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OPTIMISING WATER SUPPLY AND DEMAND ALLOCATION USING DYNAMIC PROGRAMMING: ADDRESSING DISTRIBUTION INEFFICIENCIES

Aiman Syazwan Ahmad[1,*], Rossidah Wan Abdul Aziz[1]

[1]College of Computing, Informatics and Mathematics,
Universiti Teknologi MARA, Cawangan Negeri Sembilan,
Kampus Seremban, 70300 Seremban Negeri Sembilan,
Malaysia
Email: rossidah@uitm.edu.my

Abstract

Water resource allocation is a critical issue for domestic, agricultural, and industrial sectors, particularly in regions facing water scarcity. This study aims to address the problem of inefficiency in water distribution among competing sectors, leading to unmet demands and resource wastage. The main objective of this study is to find an optimal allocation of water resources using a Dynamic Programming (DP) model to enable the available water supply to be distributed efficiently to meet the demand of each sector. A structured approach was employed to achieve this aim, beginning with a broad literature review of the prevailing strategies for allocation and how DP has been applied. Secondary data on the supply and demand of water resources from credible sources over the past five years were used. The DP model was then used to decompose this complex problem into more manageable subproblems to define optimal allocation strategies. The results obtained indicate that the proposed DP model effectively optimizes water distribution in every sector. Furthermore, data analysis results reflect that water demand is highly responsive to available water. This can be inferred from the fact that the maximum water demand is more sensitive to changes in the available water supply than the minimum demand, hence requiring adaptive management. In conclusion, this study provides valuable insights into the dynamics of water allocation and offers a robust framework that could enhance efficient distribution. Therefore, the adoption of the DP model in water management strategies by policymakers is highly recommended since it was complemented by real-time data monitoring to adapt to changing water availability and demands.

Keywords: Water resource allocation, Dynamic Programming model, Bellman equation, supply and demand

1. Introduction

The term "water resources" refers to the amount of water that is available for ecosystems and people. Humans have access to and use water for a variety of purposes, including domestic use, industry, agriculture, recreation, and environmental conservation. Another definition states that water is an essential natural resource required to raise the standard of living for urban residents and promote economic growth at the local level. The availability of water is often uncertain, and it can be considered a limited resources; therefore, distribution control must be done in an orderly manner [1]. Water allocation is a way of distributing limited resources among multiple consumers who require it at the same time. It is crucial to allocate time effectively when distributing water, as this can ensure its security, reduce its wastage and maximize its community benefits [2].

Even so, there are several major factors influence the allocation of this water, including ongoing population growth, which increases demand for water supplies. Furthermore, water pollution caused by community and industrial activities prevents the even distribution of clean water. One way to allocate resources effectively is to assess the quantity and quality of water that is available. Water must first be allocated to meet basic needs before it can be used for other purposes. Therefore, an equitable allocation of water resources across regions is essential for reducing poverty sustainably as well as promoting economic growth, particularly in areas where water demands are rising rapidly [3]. However, there are still significant differences in water accessibility.

Despite technological advancements, the challenge of balancing water resource allocation with the growing human population and industry remains unresolved. Increasing demand due to population growth leads to water shortages and causing difficulties in distribution, particularly in highly populated and industrialized regions. This imbalance forces communities to compete for quality water resources, threatening long-term sustainability and future water security. Water scarcity is a critical issue that hampers social and economic progress, exacerbated by continuous urbanization and industrial activities [4]. Addressing this issue requires optimizing water allocation while considering demand fluctuations and response uncertainties. Therefore, this study focuses on Dynamic Programming (DP) model to maximize the allocation of water supply for each sector. This study also aims to analyze the total water supply allocated according to demand for each sector by using linear regression model.

2. Literature Review

Water resources management is always indispensable in terms of the allocation of the available amount among different consumers and industries with the purpose of ensuring efficiency for long-term use. [5] evaluated the hydrological effects of climate change on reservoir operation by using the hybrid model which Stochastic Dynamic Programming (SDP) model and the Soil and Water Assessment Tool (SWAT). In light of climate change, this hybrid model optimizes reservoir operations to enhance agricultural water supply and distribution.

Integrating various goals could help preserve sustainable water consumption while also attending to different priorities of stakeholders [6]. The ability of algorithms to reproduce optimal equilibrium solutions under a range of risk circumstances helps the watershed managers in process of making decisions. Moreover, the dynamic scheduling approach was employed to take decisions with respect to the current circumstances for proper management of water resources when there is drought or unpredictable climatic changes at certain seasons [3]. The Multi-Objective Dynamic Programming model that used in that research enable to ensures the availability of water resources in the future while meeting the current sector of water demands by optimizing water utilization efficiency.

Richard Bellman first presented DP in 1950, and it has been widely used in a variety of fields, including computer science, mathematics, operations research, economics, and engineering. This method is an effective approach to address complex problems by breaking the problem down into smaller problems that are solved once. But in order to prevent recomputing, their solutions are stored and used in recursion.

Xu, [7] involved special focus on agricultural consumption optimization during water management in linking reservoirs. They solve the difficulties related to multidimensional optimization problems in reservoirs by using the improved Decomposition and Dynamic Programming Aggregation model. They found that water shortage can be minimize through optimization of water supply and reduction in system energy consumption. Other than that, [8] was focused on the optimization of water resource management in the river basin. The SDP model and water value provide a comprehensive approach to water allocation methods in an area where competing demands and growing water shortages coexist. Their study looks at water consumption as well as pricing evaluation in the domestic, agricultural and industrial sectors.

In conclusion, the applicability and efficiency of these models in solving complicated allocation problems are analyzed. This review aims to provide a theoretical basis and identify knowledge gaps that justify the adoption of dynamic programming in this study.

3. Methodology

3.1. *Data Collection*

This study utilised secondary data from [6]. The data collected for Major River Basins (MRB) are water transport loss ratio, available water resources for each period, and minimum and maximum water demands by region in the MRB for all three sectors.

3.2. *DP model*

In the modelling of the water resource allocation, we divide the work involved into two phases that are developing conceptual framework based on [9] as present in Figure 1, and modeling the DP model for water resources allocation.

Forward Dynamic Programming is used to solve the main problems which would start from the first stage and move forward stage by stage to find the optimal allocation for each stage. Initially, this model is solved for the first period by obtaining the total allocation of water supply in that period and then employing forward recursion by adding the total allocation from period 1 with total allocation for period 2. This process has been repeated until periods 3 and 4.

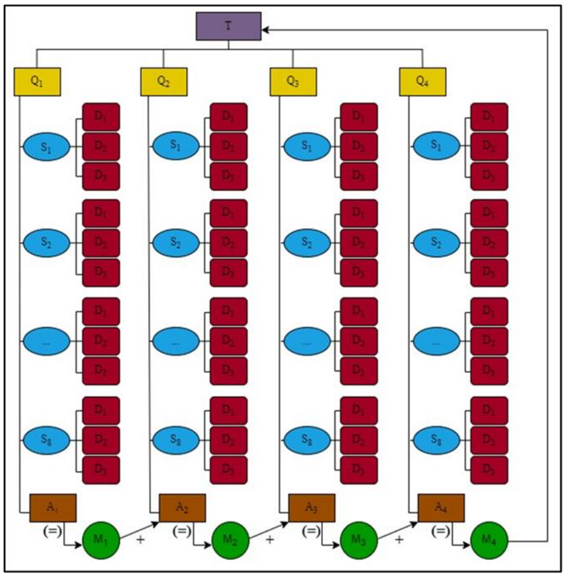

Figure 1: DP Structure

3.3. *Notations of the model*

Indices

t: Index of period t of the regional water resources allocation where $t = 1, 2, 3, 4$, $t = 1$: Spring; $t = 2$: Summer; $t = 3$: Autumn; $t = 4$: Winter.

i: Index of subarea or region i in the river basin where $i = 1, 2, \dots, 8$, $i = 1$: Chengdu; $i = 2$: Zigong; $i = 3$: Neijiang; $i = 4$: Leshan; $i = 5$: Meishan; $i = 6$: Yibin; $i = 7$: Ya'an; $i = 8$: Aba.

j: Index of water sector in the subarea i where $j = 1, 2, 3$, $j = 1$: Domestic sector; $j = 2$: Agriculture sector; $j = 3$: Industry sector.

Parameters

Q_t: Allocation of water supply from the river basin in period t.
$S_{t,i}$: Allocation of water supply for subarea i in period t.
$D_{t,i,j}^{min}$: Minimum demand of water supply in period t for subarea i for sector j.
$D_{t,i,j}^{max}$: Maximum demand of water supply in period t for subarea i for sector j.
A_t: Actual allocation of water supply in period t.
M_t: Total amount of water supply allocated for use in period t.
$\rho_{t,i}$: Water-transfer loss ratio from river basin in period t for subarea i.
$\alpha_{t,i}^{e}$: Water allocated to the sustainable development of the entire basin ecological environment of subarea i in period t
s_t: Available water supply in the river basin at the beginning of each period t.
s_{t+1}: Available water supply in the river basin for the next period t.
s_t^{max}: Maximum volume of stored water in period t.

Decision variable

$D_{t,i,j}$: Demand of water supply in period t for subarea i for sector j.
T: Total allocation of water supply from river basin in a year.

Based on DP structure, this study has derived the objective function of the model. Objective function for water allocation of DP is to find the maximum allocation of water supply for three sectors in eight subareas in a year. After employed the recursive function, the general Bellman's equation or the objective function for the model can be formulated as follows:

$$Maximise\ T = \begin{cases} \sum_{t=1}^{4}\sum_{i=1}^{8}\sum_{j=1}^{3} D_{t,i,j} & , t = 1,2,3,4, i = 1, 2, \dots, 8\ and\ j = 1,2,3 \\ \sum_{t=1}^{4}\sum_{i=1}^{8}\sum_{j=1}^{3} D_{t,i,j} + M_{t-1} & , t = 2,3,4, i = 1, 2, \dots, 8\ and\ j = 1,2,3 \end{cases} \tag{1}$$

3.4. *Formulation of Constraints*

(1) Limitation on the water supplies that are available

The total amount of water supply that allocated to each sector in each subarea in one period, $D_{t,i,j}$ cannot be more than the total amount of water supply that allocated from the river basin to that period, Q_t.

$$0 < \sum_{t=1}^{4}\sum_{i=1}^{8}\sum_{j=1}^{3} D_{t,i,j} \le Q_t \tag{2}$$

(2) Restriction of water consumption for each subarea per period

The summation between total amount of water supply that allocated to each sector, $D_{t,i,j}$ and the water allocated to the ecological environment in that subarea, $\alpha_{t,i}^{e}$ cannot be greater than the amount allocated to that subarea, $S_{t,i}$.

$$\sum_{t=1}^{4}\sum_{i=1}^{8}\sum_{j=1}^{3} D_{t,i,j} + \alpha_{t,i}^{e} \le S_{t,i} \tag{3}$$

(3) Water demand limitations

The amount of water allocated to each sector should fall between the minimum water demand, $D_{t,i,j}^{min}$

and the maximum water demand, $D_{t,i,j}^{max}$.

$$D_{t,i,j}^{min} \leq D_{t,i,j} \cdot \left(1 - \rho_{t,i}\right) \leq D_{t,i,j}^{max} \tag{4}$$

(4) Equation describing state transfer

The available water supply for the next period is equal to available water supply at the beginning of each period, s_t minus with the total amount of water supply that allocated to each sector in each subarea, $D_{t,i,j}$.

$$s_{t+1} = s_t - \sum_{t=1}^{4} \sum_{i=1}^{8} \sum_{j=1}^{3} D_{t,i,j} \tag{5}$$

(5) The non-negative restriction must also be met by each individual variable.

$$D_{t,i,j} \geq 0 \quad , \quad T \geq 0 \tag{6}$$

4. Result and Discussions

4.1. Total Water Allocation for each Period

The MRB has variable water availability throughout the year due to rainfall and other natural factors. Water management is thus necessary to supply the demand for the domestic, agriculture and industry sectors without harming the environment. Table 1 shows the result for total amount of water allocated during spring, summer, autumn and winter.

Table 1: Results for total allocation of water supply for each period

Period	Water availability ($\mathbf{10^4 m^3}$)	Total water allocation ($\mathbf{10^4 m^3}$)
Spring	60931.86	55770.49
Summer	290940.52	267839.30
Autumn	750859.10	653132.80
Winter	125009.10	117848.00

During spring season, the water availability in spring is relatively low at 60,931.86 × 10^4m^3 with most of it 55,770.49 × 10^4m^3 allocated according to the water demand from each sector. This indicates that there was leaving little surplus at this period due to a high reliance on the water resources that were available. Similarly in summer, water availability increases significantly to 290,940.52 × 10^4m^3 and water allocations increase correspondingly to 267,839.30 × 10^4m^3. The comparatively greater surplus in comparison to spring indicates that there may be more water resources available to satisfy peak demands especially for irrigation which frequently occur during this season.

In autumn, this season stands out with the highest water availability which 750,859.10 × 10^4m^3 and highest total water allocation which 653,132.80 × 10^4m^3 due to increased rainfall and storage capacity. Even with the large allocation, there is still a substantial surplus which may be the result of decreased demand or inability to meet the water demand in particular sectors. For winter, this season sees the lowest water availability 125,009.10 × 10^4m^3 but maintains relatively high allocation levels 117,848 × 10^4m^3 leaving only a small surplus of water resource. This pattern can be the result of a sustained demand due to domestic and industrial consumption with reduced natural inflow due to dry weather. Overall, the water allocation strategy seems well-judged as that would meet seasonal demands but at the same time leaving a consistent surplus to help meet the long-term water sustainability.

4.2. Total Water Allocation for Eight Subareas during All Period

The results of the water allocation throughout the regions and sectors during all seasons where spring, summer, autumn and winter have to be examined in order to ensure a fair and efficient distribution that meets all water demands. The results of this study will be discussed in the context of the comparison of allocated water resources to minimum required water demands of the domestic, agricultural and industrial sectors of each of the eight subareas, assessing the efficiency and sufficiency of the allocation model. The objective is to identify the locations where allocations have not been able to meet the demands and need further optimization. The results of water allocation for all subareas are as shown in Table 2.

Table 2: Results for water allocation in eight subareas during different period ($10^4 m^3$)

Period	**Sector**	Chengdu	Zigong	Neijiang	Leshan	Meishan	Yibin	Yaan	Aba
Spring	Dom	5795.39	768.58	1017.10	835.28	653.72	2376.46	421.92	264.67
	Agr	12597.51	1998.31	2028.99	1994.23	2918.41	5660.78	1149.82	480.45
	Ind	5795.39	886.14	920.74	971.02	870.53	4548.21	563.65	253.22
Summer	Dom	24423.80	3959.74	6560.95	5985.54	3673.17	7833.08	2626.23	2709.34
	Agr	53090.33	10295.32	15794.14	14290.49	16398.08	18658.57	7156.99	5015.93
	Ind	24423.80	4565.34	7319.85	6958.19	4891.31	14991.40	3508.32	2709.34
Autumn	Dom	82806.70	7909.75	6250.16	17774.22	10426.08	8937.94	6679.97	2437.67
	Agr	179997.90	20565.33	14673.71	42435.95	46544.97	21290.38	18204.20	4512.98
	Ind	82806.70	9119.47	6745.30	20662.54	13883.70	17105.94	8923.62	2437.67
Winter	Dom	11415.02	1721.56	2085.74	2523.87	1642.90	3695.86	1434.58	812.20
	Agr	24812.99	4476.05	5124.95	6025.73	7334.42	8803.65	3909.49	1503.68
	Ind	11415.02	1984.85	2201.71	2933.99	2187.75	7073.37	1916.42	812.20

The water allocation strategy is generally well-distributed across seasons but several allocations fail to meet the minimum demand with the autumn season showing the worst performance in meeting water demand compared to other seasons. Several subareas were generally going through a serious shortage especially in Chengdu, Zigong, Neijiang and Meishan which highlighted as critical areas for improvement in allocation strategies. Agriculture is the most affected sector and it experiences a shortfall every other season, which will impact decreased crop yields and economic losses to farmers with the potential to cause harm to local food security. Although the domestic sector often receives allocations near the minimum demand, shortages still occur in every season. This can affect the access of household to a sufficient amount of water supply for drinking, cooking and hygiene purposes during peak periods of demand. Shortage of water is comparatively more severe during the autumn season compared to spring, summer and winter. However, the water allocation in all seasons needs better management and conservation of infrastructure to ensure all sectors get adequate water.

5. Conclusion and Recommendations

In summary, this study provided an essential insight in the management of water resource in terms of maximizing distribution to the industrial, agricultural, as well as domestic sectors. This indicates that the study successfully achieved its intended goals and providing valuable insights to water resource allocation. The DP method provided a useful framework for allocating water resources in the most efficient way possible, besides giving an in-depth examination into the dynamics of water supply and demand in modelling water resources allocation under different seasonal constraint. The findings validate the effectiveness of the DP method as a robust tool for optimizing water allocation especially in scenarios with fluctuating supply and demand across seasons.

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