UNIVERSITI TEKNOLOGI MARA

MATHEMATICS AND STATISTICS

UNDERGRADUATE RESEARCH PROCEEDINGS 2025

UiTM CAWANGAN NEGERI SEMBILAN

Thermal Instability Analysis of Convection in a Horizontal Nanofluid Layer with Free-Free Boundary Condition

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Abstract

Convective flows are commonly observed in geophysical systems and hold both fundamental and practical significance in engineering and technological applications. The initiation of convective flow whether stationary or oscillatory is known to cause defects such as striations, dendrites and bubbles in manufactured products. This study investigates the influence of viscous elasticity and an applied magnetic field on convective instabilities in a deep horizontal layer of viscoelastic nanofluid with free-free boundary conditions. A linear stability analysis was employed to identify the onset of both stationary and oscillatory instabilities. A closed-form expression for the critical Rayleigh number was derived using the Galerkin-typed weighted residuals method. The impact of the scaled stress relaxation parameter, scaled strain retardation parameter and Chandrasekhar number on system stability was analyzed. The results indicate that the magnetic field effectively delays the onset of both stationary and oscillatory convective instabilities in viscoelastic nanofluids.

Keywords*:* Stress Relaxation, Strain Retardation, Chandrasekhar Number, Free-Free Boundary Condition

Introduction

Nanofluids have attracted significant attention in recent years due to the growing need for high-performance thermal management systems. As a result, numerous studies have been carried out to better understand their thermophysical behavior [1]. The onset of convective instability in nanofluids has been explored in the works of Nield and Kuznetsov [2,3] and Yadav et al. [4]. The concept of nanofluids was first introduced by Choi [5], who defined them as engineered suspensions of nanoparticles (typically ranging from 1 to 100 nanometers) within a base fluid, such as water, ethylene glycol, or oil [6]. Ganguly et al. [7] similarly emphasized that nanofluids consist of uniformly distributed nanoparticles in various carrier liquids, including water and organic solvents.

The nanoparticles used in these fluids whether metallic or non-metallic such as Al_2O_3, CuO, Cu, SiO_2, and TiO_2, considerably enhance the heat transfer performance of the base fluid. This enhancement is primarily due to improved thermal conductivity, as discussed by Kakaç and Pramuanjaroenkij [8], making nanofluids highly effective in advanced thermal applications. A novel category of fluids known as nanofluids has emerged, offering significant potential for enhancing thermophysical characteristics and improving heat transfer performance. Despite this, most investigations into non-Newtonian nanofluids have been experimental in nature [9], while theoretical analyses concerning flow instability have largely treated nanofluids as Newtonian fluids [10]. Experimental evidence, however, suggests that the Newtonian framework may not sufficiently capture the behavior of certain nanofluids [11]. Specifically, nanofluids with low nanoparticle concentrations tend to behave in a Newtonian manner, whereas higher concentrations lead to shear-thinning behavior, characteristic of non-Newtonian fluids.

The onset of convective instability in nanofluids was examined through linear stability analysis. Prior investigations by Yadav et al. [4], Nield and Kuznetsov [2,3] have explored this phenomenon in detail. Dastvareh and Azaiez [12] reported that the presence of nanoparticles and applied magnetic fields can significantly influence the stability characteristics of an initially unstable flow, either by amplifying or suppressing the onset of convection. Tahir and Kechil [13] conducted a linear stability analysis to examine the elasticity effects on hydromagnetic convective instability in a viscoelastic nanofluid layer as well as the magneto-convective instability in a horizontal viscoelastic nanofluid saturated porous layer by Tahir et al., [14]. Their findings also showed that magnetic fields contribute to the stabilization of the nanofluid layer in both stationary and oscillatory convection regimes. Additionally, it was observed that the intrinsic rheological properties of the fluid play a critical role in shaping the convective behavior.

Previous research has explored the thermal stability of nanofluid convection under various boundary conditions, including rigid–rigid and rigid–free configurations. However, the free–free boundary condition, which closely resembles conditions encountered in practical applications, has received comparatively limited attention [15]. In this configuration, both the upper and lower surfaces of the nanofluid layer are free to deform, resulting in distinct stability behavior. This added freedom introduces a broader spectrum of system responses and potential flow structures, which significantly influence heat transfer characteristics. Such behavior is particularly relevant for engineering applications involving solar thermal collectors, electronic cooling systems, and heat exchangers

Considering these observations, the present study focuses on exploring the non-Newtonian characteristics of nanofluids. It investigates how factors such as nanoparticle volume fraction, fluid viscosity and various external forces for both surface and body affect convective instability. Through theoretical analysis, this work aims to deepen the understanding of how these parameters influence the onset of stationary and oscillatory instabilities and to identify mechanisms that may either suppress or enhance such convective behaviors.

Problem Formulation

Considers a horizontal layer of viscoelastic non-Newtonian nanofluid subjected to a magnetic field, under free-free boundary conditions as shown in Figure 1. Models for nanofluids with negligible external forces were proposed by Buongiorno [16], while Nield and Kuznetsov [17] introduced a model for the natural convection onset in constant viscosity nanofluids. This study extends the framework of Narayana et al. [18] by considering the fluid as a non-Newtonian nanofluid.

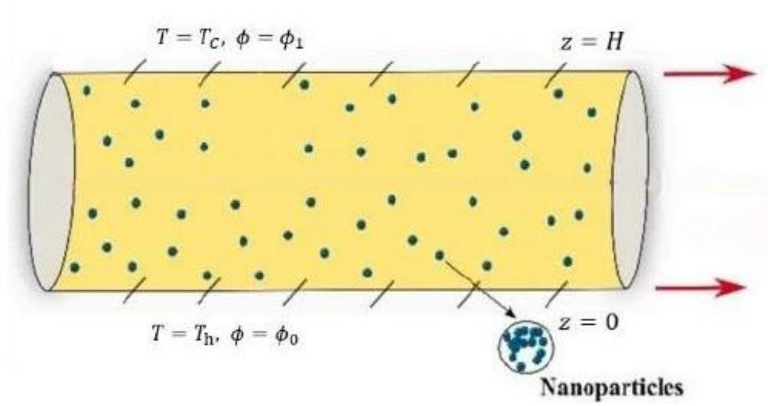

Figure 1: Schematic diagram of the physical system.

The formulation of the problem is based on the models proposed by Buongiorno [16], Narayana et al. [18] and Nield and Kuznetsov [17]. The basic governing equations for the mass, momentum, energy and nanoparticle conservation for the non-Newtonian nanofluid are

$$\nabla \cdot \mathbf{v} = 0, \tag{1}$$

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\left[\rho_{f_0}\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}\right) + \nabla p - \rho \mathbf{g} + \sigma u \mu_m^2 H_0^2 \boldsymbol{i} + \sigma v \mu_m^2 H_0^2 \boldsymbol{j}\right] = \mu\left(1 + \lambda_2 \frac{\partial}{\partial t}\right)\nabla^2 \mathbf{v}, \tag{2}$$

$$(\rho c)_f \left(\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T\right) = k \nabla^2 T + (\rho c)_p \left[D_B \nabla \phi \cdot \nabla T + \left(\frac{D_T}{T_c}\right) \nabla T \cdot \nabla T\right], \tag{3}$$

$$\frac{\partial \phi}{\partial t} + \boldsymbol{v} \cdot \nabla \phi = D_B \nabla^2 \phi + \left(\frac{D_T}{T_c}\right) \nabla^2 T, \tag{4}$$

where ∇ is divergence of a vector field, $\mathbf{v} = (u, v, w)$ is nanofluid velocity, λ_1 is relaxation time, λ_2 is retardation time, t is corresponds to time, ρ_{f_0} is nanofluid density at reference temperature, T_c, p is pressure, $\mathbf{g}$ is gravity, σ is electrical conductivity, μ_m is magnetic permeability, μ is coefficient viscosity, $(\rho c)_f$ is effective heat capacity of the fluid, k is thermal conductivity, $(\rho c)_p$ is effective heat capacity of the nanoparticle, D_B is Brownian diffusion coefficient, ϕ is nanoparticle volume fraction, T is temperature, D_T is thermophoretic diffusion coefficient and T_c is reference temperature. The nanofluid density, ρ is

$$\rho \cong \phi \rho_p + (1 - \phi)[\rho_{f_0} - \rho_{f_0} \beta (T - T_c)], \tag{5}$$

where ρ_p is nanoparticle mass density and β is volumetric coefficient of thermal expansion.

The boundary conditions are free at both lower and upper boundaries, given by

$$w = 0, \frac{\partial w}{\partial z} + a_1 H \frac{\partial^2 w}{\partial z^2} = 0, T = T_h, \phi = \phi_0 \text{ at } z = 0, \tag{6}$$

$$w = 0, \frac{\partial w}{\partial z} - a_2 H \frac{\partial^2 w}{\partial z^2} = 0, T = T_c, \phi = \phi_1 \text{ at } z = H. \tag{7}$$

Linear Stability Analysis

Linear stability analysis is applied to the system (1) – (7) to investigate the initiation of both stationary and oscillatory instabilities. Non-dimensionalization is carried out by choosing appropriate scaling parameters based on the fundamental dimensions present in the original governing equations. Following Narayana et al. [19] and Nield and Kuznetsov [17], the scaling variables for length, time, velocity, pressure, nanoparticle volume fraction and temperature are

$$(x^*, y^*, z^*) = \frac{(x, y, z)}{H}, t^* = \frac{\alpha}{H^2} t, (u^*, v^*, w^*) = (u, v, w)\frac{H}{\alpha_f}, p^* = \frac{H^2}{\mu \alpha_f} p,$$
$$\phi^* = \frac{\phi - \phi_0}{\phi_1 - \phi_0}, T^* = \frac{T - T_c}{T_h - T_c}, \tag{8}$$

where $\alpha_f = \frac{k}{(pc)_f}$, is the thermal diffusivity of the fluid, and asterisks denote dimensionless quantities. The resulting equations and boundary conditions are in the form of dimensionless partial differential equations. The non-dimensional equations after dropping the asterisks are

$$\nabla \cdot \mathbf{v} = 0, \tag{9}$$

$$\left(1 + \Lambda_1 \frac{\partial}{\partial t}\right)\left[\frac{1}{P_r}\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}\right) + \nabla p + Rm\boldsymbol{k} - RaT\boldsymbol{k} + Rn\phi\boldsymbol{k} + Q(u\boldsymbol{i} + v\boldsymbol{j})\right]$$
$$= \left(1 + \Lambda_2 \frac{\partial}{\partial t}\right)\nabla^2 \mathbf{v}, \tag{10}$$

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \nabla^2 T + \frac{N_B}{Le} \nabla\phi \cdot \nabla T + \frac{N_A N_B}{Le} \nabla T \cdot \nabla T, \tag{11}$$

$$\frac{\partial \phi}{\partial t} + \mathbf{v} \cdot \nabla \phi = \frac{1}{Le} \nabla^2 \phi + \frac{N_A}{Le} \nabla^2 T, \tag{12}$$

subject to the dimensionless boundary conditions

$$w = 0,\ \frac{\partial w}{\partial z} + a_1 \frac{\partial^2 w}{\partial z^2} = 0,\ T = 1,\ \phi = 0 \text{ at } z = 0, \tag{13}$$

$$w = 0,\ \frac{\partial w}{\partial z} - a_2 \frac{\partial^2 w}{\partial z^2} = 0,\ T = 0,\ \phi = 1 \text{ at } z = 1. \tag{14}$$

The nondimensional parameters are

$$\Lambda_1 = \lambda_1 \frac{\alpha_f}{H^2},\ \Lambda_2 = \lambda_2 \frac{\alpha_f}{H^2},\ P_r = \frac{\mu}{\rho_{f_0} \alpha_f},\ Le = \frac{\alpha_f}{D_B},\ Q = \frac{\sigma \mu_m^2 H_0^2 H^2}{\mu},$$

$$N_A = \frac{D_T (T_h - T_c)}{D_B T_c (\phi_1 - \phi_0)},\ N_B = \frac{(\rho c)_p}{(\rho c)_f}(\phi_1 - \phi_0),\ Ra = \frac{[\rho_{f_0} \beta (T_h - T_c)] g H^3}{\mu \alpha_f}, \tag{15}$$

$$Rn = \frac{[(\rho_p - \rho_{f_0})(\phi_1 - \phi_0)] g H^3}{\mu \alpha_f},\ Rm = \frac{[\rho_p \phi_0 + \rho_{f_0}(1 - \phi_0)] g H^3}{\mu \alpha_f},$$

where Λ_1 is the scaled stress relaxation parameter, Λ_2 is the scaled strain retardation parameter, P_r is the Prandtl number, Le is the Lewis number, Q is the Chandrasekhar number, N_A is the modified diffusivity ratio and N_B is the modified particle density increment.

Perturbation and Linearization

According to linear stability theory, minor perturbations are applied to the nanofluid layer to analyze its behavior. If these disturbances diminish with time, the system returns to its equilibrium state. However, if they grow, the system diverges from its steady state. The resulting linearized equations are presented below

$$\mathbf{v} = \mathbf{v}',\ p = p_b + p',\ T = T_b + T',\ \phi = \phi_b + \phi', \tag{16}$$

where prime denotes the perturbation quantity and p_b, T_b and ϕ_b are the pressure, temperature and concentration at the equilibrium state. Substituting the above expressions into the nondimensionalized partial differential equations (9) – (14) and linearizing the system yields a set of linear partial differential equations,

$$\left(1 + \Lambda_1 \frac{\partial}{\partial t}\right)\left[\frac{1}{P_r}\frac{\partial}{\partial t}\nabla^2 w' - \nabla_{\mathrm{H}}^2 Ra T' + \nabla_{\mathrm{H}}^2 Rn \phi' + Q \nabla_{\mathrm{z}}^2 w'\right] = \left(1 + \Lambda_2 \frac{\partial}{\partial t}\right)\nabla^4 w', \tag{17}$$

$$\frac{\partial T'}{\partial t} - w' = \nabla^2 T' + \frac{N_B}{Le}\left(\frac{\partial T'}{\partial z} - \frac{\partial \phi'}{\partial z}\right) - 2\frac{N_A N_B}{Le}\frac{\partial T'}{\partial z}, \tag{18}$$

$$\frac{\partial \phi'}{\partial t} + w' = \frac{1}{Le}\nabla^2 \phi' + \frac{N_A}{Le}\nabla^2 T', \tag{19}$$

subject to boundary conditions,

$$w' = 0,\ \frac{\partial w}{\partial z} + a_1 \frac{\partial^2 w}{\partial z^2} = 0,\ T' = 0,\ \phi' = 0 \ \text{at}\, z = 0, \tag{20}$$

$$w' = 0,\ \frac{\partial w}{\partial z} - a_2 \frac{\partial^2 w}{\partial z^2} = 0,\ T' = 0,\ \phi' = 0 \ \text{at}\, z = 1. \tag{21}$$

where ∇^2 is the three-dimensional Laplacian operator, ∇_H^2 is the two-dimensional Laplacian operator, ∇_z^2 is the one-dimensional Laplacian operator with respect to $z -$ plane and ∇^4 is the three-dimensional biharmonic operator.

Normal Modes

To transform the system of linear partial differential equations (17) – (21) into ordinary differential equations, the normal mode method is employed. The normal mode analysis utilizes two-dimensional periodic disturbances in an infinitely extended fluid layer [2,3,18], where the general solution is represented as a superposition of normal modes given by

$$(w', T', \phi') = [W(z), \Theta(z), \Phi(z)] e^{-(st + i\alpha_x + i\alpha_y)}, \tag{22}$$

where $W(z), \Theta(z)$ and $\Phi(z)$ are the amplitudes of the velocity, temperature and nanoparticles volume fraction, respectively. $\alpha = \left(\alpha_x^2 + \alpha_y^2\right)^{\frac{1}{2}}$ is the total wave number and s is a dimensionless complex growth rate where $\mathrm{Re}(s)$ is the growth rate and $\mathrm{Im}(s)$ is the frequency. For the stability at the marginal state $\mathrm{Re}(s) = 0$ when $\mathrm{Im}(s) = 0$, the stationary occurs and the convection is oscillatory when $\mathrm{Im}(s) \neq 0$. The system (17) – (21) in the form of the normal mode (22) are

$$(1 + \Lambda_1 s)\left[\frac{s}{P_r}(D^2 - \alpha^2)W + QD^2W + Ra\alpha^2\Theta - Rn\alpha^2\Phi\right] = (1 + \Lambda_2 s)(D^2 - \alpha^2)^2 W \tag{23}$$

$$W + \left(D^2 + \frac{N_B}{Le}D - 2\frac{N_A N_B}{Le}D - \alpha^2 - s\right)\Theta - \frac{N_B}{Le}D\Phi = 0, \tag{24}$$

$$W - \frac{N_A}{Le}(D^2 - \alpha^2)\Theta - \left[\frac{1}{Le}(D^2 - \alpha^2) - s\right]\Phi = 0. \tag{25}$$

The boundary conditions at $z = 0$ and $z = 1$ are thereby transformed into,

$$W = 0,\ DW + a_1 D^2 W = 0,\ \Theta = 0,\ \Phi = 0 \ \text{at}\, z = 0, \tag{26}$$

$$W = 0,\ DW - a_2 D^2 W = 0,\ \Theta = 0,\ \Phi = 0 \ \text{at}\, z = 1, \tag{27}$$

where $D = \frac{d}{dz}$. When $\Lambda_1 = \Lambda_2 = Q = 0$, the system (23) – (25) reduced to the problem of Nield and Kuznetsov [18]. In this study, stress-free upper and lower boundaries will consider at $z = 0$ and $z = 1$,

$$W = 0, D^2W = 0, \Theta = 0, \Phi = 0. \tag{28}$$

A feasible approach for solving the differential equations is the Galerkin-type weighted residual method. This technique addresses the linearized governing equations by representing the solution as a series expansion of selected trial functions. The functions W, Θ and Φ are expressed in the following form,

$$W = \sum_{n=1}^{N} A_n W_n , \Theta = \sum_{n=1}^{N} B_n \Theta_n, \Phi = \sum_{n=1}^{N} C_n \Phi_n, \tag{29}$$

where A_n, B_n, and C_n are unknown coefficients and $n = 1,2,3,\dots,N$. Equations (23) – (25) are multiplied by W, Θ and Φ, respectively of (29). Applying integration by parts over the interval $z \in [0,1]$ results in a system containing $N-1$ unknowns. The trial functions that satisfied the boundary conditions (28) for free lower and upper boundaries are

$$W_1 = \Theta_1 = \Phi_1 = sin\,\pi z\,. \tag{30}$$

Results and Discussions

The thermal Rayleigh number, Ra serves as a critical parameter indicating the bifurcation point at which the system transitions from steady convection to mildly chaotic behavior and eventually to fully developed turbulence. The threshold for the onset of stationary convection $(\omega = 0)$ is given by

$$Ra_{stat} = \frac{(\pi^2 + \alpha^2)}{\alpha^2}[\pi^4 + 2\pi^2\alpha^2 + \alpha^4 + \pi^2 Q] - (N_A + Le)Rn. \tag{31}$$

The stationary instability is independent of Λ_1 and Λ_2 as the time-dependent terms become negligible in this regime. In the Newtonian fluid, where $Q = 0$ Eq. (31) simplifies to the stationary convection result reported by Nield and Kuznetsov [2]. For oscillatory convection $(\omega \neq 0)$, a complex growth rate $s = i\omega$ is considered. Consequently, the Rayleigh number takes the form $Ra = Ra_r + iRa_i$ where Ra_r and Ra_i denote the real and imaginary components, respectively. Enforcing the condition $Ra_i = 0$ yields the corresponding frequency ω associated with oscillatory instability given by

$$Ra_{osc} = \frac{1}{\alpha^2}\left\{\frac{\pi^4 + 2\pi^2\alpha^2 + \alpha^4}{1 + \Lambda_1^2\omega^2}[(\pi^2 + \alpha^2)(1 + \Lambda_1\Lambda_2\omega^2) - \omega^2(\Lambda_1 - \Lambda_2)] - \frac{\omega^2}{P_r}(\pi^2 + a + \pi^2 Q(\pi^2 + \alpha^2)\right\} - \frac{(\pi^2 + \alpha^2)^2(N_A + Le) + \omega^2 Le^2}{(\pi^2 + \alpha^2)^2 + \omega^2 Le^2}Rn. \tag{32}$$

Ra_{osc} (32) reduces to the form obtained by Nield and Kuznetsov [2] if $\Lambda_1 = \Lambda_2 = Q = 0$. This section highlights the influence of stress relaxation, strain retardation and magnetic field on both stationary and oscillatory instabilities. The neutral stability curves for the thermal Rayleigh number are presented in the corresponding parameter space, (α, Ra).

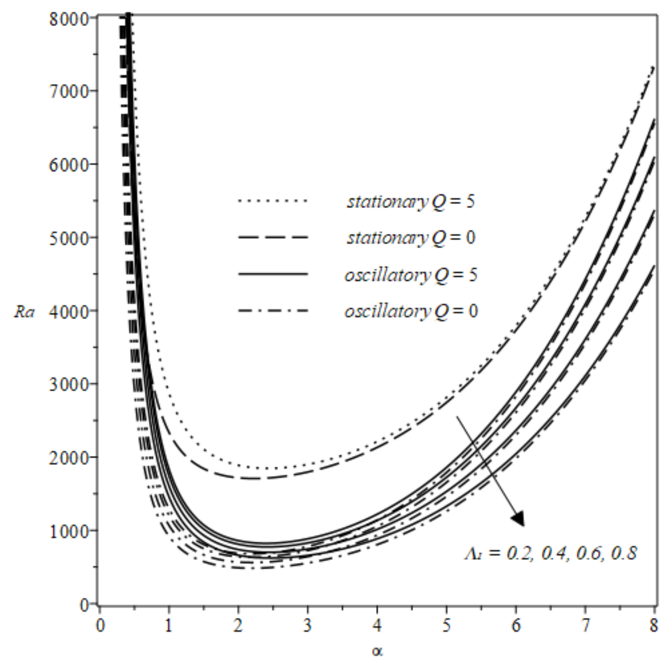

Figure 2: The neutral stability curves corresponding to stationary and oscillatory convection are presented for varying values of Λ_1, with fixed parameters $\Lambda_2 = 0.1, P_r = 5, Le = 100, N_A = 5, R_n = -10$ and $\omega = 1$.

Figure 2 illustrates the neutral stability curves representing the variation of the Rayleigh number, Ra with the wavenumber, α for different values of the stress relaxation parameter, Λ_1 under two magnetic field strengths, namely $Q = 0$ and $Q = 5$. The analysis includes both stationary and oscillatory modes of convective instability. The findings indicate that increasing Λ_1 leads to a reduction in the critical Rayleigh number for oscillatory convection, suggesting that higher values of Λ_1 promote an earlier onset of instability in the viscoelastic nanofluid layer. This behavior is attributed to the prolonged influence of applied stress on fluid elements, which diminishes the fluid's resistance to deformation. As Λ_1 increases, viscoelastic friction is reduced, lowering internal resistance and enabling convection to initiate at lower Rayleigh numbers.

Additionally, the critical Rayleigh number associated with stationary convection is observed to be higher than that of oscillatory convection. The enhanced stress relaxation effect intensifies the fluid's time-dependent response, making it more susceptible to oscillatory instability due to the reduced energy requirement for perturbation growth. In contrast, initiating stationary convection requires overcoming both viscous and elastic resistances, necessitating a higher Rayleigh number to trigger instability. Another key observation is the stabilizing influence of the magnetic field, the application of a magnetic field increases the critical Rayleigh number, indicating a suppression of convective onset.

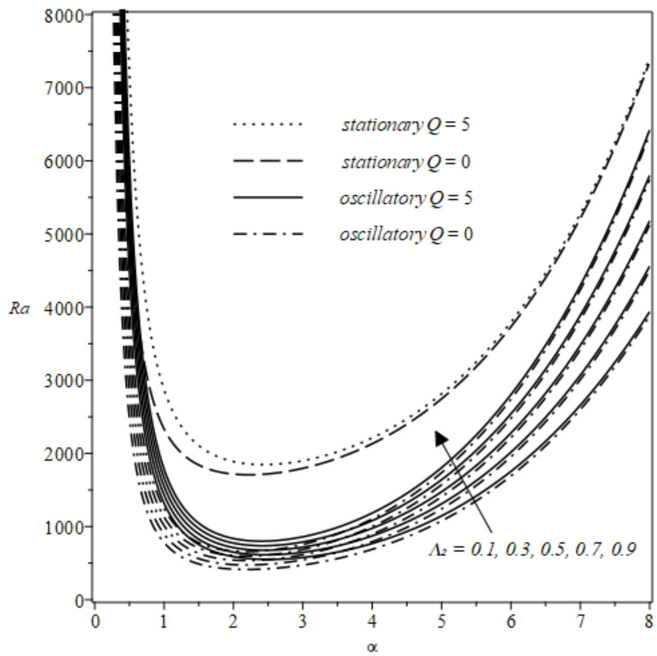

Figure 3: The neutral stability curves corresponding to stationary and oscillatory convection are presented for varying values of Λ_2, with fixed parameters $\Lambda_1 = 1.0, P_r = 5, Le = 100, N_A = 5, R_n = -10$ and $\omega = 1$.

Figure 3 displays the neutral stability curves illustrating the influence of the scaled strain retardation parameter, Λ_2 on the thermal Rayleigh number, Ra as a function of the wavenumber, α for various values of Λ_2. The results demonstrate that the critical Rayleigh number for oscillatory convection increases with an increase in Λ_2. This parameter, Λ_2 characterizes the dimensionless retardation time, representing the fluid's delayed stress response to deformation and accounting for its memory effect. A higher Λ_2 implies greater resistance to rapid changes in deformation, thereby exerting a stabilizing influence on the system. This stabilization delays the onset of convection, indicating that a greater thermal input is required to trigger instability in the viscoelastic nanofluid layer.

For both magnetic field strengths considered $Q = 0$ and $Q = 5$, the oscillatory mode consistently exhibits a lower critical Rayleigh number compared to the stationary mode. This suggests that oscillatory instabilities are more easily initiated, requiring less thermal energy, while the stationary modes demand a higher Rayleigh number, indicating comparatively greater stability in the fluid system.

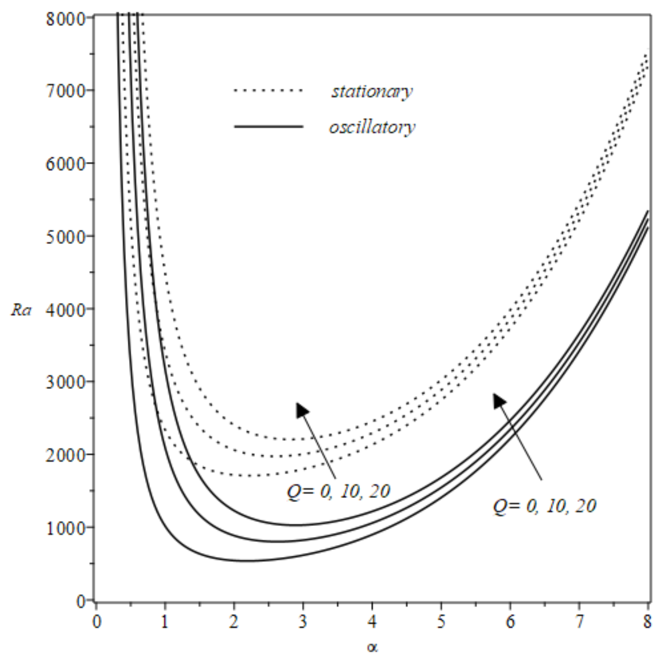

Figure 4: The neutral stability curves corresponding to stationary and oscillatory convection are presented for varying values of Q, with fixed parameters $\Lambda_1 = 1.0, \Lambda_2 = 0.1, P_r = 5, Le = 100, N_A = 5, R_n = -10$ and $\omega = 1$.

Figure 4 demonstrates that the thermal Rayleigh number increases with the Chandrasekhar number, Q for both stationary and oscillatory convection modes. As Q increases, corresponding increases in the Rayleigh number indicate enhanced thermal stability, implying that a stronger temperature gradient is required to initiate convective motion. This stabilizing effect is primarily attributed to the influence of the magnetic field, quantified by the Chandrasekhar number, which governs the strength of the Lorentz force acting within the fluid.

The Chandrasekhar number represents the ratio of magnetic to viscous forces, and a higher value signifies a more dominant magnetic influence. An intensified Lorentz force generates additional resistive drag, thereby suppressing fluid motion and increasing the threshold for the onset of convection. Consequently, the presence of a stronger magnetic field leads to reduced flow velocity and greater resistance to convective instability.

Conclusion

This study examines the convective instability in a horizontal layer of nanofluid bounded by free surfaces. By employing linear stability analysis, the governing nonlinear partial differential equations (PDEs) are transformed into a system of ordinary differential equations (ODEs) through normal mode analysis. The investigation focuses on the influence of key dimensionless parameters namely, the scaled stress relaxation parameter, the scaled strain retardation parameter and Chandrasekhar number on both stationary and oscillatory convection modes. To verify the validity of the results, comparisons were made with existing literature, and the outcomes show strong agreement with previous findings, thereby confirming the accuracy of the present analysis. The main findings are summarized as follows,

- For fixed values of $\Lambda_2 = 0.1, P_r = 5, Le = 100, N_A = 5, R_n = -10$ and $\omega = 1$, an increase in the scaled stress relaxation parameter, Λ_1 leads to a decrease in the critical Rayleigh number for oscillatory convection, indicating earlier onset of instability.
- Under the same conditions, increasing the scaled strain retardation parameter, Λ_2 results in a higher oscillatory thermal Rayleigh number, suggesting enhanced system stability due to delayed convective onset.
- For both stationary and oscillatory convection modes, the thermal Rayleigh number increases with the Chandrasekhar number, Q given $\Lambda_1 = 1.0, \Lambda_2 = 0.1$ and the same values for the remaining parameters. This highlights the stabilizing effect of the magnetic field through the influence of the Lorentz force.

Acknowledgments

The authors would like to express their sincere gratitude to Universiti Teknologi MARA (UiTM) for the facilities, resources and continuous support provided throughout the completion of this research. Appreciation is also extended to colleagues and peers for their valuable insights and constructive discussions that contributed to the improvement of this work.

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