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AN ASSESSMENT OF COLLEGE STUDENTS' UNDERSTANDING IN MATHEMATICAL PROBLEM SOLVING

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ABSTRACT

The aim of this study was to develop a comprehensive description of college freshman's understanding of basic mathematical concepts and heuristic actions in application to problem solving. The methodology used was both qualitative and quantitative in nature. This study accumulates evidence regarding the nature of students' understanding of basic mathematical concepts and some critical factors to be taken into account in facilitating their development. How well do the national examination grades reflect the mathematical knowledge of students? The grades obtained in the national examination of Sijil Pelajaran Malaysia (SPM) for mathematics do not correlate with their mathematical knowledge in problem solving. The findings indicate that these students have an instrumental understanding rather than a relational understanding as the data shows that they were not able to apply mathematical knowledge to new contextual situations. This study also indicates that there was no difference in the performance of the "A Math Students" and "Non A Math Students" in the problem solving test. The result of this study shows that majority of college freshmen could not do problem solving question. They have learnt how to do numerical computation at the expense of learning how to think and solve problems.

BACKGROUND OF THE STUDY

When we trace the history of problem solving in school mathematics, we find that it has often meant solving highly structured word problems appearing in texts. Often, the word problems were developed by the teacher to provide practice for prescribed computational procedures rather than to encourage problem solving in a broader sense. Steffe (1994) states that :

> *The current notion of school mathematics is based almost exclusively on formal mathematics procedures and concepts that, of this nature, are very remote from the conceptual world of the children who are to learn them* (p. 5)

A study by Parmjit (1998) found that only a small percentage of students who did well in the Penilaian Menengah Rendah (PMR) were able to solve complex proportional problems and the grades obtained in this exam were not indicative of their knowledge of ratio and proportion. He noted that :

> *the more we focus on raising test scores, the more instruction is distorted and the less credible are the scores themselves. Rather than serving as accurate indicators of students knowledge and performance, the tests become indicators of the amount of instructional time and attention paid to the narrow range of skills assessed.* (p.107)

In fact, basic computation skills have been the focus for competency tests through the years, spawning textbooks and instructional emphases aimed at developing these skills. Students have learned how to do numerical computations at the expense of learning how to think and solve problems. If one has ready access to a solution scheme for a mathematical task, that task is an exercise and not a problem. *Problem* is a question that exercises the *mind.* We should be cautioned against claiming to emphasize on problem solving when in fact the emphasis is on routine exercises. From various studies involving problem solving instruction, Suydam (1987) concluded that :

> *If problem solving is treated as "apply the procedure," then the students try to follow the rules in subsequent problems. If you teach problem solving as an approach, where you must think and can apply anything that works, then students are likely to be less rigid. (p. 104)*

An emphasis on problem solving as a learning tool must take place, for today's students are the people who will lead the world tomorrow. The tools they learn in school now will be applied to the many serious problems they will face in the "real world". In view of this, research in problem solving has a special importance in the study of mathematics.

Significance and Purpose of the Study

The purpose of this study is to investigate college freshman's understanding and heuristic actions in problem solving. Successful mathematics students do indeed construct a fairly large number and variety of algorithms in order to continue to achieve good results in mathematics examinations. However, have we tried to determine the quality of this knowledge? This study will give us an insight into whether these students have good reasoning abilities or are just performing algorithmic procedures without making any sense of why these procedures worked.

This study could be useful for curriculum developers and educators, especially in Malaysia, in evaluating strategies and methods used in teaching and learning of mathematics in schools. Furthermore, the importance of problem solving, coupled with difficulties in acquiring the skill gives a rationale for studying problem solving in mathematics learning.

OBJECTIVES OF THE STUDY

This study aims to develop a comprehensive description of college freshman's thinking and reasoning capabilities in solving non-routine problems in a quantitative and qualitative manner. The quantitative study aims to establish some facts and figures on college freshman's performance in problem solving situations. Specifically, the questions addressed are:

1. What are the levels of college freshman's understanding in solving non-routine problems?

2. Do college freshmen who has obtained a high grade in SPM mathematics also score likewise on the problem-solving test?

3. Are there differences among the mean scores of A Math students and non A Math students' performance in the problem-solving test?

4. What difficulty do college freshmen encounter in solving non-routine problems?

DESIGN OF THE STUDY

This study used an instrument to assess students' relational understanding of mathematics. Its main goal was to provide key information concerning students' functioning in the area of problem solving, especially their ability to cope with different problem contexts. This written assessment provided both quantitative and qualitative data. Additionally, qualitative data from clinical interviews with college freshmen gave the researchers an in-depth understanding of these students' heuristic actions and tacit mathematical understanding that constitute thinking mathematically in problem solving.

There were all together 12 items in the test (for this paper only five of the items will be discuss) and the responses were grouped into categories according to the criterion behavior exhibited. A numerical value was assigned to each of these criterion behaviours. Students' responses were categorized on a 4 point scale based on the reasoning employed. The 4 point scale used was:

4. All correct, good reasoning
3. Good reasoning, small error(s)
2. Some promising work but it is not clear on whether a solution would be reache
1. Some work but unlikely to lead to a solution
0. Blank

For testing the reliability of the instrument, Cronbach's Alpha was used. The result of the reliability for the overall instrument was 0.70.

This study was conducted during the May/June semester in 2001 at a college in Malaysia. Five hundred thirty six college freshmen, ages 18-19, participated in this study.

Of the 536 subjects in this study, 345 students obtained a grade 1A, 108 a grade 2A, 53 a grade 3B, 16 a grade 4B, 7 a grade 5C and 7 others a grade 6C in their SPM mathematics examination. For this study, the 453 students who obtained 1A and 2A grades in SPM mathematics were clustered as "A Math Students" compared to the balance of 83 as "Non A Math Students". The distribution of these two groups of students is shown in Table 4.1.

Table 4.1
Clustering Students by SPM Mathematics Grades

Type of Student	Frequency	Percentage
A Math	453	84.5
Non A Math	83	15.5
Total	536	100.0

Item Analysis of Test Items

An item analysis of the twelve test items in the instrument were conducted but for this paper only five of the items will be discuss. The data can be an important indication of college freshman's fundamental relational understanding of mathematical concepts in problem solving.

Table 4.2
Students' Performance on Test Items

Item Number	Mean Score	Standard Deviation
Item 1	1.76	0.87
Item 2	2.69	1.38
Item 3	2.25	1.19
Item 4	2.59	1.46
Item 5	2.22	1.38
Item 6	1.85	1.00
Item 7	1.80	1.23
Item 8	2.03	0.80
Item 9	1.33	1.00
Item 10	1.21	0.99
Item 11	2.44	1.21
Item 12	2.46	1.41

Item 3

Seekor anjing sedang mengejar seekor arnab yang berada 45m dihadapan. Anjing tersebut melompat sejauh 3m setiap kali, arnab melompat sejauh 2m setiap kali. Dalam berapa lompatankah anjing dapat memintas arnab?

In this item, a mean of 2.25 with approximately 39.6% gave a correct response. This is an algebraic task and some students were observed using interesting heuristic action to solve the problem such as: *difference of distance of every leap is 1 m. To cover the difference of 45 m, it requires 45 leaps. Therefore, the dog will overtake the rabbit in the 46th leap.*

Majority of them saw the problem as a difference of 1 meter between each jump of the rabbit and dog. Then they classified the problem as sequences: *2, 4, 6, 8, ... as the sequence for the distance traveled by the*

rabbit and 3, 6, 9, 12, ... as the sequence for the distance traveled by the dog.

Surprisingly, students faced difficulty in expressing the problem into a mathematical algebraic equation of $45 + 2(x) = 3x$ where x is the number of jumps. None of the students use this algebraic equation to solve the problems, as one will expect from college students.

Table 4.3 summarizes the cross tabulation data for Item 3. Only 39.6% of the sample manage to exhibit good to excellent reasoning while approximately 57.6% (261) of the "A Math" students were not able to solve Item 3.

Table 4.3
Performance of Two Groups of Students in Item 3

Types of reasoning	"A Math" Students	"Non A Math" Students	Total
No reasoning	7	2	9
Very poor or poor reasoning	254	61	315
Good to excellent reasoning	192	20	212
Total	453	83	536

Item 4
Tolak, tambah, tolak tambah.....???
Jumlahkan 1 – 2 + 3 – 4+ 5 – 6 + ...- 100.

Majority of students tried to classify the sequences as a geometric series or arithmetic series by finding the common ratio or the common difference, which led to the wrong answer that did not make sense. They did not realize that this series was neither a geometric nor an arithmetic progression.

However, the majority of the 51.5% of students who got this item correct, separated this series as sum of 1 + 3 + 5 + 7 + ... and -2 + - 4 + -6 ... . After finding the sum of each series, they total the sum to get the answer.

Table 4.4 shows the cross tabulation data for Item 4. The mean score is 2.59 with approximately 51.5% (276) showing good to excellent reasoning. The encouraging sign is that more than half of the "A Math" students are among the 51.5%. However, there were 46.8% (212) of "A Math" students who were not able to achieve a high level of reasoning.

Table 4.4
Performance of Two Groups of Students in Item 4

Types of reasoning	"A Math" Students	"Non A Math" Students	Total
No reasoning	36	10	46
Very poor or poor reasoning	176	38	214
Good to excellent reasoning	241	35	276
Total	453	83	536

Item 7

Elina dan Amy bercadang untuk mengecat pintu rumah mereka. Pada mulanya mereka membancuh (mencampur) 2 tin cat putih dengan 3 tin cat hitam untk mendapatkan cat berwarna kelabu. Kemudian mereka menambahkan satu tin setiap cat lagi dalam bancuhan tersebut. Berbanding dengan bancuhan asal, adakah warna kelabu pada bancuhan baru ini lebih terang, lebih gelap atau sama?

Approximately 76.4 % of the students got this item wrong with a low mean of 1.80. They reasoned that if one adds 1 can of black paint and 1 can of white paint, their colour should still be the same. Their reasoning was based on the assumption that since one can of each paint were added and the quantity of both colours were the same, the outcome of the new mixture should have the same shade. Their reasoning was based on the primitive additive reasoning and approximately 42.2 % of the students gave this additive reasoning. In short, these students failed to construct a coordination of two ratios simultaneously as: 2 white to 3 black and 3 white to 4 black. Analysis of Table 4.5 indicates that only a small number of students (127) managed to solve Item 7, although a big portion of them were "A Math" students. Approximately 74.8% (339) of the "A Math" students were unable to exhibit good to excellent reasoning.

Table 4.5
Performance of Two Groups of Students in Item 7

Types of reasoning	"A Math" Students	"Non A Math" Students	Total
No reasoning	6	2	8
Very poor or poor reasoning	333	68	401
Good to excellent reasoning	114	13	127
Total	453	83	536

Item 8

Jika 9 orang pekerja mengambil masa 5 jam untuk memotong rumput di atas sebidang tanah, berapakah masa yang diambil oleh 6 orang pekerja untuk melakukan kerja yang sama? (Andaikan bahawa semua pekerja dapat melaksanakan kerja pada kadar yang sama dan bekerja sepanjang masa)

In this item, 78.7 % of the students failed to see an inverse proportion relationship and solved the question by utilizing a cross multiplicative structure. Many of them used the following heuristics: *9 workers = 5 hours and 6 workers = X*

$$X/5 = 6/9; \quad 9X = 30; x = 30/9 = 3\ 1/3 \text{ hours.}$$

In fact, 57.8 % of the students utilized mechanical reasoning as shown above. Here, they did not reason what each number represents and what they were actually computing. Logically, they should have realized that the answer they produced (3 1/3 hours) implied that more people take a longer time to finish up the job!

The mean score for this item was 2.03. Table 4.6 shows the majority of the students exhibit poor reasoning in this item.

Table 4.6
Performance of Two Groups of Students in Item 8

Types of reasoning	"A Math" Students	"Non A Math" Students	Total
No reasoning	11	2	13
Very poor or poor reasoning	384	78	462
Good to excellent reasoning	58	3	61
Total	453	83	536

Item 9

Sebuah basikal antik mempunyai saiz roda yang berbeza. Roda depannya berukurlilit 8 kaki manakala roda belakangnya berukurlilit 10 kaki. Berapa jauhkah basikal ini telah bergerak ketika roda depannya telah berputar 20 pusingan (revolusi) lebih dari roda belakang?

Observations of students' responses to this item indicate that approximately 90% of the students could not even get to a stage to show any promising heuristic reasoning. They faced great difficulties in expressing the problem into a mathematical expression. From the interviews it revealed that a majority of them did not realize that both tyres traveled the same distance.

The mean score for this item was 1.33 with only 8.4 % getting it correct. Table 4.7 shows that 91.6% (491) of the students showed poor, very poor or no reasoning in attempting this problem. Consequently, it explained the low mean score of 1.33. It is disappointing that approximately 90.1% (408) of "A Math" students did not show good reasoning abilities here.

Table 4.7
Performance of Two Groups of Students in Item 9

Types of reasoning	"A Math" Students	"Non A Math" Students	Total
No reasoning	70	15	85
Very poor or poor reasoning	338	68	406
Good to excellent reasoning	45	0	45
Total	453	83	536

Comparison of Problem Solving Ability of "A Math" Students and "Non A Math" Students

The cross tabulation tables of each item discussed above indicate that there is some interesting relationships between the proportions of different types of responses from the two groups of college freshmen. Indeed, when the difference of mean scores for the "A Math" and "Non A Math" students

for each item was calculated, there were significant differences between the two groups of students in several items.

Table 4.8 indicates the mean difference between the "A Math" students and "Non A Math" students in each of the items.

Table 4.8
Performance of "A Math" Students Versus "Non A Math" Students

Item	Mean of "A Math" Students'	Means of "Non A-Math" Students	Mean Difference	Sig.
1	1.78	1.61	.17	.106
2	2.78	2.20	.57*	.000
3	2.32	1.87	.45*	.001
4	2.64	2.30	.34	.051
5	2.32	1.67	.64*	.000
6	1.89	1.69	.20	.098
7	1.85	1.53	.32*	.031
8	2.06	1.90	.15	.107
9	1.37	1.13	.24*	.047
10	1.22	1.17	.05	.673
11	2.50	2.12	.38*	.008
12	2.52	2.18	.24*	.046

*The mean difference is significant at the .05 level.

There was a significant difference at the 0.05 level between the means of the "A Math" students and the means of "Non A Math" students for item 2, item3, item 5, item 7, item 9, item 11 and item 12, while in Item 1, Item 4, Item 6, Item 8, Item 10, there was no significant difference between the means of the two groups of students. It is quite surprising that most "A Math" students could not solve some of these problems, especially the items which showed quite a low mean score. Those items are Item 1 which requires the ability to construct and solve simultaneous equations, Item 6 which requires the skill of interpreting ratio problem and Item 10 which require some skills in interpreting and organizing the data.

CONCLUSION AND DISCUSSION

The results of this study show that college freshmen just could not solve these types of problems. This is not a surprising result, given the paucity of the curriculum and instruction on problem posing and problem solving they encountered in high school. It seems that students have learnt how to do numerical computation at the expense of learning how to think and solve problems. Specifically, the following general conclusions were obtained from analyses of the quantitative and qualitative data.

1. Students obtained a very low mean score in the problem solving test.
2. There are no significant differences among the mean score of "A Math" students and "Non A Math" students' performance in the problem solving test.

3. The Mathematics grades obtained in the Sijil Pelajaran Malaysia do not correlate with a student's mathematical knowledge in the problem solving test.
4. Students who obtained a high grade on the SPM Mathematics examination do not score likewise on the problem solving test.
5. The primary goal for most of these students is to see an algorithm that will give them the answer quickly.
6. Many students know how to carry out basic mathematical procedures when problems are presented in symbolic form but are not able to apply these procedures to solve problems presented in words. In short, this study suggests that the difficulty of students lies in understanding the problems, rather than executing procedures.

Results from this research on students understanding in problem solving indicate that certain incorrect ideas about the fundamental mathematical concepts are common to the college freshmen. There is evidence that college freshmen often have many of the same conceptual and reasoning difficulties that are common among younger children. There is often little change in conceptual understanding before and after formal instruction. Moreover, students are often unable to apply the concepts that they have learnt to the task of solving non-routine quantitative problems.

The quantitative data obtained from posing these problems seem to show that for most students in this study, school mathematics instruction was procedural without sense-making: one learns to read the problem, extract the relevant numbers and the operation to be used, to perform the operation, and to write down the result—without even thinking about what it all means. Utilising the cross multiplication approach in solving these proportion problems simply becomes an act of symbolic manipulation without requiring that individuals make sense of what they are doing. This clearly shows students' memorising procedures and rules in applying the cross-multiplication procedure rather than emphasising on making sense of the outcome of such procedures. This cross-multiplication algorithm is an efficient way of getting answers but is often used without meaning. In order for students to develop structural understanding, we should perhaps emphasize on giving them experiences that can create a solid foundation for these concepts (Kieran, 1994).

In summary, the findings of this study have potential significance for the teaching and learning of mathematics in school in general and the Education Ministry of Malaysia in particular. We must change our perspective on mathematics learning and make problem solving the focus of school mathematics. Problem solving should be used to accomplish the instructional goals of learning basic facts, concepts, and procedures, as well as goals for problem solving within problem contexts. In other words, mathematics instruction should be designed so that students experience mathematics as problem solving.

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