SIR MODEL OF COVID-19 WITH QUARANTINE MEASURE IN MALAYSIA

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ABSTRACT

In this study, the susceptible-infected-recovered (SIR) model of COVID-19 infectious disease is constructed. The basic SIR model is modified to incorporate the quarantine measure. The basic reproduction number denoted by R_0 is computed using the next-generation method. The analysis showed that the disease-free equilibrium point is locally asymptotically stable when $R_0 < 1$, and the infection eventually dies out. The numerical results revealed that quarantine measures significantly reduce the number of infected individuals. As the proportions of the quarantined individual increased, the number of infections subsided rapidly, suggesting that quarantine measure is one of the effective control strategies to combat the spread of COVID-19 viruses.

***Keywords**: COVID-19, SIR Model.*

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1. Introduction

The COVID-19 epidemic began in December 2019 in Wuhan, China. The disease has spread throughout the world and the number of infected increases exponentially. As of June 1st 2021, the disease has effected 170 million people worldwide (WHO, 2021). This has led the governments around the world to take serious actions such as social distancing, self-quarantine and total lockdown to stop or slow down the spread of the virus. The first action taken by the Malaysian government was to enforce the Movement Control Order (MCO) in March 2020 which minimizes the number of people that are allowed to go outside. The purpose for this action was to reduce the contact rate among the people and subsequently reduce the infection rate.

The mathematical model, SIR, is widely used to model the spread of contagious diseases such as dengue fever and malaria (Hamdan & Kilicman, 2021). The existing SIR model can be further modified to adapt to certain parameters to increase the accuracy of predictions. The spread of Covid-19 can also be studied through various mathematical models such as SIR, SEIR and so on (Ansumali et al., 2020, Chancharoenthana et al. 2020, Gill et al., 2020, Sharov, 2020).

The standard SIR model assumes that all infected individuals contribute to the spread of the disease because it does not take into account quarantine measures and social distancing. (Nussbaumer-Streit et al., 2020) said that self-quarantine has been proven to be an effective measure to slow down the infection rate of the Covid-19 disease. Health experts suggest that 14 weeks of self-quarantine is required to ensure that the virus is no longer contagious. Thus, in this paper, the SIR model is constructed by incorporating the quarantine measure to further understand the dynamic of the spread of COVID-19 and the impact of self-quarantine to the growth trend of the disease in Malaysia.

This paper is organized as follows. In section 2, the methodology of the research is presented. The numerical results and discussion are given in section 3. Meanwhile, section 5 concludes the finding.

2. Methodology / Implementation

2.1 Model formulation

The proposed mathematical model of COVID-19 is based on the SIR model developed by Kermack and McKendrick (1927). The SIR model is constructed as follows:

$$\begin{aligned} \frac{dS}{dt} &= -\beta IS \\ \frac{dI}{dt} &= \beta IS - \gamma I \\ \frac{dR}{dt} &= \gamma I \end{aligned} \tag{1}$$

where the description of the parameter in the model is given in Table 2.1. The schematic diagram is presented in Figure 2.1.

Table 2.1 SIR Variables and Parameters Description

Variable/Parameter	**Description**
S(t)	Susceptible population
I(t)	Infected population
R(t)	Recovered population
β	Transmission coefficient
γ	Recovery rate

Figure 2.1 Schematic Diagram of SIR

In this model, the total population are divided into three components which are susceptible (S), infected (I) and recovered (R). This model describes the movement of individuals from susceptible to infected and to recover. The SIR model is useful for a disease that grant permanent immunity for those who have recovered. (Egbetade et al. 2018)

Since medical treatment cannot be fully utilized to control the spread of COVID-19, quarantine is among the most effective tools in preventing and controlling infectious. Hence, in this study, we incorporate the quarantine compartment into the SIR model. Quarantine is a measure that isolates infected individuals from infecting others. Therefore, the transmission process can be slow down. Here, the new compartment Q is introduced for the quarantine individual, and assume that quarantine is carried out such a way that fraction $0 < p \leq 1$ of infectious individuals will be isolated. A simple schematic diagram of the proposed model is depicted in Figure 2.2.

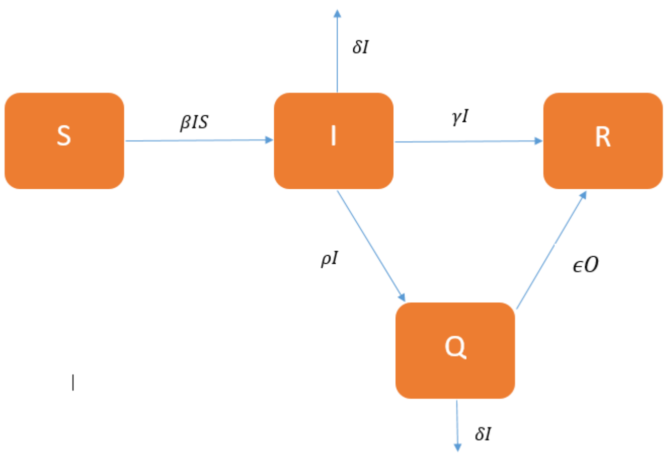

Figure 2.2 An SIR model of COVID-19 Pandemic with quarantine.

The systems of differential equations (model) based on the schematic diagram in Figure 2.2 are as follows:

$$
\begin{aligned}
\frac{dS}{dt} &= -\beta IS \\
\frac{dI}{dt} &= \beta IS - \gamma I - \rho I - \delta I \\
\frac{dQ}{dt} &= \gamma I - \epsilon Q - \delta Q \\
\frac{dR}{dt} &= \gamma I + \epsilon Q
\end{aligned}
\tag{2}
$$

where the variable and parameter used in the model are described in Table 2.2.

Table 2.2 Variables and Parameters Description

Variable/Parameter	Description	Value
S(t)	Susceptible population	-
I(t)	Infected population	-
Q(t)	Quarantined population	-
R(t)	Recovered population	-
β	Transmission coefficient	0.35
γ	Recovery rate	0.1
ρ	A fraction of infectious individuals will be quarantined	0.28
δ	Death rate due to COVID-19	0
ϵ	Recovery rate of the quarantined individuals	0.4

2.2 Basic Reproduction Number (R_0)

The basic reproduction number is the total number of individuals directly infected by a single infected individual, when introduced to totally susceptible population. The basic reproduction number can be interpreted as follows:

- $R_0 < 1$ Infectious dies out (unable to generate a major epidemic)
- $R_0 = 1$ Infectious is maintained
- $R_0 > 1$ Infectious takes over (disease spread)

In this study, the method that is used to find the basic reproduction number is the next-generation matrix approach. This method was introduced in (Diekmann et al., 1990) and it was further elaborated by Van den Driessche and Watmough (Van den Driessche and J Watmough, 2002). The next-generation matrix breaks down the Jacobian matrix for the infected population into the sum of two compartment; the transmission matrix (F) and the transition matrix (V). F represents the number of new infections and V represents the transfer rate of individuals into compartment i. Thus the basic reproduction number of system (2) is given by

$$R_0 = \frac{\beta}{\gamma + \rho + \delta} \tag{3}$$

2.3 Stability Analysis of Disease-Free Equilibrium

The local stability of the disease-free equilibrium point of system (2) is investigated using the following theorem.

Theorem 2.1 Suppose all the eigenvalues of $Df(x)$ have negative real parts. Then, the equilibrium point $x = \bar{x}$ of system (4) is locally asymptotically stable, and unstable if at least one of the eigenvalues has a positive real part. (Hamdan & Kilicman, 2021)

To find the disease-free equilibrium point, $\frac{dS}{dt} = \frac{dI}{dt} = \frac{dQ}{dt} = \frac{dR}{dt} = 0$.

Hence, the following equations are solved for a disease-free equilibrium point

$$-\beta IS = 0 \quad (5)$$

$$\beta IS - \gamma I - \rho I - \delta I = 0 \quad (6)$$

$$\gamma I - \epsilon Q - \delta Q = 0 \quad (7)$$

$$\gamma I + \epsilon Q = 0 \quad (8)$$

By using the simultaneous equation, the disease-free equilibrium denoted by $\bar{x}$ of system (2) is obtained as follows

$$\bar{x} = (S, I, Q, R) = (\frac{\gamma+\rho+\delta}{\beta}, 0, 0, 0) \qquad (9)$$

The Jacobian matrix at the DFE, $\bar{x}$ is given by

$$J_0 = \begin{bmatrix} 0 & -\gamma-\rho-\delta & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \rho-\delta & -\epsilon & 0 \\ 0 & \gamma & \epsilon & 0 \end{bmatrix} \qquad (10)$$

By solving $|J_0 - \lambda I| = 0$ where λ is the eigenvalue, we obtained

$$\lambda^3(\lambda + \epsilon) = 0 \qquad (11)$$

Hence, $\lambda = -\epsilon$.

This shows that the eigenvalues have negative real parts. It follows then by Theorem 2.1, that the disease-free equilibrium point $\bar{x}$ is locally asymptotically stable. We obtained the following theorem using the basic reproduction number R_0.

Theorem 2.2 The DFE, $\bar{x}$ of system (2) is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

Proof

From (3), the basic reproduction number can be expressed as:

$$R_0 = \frac{\beta}{\gamma + \rho + \delta}$$

Using the parameter values in Table 2.2:

$$\beta = 0.35,\ \gamma = 0.1,\ \rho = 0.28,\ \delta = 0$$

$R_0 = 0.921 < 1$. This shows that the disease free equilibrium is locally asymptotically stable.

3. Results and Discussion

By 28th February 2021, the total COVID-19 Cases in Malaysia accumulated is 300744 cases, while recovered cases are 273416 in number. Figure 3.1 showed both daily cases and removed daily of COVID-19 for Malaysia stating from 1st February 2020 until 28th February 2021 (MOH, 2021).

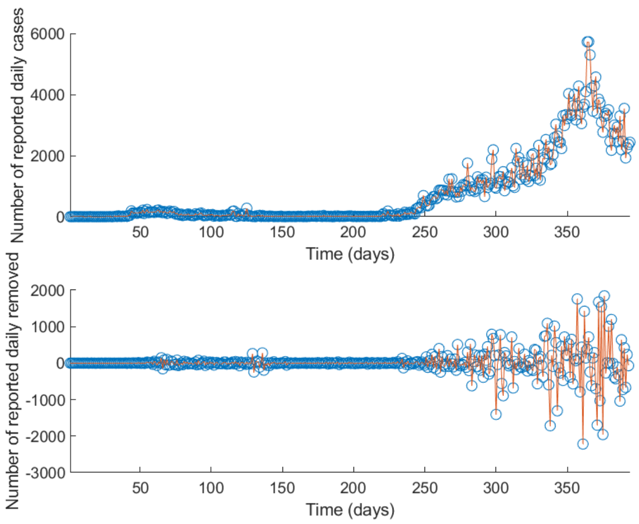

Figure 3.1 Daily cases and Removed Daily Cases of COVID-19 in Malaysia

This study aimed to develop SIR model of COVID-19 in Malaysia with quarantine measures. The transmission model was made by using MATLAB that implement the usage of Runge-Kutta method which help in solving Ordinary Differential Equation (ODEs). Hence, there are a certain parameters and values assign for the SIQR model. Table 4.1 described the parameters and the values used in the model.

Table 3.1 Definition and Values Assigned for Each Parameter

Variable/Parameter	Description	Value
β	Transmission coefficient	0.35
γ	Recovery rate	0.1
ρ	A fraction of infectious individuals will be quarantined	0.28
δ	Death rate due to COVID-19	0
ϵ	Recovery rate of the quarantined individuals	0.4

To determine if quarantine measures is effective in restraining COVID-19 transmission, we show both Figure 3.2 (SIR model) and Figure 3.3 (SIQR model). The parameters are as shown in table 3.1 except for SIR model the value for ρ which indicated fraction of infectious individuals undergoing quarantine will be 0.

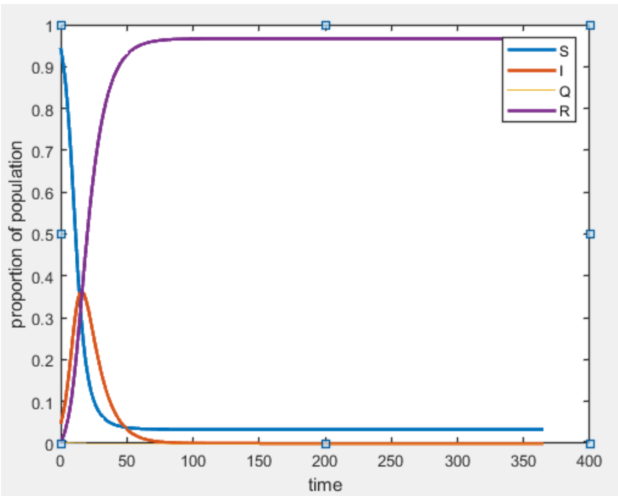

Figure 3.2 SIR Model

In Figure 3.2, the infected population are high since there is no counter measures taken to prevent susceptible population to be infected. Furthermore, most susceptible population are expected to get infected based on Figure 3.2. As for Figure 3.3, the infected population are low in number which explain why the infected population graph are low. This is due to the quarantine measures which help in preventing the spreading of COVID-19. In addition, with quarantine measures added, majority of susceptible population can be prevented from being infected from COVID-19. That explained Figure 3.3 which shows that the susceptible population settled at a constant rate.

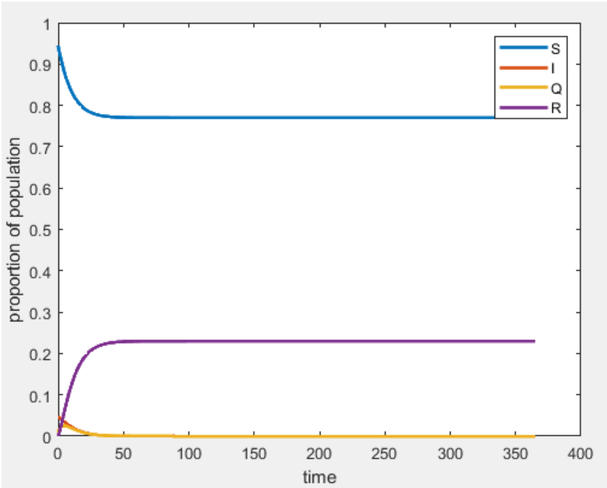

Figure 3.3 SIQR model

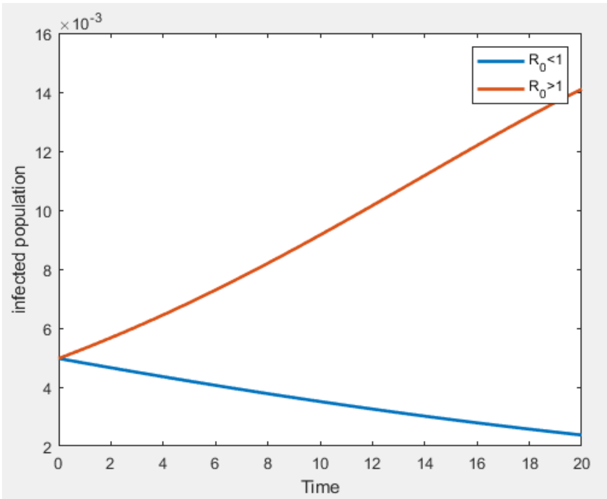

Figure 3.4 Infected Population from SIQR Model with different R_0 value

The basic reproduction number is the total number of individuals directly infected by a single infected individual, when introduced to totally susceptible population. Figure 3.4 is the infected population from SIQR model. As we can see in Figure 3.4, the infected population when the reproduction number, R_0 is less than 1, the graph for the infected population is constantly decreasing. This indicates the infected population will approaching 0 by the end of the model. However, in the same figure we can see that when R_0 is bigger than 1, the infected population will keep increasing in number.

In Figure 3.4, the stability of the equilibrium point known as the disease-free equilibrium point is reached when the basic reproduction number, R_0 is less than 1 since the graph reach a steady state. This numerically verified the results on the local stability of disease-free equilibrium point.

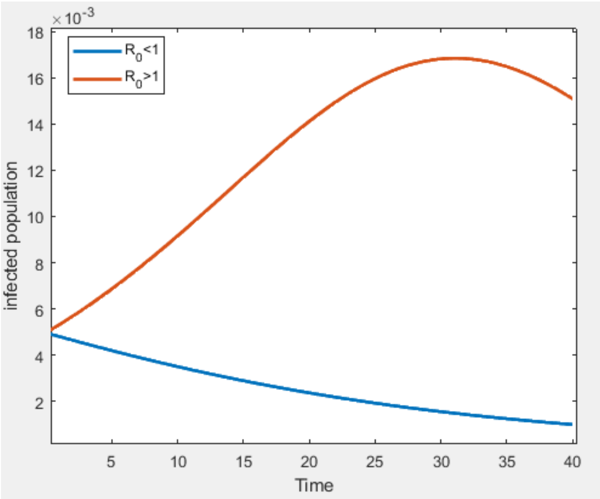

Figure 3.5 Infected Population from SIQR model with different R_0 value with longer time period

In Figure 3.5, the graph for infected population with R_0 is bigger than 1 starts to decrease. This is because, as time goes by, the disease will slowly subside as the quarantine measures that is being added has taken effect on the infected population.

Figure 3.2 and 3.3 proved that quarantine can help to prevent COVID-19 from spread. Next, we would like to see if higher values of ρ which indicated fraction of infectious individuals undergoing quarantine will affect the infected population. Figure 3.6 shows the infected population with different set of ρ value which is 0, 0.1, 0.5 and 0.8.

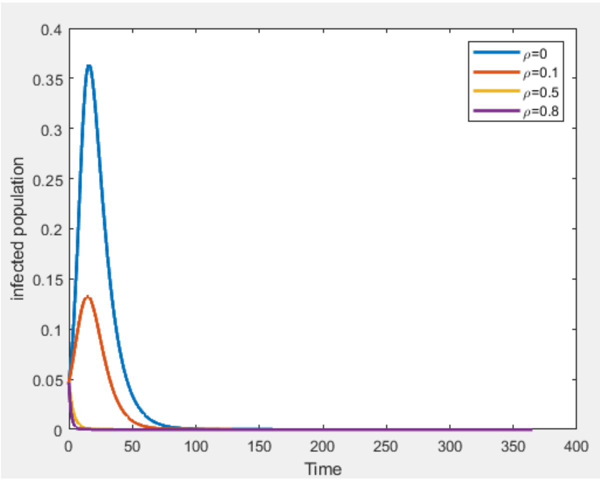

Figure 3.6 Infected Population with Different Values of ρ

From figure 3.6, we can conclude that the higher the percentage of infected people to quarantine, the lower the number of infected populations. By the end of our discussion and results, it is proven that quarantine measures help to reduce the infected population from increasing exponentially and prevent the susceptible population from being infected.

4. Conclusion and Recommendation

This paper concludes that by taking enough precaution, the spreading of COVID-19 in Malaysia can be reduced to a certain level. From the result obtained, we can see the significance of quarantine measures in restraining the escalation of infected cases. The higher the number fraction of individuals infected by COVID-19 quarantine, the lower the number of infected populations will be. We have also managed to analyze the stability of the disease-free equilibrium. We have proven that the model is stable when $R_0 < 1$. The limitation in this dissertation is the parameters are mostly assumed. This is because lack of sufficient data especially the parameter values related to COVID-19 cases in Malaysia

As for future research, we would recommend to add vaccination measures to the model. Since quarantine is proven that it help to reduce COVID-19 from spreading, by adding vaccine measures, new possibilities might happen either it helps to restrain the COVID-19 from outbreak or nothing will happen to the graph of infected population. Next, we would recommend to consider SEIR model which helps in dividing the infected individual into two categories which is symptomatic and asymptomatic. As a conclusion, the quarantine measures are surely helpful as it prevent the contact of infected individual with susceptible population and reduce the chances for COVID-19 to infect others.

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