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Torsional Springback Analysis in Thin Tubes with Non-linear Work Hardening

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ABSTRACT

A theoretical analysis of the springback of thin tubular sections of non linear work-hardening materials under torsional loading has been carried out. The non-linear behavior of the material is approximated by using Modified Ludwik type stress-strain relation. The theoretical analysis is supported by experimental results for different tubular section viz. square, triangular and rectangular sections of different thicknesses. Finally analytical generalized expressions relating angle of twist to twisting moment and residual/springback angle of twist per unit length for thin tubular bars under plastic torsion are obtained in non-dimensionalized form. A comparison between the results obtained for thin tubes on non-linear and linear work-hardening material loaded under torsion is also made.

Keywords: *Metal forming, Springback, Torsional springback, Thin tubes, Non-linear work hardening materials*

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Nomenclature

$u, v,$ and w	Small displacements in x, y, and z direction, respectively
θ	Angle of twist per unit length of the tube
θ_e	Elastic angle of twist per unit length
θ_R	Residual angle of twist per unit length
θ_S	Springback angle of twist per unit length
$\bar{\theta}$	Non-dimensionalized angle of twist per unit length
$\bar{\theta}_R$	Non-dimensionalized residual angle of twist
$\bar{\theta}_S$	Non-dimensionalized springback angle of twist
γ	Shear strain
τ_{xz}, τ_{yz}	Shear stress
G	Modulus of rigidity
$\emptyset_e$	Elastic stress function
∇	Gradient
T	Twisting moment (torque)
T_e	Elastic torque
$\bar{T}$	Non-dimensionalized torque
ξ	Deflection of membrane
σ	Yield strength in tension
β	Numerical factor; 1/2 for Tresca and $1/\sqrt{3}$ for Mises yield criterion.
$\emptyset_P$	Plastic stress function
A	Mean of the areas enclosed by the outer and inner boundaries of the cross-section of the tube
S	Length of the center line of ring section of the tube
t	Thickness of the tube
τ_o	Yield shear stress
α	Plastic modulus of rigidity
ϵ_x, ϵ_y	Longitudinal strain
μ	Poisson's ratio
I_α	Refers to the annular area over which $\iint \phi_e$ is taken
I_1	Area enclosed by inner boundary
$\emptyset_{e_1}$	Constant value of stress functions along the internal boundary
n	Non-linear work hardening index

Introduction

Thin walled tubular components produced by different sheet metal forming processes have been widely used in automotive industries for different applications. Tubes with circular, square, or rectangular cross-sections are part of the assemblies. One of the major technical issues associated with these tubes is elastic recovery of material (springback) after completion of forming process.

Springback is one of the principal physical characteristics in the metal forming processes. It can be defined as natural tendency of material to regain its original shape after the removal of externally applied forming loads. At the end of metal forming operations, the elastic deformation disappears on the removal of the applied load due to reduction in stress and the elastic strain energy is released. While designing the die-set, springback factor should be taken into consideration so that the shape after springback is such that it avoids mismatch while assembling different formed sections. Based on the part geometry and deformation area, different types of springback in sheet metal exist: bending, membrane, twisting and combined bending and membrane [1]. The twisting or torsional type of springback is the measure of elastic recovery of angle of twist on the removal of applied torque after twisting the section beyond elastic limit. Uneven elastic recovery in different directions is the main cause for this type.

An accurate analysis of springback has been made in the past on sheet bending operations through experiments [2] - [16] and simulation [17] - [24]. In all these works, issues related to accurate prediction and effective compensation of springback for different type geometries are discussed. In recent years, much attention has been placed on springback of tubes [25] - [29]. Most of the researchers use simplified models and different stress-strain relationship in finding out the amount of springback in tube bending operations. Torsional springback of bars of different cross-sections has been analyzed by Dwivedi et al. [30] - [35]. Springback of narrow rectangular strips of linear work-hardening materials under torsional loading [30] - [31] and general cross-sections with the torsional springback for work-hardening materials were estimated [32] - [34] by using Ramberg-Osgood stress-strain relation and deformation theory of plasticity. A numerical scheme based on finite difference approximation was used. The elastic-plastic boundary in the bars of square section and L-section were also determined in addition to springback. Theoretical results were verified by experiments on mild steel bars. In a study [35], springback analysis of narrow rectangular bar in torsion for non-linear work-hardening materials was considered. Non-linear behavior of the material was approximated by assuming Modified Ludwick type stress-strain relationship. The result of bi-linear case [30] was compared with by considering non-linear behavior of the material of tubular bars. It was found that experimental values had excellent match for materials having non-linear behavior.

It is always desirable to develop analytical expressions which can readily be used by the designers. From the literature it appears that no attempt has been made to analyze the springback in torsion of thin tubular sections. Therefore, the present paper is concerned with a theoretical analysis of springback in thin tubular section of non-linear work-hardening materials under torsional loading. Work hardening behavior of the material is approximated by assuming Modified Ludwik [36] type of stress-strain relation. In this analysis, analytical expression connecting the angle of twist and residual angle of twist to the given twisting moment applied to the tubular bars have been derived in non-dimensional form.

From the derived relationship, the angle of twist provided to a tubular bar of any material and cross-section can be calculated to obtain the twisting moment and residual angle of twist if its non-linearity index is known.

Theoritical Framework

A prismatic bar undergoing torsion and elastic deformation is considered. Let u, v and w be the small displacements of the point (x, y, z) relative to its initial position, in the x, y and z directions respectively. At a section, where z is constant, the cross-section rotates about the z-axis and so [37], [38].

$$u = -yz\theta, \qquad v = xz\theta, \qquad \text{and} \qquad w = \theta f(x, y) \tag{1}$$

where θ is the angle of twist per unit length of the bar.

For elastic states of stress considered,

$$\frac{\partial \tau_{xz}}{\partial y} - \frac{\partial \tau_{yz}}{\partial x} = 2G\theta = \text{constant} \tag{2}$$

If a stress function $\emptyset_e$ is taken such that

$$\tau_{xz} = \frac{\partial \emptyset_e}{\partial y} \text{ and } \tau_{yz} = -\frac{\partial \emptyset_e}{\partial x} \tag{3}$$

the equilibrium equation

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} = 0 \tag{4}$$

is automatically satisfied. Hence from equations (2) and (3)

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\emptyset_e = \nabla^2 \emptyset_e = -2G\theta \tag{5}$$

Along a boundary curve $\tau_{yz}/\tau_{xz} = dy/dx$, the boundary is free from any stress, so

$$-\tau_{yz}dx + \tau_{xz}dy = \frac{\partial \emptyset_e}{\partial x}dx + \frac{\partial \emptyset_e}{\partial y}dy = d\emptyset_e = 0 \tag{6}$$

and along a boundary, the twisting moment T is given by

$$T = \iint \left(\tau_{yz}x - \tau_{xz}y\right) dx\,dy = 2 \iint \emptyset_e\, dx\,dy \tag{7}$$

According to Prandtl's membrane or soap film analogy [39] & [40], a thin membrane clamped around a bounding curve is considered identical to that of the cross-section of the twisted bar loaded by a constant lateral pressure. It can be shown that the deflection ξ of such a membrane satisfies a differential equation of the form

$$\nabla^2\xi = \text{a constant} \tag{8}$$

Along the contour line of the surface, ξ is constant, that is equivalent to $\emptyset_e$ = constant. But along a contour line $\tau_{yz}/\tau_{xz} = dy/ds$, which means that the resulting shear stress direction is along the tangent to the contour line and the magnitude of the shear stress is proportional to the greatest slope of the surface at that point.

For a prismatic bar with hollow section [38], twisting moment

$$T = 2[\emptyset_{e1}\, I_1 + I_\alpha] \tag{9}$$

Where, I_α refers to the annular area over which $\iint \emptyset_e \, dx\, dy$ is taken and I_1 is the area enclosed by the internal boundary of the section, $\emptyset_{e1}$ being constant value of stress function along the internal boundary.

The membrane analogy may be applied to hollow sections with slight modification. The first term of the right hand side of equation (9) clearly represents a cylindrical prism formed beneath the 'hollow' and second term is the volume directly under the film. If a light rigid plate having the shape of the inner boundary is constrained to move vertically by any amount, the soap film between the plate and the outer boundary is stretched and the plate finds its own height due to air-pressure beneath. However, in case of a thin tube the variation of the slope of the membrane is negligible along the thickness and the slope may be taken as constant.

In case of plastic deformation [39],

$$\tau_{xz}^2 + \tau_{yz}^2 = \left(\frac{\alpha}{\beta}\right)^2 \tag{10}$$

Where σ may be taken as current yield strength in tension and β is a factor equal to $1/2$ for the Tresca and $1/\sqrt{3}$ for the Mises yield criterion. Equation (10) with the help of equation (3) may be written as

$$\left.\begin{array}{c} \nabla\emptyset_p + \nabla\emptyset_p = \left(\frac{\alpha}{\beta}\right) = [\tau(\gamma)]^2 \\ \text{or} \\ |\nabla\emptyset_p| = \tau(\gamma) \end{array}\right\} \tag{11}$$

Where $\emptyset_p$ is the plastic stress function, $\tau(\gamma)$ is the current yield shear strength expressed as a function of shear strain γ. For a perfectly-plastic material,

$$|\nabla\emptyset_p| = \tau = \text{constant} \tag{12}$$

From equation (12) it is clear that if the material is elastic-perfectly plastic then $\emptyset_p$ forms a surface of constant slope. So to determine the limiting torque in such a case, Nadai's Sand heap analogy [39] & [40] may be adopted.

However, when the material is strain hardened, such a procedure is not feasible. But, in such cases the roof representing the plastic state of stress will be an ever-changing one with the slope at a point increasing with increasing θ for a particular value of θ. The roof and the membrane representing the plastic region will be touching each other, i.e., they will have the same slope at a point undergoing plastic deformation.

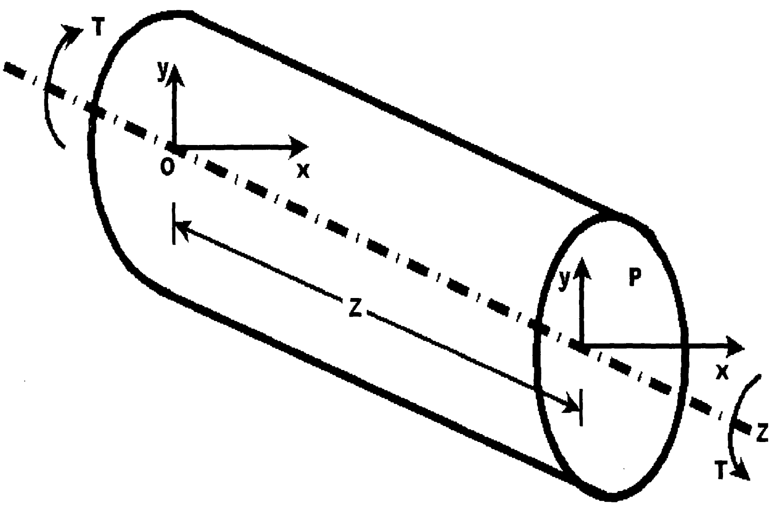

Figure 1: Geometry of the Problem

Consider a hollow tube of arbitrary section having constant thickness t under torsion (Figure 1). The tube dimensions are such that there is no chance of buckling. Let it be assumed that the shear stress-strain curve for the material of the bar is non-linear as shown in Figure 2 in which

$$\tau = \begin{cases} G\gamma & \tau \le \tau_0 \text{ or } \gamma \le (\tau_0/G) = \gamma_0 \\ \tau_0\left(\frac{G\gamma}{\tau_0}\right)^n & \tau \ge \tau_0 \text{ or } \gamma \ge \gamma_0 \end{cases} \tag{13}$$

Where τ_0 and γ_0 are the yield stress and strain respectively, and 'n' is Non-linearity index which is generally less than 0.5. Elastic analysis of the tube under torsion using soap film analogy [37] & [38] gives,

$$|\tau| = \frac{T}{2At} \tag{14}$$

$$\theta = \frac{TS}{4A^2Gt} \tag{15}$$

also,

$$|\tau| = \frac{2A}{S}G\theta \tag{16}$$

Where A is the area enclosed by the outer and inner boundaries of the cross-section of the tube and S is the length of the center line of the ring concentration section of the tube. The effect of re-entrant corner, if any, produces stress concentration but its influence in torque deformation characteristic is negligible.

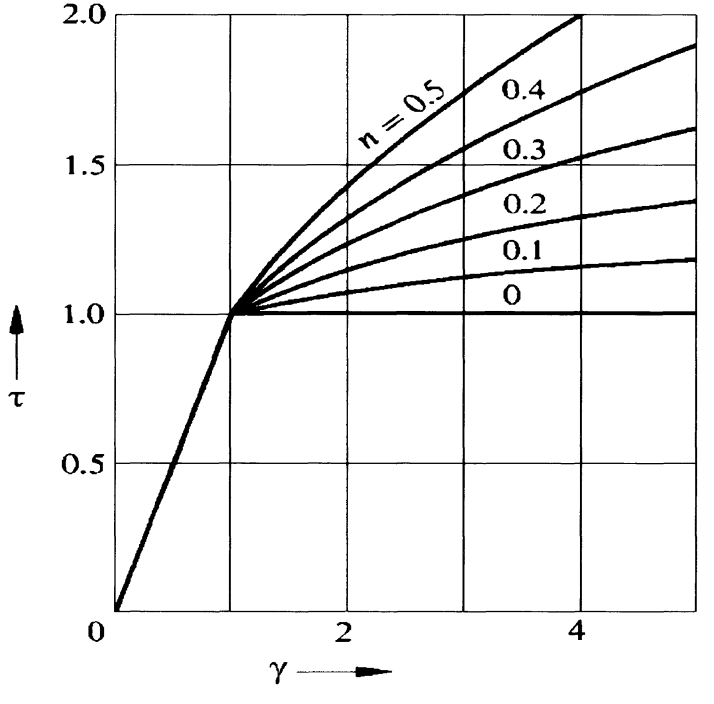

Figure 2: Modified Ludwik stress strain curve [36]

From equation (13) the shape of shear stress-strain curves $d\tau/d\gamma$ (modulus in plastic range) can be obtained. It is known that,

$$\tau = \tau_0 \left(\frac{G\gamma}{\tau_0}\right)^n \tag{17}$$

so,

$$\frac{d\tau}{d\gamma} = \tau_0 n \left(\frac{G}{\tau_0}\right)^n \gamma^{n-1}$$

For a tube undergoing plastic deformation, as the shear stress is constant over the cross-section, from equation (16)

$$\tau = \frac{2A}{S} G\theta \tag{18}$$

so,

$$d\tau = \frac{2A}{S}\left(\frac{d\tau}{d\gamma}\right) d\theta$$

Substituting the values of $d\tau/d\gamma$ from equation (17) into equation (18), we obtain

$$d\tau = \frac{2A}{S}\left[\tau_0 n \left(\frac{G}{\tau_0}\right)^n \gamma^{n-1}\right] d\theta$$

$$d\tau = \frac{2A}{S}\left[\tau_0 n \left(\frac{G}{\tau_0}\right)^n \left(\frac{\tau}{\tau_0}\right)^{\frac{n-1}{n}} \left(\frac{G}{\tau_0}\right)^{1-n}\right] d\theta$$

$$d\tau = \frac{2A}{S}\left[\tau_0 n \left(\frac{G}{\tau_0}\right) \left(\frac{\tau}{\tau_0}\right)^{\frac{n-1}{n}}\right] d\theta$$

$$d\tau = \frac{2AGn}{S}\left[\left(\frac{\tau}{\tau_0}\right)^{\frac{n-1}{n}}\right] d\theta$$

$$\frac{\tau_0^{\frac{n-1}{n}}}{\tau^{\frac{n-1}{n}}} d\tau = \frac{2AGn}{S} d\theta$$

Integrating above equation, we obtain,

$$n(\tau_0)^{\frac{1-n}{n}} \tau^{\frac{1}{n}} = \frac{2AGn}{S} d\theta + C$$

When $\theta = \theta_0$, $\tau = \tau_0$, gives $C = 0$, so

$$\tau^{\frac{1}{n}} = \tau_0^{\frac{1-n}{n}} \left(\frac{2AG}{S}\right) \theta \tag{19}$$

$$\tau = \left(\frac{1}{\tau_0}\right)^{n-1} \left(\frac{2AG}{S}\right)^n \theta^n$$

from equation (14) and equation (19)

$$T = \frac{2At}{\tau_0^{n-1}}\left(\frac{2AG}{S}\right)^n \theta^n$$

or

$$\theta = \left(\frac{T}{2At}\,\tau_0^{(n-1)}\right)^{\frac{1}{n}}\frac{S}{2AG} \tag{20}$$

When the tube is unloaded from plastic state, the unloading is elastic in nature. Thus, the residual angle of twist per unit length (θ_R) is be given by

$$\theta_R = \theta - \theta_e$$

$$= \left(\frac{T}{2At}\,\tau_0^{(n-1)}\right)^{\frac{1}{n}}\frac{S}{2AG} - \frac{TS}{4A^2Gt} \tag{21}$$

Writing in non-dimensional form,

$$\bar{\theta} = \frac{\theta}{\theta_0} = \frac{\theta}{T_0\,S/4A^2Gt}, \text{ and } \bar{T} = \frac{T}{T_0} \tag{22}$$

From equations (15), (20) & (22), we get

$$\bar{\theta} = \frac{\left\{\frac{T}{2At}\tau_0^{(n-1)}\right\}\frac{S}{2AG}}{\frac{T_0}{4A^2Gt}}$$

$$\Rightarrow \bar{\theta} = (\bar{T})^{(1/n)}$$

or (23)

$$(\bar{T}) = (\bar{\theta})^n$$

Again from equation (21) non-dimensionalized residual angle of twist θ_R can be obtained as,

$$\bar{\theta}_R = \frac{\theta_R}{\theta_0} = \frac{\theta - \theta_e}{\theta_0}$$

$$\bar{\theta}_R = \bar{\theta}\left\{1 - (\bar{\theta})^{n-1}\right\} \tag{24}$$

Spring back percentage in twist (θ_s), can be obtained as

$$\%\,\bar{\theta}_s = \left[1 - \frac{\bar{\theta}_R}{\bar{\theta}}\right] \times 100 = \left[1 - \frac{\bar{\theta} - \bar{\theta}^n}{\bar{\theta}}\right] \times 100$$

$$\%\,\bar{\theta}_s = (\bar{\theta})^{n-1} \times 100 \tag{25}$$

Experiments

The mild steel tubular bars of square and triangular cross-sections (25mm x 25mm) and rectangular cross-section (30mm x 20mm), of different lengths (178-280 mm) and of different wall thickness (2 and 3 mm) are subjected to torsion test on "Avery Torsion Testing Machine" that has accuracy up to 0.1 degree. The angle of twist, torque and residual angle of twist was noted. Experimental points were plotted along with the theoretical curves drawn for different values of work-hardening index. Details are given in Appendix.

Results and Discussion

The amount of springback in terms of angle of twist, depends on

- the point on T - θ curve from where unloading is initiated
- the slope of the elastic unloading line.

Since, the material is strain hardened, the amount of springback will be a function of the angle of twist, the elastic modulus of rigidity and the work hardening index. This has been obtained theoretically in equations (24 & 25) and verified experimentally.

Figures 3, 4 and 5 shows the variation of springback /residual angle of twist in percentage with the variation in angle of twist, respectively, for square, triangular and rectangular tubular section for $n = 1/3$. It is found from the figures that the amount of springback decreases as the angle of twist increases. This is quite expected because initially for smaller values of twist, bulk of deformation is elastic and, as such, recoverable percentage of deformation is high. However, as the angle of twist increases the share of non-recoverable plastic deformation increases and the recoverable elastic deformation (springback) as percentage of total deformation decreases. This is true for all tubular sections. It is also clear from the figures that springback decreases as thickness increases.

It is also clear from the figures that experimental values which are marked in the figures for mild steel bar $n = 1/3$ are well in agreement with the theoretical predictions, confirming the validity of the theoretical analysis. Figure 6 shows the variation of torque $(\overline{T})$ with angle of twist $(\overline{\theta})$ for different values of strain hardening index (n). It is evident from the figure that for particular angle of twist the required torque is more for higher strain-hardening index (n) and it decreases as n decreases.

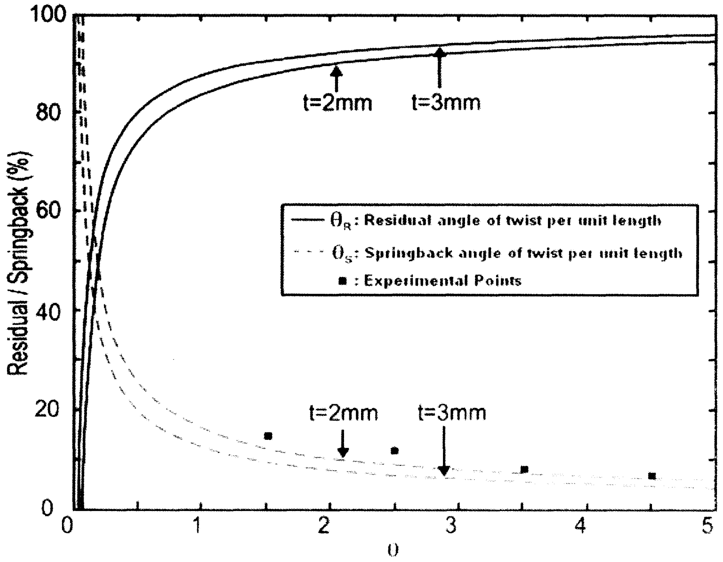

Figure 3: Springback/Residual Angle in % vs. Angle of Twist for Square (25 mm x 25 mm) Boundary

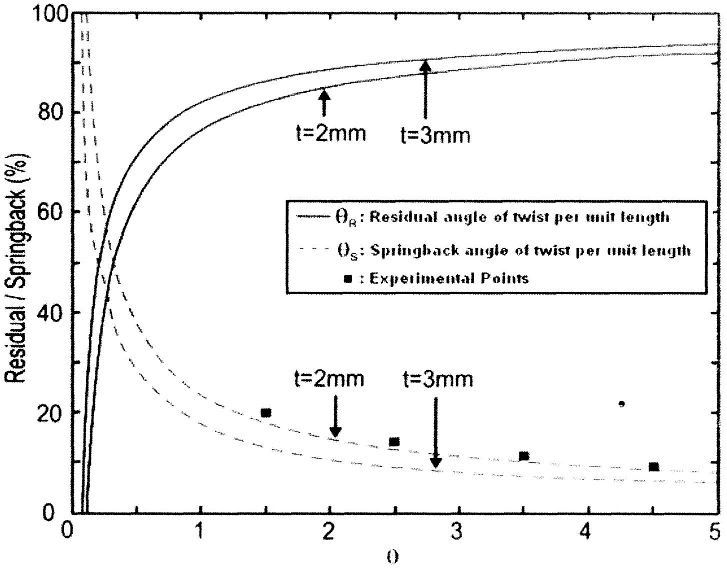

Figure 4: Springback/Residual Angle in % vs. Angle of Twist for Triangular (25 mm x 25 mm) Boundary

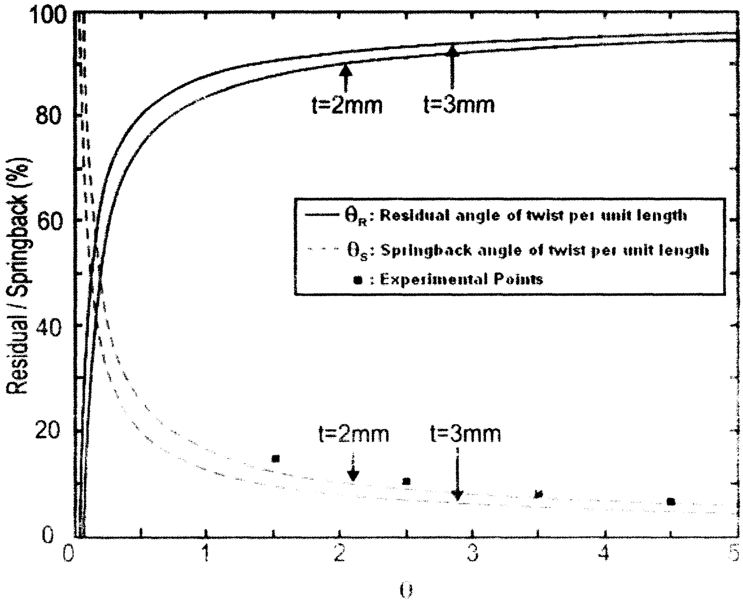

Figure 5: Springback/Residual Angle in % vs. Angle of Twist for Rectangular (30 mm x 20 mm) Boundary

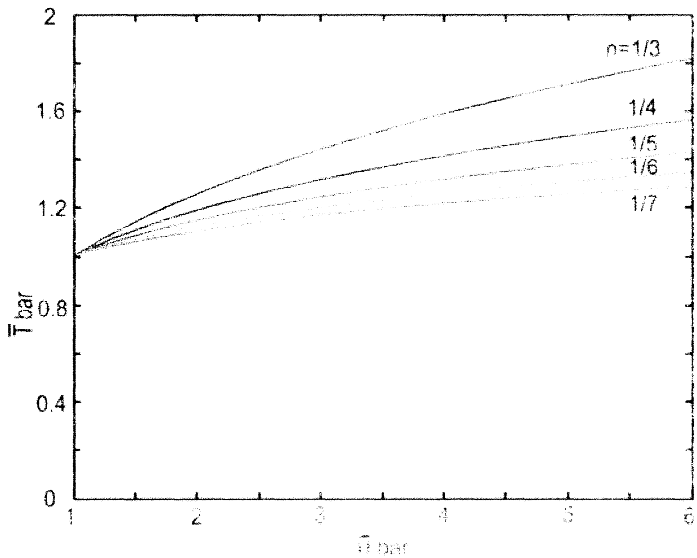

Figure 6: Non-Dimensionalized Torque $(\overline{T})$ in % vs. Non-Dimensionalized Angle of Twist $(\overline{\theta})$ with (n) as Parameter

The nature of variation of springback percentage $(\bar{\theta}_s)$ with angle of twist $(\bar{\theta})$ for different values of strain-hardening index is shown in Figure 7. It is observed from the figure that as n decreases springback percentage decreases.

So, from the analysis, it is possible to ascertain theoretically the value of angle of twist of tubular section bars or the torque applied to the general cross-section bars after knowing the value of strain-hardening index (n). The relationship between $(\bar{\theta})$ & $(\bar{T})$ and $(\bar{\theta})$ & $\bar{\theta}_R/\bar{\theta}_S$ for different values of n have been presented, respectively, in Figures (6 & 7).

The required values can be determined either from equations (23) and (25) or from these figures. Usually, in practical situation for given value of $(\bar{\theta})$, the value of $(\bar{\theta}_R)$ is to be determined if the loading is kinematic and $(\bar{T})$ is to be determined if loading is kinetic.

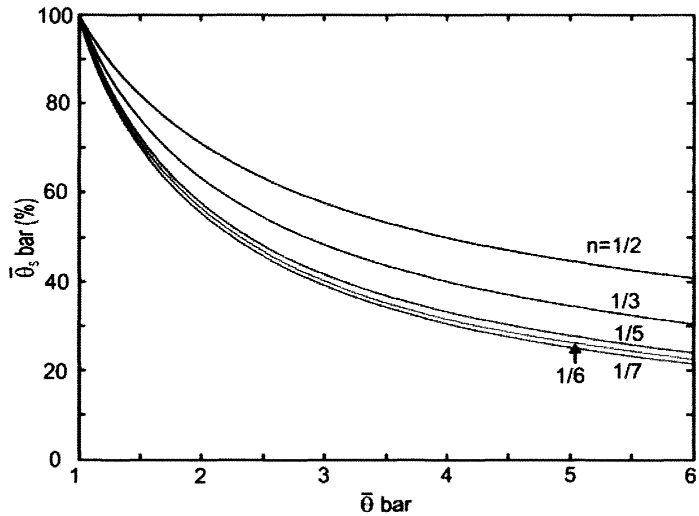

Figure 7: Non-Dimensionalized Springback $(\bar{\theta}_s)$ in % vs. Non-Dimensionalized Angle of Twist $(\bar{\theta})$ with (n) as Parameter

Figure 8 shows the comparison of torque vs. angle of twist curves for the non-linear work-hardening index n = 1/3 and the bi-linear work-hardening approximation $\alpha/G = 0.074$. It is clear from the figure that the experimental values have excellent match when non-linear material behavior is taken though the experimental points fall between these two approximations. Bi-linear behavior approximated (with $\alpha/G = 0.074$) is not found to be satisfactory.

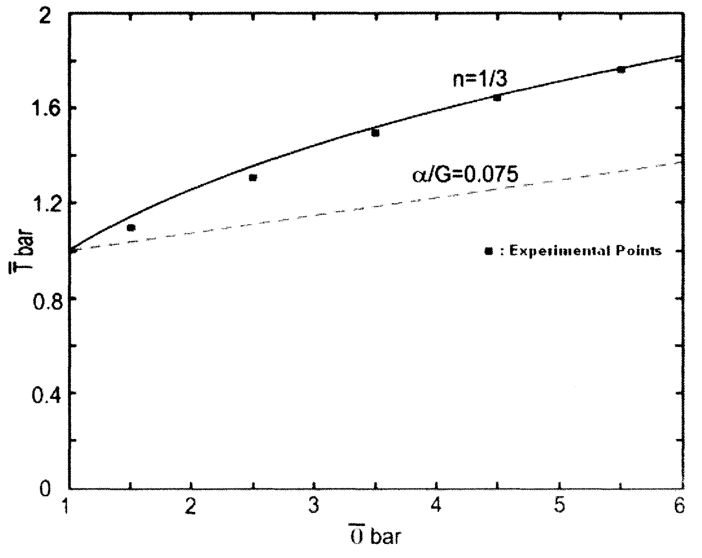

Figure 8: $(\bar{T})$ vs. $(\bar{\theta})$ for Bilinear Work Hardening $\alpha/G = 0.074$ and Non-Linear Work Hardening $n = 1/3$ Case

Conclusions

Based on the results presented above the following conclusions have been drawn:

1. Torque, Residual/Springback angle of twist per unit length are same for all tubular sections for same values of A, S and n.
2. The resulting formulae are in good agreement with experimental data of different section bars for mild steel.
3. An analysis relating the angle of twist to the twisting moment and the angle of twist to the Residual/Springback angle of twist for tubular bars under plastic torsion has been presented in non-dimensionalized form, so that same curves could be used for different material.
4. The experimental points lie closer to the curve drawn for non linear relationship as compare to that for the linear relationship. So it can be inferred that the non-linear theory gives more accurate results than those obtained by assuming linear relationship.

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Appendix

(A) Experimental Procedure

The present work is concerned with experimentally quantifying the torque, springback and residual angle of twist per unit length of mild steel tubular bars of square and triangular cross-sections (25 mm x 25 mm) and rectangular cross-section (30 mm x 20 mm), of different lengths (178-280 mm) and of different wall thickness (2 and 3 mm). Figure A shows the pictorial view of "Avery Torsion Testing Machine". The strain was applied to the specimen by worm and spur gearing was so arranged that the full load might be applied by hand without undue effort. The load was transmitted from the weighing spindle by means of a horizontal torque arm, mounted on antifriction bearings, which transmitted a vertical pull to the indicating unit. The load indicator was of self indicating cam-resistant type and the die carried two sets of graduations with pointer and chart edge to edge to avoid parallax. Capacity could be controlled by means of hand lever and a maximum load pointer mounted on a separate spindle provided a record of breaking loads. For the purpose of testing and gripping of tubes, a holder had been designed and fabricated which could be fitted in four-jaw type self gripping chuck and attached with the face plate holder.

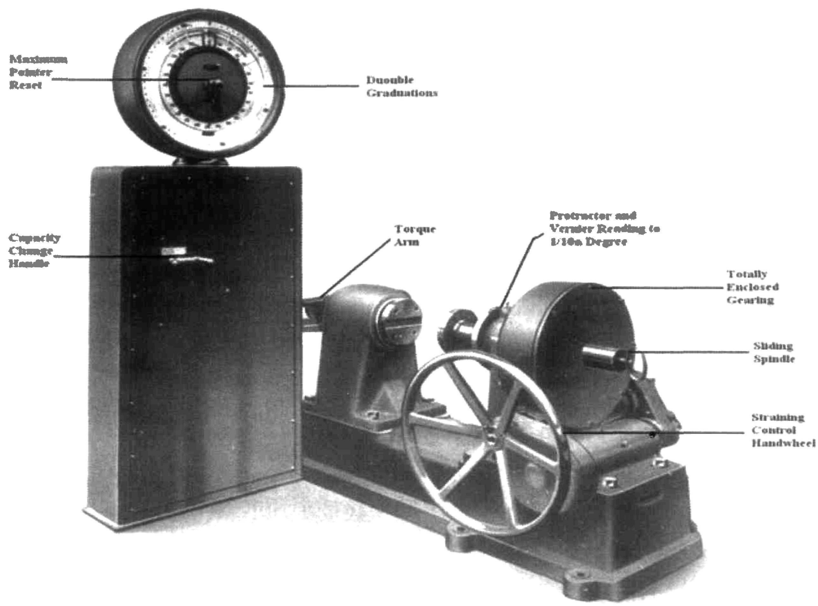

Figure A: Avery Torsion Testing Machine

With the help of hand wheel, a small torque was applied to the specimen that was noted directly from the indicator and corresponding angle of twist was also noted with the help of Vernier and protractor fitted in the machine. The torque was gradually increased up to elasto-plastic regions and corresponding angle of twists were noted. Now the torque was released slowly to zero and the residual angles of twist (θ_R) of the deformed tubes were noted. These deflection readings were again checked with the help of combination set just after setting the torque to zero. The procedure was repeated for number of specimens. Springback percentage and residual angle to twist per unit length in percentage were calculated for each specimen with the help of equations (24) and (25).

(B) Work-hardening index

(I) Linear work-hardening Index:

Linear work-hardening index has been obtained by the procedure mentioned in the paper presented by Dwivedi, et al., [15]. The values of modulus of rigidity (G) and plastic modulus of rigidity (α) of the material of the tube has been found and given in Table 1.

(II) Non-linear Work-hardening Index (n):

To determine the value of strain-hardening index (n) of the strip material, several values of shear stress (τ) and shear strain (γ) in the plastic range are considered from the shear stress-shear strain curve. Further, the value of n is computed as mentioned below.

From Modified Ludwik equation we know that

$$\tau = \tau_0 \left(\frac{G\gamma}{\tau_0}\right)^n \qquad \Longrightarrow \qquad \log \tau/\tau_0 = n \log (G\gamma/\tau_0)$$

therefore,

$$n = \log \tau/\tau_0/\log (G\gamma/\tau_0)$$

After calculating the several values of n, the average value of n has been obtained as given in Table A.

Table A. Mechanical Properties of Tube Material

Modulus of rigidity (G) (N / mm²)	Yield shear stress (τ_0) (N / mm²)	Plastic modulus of rigidity (α) (N / mm²)	α/G	n
8.24×10^4	108	6.1×10^3	0.074	1/3