Research Exhibition in Mathematics and Computer Sciences (REMACS 6.0)/
ISBN: 978-629-97440-5-4

COMPARATIVE ANALYSIS OF TAYLOR SERIES AND RUNGE-KUTTA FEHLBERG METHODS IN SOLVING THE LOTKA-VOLTERRA COMPETITIVE MODEL

Muhammad Najmi Abdul Manaf and Nur Fatihah Fauzi
College of Computing, Informatics and Mathematics,
Universiti Teknologi MARA, Perlis Branch
najmiey.miey00@gmail.com and fatihah@uitm.edu.my

ABSTRACT - This research mainly focuses on the comparisons between numerical methods, such as Taylor Series and Runge-Kutta Fehlberg (RKF) methods with the exact solution. The advantage of these methods is that they can reliably and precisely solve the non-linearity of the model. Based on the results, both methods offer an accurate way to solve the model by comparing the results. RKF proves to be the most accurate approximation method for solving the Lotka-Volterra competitive model when compared to the exact solution than Taylor Series method by using software *Mathematica 13.2* programming. This research will use data of two species, *Paramecium Caudatum* and *Stylonychia Pustulata* from Gause's experiment. Both species undergo intraspecific interaction and their population is increasing day by day until a level where they remain constant. For *P. Caudatum*, the population sizes are increasing day by day until day 16^{th}, which is at 202 cells. Meanwhile for *S. Pustulata* the population sizes are increasing day by day until day 8^{th}, which is at 41 cells. The equilibrium and stability analysis are performed to give critical insights into the long-term behaviour of the system and indicate how the system responds to perturbations, respectively. For mixed population, when the value of both species' carrying capacities is lower than the value of other species carrying capacity divided by the competition coefficient, the two species are able to coexist in this stable equilibrium. Meanwhile, when the value of both species' carrying capacities is lower than the value of other species carrying capacity divided by the competition coefficient, one of the species will undergo competitive exclusion which eventually outcompete the other species that leads to an unstable equilibrium or the extinction of the weaker species. By understanding the behavior of competition and survival, the research's outcomes shed light on the population behavior of these species and have far-reaching implications in the fields of ecology, conservation, and environmental management.

Keywords: Lotka-Volterra competitive model, RKF, Taylor Series, stability.

1. INTRODUCTION

There are several numerical methods that can be used to solve the Lotka-Volterra competitive model. In this research, numerical methods, such as Taylor Series and Runge-Kutta Fehlberg methods are used to solve the model. By using these numerical methods, the competition on intraspecific interaction can be observed for both species. Interspecific interaction will be observed using the Lotka-Volterra competitive model and their stability will be studied to determine if they can coexist or will they undergo competitive exclusion. The RKF method prove to be a reliable and appropriate method for solving population models of linear and nonlinear differential equations than Laplace Adomian Decomposition Method (LADM) (Paul et al., 2016). The purpose of this research is to compare the numerical methods, such as Taylor Series and RKF methods with the exact solution of the Lotka-Volterra competitive model.

2. METHODOLOGY

The data used in this research is from Gause's experiment on two species which are *Stylonychia Pustulata* and *Paramecium Caudatum*. The logistic equation of both species is used to find the exact solution and will be compared to numerical methods, such as Taylor Series and Runge-Kutta Fehlberg methods. The Taylor series is a mathematical tool for representing a function as an infinite sum of terms, with each term derived by differentiating the function at a specified point. It provides a way to approximate a function using its derivatives at a given point. The RKF method is a numerical method used for solving ordinary differential equations (ODEs). A more accurate numerical approximation of the ODE solution can be obtained by iteratively applying the RKF method with smaller step sizes.

3. RESULTS AND DISCUSSION

Based on the results, both methods offer accurate ways to solve the model by comparing the results. RKF proves to be the most accurate method when compared to the exact solution than the Taylor Series method on solving the model. For intraspecific interaction, *P. Caudatum* and *S. Pustulata* population are increasing day by day until they remain constant at day 16th, which is at 202 cells and at day 8th, which is at 41 cells respectively. For interspecific interaction, the results show that *P. Caudatum* are a strong competitive species that driven out the weak species, *S. Pustulata* to the extinction. There are several cases of stability and equilibrium which are either species 1 or 2 wins, coexistence of both species and competitive exclusion. Based on the interspecific interaction, the species undergo the competitive exclusion since one of the species extinct which is *S. Pustulata*.

4. NOVELTY OF RESEARCH/PRODUCT

The research compares two numerical methods for solving the Lotka-Volterra competitive model, namely the Taylor Series and the RKF methods. This comparative analysis provides insights into the positive and negative aspects of each method, allowing researchers to choose the best methods for similar ecological modeling scenarios. The research utilizes data acquired from an experiment conducted in 1934 by Gause involving the species *Parameciun Caudatum* and *Stylonychia Pustulata*. The application of real-world data gives a great opportunity to evaluate numerical methods against established experimental outcomes, enhancing reliability of the results and their practical utility. This research examines the dynamics and effects of both intraspecific and interspecific interactions using the LotkaVolterra competitive model. Hence, this research examines into the stability and equilibrium of the Lotka-Volterra competitive model, whether the populations of both species remain constant over time or undergo competitive exclusion, in which one species outcompetes and eliminates the other.

5. CONCLUSION

In conclusion, RKF method has proved to be more effective than the Taylor Series method for approximation when compared to the exact solution. The data obtained through Gause's experiment for *P. Caudatum* and *S. Pustulata* are used to study the intraspecific and interspecific interaction. For intraspecific interaction, both species' populations exhibit growth until reaching a stable equilibrium, where their population sizes remain constant. However, in mixed populations, *P. Caudatum* consistently outnumbers *S. Pustulata*. The research proves that there are stable equilibrium points, which represent a balanced coexistence, where the populations of both species remain constant over time. There is also a situation called competitive exclusion, which one species outcompetes the other, resulting the extinction of the weaker species. In the future, researchers could use others available numerical methods, such as Multistep Method and Boundary Value Method to make a comparison and solve the model.

REFERENCES

Gause, G. F. (1934). *The Struggle for Existence*. Baltimore, Williams and Wilkins Company.

Paul, S., Mondal, S. P., Bhattacharya, P., & Chaudhuri, K. (2016). Some comparison solutions by different numerical techniques on mathematical biology problem. *International Journal of Differential Equations*, *2016*, 1–14. https://doi.org/10.1155/2016/8921710

Razali, N. S. A. B., & Abdullah, F. A. (2013). Numerical methods for competitive hunters model. *AIP Conference Proceedings*, *140*(2013), 1522. https://doi.org/10.1063/1.4801116