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# NUMERICAL SOLUTION FOR MHD FLOW OVER A SHRINKING WEDGE USING BVP4C

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***Abstract***

*This study investigates MHD flow over a shrinking wedge using BVP4C. The findings were significant for several industrial applications in MHD pumps, liquid metal processing, cooling and heating systems and aerodynamic designs. This study aims to find an alternative method for solving the MHD flow problem over contracting wedges. The objective includes to solve MHD flow over a shrinking wedge using BVP4C. The governing partial differential equations (PDE) for heat transfer and flow are converted to a system of ordinary differential equations (ODE) using similarity transformation. BVP4C in MATLAB software is then used to solve this equation numerically. The study then compares the result between shooting method and BVP4C, thus analyse the effect of shrinking parameter, $\lambda$ and Hartree pressure gradient, $\beta$ on velocity profile, temperature profile, skin friction coefficient and local Nusselt number. The comparison shows good agreement between the compared value of skin friction coefficient and local Nusselt number obtained in BVP4C. The result presents that an increase in $\lambda$ will raise the velocity profile and decreased the temperature profile. There is also a second solution for cases of $\lambda = -1.30, -1.31, -1.3994$ when $\beta = 0.1$.*

**Keywords*:*** Magnetohydrodynamic, shrinking wedge, BVP4C

## 1. Introduction

This study explores boundary layer flow in various daily life situations, particularly in industries. Boundary layers are small fluid layers that develop on a bounding surface as fluid moves. The fluid interacts with the wall, creating a non-slip boundary region condition. The study also explores Magnetohydrodynamics, which involves the flow of electrically conducting fluids through a magnetic field. This fluid flow have practical applications in various technological processes, including plasma research, petroleum industry, power generator design, nuclear reactor cooling, and heat exchanger construction [1]. The flow of electrically conducting fluids over an external voltage-driven electric current or magnetic field is known as magnetohydrodynamics [2], and it is covered in this subject. This study focuses on MHD fluid flow towards shrinking wedges, a topis less explored in prior research.. Heat transfer is included in the flow process of the investigation, which may affect the methodology and findings. Convection, conduction, and thermal radiation are additional parameters included in the study.
According to [7], a wedge is a simple machine with a triangular shape and a sloping surface that is thicker at one end and narrows to a sharp edge at the other. A diminishing wedge is a

situation in which the inclined planes converge to a single point. Normally, a wedge is constructed by consisting of one or two inclined planes. The flow towards a wedge surface in fluid dynamics involves several factors, including partial slip and heat transfer, which can have a big impact on the methodology and findings of the study. Since shrinking wedge flow is the subject of this investigation, thermal energy transfer occurs naturally as the fluid passes over the wedge surface. The several ways that energy and entropy are transferred from one area to another are referred to as heat transfer [11].

The study about MHD boundary layer flow of a nanofluid across a dynamic stretched surface by using Rungge-Kutta methods was conducted by [3]. The results of the study showed that the viscoelastic and magnetic parameters affect the non-dimensional velocity, temperature and concentration profile, where there is an increase in the velocity while the temperature decreased. Meanwhile, [2] used Runge-Kutta Fehlberg (RFK45) to examine the heat transfer process of MHD Powell-Eyring fluid and the mass via an absorptive flat surface while considering the radiation and mass diffusion effects. A study used SQLM to examine a wedge-shaped MHD boundary layer and a continuous flow of nanofluid submerged in porous media with viscous dissipation was conducted by [1]. A study by [4] analyzed the effects of a magnetic field on the mass and heat transfer of a liquid form over an unstable stretched cylinder using BVP4C. The curvature parameter increased the temperature and velocity profiles, while the instability parameterreduced the fluid thickness. A study implementing the spectral quasi linearization method (SQLM) was carried out to analyze stable laminar MHD flow in nanofluid over a wedge in the presence of changing magnetic fields [13]. The study found that nanoparticle volume fractions affected heat transfer in wedge shaped fluids with different angles and caused an increase in thermal conductivity and temperature.

Whilst [5] studied the flow of a thick, incompressible fluid in a magnetic field through a continuously stretched sheet in two dimensions. Using BVP4C, the study solved the boundary value problem and discovered that an increase in the thermal slip parameter ($\gamma$) it will decreased the skin friction and heat transfer rate, while the increase in magnetic parameter ($M$) it will increased the heat transmission. When the velocity parameter ($\delta$) increased, the rate of heat transfer at the surface decreased. The heat transfer from MHD mixed convection flow to a penetrable stretched wedge implementing thermal radiation and ohmic heating using BVP4C was studiedby [12]. The numerical study showed that the skin friction, velocity, temperature and Sherwood number will all increased as the Hartree pressure gradient parameters increased. A study implementing the spectral quasi linearization method (SQLM) was carried out to analyze stable laminar MHD flow in nanofluid over a wedge in the presence of changing magnetic fields [13]. The study found that nanoparticle volume fractions affected heat transfer in wedge shaped fluids with different angles and caused an increase in thermal conductivity and temperature.

Thus, the focus of this study is to find a different approach to solving the MHD flow problem across contracting wedges while investigating it with MATLAB's BVP4C.

## 2. Methodology

Figure 1 represents the coordinates system of shrinking wedge and its physical model in 2-Dimensional Cartesian coordinates, $x$ and $y$ along the surface with a parameter $u$ and $v$ represent the velocity variables respectively. Assuming the shrinking wedge velocity is $u_w(x) = U_w x^m$, where $U_w < 0$ represents the wedge is shrinking, while $m$ and $U^\infty$ are illustrated as a positive constant. A variable magnetic field, $B(x)$ with assuming a small induced magnetic field and will not be considered is applied along the positive $y$-axis. The Hartree pressure gradient parameter, $\beta$ corresponds to $\beta = \frac{\Omega}{\pi}$ for a total angle, $\Omega$ of the wedge. $0 \leq m \leq 1$ is the interval of the boundary layer flow for a case that across the stationary flat plate in Blasius problem where $m = 0$ represent the boundary layer of the flat plate and $m = 1$ is for the flow towards a stagnation point in the boundless wall [6].

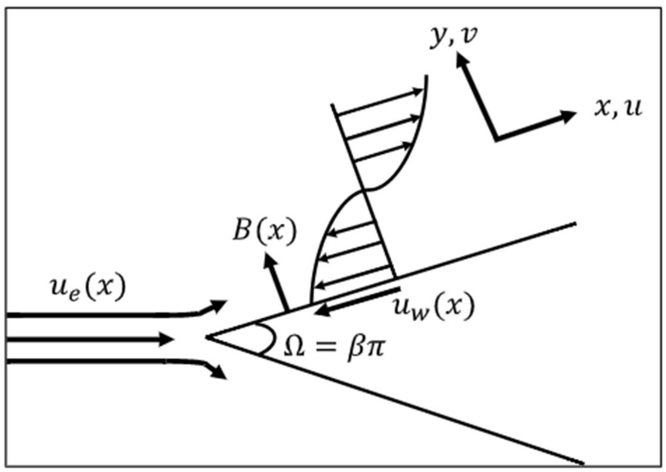


Figure 1: Coordinate system and physical model of shrinking wedge

Continuity equation, describing the conservation of mass in the boundary layer, can be expressed as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum equation, describing the balance of forces in the boundary layer, can be expressed as

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{\partial u_e}{\partial x} + v\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho}(u - u_e). \tag{2}$$

Energy equation governed by,

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

where $u$ is a velocity for $x$-axis, $v$ is a velocity for $y$- axis, $u_e$ is velocity of external flow, $B(x)$ is the magnetic field, $T$ is fluid temperature, $\alpha$ is thermal diffusivity, $\rho$ is the fluid density and $\beta$ is the Hartree pressure gradient parameter.

While the boundary conditions are

$$v=v_w,\quad u=u_w(x),\quad T=T_w \quad at\ y=0 \tag{4}$$

$$u\to u_e(x),\quad T\to T_\infty\ as\ y\to\infty. \tag{5}$$

The similarity equation variables given by

$$\psi=\sqrt{\frac{2vxu_e}{1+m}}f(\eta),\ \eta=\sqrt{\frac{(1+m)u_e(x)}{2vx}}y,\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \tag{6}$$

which $u$ and $v$ are defined as,

$$u=\frac{\partial\psi}{\partial y},\ v=-\frac{\partial\psi}{\partial x} \tag{7}$$

That satisfies the continuity equation in (1) and obtained as

$$u=u_e(x)f'(\eta), \tag{8}$$

$$v=-\sqrt{\frac{(1+m)vu_e(x)}{2x}}\left[f(\eta)+\frac{m-1}{m+1}\eta f'(\eta)\right], \tag{9}$$

where the prime represents the differentiation with respect to $\eta$, when the boundary $\eta=0$, then the transpiration rate is given by,

$$v_w=-\sqrt{\frac{(1+m)vu_e(x)}{2x}}s, \tag{10}$$

with $s = f(0)$, where the parameter $s > 0$ is for suction cases and $s < 0$ is for injection cases. The variables can be defined as $\nu$ is the kinematic viscosity, $T_w$ is the uniform surface temperature, $\sigma$ is electrical conductivity, $\psi$ is stream function, $y_w(x)$ is the mass flux velocity, $T_\infty$ is a free stream temperature and $\eta$ is the boundary layer thickness.

Additional constraints, referred to as boundary conditions, must be applied to guarantee that the solutions of PDEs or ODEs satisfy requirements at the domain boundaries and to identify a single solution that matches the physical problem. At certain locations along the edge of the spatial region where the differential equation is held, the solution must meet certain requirements known as boundary conditions. The equations in this study indicate five boundary conditions as stated (4) and (5), where $\lambda$ is defined as shrinking parameter, $Pr$ is defined as the Prandtl number and $s$ is define as the suction or injection parameter as below:

$$\lambda = \frac{U_w}{U_\infty}, \beta = \frac{2m}{m+1}, M = \sqrt{\frac{2\sigma}{(m+1)\rho U_\infty}} B_0, Pr = \frac{\nu}{\alpha}. \tag{11}$$

where $U_w$ is a velocity and $\lambda < 0$ is both condition for shrinking case. Since there is no magnetic field, $M = 0$.and when $\beta = 0$ and $\beta = 1$, (12) will reduces to the Blasius and Hiemenz equations respectively. The PDE will be transformed by using a similarity transformation where the differentiated variable will be used to substitute into the PDE in (1), (2) and (3) to obtain the ODE as follows

$$f''' + ff'' + \beta\left(1 - f'^2\right) + M^2(1 - f') = 0\,, \tag{12}$$

and

$$\frac{1}{Pr}\theta'' + f\theta' = 0\,, \tag{13}$$

which subject to the boundary condition below

$$f(0) = s, f'(0) = \lambda, \theta(0) = 1. \tag{14}$$

$$f'(\eta) \to 1, \theta(\eta) \to \infty \ as\ \eta \to \infty\,. \tag{15}$$

This study only focuses on two physical quantities which is local Nusselt number, $Nu_x$ and skin friction coefficient, $C_f$ which both of it can be written as equation below:

$$C_f = \frac{\tau_w}{\rho {u_e}^2(x)}, \qquad Nu_x = \frac{x q_w}{k(T_w - T_\infty)}, \tag{16}$$

Then wall shear stress along a shrinking surface, $\tau_w$ and surface heat flux, $q_w$ can be expressed as

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{17}$$

By substituting (17) in (16), the equation below was obtained

$$Re_x^{\frac{1}{2}} C_f = \left(\frac{1+m}{2}\right)^{\frac{1}{2}} f''(0), Re_x^{-\frac{1}{2}} Nu_x = -\left(\frac{1+m}{2}\right)^{\frac{1}{2}} \theta'(0). \tag{18}$$

## 3. Result and discussion

The results are compared with two previous findings by [6], who applied the shooting method with Rugge-Kutta Fehlberg (RFK45) in Maple software and [7], who applied a BVP4C method in MATLAB. The purpose of this comparison is to verify that the outcomes produced by BVP4C are validated. Table 1 represents the comparison results for skin friction coefficient, $f''(0)$, when $M = 0,\ \lambda = 0$ and $s = 0$ for different value of $\beta$. While Table 2 shows the comparison results for local Nusselt number when $M = 0,\ \lambda = 0$ and $s = 0$ for different value of $\beta$.

Table 1: The comparison results of skin friction coefficient when $\lambda = 0$ for different value of $\beta$

| $M$ | $s$ | $\beta$ | Skin friction coefficient $(f''(0))$ | | |
|---|---|---|---|---|---|
| | | | [7] | [6] | Present study |
| 0 | 0 | 0 | 0.469600 | 0.469600 | 0.469600 |
| | | 0.5 | 0.927680 | 0.927680 | 0.927680 |
| | | 0.7 | 1.059808 | 1.059808 | 1.059808 |

Table 2: The comparison results of local Nusselt number when $\lambda = 0$ for different value of $\beta$

| $M$ | $s$ | $\beta$ | Local Nusselt Number, $Nu_x$ | |
|---|---|---|---|---|
| | | | [6] | Present study |
| 0 | 0 | 0 | -0.469600 | -0.469600 |
| | | 0.5 | -0.538978 | -0.538979 |
| | | 0.7 | -0.533660 | -0.553661 |
| | | 1 | -0.570465 | -0.570465 |

From Table 1 and Table 2, it shows a good agreement of compared value for both skin friction coefficient and local Nusselt number result between the present study with the previous study where the value obtained by using BVP4C is similar as a value obtained from previous study using Maple.

### 3.1 Effect of shrinking parameter, $\lambda$ on velocity profile, temperature profile, skin friction coefficient and local Nusselt number.

Figures 3.1 and Figure 3.2 present the velocity profiles, $f'(\eta)$ for various values of the shrinkage parameter, $\lambda$, where $M = 0.2,\ s = 1$ with Hartree pressure gradient parameter, $\beta = 0.1$ and $\beta = 0.33$. As shown in Figure 3.1, the increase in $\lambda$ contributes to an increase of $f'(\eta)$. A decrease in $\lambda$ reduces $f'(\eta)$ in Figure 3.2 while the boundary layer thickness is increasing when value of $\lambda$ decrease and the value of Hartree pressure gradient, $\beta$ increases.

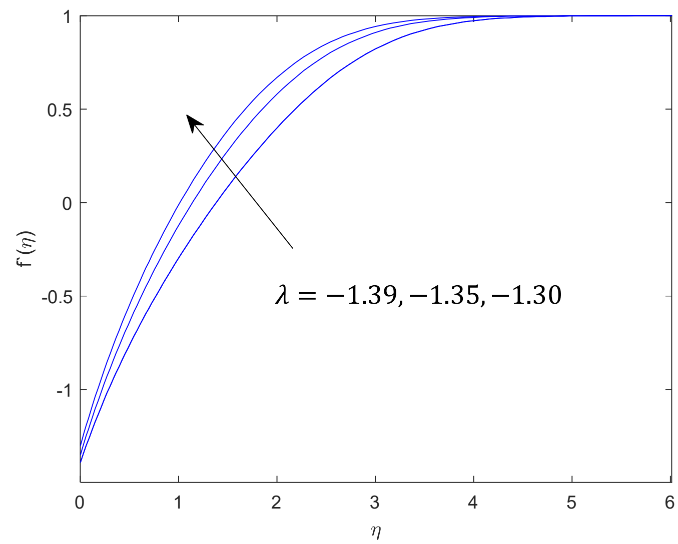


Figure 3.1: The velocity profile for different value of $\lambda$, when $\beta = 0.1$

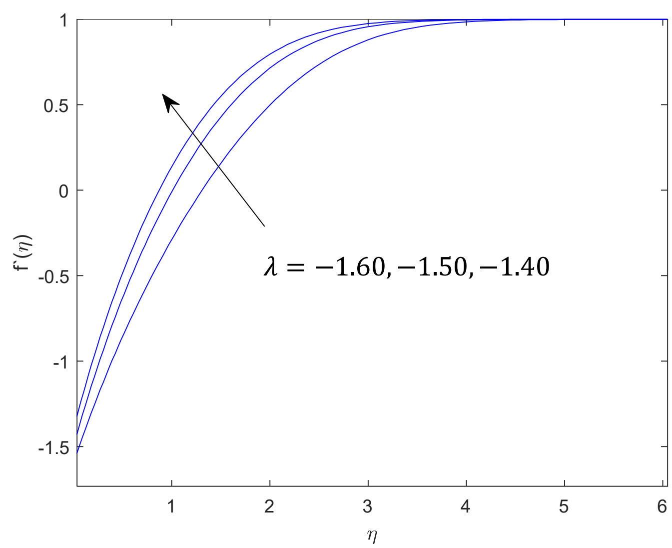


Figure 3.2: The velocity profile for different value of $\lambda$, when $\beta = 0.33$

Thus, the temperature profile for various values of $\lambda$ when $M = 0.2,\ s = 1, Pr = 1$ with $\beta = 0.1$ and $\beta = 0.33$ is shown in the figure below, as in Figures 3.3 and Figure 3.4 respectively. The temperature profile decreases as the value of $\lambda$ rises, as seen in Figure 3.3. This is due to the capacity of viscous dissipation to raise temperature. As the value of $\lambda$ increases, the temperature profile decreases, and the boundary layer thickness is high as shown in Figure 3.4 when the value of Hartree pressure gradient parameter, $\beta$ is increasing.

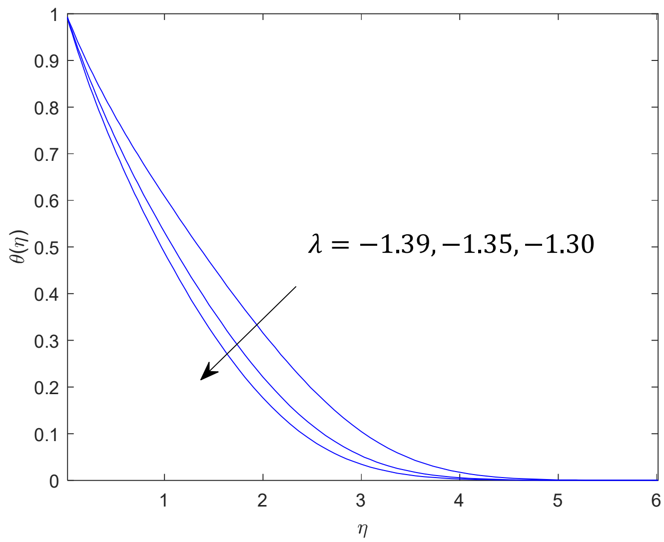


Figure 3.3: The temperature profile for different value of $\lambda$, when $\beta = 0.1$

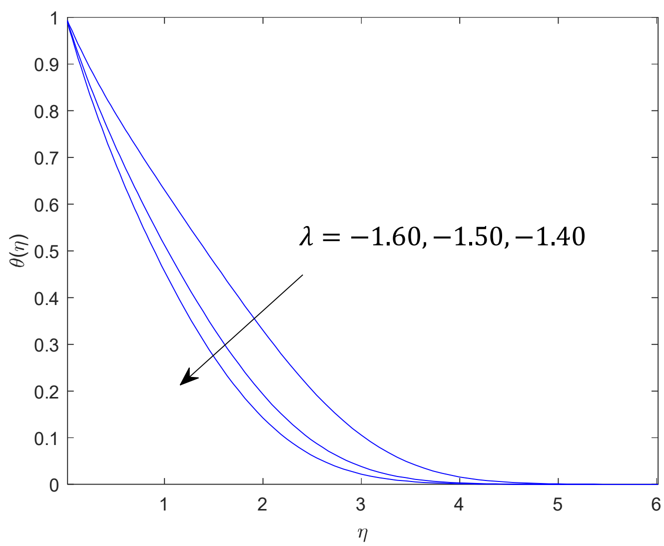


Figure 3.4: The temperature profile for different value of $\lambda$, when $\beta = 0.33$

Figure 3.5 and Figure 3.6 reveals the existence of the dual solution, subject to the boundary condition for a certain range of $\lambda$ which depends on the skin friction coefficient and local Nusselt number. For the cases in this study, which is a shrinking wedge, $\lambda < 0$, the study presents the dual solution for the cases. Figure 3.5 illustrates the relation between $\lambda$ and skin friction coefficient, $f''(0)$. It shows that the increase in value $\lambda$ will decrease the value of $f''(0)$. Figure 3.6 shows the relation between $\lambda$ and local Nusselt number, $\theta(0)$. It shows that the increase in value $\lambda$ will also increase the value of $\theta(0)$. It can be observed that the solution is not unique for a certain critical value $\lambda$ in both figures. There is no solution to the equation obtained when $\lambda < \lambda_c$ due to separation of the boundary layer from the surface.

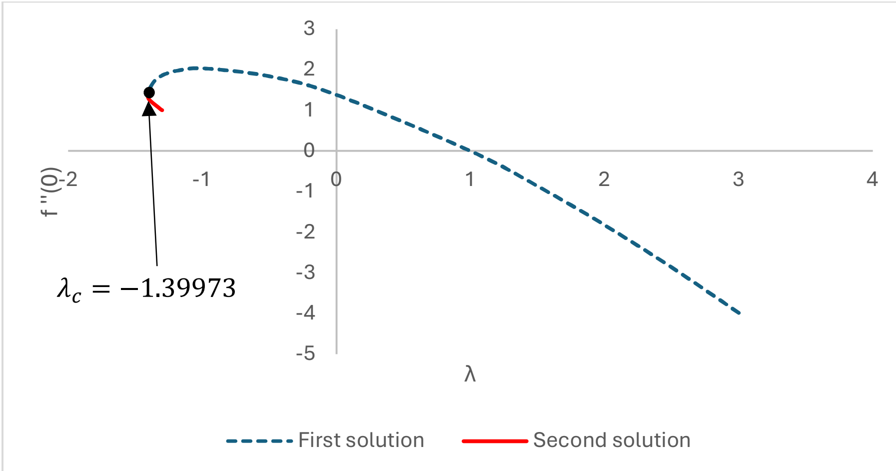


Figure 3.5: The skin friction coefficient, $f''(0)$ versus $\lambda$ when $\beta = 0.1$

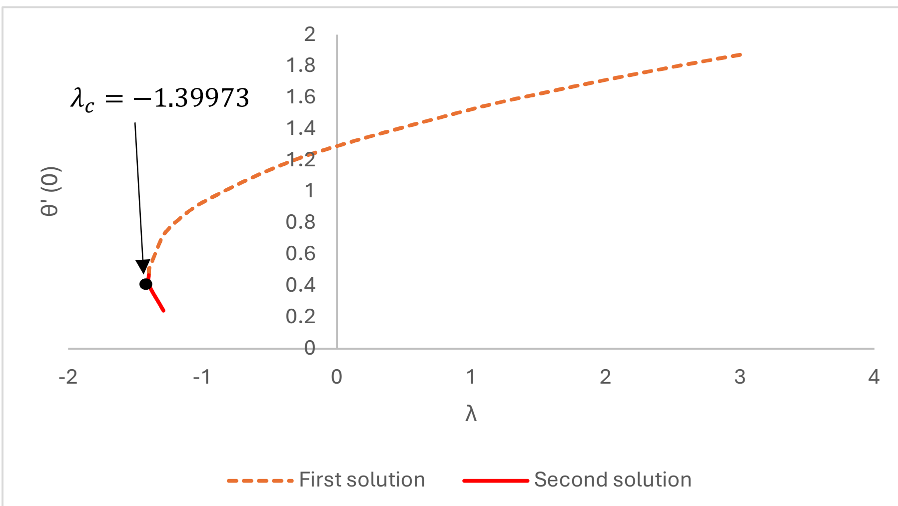


Figure 3.6: The local Nusselt number, $\theta'(0)$ versus $\lambda$ when $\beta = 0.1$

### 3.2 Effect of Hartree pressure gradient, $\beta$ on velocity profile, temperature profile, skin friction coefficient and local Nusselt number.

The velocity profile and temperature profile graphs are shown in Figures 3.7 and Figure 3.8 with $M = 0.2, s = 1$, $\lambda = -1.30$ with $\beta = 0.4, 3.0, 7.5$. The velocity profile in Figure 3.7 illustrates how increases in $\beta$ will result in higher flow velocity profiles close to the wedge's surface. When $\beta$ increases, the thickness of the boundary layer decreases, indicating a greater velocity gradient near the surface. However, in Figure 3.8, the graph shows that the local Nusselt number decreases as the $\beta$ increases. It demonstrates that $\beta$ has a significantly lower impact on temperature profiles than velocity profiles.

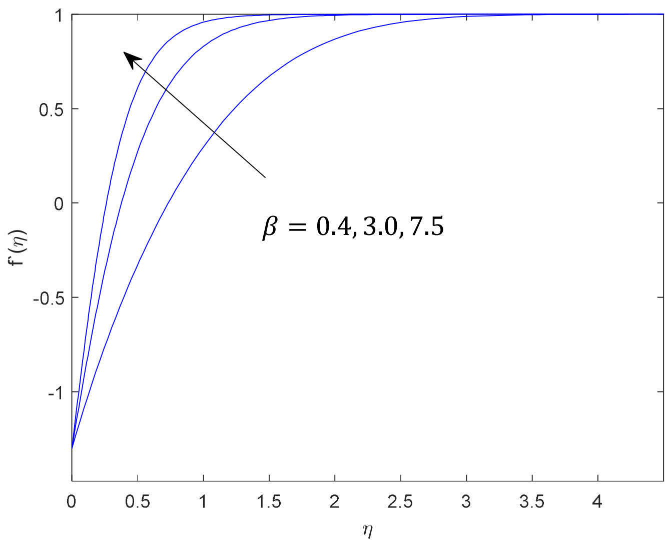


Figure 3.7: The velocity profile for different value of $\beta$, when $\lambda = -1.30$

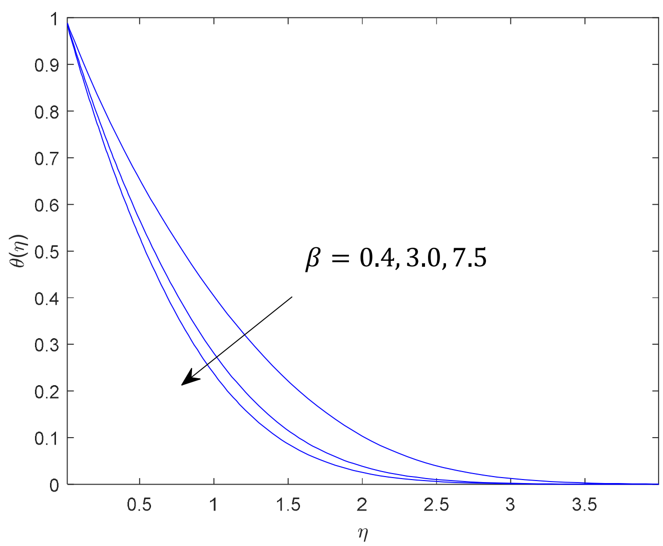


Figure 3.8: The temperature profile for different value of $\beta$, when $\lambda = -1.30$

## 4. Conclusion

This study investigated magnetohydrodynamic (MHD) flow over shrinking wedges. The use of similarity reduction variables transforms the nonlinear equations, known as PDEs, into ODEs. The equations are then numerically resolved using MATLAB's integrated BVP4C function. In addition to the effects of local Nusselt number and skin friction, temperature and velocity profiles impacted by different parameter values especially in shrinking parameter, $\lambda$ and Hartree pressure gradient, $\beta$ were observed. The results were also compared to earlier research to determine the validity of BVP4C's results. The results are validated through their lose agreement with previous studies.

## 5. Acknowledgments

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