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Thermal Instability Analysis of Convection in a Horizontal Nanofluid Layer with Rigid-Rigid Boundary Condition

Norazuwin Najihah Mat Tahir[1,*], Norshahirah Izati Jaaffar[1], Nurul Nabila Rozali[1]
[1]*Center of Mathematical Sciences, Faculty of Computer and Mathematical Sciences,*
Universiti Teknologi MARACawangan Negeri Sembilan, Kampus Seremban, Persiaran
Seremban Tiga/1, 70300 Seremban, Negeri Sembilan
[*]najihahtahir@uitm.edu.my

Abstract

This study investigates the phenomenon of double-diffusive convection in viscoelastic nanofluids under the influence of external magnetic fields. The findings provide vital insights into how temperature gradients and magnetic forces affect fluid layer stability, demonstrating that these interactions can either enhance or delay the onset of convective instabilities. The study's goal is to develop a robust mathematical model that captures the dynamics of these interactions and employs linear stability analysis to understand how they affect fluid dynamics. The main objective is to transform the governing partial differential equations (PDE) into a system of ordinary differential equations (ODE) by using the linear stability analysis. The Galerkin-type weighted residual method is used to derive analytical solutions that explain the stability characteristics of the system. The influence of the scaled stress relaxation parameter, scaled strain retardation parameter and Chandrasekhar number on the system's stability was analysed with result interpretation carried out using Maple software. The stress relaxation advance the onset of oscillatory convection while the strain retardation delay the onset of oscillatory convection. The presence of magnetic field introduces a stabilizing effect on the nanofluid layer. These findings suggest that adjusting magnetic field strength and viscoelastic parameters can effectively regulate convective behavior in nanofluid systems, with potential applications in thermal management and energy-related technologies.

Keywords*:* Stress Relaxation, Strain Retardation, Chandrasekhar Number, Rigid-Rigid Boundary Condition

Introduction

Convective instabilities in fluid systems arise from variations in temperature, concentration, and surface tension, leading to buoyancy-driven motion that significantly affects heat and mass transfer efficiency. Among these phenomena, double-diffusive convection resulting from simultaneous thermal and solutal gradients has garnered increasing attention due to its critical role in geophysical, environmental, and engineering systems [1]. Nanofluids, which consist of nanoparticles suspended in a base fluid, exhibit enhanced thermal conductivity and heat transfer performance, making them highly suitable for such applications. When subject to external magnetic fields, these nanofluids demonstrate unique behaviours, particularly in electrically conductive environments, a phenomenon known as magnetoconvection [2]. Viscoelastic nanofluids, which combine both viscous and elastic properties, add further complexity to the study of convection due to their non-Newtonian behaviour, which is influenced by factors such as stress relaxation and strain retardation [3].

The analysis of thermal instability in these complex fluids is vital for advancing technologies in fields such as nuclear cooling, biomedical transport, and energy storage. While previous studies have examined the influence of magnetic fields, nanoparticle concentration, and porous structures independently, limited research has integrated these factors with realistic boundary conditions such as rigid-rigid confinements. These rigid boundaries are especially relevant in practical settings, such as microfluidic devices and industrial reactors, where the fluid is mechanically constrained [4]. This study aims to fill this gap by employing linear stability analysis and the Galerkin-type weighted residuals method to analytically investigate the onset of stationary and oscillatory convection in a viscoelastic nanofluid layer. The findings will provide a deeper understanding of how physical parameters such as the Chandrasekhar number and viscoelastic relaxation times influence flow stability, contributing to improved designs in thermally driven fluid systems. Resulting from growing interests in heat transfer and magnetoconvective performance, research on thermal instability in nanofluids has been of immense academic interest. This review is intended to address present investigations forming the foundation of the influence of an external magnetic field, boundary conditions, and viscoelastic fluid behaviour on the onset of convection. Thermal instability normally occurs in nanofluid systems by way of buoyancy-induced convection due to temperature and solute concentration gradients. Central to such systems is the Rayleigh number, a measure of the balance between thermal diffusion and buoyancy, and whether a fluid system will be stable or convective.

A pivotal study by Yadav et al. [5] explored the role of external magnetic fields in Jeffrey nanofluid convection, showing that magnetic fields could significantly stabilise thermal and solutal instabilities. The research demonstrated that parameters like the Chandrasekhar number, Q and Lewis number, Le delay the onset of convection by counteracting the thermal gradients that drive instability. Their models incorporated Brownian motion and thermophoresis, which are critical components of nanofluid transport theory, yielding analytical solutions using Galerkin methods. These insights were further reinforced by Yadav et al. [6], who examined internal heating in nanofluid systems and highlighted that non-uniform heat sources and nanoparticle concentrations tend to destabilise the system, though this can be counteracted by increasing porosity and Darcy number.

Extending research on nanoparticle effetcs, Nield and Kuznetsov [7] studied the effect of nanoparticle dispersion and revealed that top-heavy configurations in particle concentration tend to stabilise the nanofluid layer, whereas bottom-heavy configuration promote oscillatory instabilities. This study emphasised that magnetic fields may either dampen or amplify convection depending on how they interact with the particle distribution and base thermal profiles. In a related study, Kumar et al. [3] adopted Jeffrey fluid dynamics to analyse viscoelastic nanofluids in porous media, showing that while magnetic fields stabilise, parameters like Jeffrey number and nanoparticle Rayleigh number promote convection, especially in biomedical and nuclear applications. Kuznetsov [8] expanded on Buongiorno's theory by including both Brownian diffusion and thermophoresis in analysis of the oscillatory and stationary convection modes. Their stability analysis revealed that nanoparticle-induced slip mechanisms can be harnessed to improve bioconvection, a principle that is valuable in microfluidic devices for diagnostics and drug delivery systems.

Boundary conditions also significantly affect thermal instability. Sharma et al. [9] investigated convection in Jeffrey nanofluids under rigid-rigid boundaries, utilising the Galerkin-type weighted residual method. They found that rigid boundaries elevate the critical Rayleigh number, making the system more resistant to convection, which is essential in applications such as geothermal reservoirs and medical fluid transport. Complementing this, Ahuja and Gupta [10] showed that rigid-rigid boundaries in nanofluids enhanced system stability even under magnetic influence, suggesting stronger boundary resistance to fluid motion.

Mahajan and Sharma [4] used Chebyshev pseudospectral methods to reinforce that rigid-rigid conditions reduce flow disruptions, especially when exposed to gravity variations or magnetic fields. Their findings are applicable to systems with strict physical confinement, such as microgravity environments or nuclear cooling. Moreover, Yadav [11] confirmed that the no-slip conditions inherent to rigid boundaries significantly influence temperature gradients and flow behaviours, thus acting as a strong stabilising mechanism in nanofluid layers. The techniques employed to analyse stability have evolved from, classical linear stability analysis to the Galerkin methods and eigenvalue problems. For instance, Yadav et al. [6] used a linear stability analysis combined with eigenvalue formulations to determine critical convection thresholds. Narayana et al. [12] introduced a weakly nonlinear analysis and reduced their model to a generalised Lorenz system to explore the oscillatory patterns under thermal and solute gradients. Electro-thermal influences were the focus of Chand [13], who investigated nanofluids in Brinkman porous media. Using Galerkin methods and Newton's approximation, they showed how factors like Brinkman–Darcy number and electric Rayleigh number modify the critical Rayleigh number. Yadav et al. [14] extended this by demonstrating how the presence of a magnetic field modifies flow stability, revealing physical insights into dynamic instabilities.

Studies such as Ketchate et al. [15] and Boulal et al. [16] used harmonic balance methods and Galerkin projections to model periodic solutions in ferrofluid systems. These works illustrate how vibrational phenomena and external magnetic fields influence convection in ferrofluid layers, especially those used in targeted drug delivery or thermal therapy. Finally, Govender [17] examined the role of Darcy models in determining convection onset in vertically vibrated nanofluid layers and emphasised that critical Rayleigh numbers depend on both porosity and thermal properties, making them sensitive indicators of the stability of heating systems. Garg et al. [18] analysed triply diffusive thermo-bio-convection in anisotropic porous media and concluded that microorganism motility and internal heating contribute to both stabilising and destabilising influences, depending on system symmetry. Motivated by these insights, the current study aims to investigate the non-Newtonian behaviour of nanofluids, with particular emphasis on how parameters such as nanoparticle volume fraction, fluid viscosity and external forces for both surface and body affect convective instability. By conducting a theoretical analysis, this work seeks to enhance the understanding of how these variables govern the onset of stationary and oscillatory instabilities and to uncover the underlying mechanisms that may either inhibit or amplify convective motion.

Methodology

Mathematical Model

This section provides a concise overview of the formulation and schematic representation of the case study. The study examines double-diffusive convection in a viscoelastic fluid of Oldroyd-B type with weak electrical conductivity. Figure 1 illustrates the system, which consists of a horizontal channel of infinite lateral extent and depth H, represented in a Cartesian coordinate system. The lower surface is placed at $z = 0$ and the upper surface at $z = H$. A uniform adverse temperature gradient, ΔT is maintained across the channel, driving convection. Gravity, $\boldsymbol{g} = -g\hat{e}_z$ acts vertically downwards, while an externally applied magnetic field of uniform strength H_0 is aligned along the positive z-direction.

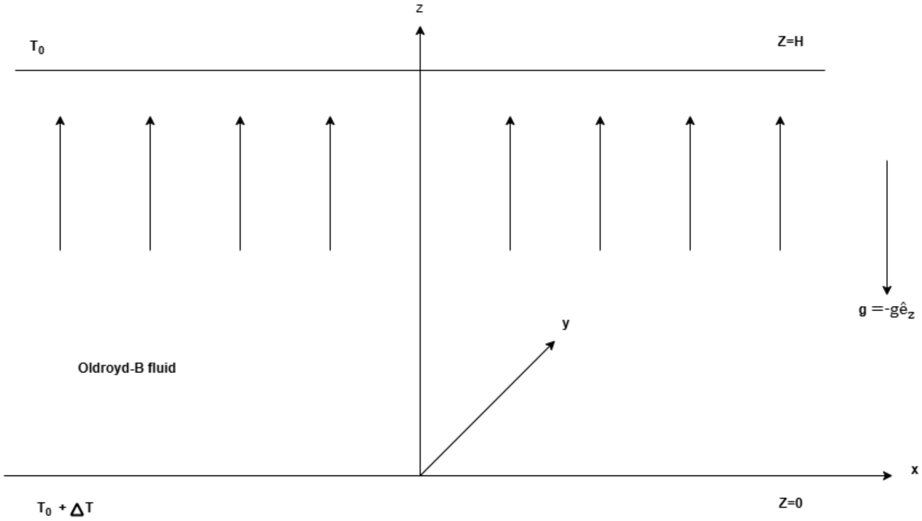

Figure 1: Schematic diagram of the physical problem

Governing Non-Linear Equation (PDE)

The governing equations for convective instabilities in a horizontal viscoelastic nanofluid layer are mathematical models that describe fluid behaviour. These equations are formulated as partial differential equations (PDEs) that involve various parameters. This study focuses on four key equations: conservation of mass, momentum, energy and nanoparticles.

$$\nabla \cdot \vec{v} = 0. \tag{1}$$

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\rho_{f_0}\left(\frac{\partial\vec{v}}{\partial t}+\vec{v}\cdot\nabla\vec{v}\right)+\nabla p-\rho\mathbf{g}+\sigma u\mu_m^2H_0^2\hat{e}_x+\sigma u\mu_m^2H_0^2\hat{e}_y\right]$$
$$=\mu\left(1+\lambda_2\frac{\partial}{\partial t}\right)\nabla^2 v. \qquad (2)$$

$$(\rho c)_f\left(\frac{\partial T}{\partial t}+\vec{v}\cdot\nabla\mathrm{T}\right)=\left(k\nabla^2T\right)+(\rho c)_p\left(D_B\nabla\phi\cdot\nabla T+\left(\frac{D_T}{T_C}\right)\nabla T\cdot\nabla T\right). \qquad (3)$$

$$\frac{\partial\phi}{\partial t}+\vec{v}\cdot\nabla\phi=D_B\nabla^2\phi+\left(\frac{D_T}{T_C}\right)\nabla^2T, \qquad (4)$$

where $\vec{v}=(u,v,w)$ is the nanofluid velocity, p is the pressure, λ_1 is the relaxation time, λ_2 is the retardation time, t is the time, σ is the electrical conductivity, μ_m is the magnetic permeability, μ is the coefficient of viscosity, ϕ is the nanoparticle volume fraction, T is the temperature, $(pc)_f$ is the effective heat capacity of the fluid, $(pc)_p$ is the effective heat capacity of the nanoparticle, k is the thermal conductivity, D_B is the Brownian diffusion coefficient and D_T is the thermophoretic diffusion coefficient. The nanofluid density, ρ is

$$\rho\cong\phi\rho_p+(1-\phi)\left[\rho_{f_0}-\rho_{f_0}\beta\left(T-T_c\right)\right], \qquad (5)$$

where ρ_p is the nanoparticle mass density, $\rho_{f_0}$is the nanofluid density at the reference temperature T_c and β are the volumetric coefficient of thermal expansion. The temperature and the volumetric fraction of the nanoparticles are constant at the boundaries. In this study, the boundary conditions are rigid lower and upper boundaries will be considered. Therefore, the boundary conditions are

$$w=0,\quad \frac{\partial w}{\partial z}=0,\quad T=T_h,\quad \phi=\phi_0,\quad \text{at } z=0, \qquad (6)$$

$$w=0,\quad \frac{\partial w}{\partial z}=0,\quad T=T_c,\quad \phi=\phi_1,\quad \text{at } z=H. \qquad (7)$$

Linear Stability Analysis

The introduction of the scaling quantities for the length, velocity, time, pressure, nanoparticles volume fraction and temperature give the non-dimensional variables by

$$(x^*,y^*,z^*)=\frac{(x,y,z)}{H},(u^*,v^*,w^*)=(u,v,w)\frac{H}{\alpha_f},$$
$$t^*=\frac{\alpha_f}{H^2}t,p^*=\frac{H}{\mu\alpha_f}p,\phi^*=\frac{\phi-\phi_0}{\phi_1-\phi_0},T^*=\frac{T-T_c}{T_h-T_c}, \qquad (8)$$

where $\alpha_f=\frac{k}{(\rho c)_f}$ is the thermal diffusivity of the fluid and asterisks denote dimensionless quantities. The resulting equations (1) – (7) in the form of the non-dimensional equations after dropping the asterisks by the scaling quantities (8) are

$$\nabla\cdot\vec{v}=0. \qquad (9)$$

$$(1+\varLambda_1\frac{\partial}{\partial t})[\frac{1}{P_r}(\frac{\partial\vec{v}}{\partial t}+\vec{v}\cdot\nabla\vec{v})+\nabla p+Rm\hat{e}_z-RaT\hat{e}_z+Rn\phi\hat{e}_z+Q(u\hat{e}_x+\vec{v}\hat{e}_y)] \\ =(1+\varLambda_1\frac{\partial}{\partial t})\nabla^2\vec{v}, \tag{10}$$

$$\frac{\partial T}{\partial t}+\vec{v}\cdot\nabla T=\nabla^2 T+\frac{N_B}{Le}\nabla\phi\cdot\nabla T+\frac{N_AN_B}{Le}\nabla T\cdot\nabla T, \tag{11}$$

$$\frac{\partial\phi}{\partial t}+\vec{v}\cdot\nabla\phi=\frac{1}{Le}\nabla^2\phi+\frac{N_A}{Le}\nabla^2 T, \tag{12}$$

subject to the dimensionless boundary conditions

$$w=0,\quad \frac{\partial w}{\partial z}=0,\quad T=1,\quad \phi=0,\quad \text{at } z=0, \tag{13}$$

$$w=0,\quad \frac{\partial w}{\partial z}=0,\quad T=0,\quad \phi=1,\quad \text{at } z=1. \tag{14}$$

The non-dimensionless parameters are

$$\varLambda_1=\lambda_1\frac{\alpha_f}{H^2},\varLambda_2=\lambda_2\frac{\alpha_f}{H^2},P_r=\frac{\mu}{\rho_{f_0}\alpha_f},Le=\frac{\alpha_f}{D_B},Q=\frac{\sigma\mu^2{}_mH^2{}_0H^2}{\mu}, \\ N_A=\frac{D_T(T^*{}_h-T^*{}_c)}{D_BT^*{}_c(\phi^*{}_1-\phi^*{}_0)},N_B=\frac{(\rho c)_p}{(\rho c)_f}(\phi^*{}_1-\phi^*{}_0)Ra=\frac{[\rho_{f_0}\beta(T^*{}_h-T^*{}_c)]gH^3}{\mu\alpha_f}, \\ Rn=\frac{[(\rho_p-\rho_{f_0})(\phi^*{}_1-\phi^*{}_0)]gH^3}{\mu\alpha_f},Rm=\frac{[\rho_p\phi^*{}_0+\rho_{f_0}(1-\phi^*{}_0)]gH^3}{\mu\alpha_f}, \tag{15}$$

where $\varLambda_1$is the scaled stress relaxation parameter, $\varLambda_2$ is the scaled strain retardation parameter, P_r is the Prandtl number, Le is the Lewis number, Q is the Chandrasekhar number, N_A is the modified diffusivity ratio, N_B is the modified particle density increment, Ra is the thermal Rayleigh number, Rn is the concentration Rayleigh number and Rm is the basic density Rayleigh number.

Perturbation and Linearization

The fluid is at rest in the reference steady basic state varying in the z-direction. The basic state is perturbation by infinitesimal perturbations

$$\vec{v}=\vec{v}',p=p_b+p',T=T_b+T',\phi=\phi_b+\phi', \tag{16}$$

where prime denotes the perturbation quantity and $p_b=p_0,T_b=1-z,\phi_b=z$ where p_0 is the constant reference pressure. The system (9) – (12) and the boundary conditions (13) and (14) are perturbed with infinitesimal perturbations (16). The linearized perturbed system is,

$$\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left[\frac{1}{P_r}\frac{\partial}{\partial t}\nabla^2 w'-\nabla^2{}_H RaT'+\nabla^2{}_H Rn\phi'+Q\nabla^2{}_H w'\right]=\left(1+\Lambda_2\frac{\partial}{\partial t}\right)\nabla^4 w, \tag{17}$$

$$\frac{\partial T'}{\partial t}-w'=\nabla^2 T'+\frac{N_B}{Le}\left(\frac{\partial T'}{\partial z}-\frac{\partial\phi'}{\partial z}\right)-2\frac{N_AN_B}{Le}\frac{\partial T'}{\partial z}, \tag{18}$$

$$\frac{\partial\phi'}{\partial t}+w'=\frac{1}{Le}\nabla^2\phi'+\frac{N_A}{Le}\nabla^2 T', \tag{19}$$

subject to boundary conditions,

$$w' = 0, \qquad \frac{\partial w}{\partial z} = 0, \qquad T' = 0, \quad \phi' = 0 \quad \text{at } z = 0\,, \tag{20}$$

$$w' = 0, \qquad \frac{\partial w}{\partial z} = 0, \qquad T' = 0, \quad \phi' = 0 \quad \text{at } z = 1\,, \tag{21}$$

where ∇^4 is the three-dimensional biharmonic operator, ∇^2 is the three-dimensional Laplacian operator and ${\nabla^2}_H$ is the one-dimensional Laplacian operator in the horizontal plane.

Normal Modes

The non-dimensional system (17) – (21) constitutes a linear boundary-value problem that can be solved using the method of normal modes given by

$$(w', T', \phi') = [W(z), \Theta(z), \Phi(z)]e^{(st + ia_x + ia_y)}, \tag{22}$$

where $W(z), \Theta(z)\ and\ \Phi(z)$ are the amplitudes of the velocity, temperature and nanoparticle's volume fraction, respectively. $\alpha = ({\alpha^2}_x + {\alpha^2}_y)^{\frac{1}{2}}$ is the total wave number and s is a dimensionless complex growth rate. The system in the form of the normal modes are

$$\left(1+\Lambda_1 s\right)\left[\frac{s}{P_r}\left(D^2-\alpha^2\right)W + QD^2W + Ra\alpha^2\Theta - Rn\alpha^2\Phi\right] = \left(1+\Lambda_2 s\right)\left(D^2-\alpha^2\right)^2 W, \tag{23}$$

$$W + \left(D^2 + \frac{N_B}{Le}D - 2\frac{N_A N_B}{Le}D - \alpha^2 - s\right)\Theta - \frac{N_B}{Le}D\Phi = 0\,, \tag{24}$$

$$W - \frac{N_A}{Le}\left(D^2-\alpha^2\right)\Theta - \left[\frac{1}{Le}\left(D^2-\alpha^2\right) - s\right]\Phi = 0\,, \tag{25}$$

with the boundary conditions at $z = 0$ and $z = 1$,

$$W = 0, \qquad DW = 0, \qquad \Theta = 0, \quad \Phi = 0, \tag{26}$$

where $D \equiv \frac{d}{dz}$. The system with normal mode (23) – (26) is reduced to Nield and Kuznetsov [2] with $Q = 0$.

Analytical Computation using Galerkin-type Weighted Residuals Method

The Galerkin-type weighted residuals method was used to obtain an approximation of the closed-form solution to the system. The functions W, Θ and Φ are in the form of

$$W = \sum_{n=1}^{N} A_n W_n\,, \Theta = \sum_{n=1}^{N} B_n \Theta_n\,, \phi = \sum_{n=1}^{N} C_n \phi_n, \tag{27}$$

where A_n, B_n and C_n are unknown coefficients and $n = 1,2,3, \ldots, N$. Equations (23) – (26) are multiplied by W, Θ and Φ, respectively (27). Performing integration by parts with respect to z from 0 to 1 will lead to a system of $N - 1$ equation with $N - 1$ unknown. For rigid-rigid boundaries, the trial functions are

$$W_1 = z^2(1-z)^2, \qquad \Theta_1 = z(1-z), \qquad \phi_1 = z(1-z), \tag{28}$$

where W_1, Θ_1 and ϕ_1 are trial functions that satisfy the boundary conditions (26) [2]. The thermal Rayleigh number Ra determines the threshold for the transition from stationary patterns to weakly chaotic evolution to a highly turbulent state. For $s = 0$, we obtain the solution for the case of stationary convection

$$Ra_{stat} = \frac{28(\alpha^4 + 24\alpha^2 + 504)(\alpha^2 + 10)}{27\alpha^2} + \frac{112Q(\alpha^2 + 10)}{9\alpha^2} - (Na + Le)Rn. \qquad (29)$$

The stationary instability does not depend on the Λ_1 and Λ_2. For the case of Newtonian fluid, where $\Lambda_1 = \Lambda_2 = 0$ (no scaled stress relaxation and scaled strain retardation parameters) and in the absence of magnetic field, Eq. (29) reduces to the result of Nield and Kuznetsov [2] for stationary convection. Setting $s = i\omega$, we obtain the solution for $Ra = Ra_r + iRa_i$. Setting $Ra_i = 0$ will give the corresponding $\omega \neq 0$ for oscillatory convection and substitution of ω on Ra_rwill give the Rayleigh number for the oscillatory convection,

$$\begin{aligned} Ra_{osc} = \left(\frac{28}{27\alpha^2}\right)\Bigg\{&(\alpha^4 + 24\alpha^2 + 504)(\alpha^2 + 10)\left(\frac{1 + \Lambda_1\Lambda_2\omega^2}{1 + {\Lambda_1}^2\omega^2}\right) \\ &-(\alpha^4 + 24\alpha^2 + 504)\omega^2\left(\frac{\Lambda_2 - \Lambda_1}{1 + {\Lambda_1}^2\omega^2}\right) - \left(\frac{12 + \alpha^2}{Pr}\right)\omega^2 \qquad (30) \\ &+12Q(10 + \alpha^2)\Bigg\} - \left[\frac{(10 + \alpha^2)^2(Na + Le) + \omega^2 Le^2}{(10 + \alpha^2) + \omega^2 Le^2}\right]Rn. \end{aligned}$$

For the case of Newtonian fluid, where $\Lambda_1 = \Lambda_2 = 0$ (no scaled stress relaxation and scaled strain retardation parameters) and in the absence of a magnetic field, Eq. (30) reduces to the result of Nield and Kuznetsov [2] for oscillatory convection.

Result and Discussion

The marginal stability curves for the onset of stationary and oscillatory instabilities were plotted using Maple software. Graphical illustrations of the marginal stability curves were analysed to assess the effects of the physical parameters. This section reports on the marginal stability curves derived using the Galerkin-type weighted residuals method. Our analysis elucidated the influence of key physical parameters, including the Chandrasekhar number, on the stability of double-diffusive convection in viscoelastic nanofluids. The results demonstrate the complex interplay between these parameters, which can either stabilise or destabilise the fluid layer.

Figure 2 presents the neutral stability curves for the variations of thermal Rayleigh number, Ra as a function of the wave number α for different values of the scaled stress relaxation parameter $\mathit{\Lambda}_1$, with values $\mathit{\Lambda}_1 = 0.2, 0.6,$ and 1.0. Stress relaxation refers to the process by which a fluid takes time to recover after a reduction in the applied deformation rate. The stress relaxation parameter does not depend on the stationary Rayleigh number. The oscillatory thermal Rayleigh number decreases with an increase in $\mathit{\Lambda}_1$. This trend indicates that the fluid becomes less resistant to convection as the stress relaxation parameter increases. The increase in $\mathit{\Lambda}_1$ allows the applied stress to influence the fluid element over a longer duration, resulting in reduced elastic memory. This reduced the internal friction, of the fluid. Consequently, convection initiates at lower values of the Darcy-Rayleigh number. Therefore, increasing $\mathit{\Lambda}_1$ advance the onset of oscillatory convection in the viscoelastic nanofluid layer, making the system more thermally unstable.

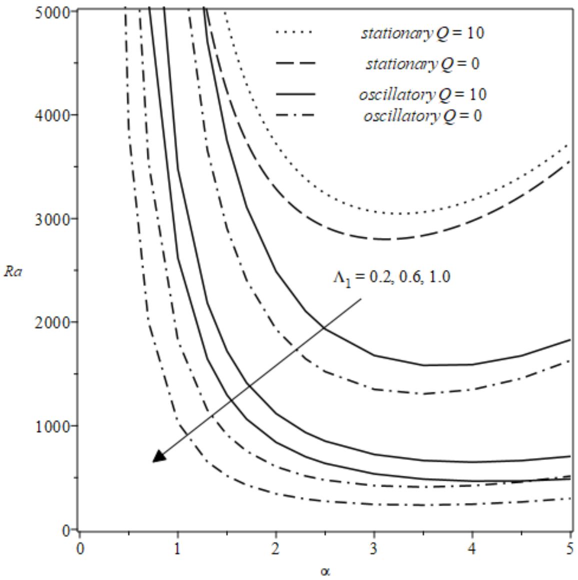

Figure 2: Neutral stability curve of the stationary and oscillatory convection for various values of Λ_1 when $\Lambda_2 = 0.1, P_r = 5, Le = 100,\ N_A = 5$ and $Rn = -10$

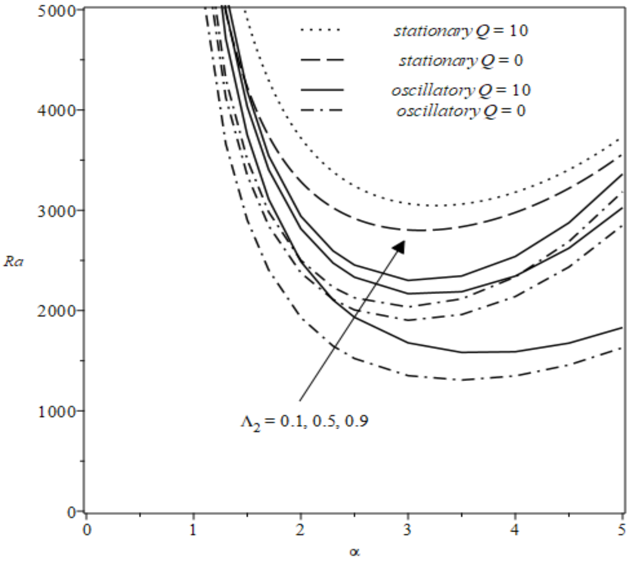

Figure 3: Neutral stability curve of the stationary and oscillatory convection for various values of Λ_2 when $\Lambda_1 = 0.2, P_r = 5, Le = 100,\ N_A = 5$ and $Rn = -10$

Figure 3 illustrates the neutral stability curves for the variations of thermal Rayleigh number, Ra as a function of the wave number α for various values of the scaled strain retardation parameter Λ_2, while other parameters are held constant. The strain retardation also does not depend on the stationary convection. This is because stress relaxation and strain retardation do not depend on time during the perturbation solution. It has been discovered that an increase in the value of Λ_2 leads to a higher critical Rayleigh number for the onset of oscillatory convection, indicating that the system becomes thermally more stable. Parameter Λ_2 represents the time taken by the fluid to respond to an applied stress. A higher value of Λ_2 means that the fluid has a stronger resistance to deformation, which increases internal friction and delays the onset of convection. This stabilising effect is particularly significant in the oscillatory mode, as indicated by the upward shift of the neutral curves. In contrast, the stationary mode remains mostly unaffected by changes in Λ_2. This behaviour occurs because the stationary instability is independent of the viscoelastic parameters, as the base state remains equivalent to pure conduction, similar to a Newtonian nanofluid.

Figure 4 illustrates the neutral stability curves, plotted against the Rayleigh number Ra, and wave number α, for different values of the magnetic parameter Q. As shown in the figure, the critical Rayleigh number increases with increasing values of Q, for both stationary and oscillatory convection, indicating that the presence of a magnetic field introduces a stabilising effect on the nanofluid layer. This is due to the Lorentz force, represented by Q, which resists fluid motion and acts as a damping mechanism. As Q increases, the damping becomes stronger, requiring more thermal energy to initiate convection. The influence of Q is more prominent in the oscillatory mode, where the curves shift upward significantly, reflecting the greater sensitivity of dynamic fluid motion to electromagnetic resistance. In contrast, the stationary mode exhibits a more gradual rise in critical Ra with increasing Q, indicating a slower onset of convection. Nonetheless, the presence of a magnetic field still contributes to thermal stabilisation even in the absence of oscillatory motion. Overall, the increasing critical Rayleigh number for both modes confirmed that the magnetic fields suppressed convection by enhancing the system stability through electromagnetic damping.

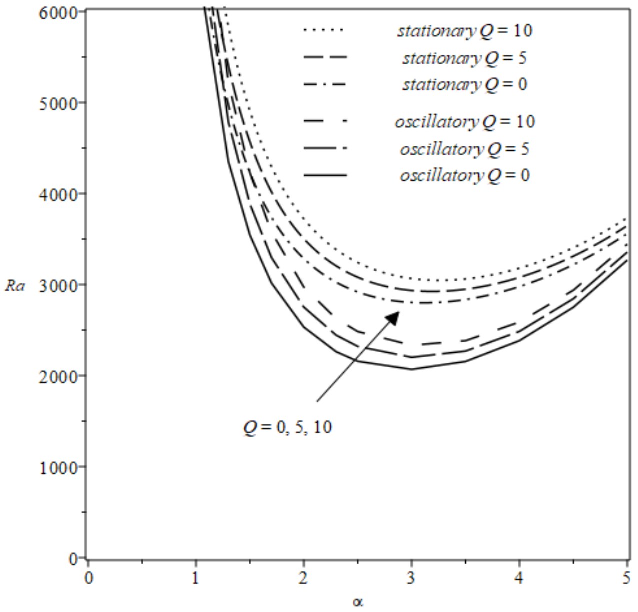

Figure 4: Neutral stability curve of the stationary and oscillatory convection for various values of Q when $\Lambda_1 = 0.2,\ \Lambda_2 = 1.0,\ P_r = 5, Le = 100,\quad N_A = 5$ and $Rn = -10$.

Conclusion

This study discusses the thermal instability of convection in a rigid-rigid horizontal nanofluid layer under the effects of viscoelastic parameters and magnetic field on the onset of convection. Linear stability and the Galerkin-type weighted residual method were used to simplify the governing partial differential equations into ordinary differential equations, for which the stability behaviour was analysed. The marginal stability curves obtained from the resulting equations proved that increases in the scaled stress relaxation parameter lowered the critical Rayleigh number, thus indicating an earlier convection onset and thus an increasingly thermally unstable system. Conversely, increases in the scaled strain retardation parameter and Chandrasekhar number increased the critical Rayleigh number, thereby stabilising the fluid system. Graphical analysis also confirmed that the stationary convection was largely insensitive to viscoelastic parameters, while oscillatory modes were highly sensitive to changes in Λ_1, Λ_2, and Q. The findings locate the complex dynamic interaction of viscoelastic and magnetic effects in controlling the convective stability in nanofluids. Furthermore, the findings coincide with previous theoretical expectations and validate the appropriateness of the Galerkin method for such fluid dynamics problems. Finally, the study demonstrates that magnetic fields are beneficial for delaying the onset of thermal convection and enhancing the thermal stability of nanofluids. This is particularly valuable in applications such as biomedical cooling, electronics cooling, and energy systems. However, this study also highlights the importance of selecting parameters and boundary conditions for predicting system behaviour, thereby providing a foundation for future experimental and computational investigations of nanofluid convection dynamics.

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