval subtraction (Dymova et al., 2013) as below:

$$d^* = \Delta_{A-B} = \frac{1}{2}((a^L - b^U) + (a^U - b^L)) \text{ for FPIS, } A^* \tag{12}$$

$$d^- = \Delta_{A-B} = \frac{1}{2}((a^L - b^U) + (a^U - b^L)) \text{ for FNIS, } A^- \tag{13}$$

Step 8: Aggregate the distance for each alternative.

$$d_i^* = \left(\frac{1}{k}\right) \otimes (d_{i1}^* \oplus d_{i2}^* \oplus ... \oplus d_{ik}^*) \tag{14}$$

$$d_i^- = \left(\frac{1}{k}\right) \otimes (d_{i1}^- \oplus d_{i2}^- \oplus ... \oplus d_{ik}^-) \tag{15}$$

Step 9: Calculate the relative closeness coefficient of each alternative by the following formula,

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-} \tag{3.44}$$

where i = 1, 2, …, n. The larger the value of CC_i the alternative will be ranked higher.

3. Implementation

As a case study, seven candidates have applied for the PIHTAS post at TH. Three panels of evaluators are appointed to evaluate the applicants. The applicants will be ranked based on the information given in the application form. Successful applicants will be called for an interview. The weight of the criteria will be determined before the evaluation of the applicants.

The evaluations by each decision-maker on the criteria are given in Table 2 –Table 4. There are four criteria under consideration in the evaluation which are academic qualification (AQ), language proficiency (LP), working experience (WE) and evidence provided related to the position. (E).

Table 2. Evaluation of criteria weight by decision-maker 1

Criteria	AQ	LP	WE	E
AQ	(1,1,1)	(1/2,1,3/2)	(1/2,2/3,1)	(1/3,2/5,1/2)
LP	(2/3,1,2)	(1,1,1)	(1/2,2/3,1)	(2/3,1,2)
WE	(1,3/2,2)	(1,3/2,2)	(1,1,1)	(2/5,1/2,2/3)
E	(2,5/2,3)	(1/2,1,3/2)	(3/2,2,5/2)	(1,1,1)

Table 3. Evaluation of criteria weight by decision-maker 2

Criteria	AQ	LP	WE	E
AQ	(1,1,1)	(1,3/2,2)	(3/2,2,5/2)	(2,5/2,3)
LP	(1/2,2/3,1)	(1,1,1)	(2/5,1/2,2/3)	(1/2,2/3,1)
WE	(2/5,1/2,2/3)	(3/2,2,5/2)	(1,1,1)	(2,5/2,3)
E	(1/3,2/5,1/2)	(1,3/2,2)	(1/3,2/5,1/2)	(1,1,1)

Table 4. Evaluation of criteria weight by decision-maker 3

Criteria	AQ	LP	WE	E
AQ	(1,1,1)	(1,3/2,2)	(3/2,2,5/2)	(2,5/2,3)
LP	(1/2,2/3,1)	(1,1,1)	(2/5,1/2,2/3)	(1/2,2/3,1)
WE	(2/5,1/2,2/3)	(3/2,2,5/2)	(1,1,1)	(2,5/2,3)
E	(1/3,2/5,1/2)	(1,3/2,2)	(1/3,2/5,1/2)	(1,1,1)

The weights are calculated using the λ-max method of FAHP proposed by Csutora & Buckley (2001) and are obtained in the form of fuzzy numbers and the α-cut representation as

$$\begin{bmatrix} AQ \\ LP \\ WE \\ E \end{bmatrix} = \begin{bmatrix} (0.270,0.279,0.283) \\ (0.187,0.192,0.222) \\ (0.296,0.303,0.305) \\ (0.219,0.225,0.229) \end{bmatrix} = \begin{bmatrix} 0.009\alpha + 0.271, -0.003\alpha + 0.28 \\ 0.005\alpha + 0.187, -0.030\alpha + 0.222 \\ 0.007\alpha + 0.296, -0.001\alpha + 0.305 \\ 0.005\alpha + 0.219, -0.004\alpha + 0.229 \end{bmatrix}$$

Next, the evaluation process of the applicants will utilize the linguistic variables with corresponding fuzzy numbers and intervals as depicted in Table 5.

Table 5. Linguistic variables and corresponding fuzzy numbers for the rating of alternatives

Criteria	AQ	LP	WE	E	Fuzzy Number	α-cuts representation
	Average (A)	Average (A)	Least Experience (LE)	Low Evidence (LE)	(0,0,3)	[0, -3α+3]
	Quite Good (QG)	Quite Good (QG)	Less Experience (L)	Average Evidence (A)	(2,5,7)	[3α+2, -2α+7]
	Good (G)	Good (G)	Marginally Experience (M)	Plenty of Evidence (P)	(6,8,9)	[2α+6, -α+9]
	Very Good (VG)	Very Good (VG)	Highly Experience (H)	Comprehensive Evidence (C)	(7,9,10)	[2α+7, -α+10]
	Excellent (E)	Excellent (E)	Very Highly Experience (VH)	Very Comprehensive Evidence (VC)	(9,10,10)	[α+9, 10]

Based on the linguisti values in Table 5, the evaluation of the applicants by the three decision-makers are given in Table 6.

Table 6. Evaluation of each candidate by decision-makers

Criteria	Alt	DM1	DM2	DM3	Criteria	Alt	DM1	DM2	DM3
AQ	A1	QG	QG	QG	**LP**	A1	G	QG	G
	A2	QG	G	G		A2	G	QG	G
	A3	A	QG	A		A3	QG	QG	QG
	A4	VG	VG	G		A4	VG	VG	VG
	A5	VG	G	G		A5	VG	VG	G
	A6	VG	G	VG		A6	E	E	E
	A7	G	G	G		A7	VG	G	VG
WE	A1	L	L	L	**E**	A1	A	A	P
	A2	M	M	L		A2	A	A	A
	A3	L	L	L		A3	A	LE	LE
	A4	M	M	M		A4	P	C	C
	A5	M	M	M		A5	VC	C	C
	A6	M	M	M		A6	P	P	C
	A7	M	M	M		A7	P	P	P

The evaluation will then be aggregated and normalized in the α-cut form as in Table 7.

Table 7. Aggregated normalized ratings of alternatives

Criteria	Alternatives	Aggregated and normalized ratings
AQ	A1	$[0.3\alpha+0.2, -0.2\alpha+0.7]$
	A2	$[0.233\alpha+0.4667, -0.133\alpha+0.833]$
	A3	$[0.1\alpha+0.0667, -0.267\alpha+0.433]$
	A4	$[0.2\alpha+0.667, -0.1\alpha+0.967]$
	A5	$[0.2\alpha+0.633, -0.1\alpha+0.933]$
	A6	$[0.2\alpha+0.667, -0.1\alpha+0.967]$
	A7	$[0.2\alpha+0.6, -0.1\alpha+0.9]$
LP	A1	$[0.233\alpha+0.4667, -0.133\alpha+0.833]$
	A2	$[0.233\alpha+0.4667, -0.133\alpha+0.833]$
	A3	$[0.3\alpha+0.2, -0.2\alpha+0.7]$
	A4	$[0.2\alpha+0.7, -0.1\alpha+1.0]$
	A5	$[0.2\alpha+0.667, -0.1\alpha+0.967]$
	A6	$[0.1\alpha+0.9, 1.0]$
	A7	$[0.2\alpha+0.667, -0.1\alpha+0.967]$
WE	A1	$[0.3\alpha+0.2, -0.2\alpha+0.7]$

	A2	[0.233α+0.4667, -0.133α+0.833]
	A3	[0.3α+0.2, -0.2α+0.7]
	A4	[0.2α+0.6, -0.1α+0.9]
	A5	[0.2α+0.6, -0.1α+0.9]
	A6	[0.2α+0.6, -0.1α+0.9]
	A7	[0.2α+0.6, -0.1α+0.9]
E	A1	[0.267α+0.333, -0.167α+0.767]
	A2	[0.3α+0.2, -0.2α+0.7]
	A3	[0.1α+0.067, -0.267α+0.433]
	A4	[0.2α+0.667, -0.1α+0.967]
	A5	[0.167α+0.767, -0.067α+1.0]
	A6	[0.2α+0.633, -0.1α+0.933]
	A7	[0.2α+0.6, -0.1α+0.9]

The integration of the weight into the aggregated evaluation will give us the decision matrix as in Table 8. It is observed that the shape of the resultant fuzzy numbers is no longer in the form of triangular since the equation of the intervals is now in quadratic forms which are non-linear. This is the desired result one would like to have so that the information in the fuzzy number is still preserved.

Table 8. Normalized decision matrix in the α-cut form

Criteria	Alternatives	Normalized decision matrix
AQ	A1	$[0.0096\alpha^2+0.2933\alpha+0.1913, 0.0023\alpha^2-0.2082\alpha+0.7]$
	A2	$[0.0075\alpha^2+0.2377\alpha+0.4463, 0.0016\alpha^2-0.1431\alpha+0.833]$
	A3	$[0.0032\alpha^2+0.0978\alpha+0.0638, 0.0031\alpha^2-0.2714\alpha+0.433]$
	A4	$[0.0064\alpha^2+0.2126\alpha+0.6379, 0.0012\alpha^2-0.1113\alpha+0.967]$
	A5	$[0.0064\alpha^2+0.2115\alpha+0.6053, 0.0012\alpha^2-0.1109\alpha+0.933]$
	A6	$[0.0064\alpha^2+0.2126\alpha+0.6379, 0.0012\alpha^2-0.1113\alpha+0.967]$
	A7	$[0.0064\alpha^2+0.2104\alpha+0.5738, 0.0012\alpha^2-0.1105\alpha+0.9]$
LP	A1	$[0.0052\alpha^2+0.2069\alpha+0.3931, 0.018\alpha^2-0.2458\alpha+0.833]$
	A2	$[0.0052\alpha^2+0.2069\alpha+0.3931, 0.018\alpha^2-0.2458\alpha+0.833]$
	A3	$[0.0067\alpha^2+0.2571\alpha+0.01685, 0.0271\alpha^2-0.2948\alpha+0.7]$
	A4	$[0.0045\alpha^2+0.1841\alpha+0.5896, 0.0135\alpha^2-0.2354\alpha+1.0]$
	A5	$[0.0045\alpha^2+0.1834\alpha+0.5618, 0.0135\alpha^2-0.2309\alpha+0.967]$
	A6	$[0.0022\alpha^2+0.1043\alpha+0.758, -0.1354\alpha+1.0]$
	A7	$[0.0045\alpha^2+0.1834\alpha+0.5618, 0.0135\alpha^2-0.2309\alpha+0.967]$
WE	A1	$[0.0073\alpha^2+0.2965\alpha+0.1944, 0.0007\alpha^2-0.2025\alpha+0.7]$
	A2	$[0.0056\alpha^2+0.2381\alpha+0.4537, 0.0005\alpha^2-0.136\alpha+0.833]$
	A3	$[0.0073\alpha^2+0.2965\alpha+0.1944, 0.0007\alpha^2-0.2025\alpha+0.7]$
	A4	$[0.0048\alpha^2+0.209\alpha+0.5833, 0.0004\alpha^2-0.1032\alpha+0.9]$
	A5	$[0.0048\alpha^2+0.209\alpha+0.5833, 0.0004\alpha^2-0.1032\alpha+0.9]$

	A6	[0.0048α²+0.209α+0.5833,0.0004α²-0.1032α+0.9]
	A7	[0.0048α²+0.209α+0.5833,0.0004α²-0.1032α+0.9]
E	A1	[0.0064α²+0.2642α+0.3195,0.0027α²-0.1796α+0.767]
	A2	[0.0072α²+0.2927α+0.1919,0.0033α²-0.2155α+0.7]
	A3	[0.0024α²+0.0976α+0.0643,0.0044α²-0.2731α+0.433]
	A4	[0.0048α²+0.2079α+0.64,0.0016α²-0.1159α+0.967]
	A5	[0.004α²+0.1787α+0.736,0.0011α²-0.0824α+1.0]
	A6	[0.0048α²+0.2071α+0.6074,0.0016α²-0.1153α+0.933]
	A7	[0.0048α²+0.2063α+0.5757,0.0016α²-0.1148α+0.9]

The FPIS and the FNIS have been determined as [1,1] and [0,0] respectively. Thus the distance of the alternatives from FPIS and FNIS is calculated using different values of α and is shown in Table 9 and Table 10 respectively

Table 9. FPIS, A^* for each alternative for different values of α-cut.

Alt	α=0	α=0.1	α=0.2	α=0.3	α=0.4	α=0.5	α=0.6	α=0.7	α=0.8	α=0.9	α=1
A1	0.488	0.485	0.482	0.479	0.475	0.472	0.468	0.465	0.461	0.505	0.453
A2	0.414	0.411	0.408	0.405	0.402	0.398	0.394	0.391	0.387	0.373	0.378
A3	0.655	0.659	0.662	0.666	0.669	0.672	0.675	0.678	0.680	0.835	0.685
A4	0.214	0.211	0.208	0.205	0.201	0.198	0.194	0.190	0.187	0.241	0.179
A5	0.214	0.211	0.208	0.204	0.201	0.197	0.193	0.190	0.186	0.273	0.178
A6	0.202	0.198	0.195	0.191	0.188	0.184	0.181	0.177	0.173	0.241	0.166
A7	0.255	0.252	0.248	0.245	0.242	0.238	0.234	0.231	0.227	0.307	0.219

Table 10. FNIS, A^- for each alternative for different values of α-cut.

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Alt	α=0	α=0.1	α=0.2	α=0.3	α=0.4	α=0.5	α=0.6	α=0.7	α=0.8	α=0.9	α=1
A1	0.512	0.515	0.518	0.518	0.525	0.528	0.532	0.535	0.539	0.495	0.547
A2	0.586	0.589	0.592	0.592	0.598	0.602	0.606	0.609	0.613	0.627	0.622
A3	0.345	0.341	0.338	0.338	0.331	0.328	0.325	0.322	0.320	0.165	0.315
A4	0.786	0.789	0.792	0.792	0.799	0.802	0.806	0.810	0.813	0.759	0.821
A5	0.786	0.789	0.792	0.792	0.799	0.803	0.807	0.810	0.814	0.727	0.822
A6	0.798	0.802	0.805	0.805	0.812	0.816	0.819	0.823	0.827	0.759	0.834
A7	0.745	0.748	0.752	0.752	0.758	0.762	0.766	0.769	0.773	0.693	0.781

Aggregation of the distance for each alternative is carried out and is obtained as in Table 11.

Table 11. Aggregated distance for alternatives from A^* *and* A^-

Alt	A^*	A^-
A1	0.47	0.52
A2	0.40	0.60
A3	0.67	0.32

A4	0.20	0.80
A5	0.20	0.79
A6	0.19	0.81
A7	0.24	0.75

Next, the CC_i is calculated to determine the order of preference of the applicants. The ordering of the applicants based on the CC_i values is given in Table 12.

Table 12. The order of preference of applicants PIHTAS post.

Alt	CC_i	Order of preference
A1	0.52569	6
A2	0.60193	5
A3	0.31989	7
A4	0.80043	3
A5	0.80041	2
A6	0.81353	1
A7	0.75942	4

The order of the applicants is decided based on the values of the CC_i . The higher the value of CC_i will result in a better preference of the applicants. The order of preference shows that applicant A6 is preferred as compared to other applicants since the value of the CC_i is the highest. From this result, the management of TH may call the desired number of top applicants that are suitable for the post.

4. Conclusion

This study aims to select the qualified PIHTAS at Tabung Haji (TH) to guide pilgrims to perform better hajj activities. Pilgrims need PIHTAS during their stay at Mecca to guide and show them the hajj process so that they can perform hajj smoothly and follow the obligation of Hajj. The PIHTAS selection consists of two stages which are the evaluation of their background obtained from the application form and the interview session. The focus of this study is to select successful applicants based on the application form to be called for an interview session. The λ-Max method of FAHP was used to determine the importance weight of the four criteria under consideration. The weight will be integrated into the FTOPSIS method for obtaining the preferred applicants to go to the next stage of selection. In both methods of FAHP and FTOPSIS, the computation is done using the α-cut method. As discussed in the literature, the α-cut method gives a better representation of fuzzy numbers and its arithmetic and aggregation operations will include all possible values in the interval as opposed to the three tuple representations that only consider the three endpoint values and approximate others. The α-cut method gives a true shape of the triangular fuzzy numbers in particular for the multiplication and division operation since the normal multiplication between triangular fuzzy numbers will only give an approximation to the resultant triangular fuzzy number. The result in this study shows that fuzzy evaluation using the fuzzy hierarchical TOPSIS, a hybrid of FAHP and FTOPSIS is suitable to be used as compared to normal numerical evaluation.

The method is also suitable to be used to solve other decision-making problems such as supplier selection, evaluating financial performance and others. The method can also be improved and enhanced in many ways. Alternatively, different ideal solutions and distances may be used in the method. Furthermore, a system with if-then rules may also be developed for obtaining better results and consistency.

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