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MERGING LANES: WHERE E-LEARNING DIVERSITY MEETS FUTURE TRENDS

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ANALYSIS OF COGNITIVE BARRIERS IN GRAPHICAL REPRESENTATION FOR TRIPLE INTEGRAL PROBLEM-SOLVING AMONG ENGINEERING UNDERGRADUATES

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ABSTRACT

Spatial visualization skills are a critical element in mastering multivariate calculus, particularly in solving triple integral problems. However, many engineering undergraduates face significant challenges in transforming complex algebraic equations into accurate three-dimensional (3D) graphical sketches. This study aims to identify systematic errors committed by students when sketching the integration domain and analyze the impact of visualization failure on the determination of integration limits. The research methodology involves a diagnostic test administered to 50 engineering students at Universiti Teknologi MARA Cawangan Pulau Pinang, followed by an error analysis based on the phases of equation deconstruction in 3D sketching. Preliminary findings indicate that the majority of students experience cognitive errors during the sketching phases based on the categorised errors. The study concludes that a significant gap exists between procedural calculation skills and conceptual visual understanding. The implications of this study suggest the need for integrating computer-aided design (CAD) or Geogebra tools into engineering mathematics pedagogy to strengthen students' visual intuition before transitioning to theoretical calculation phases.

Keywords: *engineering mathematics, error analysis, graphical representation, spatial visualization, triple integrals*

Introduction

The integration of multi-dimensional calculus into the engineering curriculum is fundamental for developing the analytical skills required to model physical phenomena. In Engineering Mathematics, the concept of triple integrals ($\iiint f(x, y, z) dV$) serves as a cornerstone for calculating properties such as center of mass, moments of inertia, and fluid dynamics within a three-dimensional boundary (Martínez-Planell et al., 2025). As engineering problems transition from theoretical exercises to real-world applications, the ability to define a spatial domain becomes a prerequisite for any successful numerical computation (Anita et al., 2025).

However, a significant pedagogical challenge arises when students move from basic calculus to multivariable systems. While students are generally proficient in basic integration techniques, they often encounter a cognitive border when required to synthesize algebraic equations into a coherent geometric structure. This transition from a symbolic representation to a spatial one is not merely a mechanical step but a sophisticated mental process that involves interpreting boundaries, intersections, and orientations in 3D space (Bazarbayeva & Ye, 2024).

The purpose of this study is to investigate the specific graphical difficulties faced by engineering undergraduates when dealing with triple integrals. By focusing on the drafting phase, where a student must sketch the domain before setting up the integral, this research aims to pinpoint where the disconnect between algebra and geometry occurs. Understanding these hurdles is vital for improving engineering education and ensuring that future engineers possess the spatial intuition necessary for complex design and analysis.

Problem Statement

The primary issue identified in undergraduate engineering mathematics is the persistent Visualization Gap. Many students can perform the mechanical steps of integration, such as applying the power rule or substitution, but they struggle significantly when a problem is presented as a set of boundary equations without a pre-drawn diagram. This lack of spatial reasoning leads to a trial-and-error approach in setting up the limits of integration, which is mathematically inefficient and often leads to erroneous results in engineering contexts.

Furthermore, students frequently demonstrate a lack of strategic decision-making when selecting an appropriate coordinate system. In many triple integral problems, transforming from Cartesian coordinates to Cylindrical or Spherical systems can simplify the calculation from an impossible task to a straightforward one. However, due to a failure in visualizing the symmetry of the solid (e.g., recognizing a cone or a paraboloid), students often persist with Cartesian coordinates, leading to overly complex radical expressions and a higher probability of calculation errors.

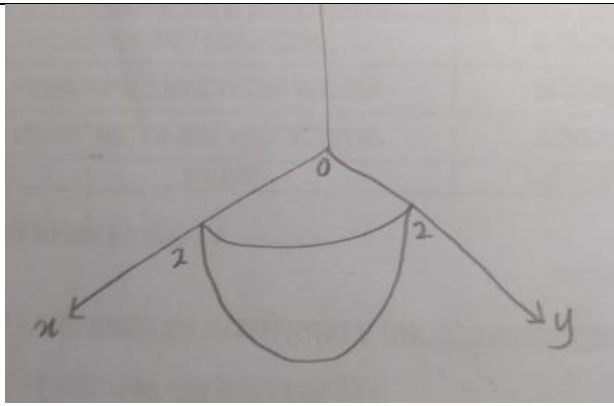
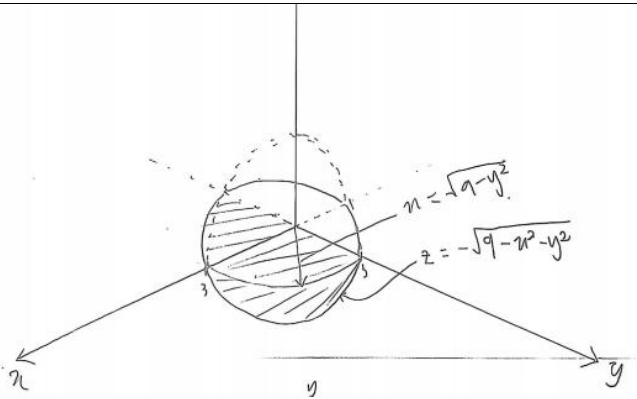
Lastly, the process of 2D Projection (projecting a 3D solid onto the xy , xz , or yz planes) remains a major stumbling block. Even if a student can vaguely imagine the 3D shape, they often fail to accurately determine the shadow or projection of that shape on a coordinate plane, which is essential for defining the outer limits of the integral. This inability to transition between dimensions (3D to 2D) suggests a deeper deficiency in graphical literacy that needs to be addressed through targeted pedagogical interventions.

Methodology

This research utilizes a qualitative error analysis framework to examine the sketching processes of engineering students during an assessment in solving triple integrals questions. The study focuses on 35 engineering students in UiTM Cawangan Pulau Pinang that have already received formal instruction on triple integrals and coordinate transformations. By analyzing the rough sketches and scratch work produced by these students, this study can categorize errors into distinct themes: conceptual (misunderstanding the shape), procedural (incorrect plotting), and strategic (choosing the wrong coordinate system).

The primary data collection tool is an assessment consisting of two equations of triple integral problems. These problems are formulated as text-based descriptions of solid (spherical coordinates) rather than provided diagrams (Table 1). Students are explicitly required to show their sketching process before attempting the integration. This allows the study to observe the direct correlation between the accuracy of the initial graph and the subsequent mathematical setup of the integral limits.

Table 1: Assessment questions on triple integrals

Questions	Triple integrals (Equations)	Correct sketches
1	$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_{-\sqrt{4-x^2-y^2}}^0 z \, dz \, dy \, dx$	
2	$\int_{-3}^3 \int_0^{\sqrt{9-y^2}} \int_{-\sqrt{9-x^2-y^2}}^0 dz \, dx \, dy$	

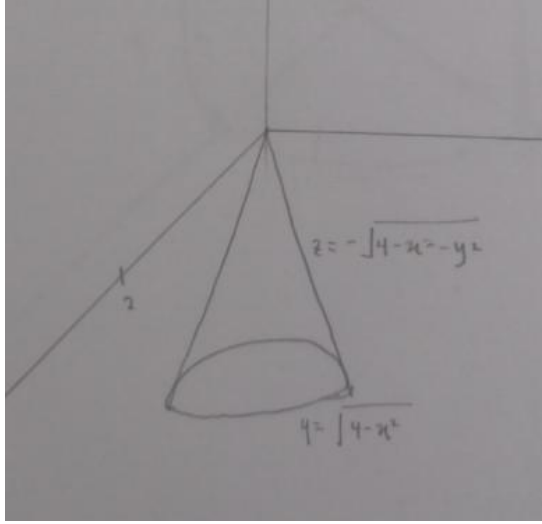
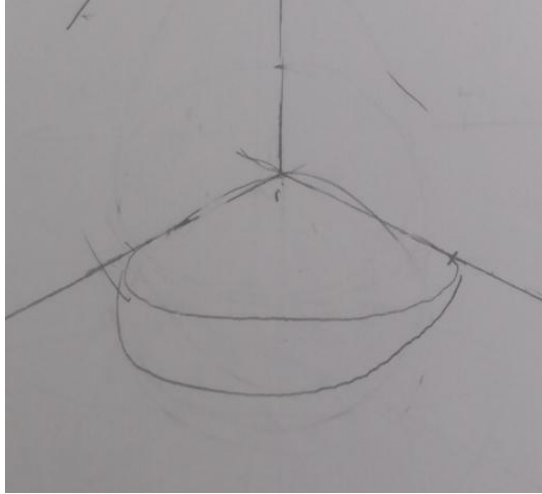
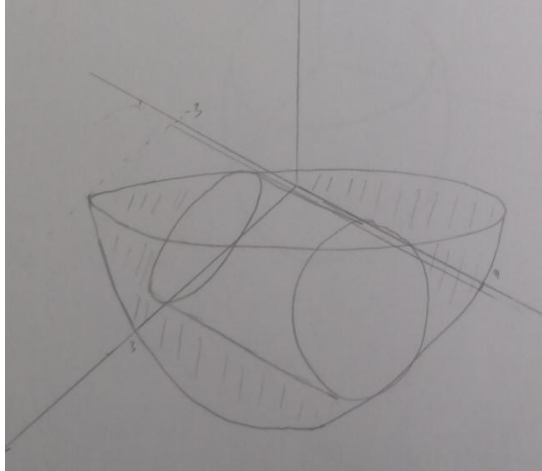
Results

The result show some of the students' samples and categorised errors based on distinct themes: conceptual (misunderstanding the shape), procedural (incorrect plotting), and strategic (choosing the wrong coordinate system) (Table 2 and Table 3).

Table 2: Sample of Errors for Sketching Question 1

Sample	Sketch	Categorised Errors		
		Conceptual	Procedural	Strategic
1		√	√	√
2		√	√	√
3		√	-	-

Table 3: Sample of Error for Sketching Question 2

Sample	Sketch	Errors		
		Conceptual	Procedural	Strategic
1		√	√	√
2		√	-	-
3		√	√	√

Discussion

The analysis of student works reveals that many undergraduates rely on memorized templates rather than true geometric understanding. For instance, while a student might successfully sketch a sphere centered at the origin, they often falter when the sphere is shifted or truncated by another surface, such as a plane or a cone. This suggests that their graphical skills are frail and highly dependent on seeing familiar patterns, rather than being able to derive a shape from their underlying mathematical constraints.

Another critical finding involves the Cognitive Load Theory in the context of multivariable calculus (Aditomo, 2009; Barbieri et al., 2025). Solving a triple integral requires the student to simultaneously manage algebraic manipulation, geometric visualization, and coordinate transformation. When the sketching phase is difficult, it consumes a large portion of the student's thinking capacity, leaving little room for accurate calculation. This often results in incorrect limits, where students use variables in the outermost integral, a fundamental conceptual error that violates the rules of definite integration (Martinez & Silva, 2024).

Finally, the discussion highlights the potential role of Technology-Enhanced Learning (TEL) in bridging this gap (Wiwatanapataphee et al., 2010; Milenković et al., 2024). The data suggests that students who are exposed to dynamic visualization tools, such as GeoGebra or MATLAB, tend to develop a better view for 3D surfaces (Saad et al., 2025). However, there is a risk that over-reliance on software might atrophy the student's ability to perform quick hand-sketches. Therefore, the study argues for a balanced approach where digital tools are used to build intuition, while manual sketching remains a core competency for engineering problem-solving.

Conclusion

In conclusion, this research confirms that the ability to draft a graph from a mathematical description is not a secondary skill, but a fundamental prerequisite for mastering triple integrals. The findings indicate that the Visualization Gap is a multi-layered issue involving conceptual, strategic, and dimensional hurdles. For engineering students, the failure to visualize is often the root cause of failure in the entire integration process, regardless of their proficiency in basic calculus.

The implications for engineering education are significant. It is recommended that mathematics departments place a greater emphasis on graphing from equations as a standalone skill before introducing the integration process. Incorporating more sketching exercises and teaching students how to identify the key features of 3D surfaces (such as intercepts and traces) can provide them with the scaffolding needed to tackle complex multivariable problems with confidence.

Ultimately, strengthening the link between the visual and the symbolic will produce more capable engineers. A student who can accurately view a problem before they solve it is far less likely to make catastrophic design errors in their professional career. By addressing these graphical difficulties at the undergraduate level, we ensure that the next generation of engineers possesses the spatial literacy required to navigate the increasingly complex physical and digital worlds.

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