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APPLICATION OF THE RUNGE-KUTTA FOURTH ORDER (RK4) AND RUNGE-KUTTA FIFTH ORDER (RK5) IN DYNAMIC SYSTEMS: A FOCUS ON ELECTRICAL CIRCUIT ANALYSIS

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Abstract

This study examines the application of the Runge-Kutta Fourth Order (RK4) and Runge-Kutta Fifth Order (RK5) numerical methods in analyzing dynamic electrical systems, with a particular focus on both series and parallel RLC circuits. These numerical techniques are renowned for their ability to solve ordinary differential equations (ODEs) with a high level of accuracy and computational efficiency, especially in scenarios where analytical solutions are difficult or impossible to derive. The research aims to compare the effectiveness of RK4 and RK5 in simulating the transient and steady-state behaviours of electrical circuits under various damping conditions, including underdamped, overdamped, and critically damped responses. The study employs Mathematica as the primary tool to compare the accuracy of RK4 and RK5 in simulating both the transient (short-term) and steady-state (long-term) responses of electrical circuits under various damping conditions. The performance of each method is evaluated based on accuracy, stability, and computational efficiency. The results show that RK4 is a solid choice for balancing simplicity and performance, while RK5 stands out for its higher accuracy, especially important in situations where precision matters most. By applying both methods to series and parallel circuit setups, the study gives a more complete picture of how these numerical approaches perform in real-world circuit analysis. Overall, this work helps engineers, researchers, and students make better decisions about which method to use based on the type of circuit and how precise the simulation needs to be. It highlights the trade-offs between RK4 and RK5 and shows how choosing the right method can improve the accuracy and efficiency of electrical circuit modelling.

Keywords*:* Runge-Kutta, RK4, RK5, Mathematica, Circuit Analysis

Introduction

Numerical methods help solve ordinary differential equations (ODEs), especially when it is hard to find exact solutions. In electrical engineering, these methods are used to study how circuits behave over time, particularly during sudden changes. Although Euler's method is simple to use, it often lacks the accuracy required for analyzing complex or sensitive systems [1].

Runge-Kutta methods, such as RK4 and RK5, are widely recognized for offering improved accuracy and stability. These methods approximate slopes at multiple points within a step, making them highly suitable for studying how circuits respond in real-time [2],[3]. This study focuses on employing RK4 and RK5 to simulate the behavior of RLC circuits over time. Although both methods are commonly applied, there is limited research that directly compares them in detail for electrical circuit applications. Such a comparison is significant because electrical circuits, especially RLC systems, are susceptible to numerical precision when modeling transient oscillations, damping effects, and steady-state responses. Small differences in accuracy or stability between RK4 and RK5 can impact on how reliably simulations capture circuit behavior, which in turn affects design and performance decisions in areas such as power electronics, signal processing, and control systems.

Therefore, by analyzing RK4 and RK5 side by side using Mathematica software, this study aims to evaluate their accuracy and efficiency. The goal is to provide engineers and researchers with practical guidance on selecting the appropriate method based on circuit type, computational constraints, and precision requirements [5-6].

Methodology

To evaluate the performance of the Runge-Kutta Fourth Order (RK4) and Fifth Order (RK5) methods in electrical circuit analysis, this study applies both techniques to the governing differential equations of RLC circuits. Since RLC systems are modeled using second-order ordinary differential equations (ODEs), analytical solutions are often difficult to obtain, particularly for nonlinear or time-varying cases. Therefore, provide a practical way to approximate circuit behavior over time.

The implementation was carried out in Mathematica, using specified parameters for resistance (R), inductance (L), and capacitance (C). Simulations were performed for series and parallel RLC circuits under different damping conditions to capture both transient and steady-state responses. The performance of RK4 and RK5 was then compared in terms of accuracy and computational effort.

Equation (1) is the formula of the Runge-Kutta fourth-order method given by [5] as below:

$$y'' = f(x, y) \tag{1}$$

With initial conditions

$$y(x_0) = y_0 \ \ and \ \ y'(x_0) = y'_0$$

The RK4 formula is

$$y_{n+1} = y_n + \frac{1}{6}(k1 + 2(k2 + k3) + k4) + O\left(h^5\right)$$

$$where,$$

$$k1 = hf(t_{n,} y_n)$$

$$k2 = \ hf(t_n + \frac{1}{2}h, y_n + \frac{1}{2}R1)$$

$$k3 = \ hf(t_n + \frac{1}{2}h, y_n + \frac{1}{2}R2)$$

$$k4 = \ hf(t_n + h, y_n + R3)$$

h represents the step size, $\mathbf{t_n}$ are the values of the independent variables and $\mathbf{y_n}$ are the values of the dependent variables at the current step. k1, k2, k3, and k4 represent successive slope estimates that are combined to approximate the solution with fourth-order accuracy.

Although several RK5 formulations exist in the literature, in this study, RK5 implemented the formulation given by [4], shown in Eq.2. This specific scheme was chosen because it is well established in electrical circuit applications and provides higher-order accuracy with manageable computational effort. The alternative RK5 formulation Eq. 3 was reviewed for comparison but was not implemented in the simulations.

$$y_{n+1} = y_n + \frac{h}{90}(7k_1 + 32k_3 + 12k_4 + 32k_5 + 7k_6) \tag{2}$$

$$\textit{where,}$$

$$t_{n+1} = t_n + h,$$

$$k_1 = f(t_n, y_n),$$

$$k_2 = f\left(t_n + \frac{h}{4}, y_n + \frac{h}{4}k_1\right),$$

$$k_3 = f\left(t_n + \frac{h}{4}, y_n + \frac{h}{8}k_1 + \frac{h}{8}k_2\right),$$

$$k_4 = f\left(t_n + \frac{h}{2}, y_n - \frac{h}{2}k_2 + hk_3\right),$$

$$k_5 = f\left(t_n + \frac{3h}{4}, y_n + \frac{3h}{16}k_1 + \frac{9h}{16}k_4\right),$$

$$k_6 = f\left(t_n + h, y_n - \frac{3h}{7}k_1 + \frac{2h}{7}k_2 + \frac{12h}{7}k_3 - \frac{12h}{7}k_4 + \frac{8h}{7}k_5\right)$$

Also, the RK5 formula constructed by [7] as Eq. 3:

$$y_{n+1} = y_n + \frac{1}{192}(23k_1 + 125k_3 - 81k_5 + 125k_6) \tag{3}$$

$$\textit{where,}$$

$$k_1 = hf(t_n, y_n)$$

$$k_2 = hf\left(t_n + \frac{h}{3}, y_n + \frac{k_1}{3}\right)$$

$$k_3 = hf\left(t_n + \frac{2h}{5}, y_n + \frac{1}{25}(4k_1 + 6k_2)\right)$$

$$k_4 = hf\left(t_n + h, y_n + \frac{1}{4}(k_1 - 12k_2 + 15k_3)\right)$$

$$k_5 = hf\left(t_n + \frac{2h}{3}, y_n + \frac{1}{81}(6k_1 + 90k_2 - 50k_3 + 8k_4)\right)$$

$$k_6 = hf\left(t_n + \frac{4h}{5}, y_n + \frac{1}{75}(6k_1 + 36k_2 + 10k_3 + 8k_4)\right)$$

h is the step size, t_n are the values of the independent variables and y_n are the values of the dependent variables. k1, k2, k3, k4, k5, and k6 are intermediate calculations based on the derivative function.

This requires the use of numerical techniques to solve the governing differential equations of the circuit. Parameters for the circuit are given as resistance (R), inductance (L), and capacitance (C).

Classification of circuit response:

The damping behavior of RLC circuits was classified based on the damping factor (ξ) using Eq. (4):

$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}} \quad (4)$$

The circuit response is categorized as:
Overdamped ($\xi > 1$) : slow, non-oscillatory return to steady state.
Critically damped ($\xi = 1$): fastest return to steady state without oscillation.
Underdamped ($\xi < 1$) : oscillatory behavior with decaying amplitude.
This classification is crucial as it determines the nature of the transient response, ranging from convergence to steady state in overdamped systems to oscillatory behaviour in systems.
The accuracy of RK4 and RK5 was evaluated using the absolute error formula in Eq. (5) from [6]:

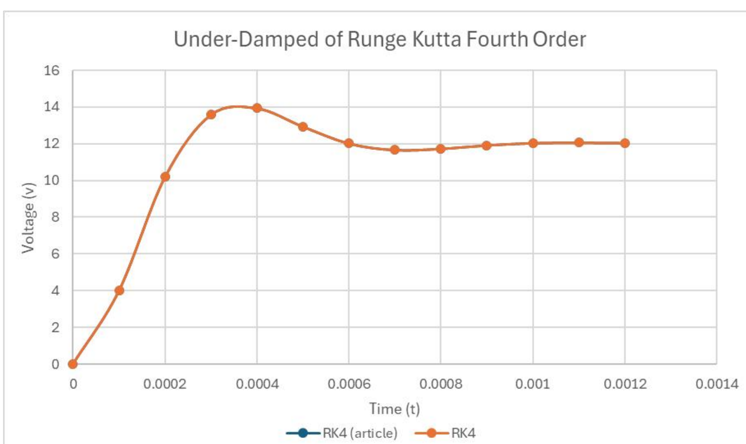
$$\text{Absolute Error} = |\boldsymbol{V_A} - \boldsymbol{V_E}| \quad (5)$$

where $\mathbf{V_A}$ is the analytical (exact) solution and $\mathbf{V_E}$ is the numerical approximation. This allows for a direct performance comparison of the two methods under the same simulation conditions.

$$\boldsymbol{V_A} = 12 - [12\cos(8660.25t) + 6.39\sin(8660.25t)]e^{-5000t}$$

Result and Discussion

Figure 1 illustrates the voltage-time response of a series RLC circuit under underdamped conditions. The oscillatory response with decreasing amplitude reflects the underdamped nature of the system. Both RK4 and RK5 captured this transient behavior; however, RK5 tracked the benchmark solution more closely. This is due to its inclusion of higher-order expansion terms, which reduce local truncation errors. In contrast, RK4, while accurate, accumulates error more rapidly over time.



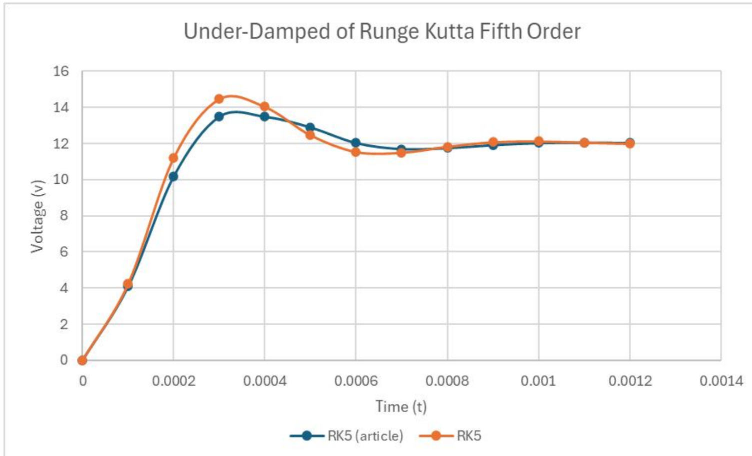


Figure 1: Voltage-Time Response of the Underdamped RLC Series Circuit

Figure 2 compares the voltage response obtained from Euler, RK3, RK4, and RK5 against the benchmark BRK5 solution. The results show that Euler and RK3 deviate significantly from the reference, especially as time progresses. RK5 remains consistently closer to the BRK5 curve than RK4, owing to its higher-order Taylor expansion terms. These additional terms reduce truncation errors and improve fidelity in capturing fast-changing signals, explaining why RK5 outperforms RK4, particularly in oscillatory systems. The x-axis represents simulation time (seconds), while the y-axis represents circuit voltage (volts). This allows direct comparison of how accurately each method tracks the reference response over time.

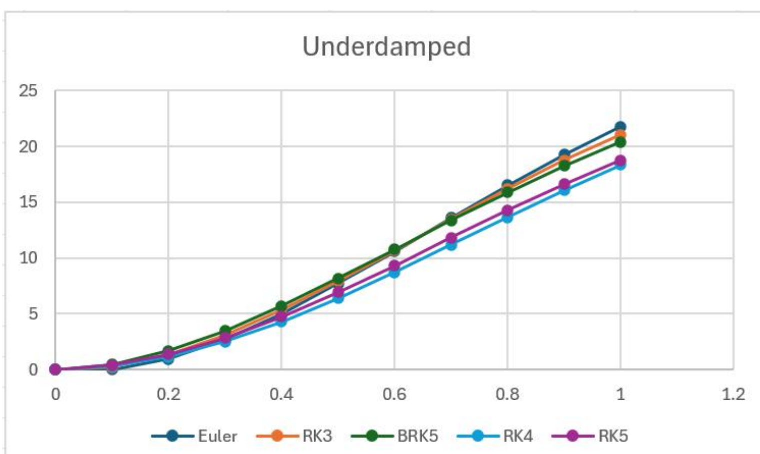


Figure 2: Voltage-Time Response of the Underdamped RLC Parallel Circuit

Table 1 presents the absolute error analysis for the underdamped series RLC circuit over a short simulation window (t = 0.0000 to 0.0012 seconds). The errors are generally small for both RK4 and RK5. Interestingly, RK4 sometimes yields smaller errors than RK5 within this short interval. This indicates that RK4 can perform well in short-duration simulations where computational efficiency is a priority, since its simpler formulation requires fewer function evaluations per step.

Table 1: Error Analysis of RK4 and RK5 of RLC Series Circuit
(Case I: Underdamped)

t	Exact Solution	Abs Error	
		RK4	RK5
0.0000	0	0	0
0.0001	4.0828	0.0828	0.1447
0.0002	10.1925	0.0050	1.0215
0.0003	13.4920	0.0869	0.9744
0.0004	13.8375	0.09532	0.1951
0.0005	12.8952	0.0401	0.4289
0.0006	12.0276	0.0153	0.4899
0.0007	11.6923	0.0350	0.1972
0.0008	11.7481	0.0240	0.0778
0.0009	11.9152	0.00452	0.1588
0.0010	12.0260	0.007187	0.0934
0.0011	12.0517	0.0082915	0.0033
0.0012	12.0310	0.00387	0.0389

Table 2 compares the absolute errors for the underdamped parallel RLC circuit over a longer simulation interval (t = 0.0 to 1.0 seconds). Here, RK5 demonstrates superior performance, with consistently lower errors than RK4 throughout the simulation. The advantage of RK5 becomes more pronounced as time increases, since its higher-order accuracy helps reduce cumulative truncation error. This makes RK5 more reliable for long-duration simulations or systems where sustained precision is necessary.

Table 2: Absolute Error Analysis of RK4 and RK5 of RLC Parallel Circuit
(Case II: Underdamped)

t	Exact Solution	Abs Error	
		RK4	RK5
0.0	0	0	0
0.1	0.73755	0.44349	0.34137
0.2	2.18195	1.03632	0.81197
0.3	4.15289	1.65848	1.30695
0.4	6.48369	2.21999	1.75112
0.5	9.02491	2.66072	2.09789
0.6	11.64659	2.94854	2.32655
0.7	14.23940	3.07625	2.43851
0.8	16.71490	3.05767	2.45295
0.9	19.00490	2.92302	2.40189
1.0	21.06029	2.71405	2.32534

The results also highlight the role of damping in numerical accuracy. In underdamped circuits, oscillations amplify the effect of numerical errors, making higher-order methods like RK5 particularly beneficial. In overdamped or critically damped cases, where responses are smoother, RK4 may provide sufficiently accurate results with lower computational cost. This demonstrates that damping characteristics directly influence the degree to which higher-order accuracy improves numerical performance. For engineers, this means RK4 is well-suited for real-time applications or embedded systems, while RK5 should be preferred in high-precision modeling tasks such as nonlinear circuits, control systems, or long-duration simulations.

Conclusion

This study evaluated the application of the Fourth-Order Runge–Kutta (RK4) and Fifth-Order Runge–Kutta (RK5) methods in modeling the dynamic behavior of RLC electrical circuits. By simulating transient and steady-state responses under underdamped, critically damped, and overdamped conditions, both methods were shown to be effective for circuit analysis. RK4 offered a balance between simplicity and accuracy, making it particularly suitable for short-duration or real-time simulations where computational efficiency is important. In contrast, RK5 consistently provided higher accuracy, especially in long-duration simulations and under oscillatory conditions where error accumulation is significant.

However, this work has several limitations. The analysis was restricted to linear RLC circuits, and simulations were conducted using Mathematica without adaptive step-size control. Future research should extend this comparison to include nonlinear and stiff circuits, investigate the performance of adaptive Runge–Kutta methods, and benchmark the results using other simulation environments

In conclusion, the findings of this study highlight the practical trade-off between computational efficiency and accuracy. RK4 remains a reliable option for fast, resource-limited applications, while RK5 is preferred when long-term precision and error minimization are critical. This guidance can help engineers and researchers make informed decisions when selecting numerical methods for circuit modeling and simulation.

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