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NUMERICAL SOLUTION OF LOTKA-VOLTERRA COMPETITIVE MODEL BY USING EULER AND FOURTH ORDER RUNGE-KUTTA METHODS

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ABSTRACT - This study compares the Euler and Fourth order Runge-Kutta numerical methods for solving the Lotka-Volterra Competitive model. The objective of the study is to analyze the competitive interactions that occurred between lions *Panthera leo* and leopards *P. pardus* in the Sabi Sand Game Reserve, South Africa. Numerical approximation tests evaluate the reliability and precision of both methods applied to the logistic equation. The findings indicate that the RK method provides a superior approximation, outperforming the Euler method. Graphical representation of the numerical approximations in Wolfram Mathematica 12.1 confirms the close alignment between the RK method with exact values. The study also explores the impact of carrying capacity on the dynamics of interspecies competition. Carrying capacity refers to the maximum population size that a species can sustain indefinitely, considering factors like food, habitat, water, and others. The results demonstrate that a larger carrying capacity corresponds to a greater ability for a species to thrive and survive in competitive environments. Moreover, this study showcases the practical application of numerical approximation as a predictive tool for understanding species interactions. By using data simulation to emulate real-world scenarios, researchers can accurately predict the outcomes of interspecific competition without the need for prolonged experiments.

Keywords: Numerical method, competitive hunter model, euler method, runge-kutta method, dynamic behavior

1. INTRODUCTION

The Lotka-Volterra Competitive model is used to study the population dynamics of species competing for common resources. The main objective of this study is to compare the exact solutions using Euler and RK methods in solving the Lotka-Volterra Competitive model. Furthermore, the impact of carrying capacity on the dynamic behavior of species competition, equilibrium and stability based on initial conditions of two species will be analyzed. This study examines competitive interaction between two top predators, lions *Panthera leo* and leopards *P. pardus* to determine if lions, as the dominant competitor, would limit the distribution and abundance of leopards. This study offers insights into competition dynamics, the theory of exclusion, and coexistence possibilities between lions and leopards.

2. METHODOLOGY

The logistic equation for the population of lions and leopards was tested using numerical approximation methods, specifically the Euler and Runge-Kutta methods. The Euler method and RK method were applied to plot orbits for the Competitive Hunters model, and the resulting simulation results were compared with the exact orbit. Numerical approximation tests were conducted to evaluate the reliability and precision of the Euler and RK methods in solving the Lotka-Volterra Competitive model. Therefore, the numerical approximations were rendered graphically using Wolfram Mathematica 12.1 programming to see which method was closest to the exact values.

3. RESULTS AND DISCUSSION

The results obtained from the numerical approximation test, applying both the Euler and RK methods to the logistic equation of lions *Panthera leo* and leopards *P. pardus* shows that RK method is a more effective for solving Lotka-Volterra Competitive model compared to Euler method. Therefore, by attaining appropriate carrying capacity using curve fitting procedure shows that the distribution and abundance of leopards were not significantly affected by the lions, the dominant predator at least in our study area. Additionally, by conducting equilibrium and stability testing by observing four outcomes whether species 1 winning, species 2 winning, the existence of an unstable equilibrium, or the coexistence of both species. The study found that the equilibrium and stability of any competition interaction were relying on the initial conditions and initial numbers of the competing population sample.

4. NOVELTY OF RESEARCH / PRODUCT

There have been numerous studies investigating the comparison of numerical techniques in solving mathematical biology problems. For example, research from Paul et al., (2016) utilized numerical methods to approximate solutions for nonlinear ordinary differential equation systems, including models for insect populations and one-species Lotka-Volterra models. Building upon this research, our study focuses specifically on the dynamics of wildlife population interactions between two different species, applying the Lotka-Volterra Competitive model.

5. CONCLUSION

In conclusion, this study compared the Euler and RK methods for solving the Lotka-Volterra Competitive model. The results showed that the RK method provided a more accurate approximation of the numerical methods. The estimation of carrying capacity revealed the importance of resource availability in species competition. Future research should explore the effects of additional factors and incorporate multiple step of numerical methods to enhance the accuracy of numerical approximations and further our understanding of species interactions in competitive environments.

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