MERGING LANES: WHERE E-LEARNING DIVERSITY MEETS FUTURE TRENDS

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RECURRING ERRORS IN INTEGRATION BY SUBSTITUTION: THE NEED FOR PEDAGOGICAL INNOVATION BASED ON MATHEMATICAL DECISION SUPPORT

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ABSTRACT

Integration by substitution is a fundamental technique in integral calculus that frequently poses difficulties for students in higher education. Although this technique is taught systematically, many students continue to make recurring errors, particularly in the selection of appropriate substitution variables and the management of differentials. This phenomenon indicates that the difficulties encountered are not merely procedural in nature but are closely related to weaknesses in students' conceptual mathematical decision-making. Therefore, this study aims to discuss the forms of recurring errors in integration by substitution and to argue for the need for pedagogical innovation based on mathematical decision support. The discussion is grounded in a review of recent literature and reflections on calculus teaching experience. The findings indicate that teaching approaches that focus primarily on procedural memorisation are insufficient to help students understand the mathematical rationale underlying the substitution technique. Accordingly, pedagogical innovations that provide structured and visual decision support are proposed to reduce recurring errors and to enhance students' conceptual understanding and confidence in learning calculus.

Keywords: integration by substitution, recurring errors, pedagogical innovation, calculus, decision support

Introduction

Integration by substitution is one of the fundamental techniques in integral calculus that serves to simplify the integration of composite functions through variable substitution. This technique is formally taught at the higher education level and constitutes an essential prerequisite for mastering more advanced methods of integration. However, teaching experience indicates that many students continue to face difficulties in effectively mastering this technique. These difficulties are often manifested through recurring errors in problem solving, particularly in the selection of appropriate substitution variables, the management of differential symbols, and the re-substitution to the original variable, even after students have undergone repeated practice.

Previous studies in mathematics education have shown that students' errors in calculus often stem from weak conceptual understanding and a tendency to memorise procedures without comprehending the underlying mathematical rationale (Hiebert & Lefevre, 1986). Students are found to develop mechanical forms of understanding, especially in interpreting calculus symbols such as differentials, which directly affect their strategies for solving integration problems (Tall & Vinner, 1981; Nilsen & Knutsen, 2023). Furthermore, studies have reported that students who rely heavily on procedural memorisation struggle to adapt the integration by substitution technique when confronted with variations in problem structure and tend to exhibit lower levels of confidence in problem solving (Fonbuena, 2022).

Despite the extensive body of research addressing students' difficulties in calculus, a significant gap remains in understanding why errors in integration by substitution persistently recur and how weaknesses in mathematical decision-making contribute to this phenomenon. Most previous studies have evaluated student performance based primarily on test scores, without examining in depth the cognitive processes and decisions students make when selecting substitution variables. Traditional teaching approaches that emphasise mechanical practice have also been found to be insufficient in helping students grasp the rationale behind each step of the solution process. Consequently, there is a need to examine the issue of recurring errors from a pedagogical perspective and to argue for the necessity of instructional innovations grounded in mathematical decision support.

Accordingly, this study aims to identify the forms of recurring errors committed by students in the integration by substitution technique and to argue for the need for pedagogical innovation based on mathematical decision support as a potential approach to reducing such errors and enhancing students' conceptual understanding in calculus learning.

Literature Review

The literature indicates that students' errors in learning calculus, particularly in the topic of integration, have long been a major focus of research in mathematics education. Tall and Vinner (1981) argued that students often construct concept images that are misaligned with formal mathematical definitions, leading to persistent conceptual misunderstandings. This issue provides a fundamental explanation for why students' errors tend to recur despite repeated practice.

Orton (1983) further demonstrated that errors in integration do not arise solely from computational weaknesses, but rather from a failure to understand underlying concepts and a tendency to memorise procedures. This finding is supported by Hiebert and Lefevre (1986), who distinguished between conceptual and procedural understanding, highlighting that learning environments that overly emphasise procedures without conceptual grounding make it difficult for students to adapt their knowledge to new situations.

In the context of modern calculus education, Nilsen and Knutsen (2023) found that students tend to interpret mathematical symbols such as differentials and integrals mechanically, without understanding their conceptual meanings. This limited interpretation directly affects how students apply the integration by substitution technique, particularly in selecting substitution variables and managing the symbol dx. Their study highlights the need for pedagogical support that enables students to interpret mathematical symbols in a more meaningful way.

Fonbuena (2022) emphasised the importance of quantitative understanding in u-substitution, showing that when students are guided through scaffolding based on quantitative reasoning, they are better able to grasp the conceptual structure of substitution. This finding supports the need for instructional approaches that go beyond procedural steps and instead emphasise the mathematical decision-making process.

Empirical studies by Hanifah (2021) and Sulistyaningtyas et al. (2023) further revealed that although students are often able to perform integration steps, they exhibit weaknesses in strategy selection and reflective thinking during problem solving. The errors identified were found to be recurring, particularly in determining appropriate substitution variables and correctly replacing differentials. These findings point to a lack of systematic guidance that supports students in making conscious and well-structured mathematical decisions.

Additionally, Shamsuddin and Abdul Rahman (2020) reported that errors in integration by substitution tend to be recurring and consistent, indicating the presence of stable error patterns that are not being effectively addressed. Their study proposed the use of visual aids and structured pedagogical approaches to help students better understand the flow of mathematical reasoning.

Overall, the literature review reveals that while numerous studies have examined students' errors in calculus, there remains a significant gap in research focusing specifically on recurring errors and the need for mathematical decision support in integration by substitution, particularly at the higher education level. Therefore, this study is proposed to address this gap by emphasising pedagogical innovation that guides students to make mathematical decisions in a systematic and meaningful manner.

Methodology

This study employed a qualitative descriptive research design to examine students' recurring errors in the integration by substitution technique and to understand the mathematical decision-making processes underlying these errors. The study sample consisted of 40 undergraduate engineering students at Universiti Teknologi MARA (UiTM) Cawangan Pulau Pinang who were enrolled in a calculus course and had been introduced to the topic of integration by substitution. Participants were selected using purposive sampling to ensure the relevance of the study context. Demographic data of the participants were reported in the methodology section for respondent profiling purposes and were not discussed in the findings section, as the focus of the study was on recurring errors and students' solution strategies in line with the research objectives.

The research instrument consisted of a set of written problems specifically designed to elicit a variety of solution strategies and potential errors in integration by substitution, with particular emphasis on the selection of substitution variables, the management of differentials, and re-substitution to the original variable. The instrument was reviewed and pilot-tested to ensure clarity, appropriate difficulty level, and content validity. Data were analysed using qualitative error analysis supported by basic descriptive statistics such as frequencies and percentages to identify patterns of recurring errors among students. This analytical approach was chosen because it aligns with the exploratory and conceptual nature of the study and serves to substantiate the need for pedagogical innovation based on mathematical decision support.

Results and Findings

Based on the analysis of students' written responses and observations of the solution strategies employed, three main findings were identified with respect to recurring errors in the integration by substitution technique. The findings focus on errors related to the selection of substitution variables, the management of differentials, and the consistency of solution steps, as summarised in Table 1 and visually illustrated in Figure 1.

Table 1: Types of Recurring Errors

Finding	Type of error	Brief Description
Finding 1	Error in selecting the substitution variable (u)	Students select u based solely on visual form without considering the appropriateness of its derivative
Finding 2	Error in managing differentials (du and dx)	Students fail to correctly replace dx or ignore the derivative factor
Finding 3	Inconsistency in solution steps	Students do not substitute back to the original variable or skip essential steps

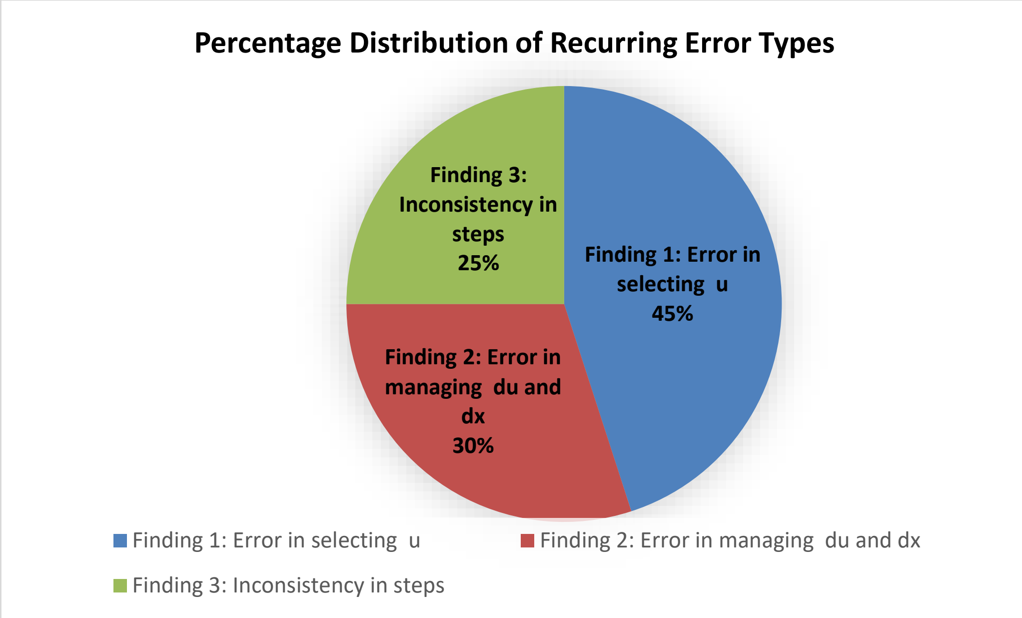

Figure 1: Pie Chart Showing the Percentage Distribution of Recurring Error Types in Integration by Substitution

As shown in Table 1, the findings indicate that the most dominant error is related to the selection of the substitution variable (u). Students tend to choose u based on similarities with previous examples or the visual form of the integrand, without evaluating the relationship between the function and its derivative. This error occurs at the initial stage of problem solving and has a direct impact on the entire integration process. This finding is supported by Figure 1, which shows that 45% of students committed errors in selecting the substitution variable.

Subsequent findings reveal that 30% of students experienced difficulties in managing differentials, particularly in consistently replacing dx with du. Students were found to ignore the derivative factor or fail to properly incorporate du into the new integrand, reflecting weaknesses in the conceptual interpretation of calculus symbols. In addition, 25% of students were identified as making errors related to inconsistencies in solution steps, especially failing to substitute back to the original variable after integration or skipping essential steps.

Overall, the combined findings presented in Table 1 and the percentage distribution shown in Figure 1 indicate that students' recurring errors are concentrated at critical points in the mathematical decision-making process when applying the integration by substitution technique. These findings reinforce the need for pedagogical approaches that provide systematic and structured decision support to help students reduce recurring errors and develop stronger conceptual understanding.

Example of Task Used for Analysis

To provide context for the analysis presented in Table 2, the following example represents a typical task given to students:

Evaluate the integral $\int 2x\cos(x^2)\,dx$

This task was designed to elicit students' strategies in selecting an appropriate substitution variable, managing differentials, and performing re-substitution to the original variable.

Table 2: Examples of Analysis of Students' Responses and Types of Errors

Student	Excerpt from Student's Response (Clarified)	Type of Error	Error Category	Related Finding
P1	Let ($u = x$), hence ($du = dx$). The integral becomes $\int 2x\cos(x^2)\,dx$	Inappropriate choice of (u)	Error in selecting the substitution variable	Finding 1
P2	Let ($u = x^2$), ($du = 2xdx$). However, the final answer is written as $\int\sin(u)$+C without substituting back.	Failure to substitute back to the original variable	Inconsistency in solution steps	Finding 3
P3	Let ($u = x^2$), but directly writes ($du = dx$).	Incorrect handling of differentials	Error in managing (du/dx)	Finding 2
P4	Proceeds to integrate directly as $\int\sin(x^2)\,dx$ without using substitution.	Incorrect choice of integration technique	Error in solution strategy	Finding 1
P5	Let ($u = x^2$), ($du = 2xdx$). The solution is completed correctly and consistently.	No error identified	Conceptual understanding	Reference comparison

Analysis of students' written responses in Table 2 indicates that the most prominent errors occur at the initial stage of problem solving, particularly in the selection of the substitution variable. Students tend to choose u based on visual elements or similarities with previous examples without considering the structure of the composite function, supporting Finding 1. Such errors compromise the entire integration process and reflect weaknesses in students' conceptual mathematical decision-making.

In addition, errors in managing differentials were identified when students failed to correctly match *du* with the original integrand or ignored the derivative factor. These findings reveal a procedural interpretation of the symbols *dx* and *du* without conceptual understanding, in line with Finding 2. Inconsistencies in solution steps were also observed when students failed to substitute back to the original variable after completing the integration, supporting Finding 3.

In contrast, the reference student's response demonstrates that conceptual understanding enables a systematic and complete solution process. Overall, this analysis confirms that students' recurring errors are concentrated at critical points in the mathematical decision-making process, thereby reinforcing the need for structured decision support in the teaching of integration by substitution.

Discussion

The combined findings of the study indicate that recurring errors in integration by substitution stem from students' weaknesses in conceptual mathematical decision-making, encompassing the selection of substitution variables, the management of differentials, and the consistency of solution steps. Students tend to apply this technique through procedural memorisation, relying on visual similarities and previous examples without understanding the rationale underlying the relationship between a function and its derivative. These findings are consistent with recent studies reporting that reliance on procedural learning causes students to fail in adapting solution strategies when problem structures change (Bingölbali & Monaghan, 2021; Nilsen & Knutsen, 2023). Furthermore, weaknesses in interpreting the symbols *dx* and *du* meaningfully suggest that students view differentials as algebraic manipulation symbols rather than representations of mathematical relationships, as also reported in analyses of students' calculus thinking (Fonbuena, 2022). The identified inconsistencies in solution steps further reflect a lack of self-monitoring and reflective thinking structure, which are essential components of higher-level mathematical problem solving (Roh & Lee, 2020).

Overall, this discussion confirms that students' recurring errors in integration by substitution are not merely technical issues but are rooted in weak conceptual understanding and the absence of systematic mathematical decision support. These findings strengthen the argument that teaching approaches based primarily on procedural memorisation are insufficient for developing meaningful understanding of calculus. Consequently, pedagogical innovations grounded in mathematical decision support structuring the processes of selection, verification, and solution in a visual and stepwise manner have the potential to help students reduce recurring errors and develop more organised and reflective mathematical thinking in calculus learning.

Recommendations

Based on the findings and discussion, recurring errors in integration by substitution are identified as arising from students' weaknesses in conceptual mathematical decision-making, particularly in the selection of substitution variables, the management of differentials, and the maintenance of consistent solution steps. Therefore, existing teaching approaches that overly emphasise procedural memorisation should be strengthened through pedagogical innovations that provide structured and meaningful mathematical decision support. Such approaches enable students to understand the rationale behind each step of the solution process, thereby reducing the recurring errors identified in the study. The implementation of these recommendations can be carried out by mathematics lecturers in higher education institutions through the integration of decision-support-based instructional tools, such as visual guides and decision flowcharts, into lectures, tutorials, and independent practice activities.

Conclusion

This study addresses the issue of recurring errors in integration by substitution frequently encountered by higher education students in calculus learning. Based on the analysis of findings and discussion, students' errors are identified not merely as stemming from weaknesses in computational skills but are closely related to deficiencies in conceptual understanding and the ability to make systematic mathematical decisions.

Specifically, the study identifies that students' recurring errors are concentrated in three main aspects: the selection of substitution variables, the management of differentials, and inconsistencies in solution steps. These findings reinforce the argument that teaching approaches based solely on procedural memorisation are insufficient to help students meaningfully master the integration by substitution technique. Therefore, this study emphasises the need for pedagogical innovation based on mathematical decision support to assist students in developing stronger conceptual understanding, reducing recurring errors, and increasing confidence in calculus learning.

As an implication of the study, future research is recommended to focus on the development and evaluation of structured pedagogical interventions that emphasise students' thinking processes, particularly in supporting mathematical decision-making during calculus problem solving.

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