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Shortest Route Optimization for Emergency Cases using Integer Linear Programming

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Abstract

Emergency cases require efficient route planning to minimize response times and ensure timely access to critical care. This study presents an optimization model for determining the shortest route in emergency cases using Integer Linear Programming (ILP). The model aims to efficiently allocate resources such as ambulances or private vehicles by optimizing their routes while considering the road networks. This study considers six cities and six hospitals in Seremban, with key nodes representing cities, hospitals, and major intersections, whiles edges representing the connecting roads. 36 case studies are conducted using real-world road network data, and the solution is obtained using QM for Windows and Excel Solver. Results demonstrate that the distance from each city to Hospital Tuanku Ja'afar which is the government hospital is in the range of 4.8km to 8.2km. However, for alternatives, the results show that the nearest hospital to Seremban is CMH Specialist Hospital. For Oakland and Seremban 2, the closest hospital is Columbia Asia Hospital. Mawar Medical Centre serves as the nearest hospital for Seremban 3 and Taman Rahang. Meanwhile, residents of Ampangan have CMH Specialist Hospital and SALAM Senawang Specialist Hospital as their nearest medical facilities. This study contributes to the development of intelligent decision-support systems for emergency management, enhancing rapid response capabilities and overall public safety.

Keywords*:* Shortest route problem, Integer Linear Programming, QM for Windows, Excel Solver

Introduction

In an emergency, every second matters. To resolve the emergency cases issue, the criteria by which drivers select the routes between their starting point and destination must be specified. In most cases, every driver may wish to minimize the route to the destination. The shortest route problem can simply be defined as finding the most efficient route between two points in a graph where this efficiency is measured by many variables such as distance, time, and cost.

This study deals with the problem of finding shortest routes in traversing some locations in Seremban for emergency cases. The shortest route will directly reduce the emergency vehicle's response time. For example, a quicker response time corresponds with a higher patient survival rate. Therefore, the response time of the rescue vehicle should be optimized by determining the shortest route to the emergency center. Integer Linear Programming (ILP) has emerged as a powerful mathematical tool to address the complexities involved in routing emergency vehicles under various constraints.

Previously, Li et al. [1] developed an integrated emergency vehicle scheduling model tailored for urban emergency rescue scenarios within constrained time frames. This model incorporates real-time traffic data and prioritizes critical cases to ensure rapid response. The study demonstrates that integrating live traffic updates can significantly improve response times. However, the reliance on real-time data introduces computational complexities, which may limit its feasibility in resource-constrained emergency response systems.

Besides that, Wang et al. [2] addressed the routing of emergency vehicles on highways by developing a mixed-integer linear programming model that considers factors such as traffic congestion, incident severity, and vehicle availability. The model aims to minimize total response time, maximize the success rate of emergency responses, and optimize the utilization of emergency vehicles. Tested on real-world datasets, the model demonstrated effectiveness in improving emergency vehicle routing on highways. While the study provides valuable insights, its focus on highway scenarios may limit its applicability to urban environments with more complex road networks and variable traffic patterns.

Similarly, García et al. [3] developed a mixed-integer linear programming model to optimize the relocation of Emergency Medical Vehicles (EMV) in Valencia, Spain. The study aimed to minimize the population not covered within the stipulated maximum response times of 12 and 15 minutes for advanced life support and basic life support units, respectively. By proposing a new distribution of EMV bases, the model reduced the coverage deficit by more than half compared to the existing arrangement. This research underscores the significance of strategic vehicle placement in enhancing emergency response efficiency. However, the study primarily focuses on static relocation strategies and does not account for real-time traffic conditions or dynamic demand fluctuations, which are critical in actual emergency scenarios.

Other than that, Nodoust et al. [4] explored the vehicle routing problem for humanitarian relief distribution under hybrid uncertainty, combining both probabilistic and possibilistic approaches to model demand uncertainty. They developed a robust scenario-based possibilistic-stochastic programming model to distribute first aid relief items in post-disaster scenarios where routes are subject to disruption. Applied to a real earthquake case study, the model demonstrated a higher demand satisfaction level compared to traditional stochastic programming models. This study highlights the importance of considering hybrid uncertainty in emergency logistics. However, the complexity of the model may pose challenges for practical implementation in time-sensitive disaster response situations.

The application of linear programming in optimizing emergency response routes has shown promising results in various contexts. By addressing existing gaps and building upon previous research, future studies can develop more robust and adaptable models that enhance the efficiency and effectiveness of emergency services across diverse scenarios.

Network Problem Definition

This study starts with identifying and defining the nodes and pathways that form the network for the emergency services. The nodes in this study are the selected cities and hospitals located in Seremban.

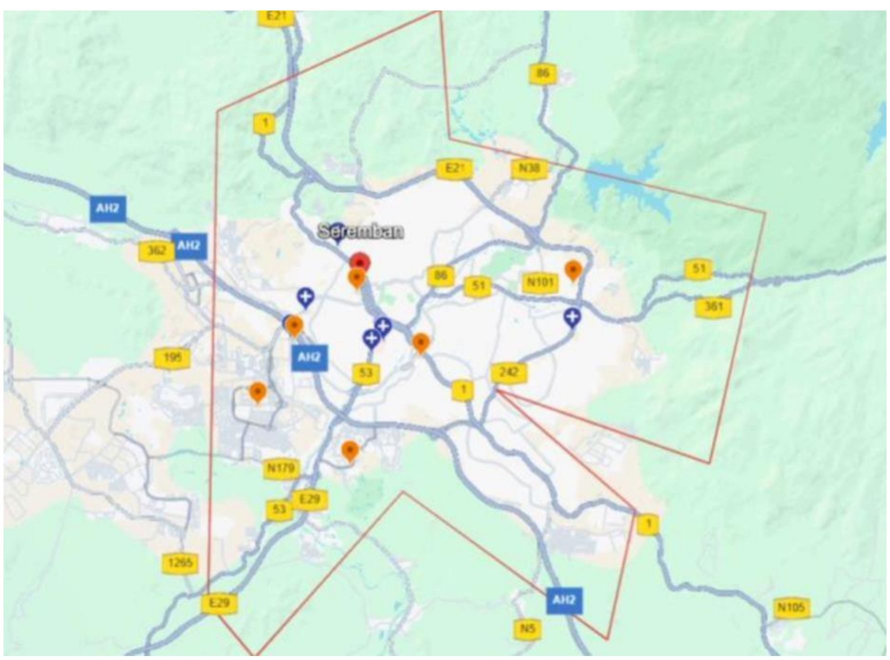

Figure 1: Map of selected area

Figure 1 illustrates the study area in Seremban, highlighting the orange nodes representing cities and blue nodes representing hospitals. These nodes serve as key points in the optimization model for determining the shortest routes for emergency services. This study focuses on six cities and six hospitals as in Table 1 and there are 36 emergency cases are considered.

Table 1: Cities and hospitals

Cities		Hospitals	
C1	Seremban	H1	CMH Specialist Hospital
C2	Oakland	H2	KPJ Seremban Specialist Hospital
C3	Seremban 2	H3	Columbia Asia Hospital
C4	Seremban 3	H4	Mawar Medical Centre
C5	Taman Rahang	H5	Hospital Tuanku Ja'afar
C6	Ampangan	H6	SALAM Senawang Specialist Hospital

Figure 2 shows the road network for Case 1 spans from C1 to H1. The key intersections marked as junctions J1 through J11. The network for Case 1 consists of 13 nodes are constructed with the goal of optimizing the route for transportation.

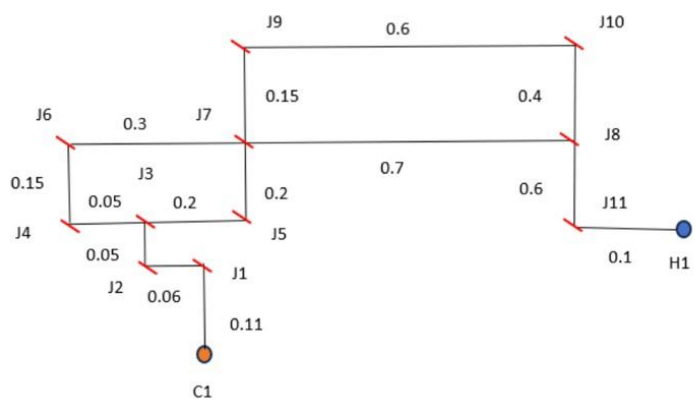

Figure 2: Road network from C1 to H1

ILP Model Formulation for Case 1

To formulate the ILP model for the optimization of the shortest route for emergency services, consider the definition of decision variables, an objective function, and constraints.

Parameters

c_j	the distance (*km*)
$A(i)$	the set of the arc that leave node *i*
$B(i)$	the set of the arc that enter node *i*
x_j	the flow on arc *j*

Decision variables

$$\textbf{Let } x_j = \begin{cases} 1, & \text{if route } j \text{ is chosen} \\ 0, & \text{otherwise} \end{cases}$$

x_1	C1 to J1	x_8	J6 to J7
x_2	J1 to J2	x_9	J7 to J8
x_3	J2 to J3	x_{10}	J7 to J9
x_4	J3 to J4	x_{11}	J9 to J10
x_5	J3 to J5	x_{12}	J10 to J8
x_6	J4 to J6	x_{13}	J8 to J11
x_7	J5 to J7	x_{14}	J11 to H1

Objective function

The objective is to minimize the total travel distance

$$\text{Min} \sum_j c_j x_j \tag{1}$$

$$\text{Min Z} = 0.11x_1 + 0.06x_2 + 0.05x_3 + 0.05x_4 + 0.2x_5 + 0.15x_6 + 0.2x_7 + 0.3x_8 + 0.7x_9 + 0.15\,x_{10} + 0.6x_{11} + 0.4x_{12} + 0.6x_{13} + 0.1x_{14}$$

Constraints

Below are the flow conservation constraints for each node in Case 1 network.

Node 1:

$$\sum_{j\in A(i)} x_j = 1 \tag{2}$$

$$x_1 = 1$$

Node 2 to Node 12:

$$\sum_{j\in A(i)} x_j - \sum_{j\in B(i)} x_j = 0 \tag{3}$$

Node 2: $x_1 = x_2$

Node 3: $x_2 = x_3$

Node 4: $x_3 = x_4 + x_5$

Node 5: $x_4 = x_6$

Node 6: $x_5 = x_7$

Node 7: $x_6 = x_8$

Node 8: $x_7 + x_8 = x_9 + x_{10}$

Node 9: $x_9 + x_{12} = x_{13}$

Node 10: $x_{10} = x_{11}$

Node 11: $x_{11} = x_{12}$

Node 12: $x_{13} = x_{14}$

Node 13:

$$-\sum_{j\in B(i)} x_j = -1 \tag{4}$$

$$x_{14} = -1$$

Results and Discussion

QM for Windows - [Data] Results

C1 to H1 solution

Total distance = 2.02	Start node	End node	Distance	Cumulative Distance
J 1	1	2	.11	.11
J 2	2	3	.06	.17
J 3	3	4	.05	.22
J 5	4	6	.2	.42
J 7	6	8	.2	.62
J 8	8	9	.7	1.32
J 11	9	12	.6	1.92
H 1	12	13	.1	2.02

Figure 3: Result of QM for Windows from C1 to H1

Figure 3 shows the result of QM for Windows from C1 to H1 by identifying the optimal junctions. The identified path includes the following junctions: C1 → J1 → J2 → J3 → J5 → J7 → J8 → J11 → H1 with the total distance of 2.02 km.

To verify the results obtained using QM for Windows, Excel Solver was used. Figure 4 presents the data layout in Excel. The decision variables are in the changing cells A2:A15, while cell D16 serves as the objective cell. Cell D16 contains the summation of the decision variables (0 or 1) multiplied by their corresponding distances. Based on Figure 4, the total distance from C1 to H1 is 2.02 km as the result obtained from QM for Windows.

	A	B	C	D
1	Select	From	To	Distance
2	1	C1	J1	0.11
3	1	J1	J2	0.06
4	1	J2	J3	0.05
5	0	J3	J4	0.05
6	1	J3	J5	0.2
7	0	J4	J6	0.15
8	1	J5	J7	0.2
9	0	J6	J7	0.3
10	1	J7	J8	0.7
11	0	J7	J9	0.15
12	0	J9	J10	0.6
13	0	J10	J8	0.4
14	1	J8	J11	0.6
15	1	J11	H1	0.1
16		TOTAL		2.02

Figure 4: Data layout in Excel

Figure 5 illustrates the constraints layout in Excel for each node from C1 to H1. As for out flow of Node 1, cell G2 represents the formula "=SUMIF($B$2:$B$15,F2,$A$2:$A$12)" while for in flow of Node 1, cell H2 represents the formula "=SUMIF($C$2:$C$15,F2,$A$2:$A$15)". The net flow of a node is determined by subtracting its inflow from its outflow.

The results obtained in Column K in Figure 5 satisfy the conditions and match the solution obtained from QM for Windows. This consistency validates the optimization model and its adherence to the defined constraints, ensuring an accurate and feasible solution.

	F	G	H	I	J	K
1	Node	Outflow	Inflow	Netflow		
2	C1	1	0	1	=	1
3	J1	1	1	0	=	0
4	J2	1	1	0	=	0
5	J3	1	1	0	=	0
6	J4	0	0	0	=	0
7	J5	1	1	0	=	0
8	J6	0	0	0	=	0
9	J7	1	1	0	=	0
10	J8	1	1	0	=	0
11	J9	0	0	0	=	0
12	J10	0	0	0	=	0
13	J11	1	1	0	=	0
14	H1	0	1	-1	=	-1

Figure 5: Constraints layout in Excel

Table 2 summarizes the resulted distances from six selected cities to six selected hospitals. H5, a government hospital, is considered the primary choice for each city due to its role in providing affordable and accessible healthcare. The distance from each city to H5 ranges from 4.8 km to 8.2 km. However, as an alternative, residents of C1 can be transported to H1, which is 2.02 km away. For residents of C2 and C3, the nearest alternative hospital is H3, located 0.45 km and 4.20 km away, respectively. Meanwhile, the alternative hospital for residents of C4 and C5 is H4, with distances of 6.20 km and 4.10 km, respectively. For residents of C6, two private hospital options are available: H1 and H6, with a minimum distance of 7.2 km.

Table 2: Shortest distance for 36 emergency cases

From	To	Shortest distance (*km*)	From	To	Shortest distance (*km*)
C1	**H1**	**2.02**	C4	H1	10.20
	H2	3.60		H2	7.90
	H3	4.80		H3	7.70
	H4	5.85		**H4**	**6.20**
	H5	6.55		H5	6.80
	H6	13.35		H6	10.70
C2	H1	8.40	C5	H1	5.20
	H2	5.10		H2	5.50
	H3	**0.45**		H3	6.10
	H4	4.80		**H4**	**4.10**
	H5	5.40		H5	4.80
	H6	12.8		H6	9.30
C3	H1	8.30	C6	**H1**	**7.20**
	H2	5.10		H2	8.80
	H3	**4.20**		H3	9.90
	H4	6.20		H4	7.50
	H5	7.30		H5	8.20
	H6	13.80		**H6**	**7.20**

Conclusion

In conclusion, this study successfully demonstrated the effectiveness of Integer Linear Programming in optimizing emergency service routes within six cities and six hospitals in Seremban. The developed model minimized travel distances, was validated through multiple tools (QM for windows and Excel Solver), and proved reliable in enhancing emergency response efficiency. These findings highlight the practical value of mathematical modeling in improving healthcare logistics and urban planning, while also providing a strong basis for future studies on emergency route optimization.

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