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Chaos in the Waters: Investigating Non-Linear Dynamics Of Rainfall In River Basin

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Abstract

Floods are a perennial natural disaster in Malaysia, having significant impacts on urban and rural society. It is essential to recognize the random nature of rainfall to enhance flood prediction and water resource planning. This research explores the evidence of chaos in rainfall patterns in the Ampang (urban) and Hulu Langat (rural) river systems through the 0-1 Test and Lyapunov Exponent (LE) techniques. A ten-year record (2013–2023) from ten chosen rainfall stations was used to identify the evidence of non-linear rainfall time series. The imputation of missing values and reduction in noise through the Single Exponential Smoothing Technique improved the quality of the data, and the chaos test was applied to the improved data. The findings affirm the evidence of chaos in rainfall in urban and rural river catchments, displaying sensitivity to the initial condition and non-linear. Against anticipation, the magnitude of chaos in the urban and rural catchments remained close to each other, indicating the influence of urbanization and land use alteration did not impact the chaos in rainfall. The findings suggest the limitation in conventional hydrological models since they are linear in nature and fail to capture the unpredictability of extreme rainfall events. Through the integration of chaos theory in the study of hydrology, the research offers useful contributions to the improvement of flood forecasting models and disaster readiness. The research underscores the necessity to develop adaptive, non-linear flood forecasting models to accommodate the innate chaos in rainfall patterns. The research findings could support the policymaker and urban planner in the formulation of effective flood protection strategies to build flood resilience to extreme weather events.

Keywords*:* chaos theory, river basin hydrology, 0-1 test, lyapunov exponent, non-linear dynamics

Introduction

Chaos theory is a mathematical theory defining dynamical behaviour as highly sensitive to the condition of dynamic systems. The sensitivity also referred to as the butterfly effect, suggests the outcomes in the system are significantly different depending on the tiny variations in the starting point. Figure 1 illustrates some of the characteristics of the chaos system. Even the random-like nature, the deterministic laws dictate the chaos, and the chaos theory asserts the chaos in the short-term horizon could be predictable, while the unpredictability intensifies in the long term [1]. The chaos theory in meteorology started when Edward Lorenz discovered variations in the conditions in the air resulting in drastically different patterns in the climate, showcasing the impossibility of meteorology's long-term forecasting [2]. The theory has also found applications in physics, economics, biology, and environmental science, among others, in the use in the research in intricate, structured, and random-like patterns in the systems [3]. The chaos theory in the research in the area of hydrology assists in the research of rainfall patterns, and the patterns are required in the determination of flood risks and water resource planning [4].

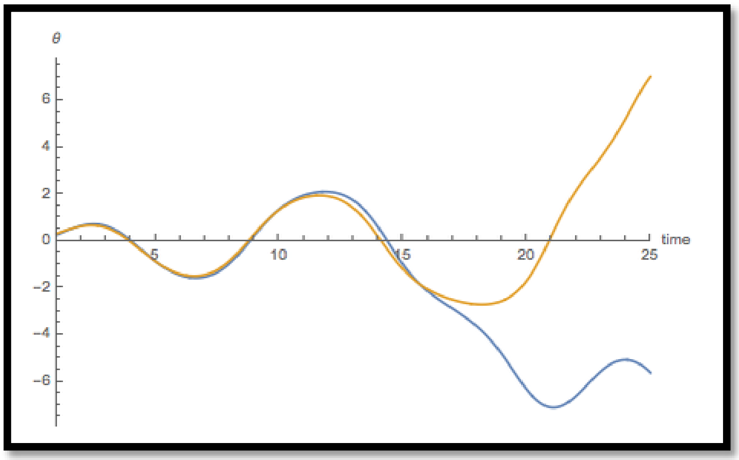

Figure 1: Pendulum that has chaotic system characteristics which are sensitivity to the initial conditions, chaotic motion difficult to forecast and the motion looks random [5].

The use of chaos theory is applied across numerous scientific fields, giving insightful information to systems that have random yet deterministic behavior. In meteorology, chaos theory has helped improve forecasting models by including the inherent unpredictability of atmospheric phenomena (Lu et al., 2010). Within the field of medicine, chaos theory is applied in analyzing physiological signals like heart rate variability and neurological function for the identification of early stages of disorders (Biswas et al., 2018). In finance, it has been used to model stock market volatility and identify patterns within apparently random price changes, assisting investors in making informed decisions and analyzing risk (Peters, 1996). Engineering disciplines use chaos theory to fine-tune control systems to introduce stability in mechanical and electronic processes (Bernardini & Litak, 2016). Environmental applications encompass the study of seismic events, climatic trends, and water cycle processes to more accurately predict natural catastrophes (Domenico et al., 2013). Chaos theory is used in hydrology to study rainfall variability and streamflow dynamics, helping policymakers and urban planners develop efficient flood control strategies (Sapini et al., 2017). Being able to detect chaos in natural phenomena provides a significant edge in predicting and managing environmental extremes.

The atmospheric and hydrological systems possess an intrinsic level of chaos due to their intricate interactions and sensitivity to initial conditions. Meteorologists utilize numerical models in the prediction of meteorological patterns; however, the predictability of the forecasts is inhibited by the non-linearity of atmospheric dynamics (Ahmed, 2023). The butterfly effect, which is a key concept of chaos theory, demonstrates that minute alterations in the weather patterns can lead to entirely disparate climatic outcomes in the long run (Drew, 2022). For example, a small change in temperature or wind speed will have radical differences in the storm's intensity and path, thus long-term prediction will carry a very high degree of uncertainty (Sapini et al., 2019). River systems and precipitation also exhibit chaotic behavior, in which small differences in atmospheric moisture content will lead to big differences in rainfall and flood occurrence (Sivakumar, 2000). The intrinsic uncertainty involved in flood forecasting models poses enormous challenges since conventional hydrological models are incapable of capturing the non-linear nature of precipitation patterns (Su, 2021). Examining the chaotic dynamics in rainfall patterns can enhance the precision of flood risk analysis, thus allowing concerned authorities to adopt proper disaster management measures (Fauzi et al., 2022). This study investigates the chaotic nature of rainfall patterns to better improve flood prediction models; in Malaysia, incidents of extreme climatic events are quite common.

In Malaysia, flood remains the most devastating climatic disaster that inflicts massive damage to infrastructure, the environment, and human life (Bubeck & Thieken, 2018). The occurrence of flash flood incidents in urban areas such as Ampang, in addition to rural places such as Hulu Langat, highlights the need to expand flood forecasting and management measures (Tella et al., 2023). Flash floods in Kuala Lumpur in January and February 2022 illustrated the unforeseeable nature and magnitude of extreme rainfall events (Tella et al., 2023). Conventional models based on the conventional theory often fail to forecast flood events accurately because rainfall patterns and rivers are inherently non-linear (Sapini et al., 2017). The ineffectiveness of forecasting tools in turn prevents the authorities from undertaking timely disaster responses, and the risks to the people in flood-affected zones increase (Fauzi et al., 2022). This research endeavours to solve this challenge by applying chaos theory to model rainfall dynamics and identifying chaotic patterns in rainfall data from urban and rural river catchments. Through the detection of chaos, this research aims to enhance flood prediction models and support effective water resource management in Malaysia.

This research consists of three objectives, to the effect, detecting the existence of chaos in the river catchments through the 0-1 Test and Lyapunov Exponent, reducing rainfall data noise through the Single Exponential Smoothing Technique, and the comparison between the urban and rural catchment rainfall patterns' behaviours. The novelty in this research lies in the ability to enhance the flood forecasting function, and through the same, inform decision-making by the authorities in flood prevention and control. The realization of the patterns in rainfall chaos helps in the improvement of the models in the hydrological discipline, enhancing the warning mechanisms, and aiding in urban planning to curb flood risks (Sivakumar, 2000). This research also advances the academic discipline by bringing forth evidence in the case of Malaysian rainfall's chaotic nature, supplementing the current research in the area of hydrology. Through the integration between the discipline in chaos theory and the discipline in the study of water in the hydrosphere, the research aims to provide useful contributions to theoretical and applied research in water resource management.

The research methodology in this research comprises the techniques in the acquisition, preprocessing, and chaos detection. Rainfall data are from ten stations, half in Ampang (urban) and half in Hulu Langat (rural) and cover the period 2013-2023. The data are pre-processed, the missing values are imputed by the use of the mean, and the noise in the data is minimized by the use of the Single Exponential Smoothing Technique to increase the precision in detecting chaos. The chaos is detected by the use of the 0-1 Test, to determine if the system follows regular or chaotic patterns, and the Lyapunov Exponent, to test the sensitivity of the system to the initial condition. The above techniques provide a quantitative measurement of the chaos in the rainfall pattern, and from the measurement, the urban and rural patterns are compared. Through the use of the above techniques, the current research aims to provide a reliable methodology in detecting chaos in rainfall, and in the long run, to increase the precision in flood prediction and water management.

This paper consists of five sections, each having a detailed explanation of the research. The introduction provides an overview of chaos theory, its applications, the problem, the research objectives, and the methodology. The research methodology provides an explanation of the data collection, the preprocessing techniques, and the chaos-detecting techniques used in the research. The result and the discussion provide the findings from the 0-1 Test and Lyapunov Exponent, and the comparison between the urban and rural rivers in the presence of chaos. The conclusion provides the findings, the implications of flood prediction and water management, and the recommendations for future research.

Research Methodology

The research methodology in the current research endeavours to systematically assess the incidence of chaos in rainfall patterns by employing a structured methodology in the acquisition, preprocessing, and processing of the data. This section elaborates the techniques employed to derive rainfall data, preprocess the same to maintain precision, and employ chaos detection algorithms to determine the incidence of the non-linear dynamics in the river systems.

Data Acquisition

This study utilizes rainfall time series data from ten stations selected from the years 2013 to 2023. The study area consists of five stations in the urban area, Ampang, and five stations in the rural area, Hulu Langat. The stations are selected because they are easily flooded and are, consequently, good places to research the random character of rainfall patterns. The ten stations under selection are shown in the list in Table 1.

Table 1: The list and the names of the ten selected stations

Station code	Name of the station
S1	Pusat Penyelidekan at JPS Ampang Selangor
S2	Pemasokan Ampang at Selangor
S3	Bukit Antarabangsa at Selangor
S4	Kg. Berembang at Keramat W. Persekutuan
S5	Gombak Damsite at W, Persekutuan
S6	Stor JPS Hulu Langat at Selangor
S7	Sg. Serai Batu 12 at Hulu Langat
S8	Sg. Raya Bt.9 Hulu Langat at Selangor
S9	Sg. Balak Hulu Langat at Selangor
S10	Balai Polis Batu 14 at Hulu Langat

The data was extracted from the Malaysian Department of Irrigation and Drainage (JPS), which maintains detailed meteorological records across the country. The rainfall measurement each day at each site was recorded and entered into a dataset, forming the basis for further analysis. Since rainfall is highly variable and determined by numerous parameters such as climate variability, land use, and topography, a long-term record was necessary to see patterns and capture chaos (Sapini et al., 2017). The employment of urban and rural sites allows comparative investigation into how the activity of people and the natural world dictates rainfall patterns and flood risks.

Data Preprocessing

Before chaos detection, the data received from the sampling sites underwent preprocessing to enhance accuracy and dependability. Since raw hydrological datasets are prone to having missing values and noise from instrument errors, imputation and reduction techniques in the data were carried out (Bhandari, 2022).

Missing data, also known as missing values, occur in a dataset where no available data is stored for specific variables or participants. A variety of reasons, including the deletion of files, dysfunctional equipment, and incorrect entry, could result in the disappearance of data (Bhandari, 2022). The missing values were addressed through the use of mean imputation, a commonly applied statistical technique that substitutes missing entries with the average of available data points. This method ensures consistency in time series analysis while maintaining data integrity. Eq. 1 is the formula for mean amputation, $\bar{X}$:

$$\bar{X} = \frac{\sum X_i}{n} \tag{1}$$

where $\sum X_i$ is the sum of available data value and n is the number of summed data.

Additionally, noise reduction was also accomplished by applying the Single Exponential Smoothing Technique (Eq. 2), smoothing the variability in the time series by assigning higher weights to the most up-to-date observations and reducing the impact of short-term fluctuations. The smoothing parameter (α) was determined at 0.3, based on the outcomes in other research, in balancing responsiveness and stability in the employment of hydrological data.

$$f_{t+m} = \alpha y_t + (1-\alpha) f_t \tag{2}$$

where f_{t+m} is the Single Exponential Smoothed value in period $t+m$, for $m = 1, 2, 3, \ldots$,.

α is the unknown smoothing parameter to be set from 0 to 1. y_t is the actual value in time, t. f_t is the forecast or smoothed value for time, t. The preprocessed data was then divided into original and smoothed datasets, enabling comparisons between raw and refined data when detecting chaotic patterns.

Chaos Detection Techniques

To analyze the presence of chaos in rainfall patterns, two widely recognized methods were employed: the 0-1 test and the Lyapunov Exponent (LE).

The 0-1 Test provides a good and simple tool to distinguish between regular and chaotic dynamics in deterministic systems. It calculates a statistical value, denoted by K, whose values approaching 1 describe the case of chaos, and whose values approaching 0 describe the case of regularity (Gottwald & Melbourne, 2016). The equation of the 0-1 test is shown in Eq. 3 to Eq. 6.

1) For $c \in (0, \pi)$, the following variable are calculated:

$$p_C(n) = \sum_{j=1}^{n} x(j) cos\,(jc) \tag{3}$$

$p_C(n)$ is test trajectory, x_j is the data point, $\cos(jc)$ a periodic weighting function applied to each data point and c is a constant from 0 to 2π to get the median of K.

2) The mean square displacement M_c will be calculated:

$$M_c = \lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} [p_c(j+n) - p_c(j)]^2 \tag{4}$$

where $[p_c(j+n) - p_c(j)]$ is the difference in trajectory position at times $j+n$ and j.
$[p_c(j+n) - p_c(j)]^2$ is the squared displacement, the spread of the trajectory.
$\lim_{N\to\infty}$ to make sure that the mean square displacement is evaluated for the entire data set.

3) The next step is to estimate the asymptotic growth rate, K_c:

$$K_c = \lim_{n\to\infty} \frac{\log M_c(n)}{\log(n)} \tag{5}$$

$\log M_c(n)$ is the logarithm of MSD, $\log(n)$ is the logarithm of time, n and $\frac{\log M_c(n)}{\log(n)}$ is the growth rate of the MSD over time

4) Repeat step 1, 2, and 3 for numerous values of c, different randomly selected where $c \in (0, \pi)$.

$$K = median\,(K_c) \tag{6}$$

If the value of $K \approx 0$, it indicates normal dynamics, whereas if the value of $K \approx 1$, it signifies chaotic dynamics.

The second technique, Lyapunov Exponent (LE), computes the rate at which small disturbances in the system grow over time. A positive LE value confirms the presence of chaos, while a negative or zero value indicates a stable or periodic system (Fauzi et al., 2022). This method is particularly useful in hydrological studies as it quantifies sensitivity to initial conditions, a key characteristic of chaotic systems (Sapini et al., 2017). The equation for the Lyapunov Exponent (LE) is presented in Eq. 7 to Eq.8 (Fauzi et al., 2022). The value of the

Largest Lyapunov Exponent (LLE) can be determined by computing the mean logarithmic rate of separation,

$$L_k = \frac{1}{2(N-k-m+1)} \sum_{n=m}^{N-k} log \sum_{j=0}^{m-1} (x_{l-j+k} - x_{n-j+k})^2 \tag{7}$$

where m is a value that represent the value of the embedding dimension, N is the total number of data available, k is the number of intervals, x_l is the value of the signal at time, $l$and L_k is the local divergence at time step, k.

To prevent inaccuracies in the results, the extremely small and large values of k should be excluded. This is because small values of k may contain noise and round-off errors in the separation calculations, as they are not aligned along the direction of greatest expansion. In the meantime, the high values of k, generate a separation that eventually gets close to the attractor's size. The Lyapunov Exponent is defined as Eq.8:

$$\lambda_1 = \frac{dL_k}{dk} \tag{8}$$

If the value λ_1 is greater than zero, the system exhibits chaos and instability. The test was conducted on both the original and smoothed rainfall datasets to assess the impact of noise reduction on chaos detection. By utilizing these two techniques, the study aims to provide a comprehensive evaluation of chaotic rainfall patterns and their implications for flood forecasting.

Results and Discussions

This chapter presents the results and discussions derived from the study on the analysis of rainfall data through the application of chaos detection algorithms. The results are organized into three broad categories: the performance of data preprocessing, chaos detection using the 0-1 Test and Lyapunov Exponent, and comparative analysis of chaotic dynamics in urban and rural river systems.

Effectiveness of Data Preprocessing

Data preprocessing is essential to ensure the quality and reliability of results from chaos detection methods. Mean imputation was successful in addressing the missing value problems within the dataset, ensuring data integrity and facilitating precise time series analysis. Table 2 shows a sample of rainfall data with the application of mean imputation. The missing value denoted by NaN was imputed with the new mean imputation value of 5.3733.

Table 2: The sample of raw rainfall data for the ten selected stations with calculated mean imputation

Date	Raw data	Imputation data
01/07/2018	6.0	0.4
.	.	.
10/07/2018	**NaN**	5.3733
11/07/2018	**NaN**	5.3733
12/07/2018	**NaN**	5.3733

Also, the application of the Single Exponential Smoothing Technique was successful in removing noise from rainfall data, thus making trends more apparent without altering the integrity of the original dataset. As observed in Table 3, the smoothed dataset was less fluctuating compared to the raw data, yielding a cleaner dataset for chaos detection. Nevertheless, although noise reduction enhances data quality, it also carries with it a risk of diminishing some chaotic nature. The analysis of raw and smoothed outcomes indicates that, while noise reduction results in record stability for rainfall, it does not remove the intrinsic chaotic dynamics of river systems (Sapini et al., 2017). The role of preprocessing in hydrologic research is therefore highlighted, specifically the proper detection of chaos in time series data sets.

Table 3: The sample of original and smoothed data for S1

Date	Original Rainfall	Smoothed Rainfall
01/01/2013	0.0	0.0
02/01/2013	0.0	0.0
03/01/2013	0.0	0.0
04/01/2013	0.5	0.15
05/01/2013	15.5	4.755
06/01/2013	0.0	3.3285
07/01/2013	0.0	2.3299
08/01/2013	0.0	1.6309
09/01/2013	0.0	1.1416
. . .	. . .	. . .
30/01/2013	1.0	2.5417

Detection of Chaos in Rainfall Data

The existence of chaos in the rainfall time series was verified by the use of the 0-1 Test coupled with the Lyapunov Exponent (LE) test. As evident in Figure 2, the 0-1 Test outcomes revealed that K values for all ten stations were near 1, hence verifying the chaotic nature of both urban and rural river systems. This validates the assertion that rainfall dynamics are non-linear, rendering extreme events such as flash floods difficult to predict (Gottwald & Melbourne, 2016).

Likewise, the Lyapunov Exponent (LE) test also provided additional evidence for the existence of chaotic dynamics because all observation stations gave positive LE values that provide evidence of sensitivity to initial conditions, as illustrated in Figure 3. Such results corroborate the hypothesis that precipitation regimes of river catchments are not in a purely deterministic or periodic form, but rather in the features of chaotic systems. The existence of chaos implies that traditional flood forecasting models, which tend to rely on linear relationships, are inadequate to describe the complexity of rainfall dynamics, and more sophisticated, non-linear hydrological models are required (Sapini et al., 2019).

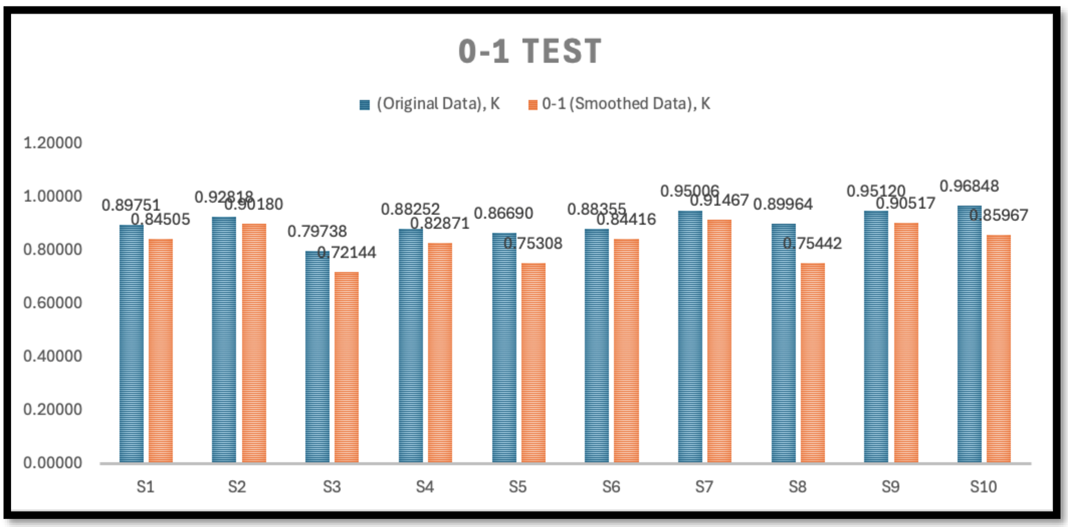

Figure 2. *K* values from 0-1 test for all ten stations

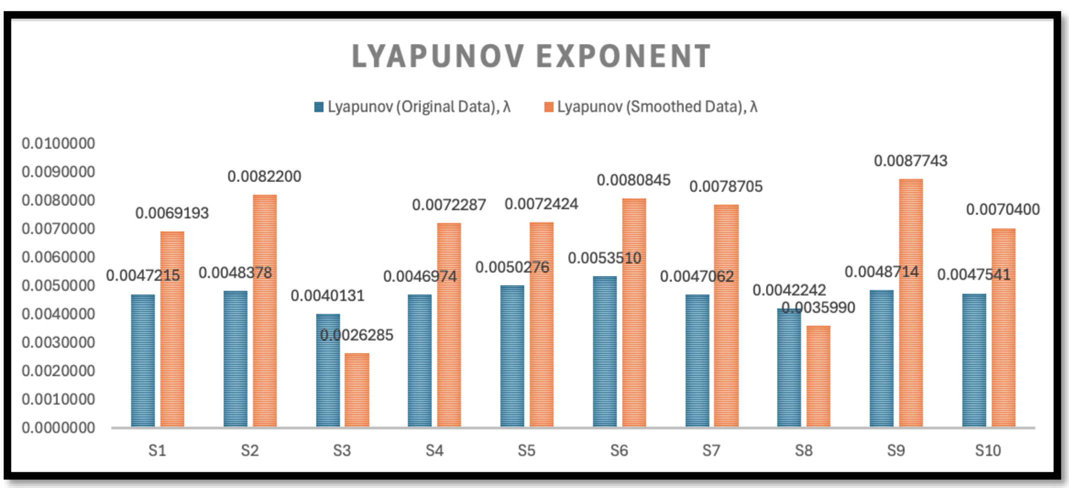

Figure 3. Lyapunov Exponents values for all ten stations

Comparison of Chaos Between Urban and Rural River Systems

This research essentially attempted to test how much chaos was present in rural and urban river basins. Results from 0-1 Test and Lyapunov Exponent (LE) analyses suggest that both urban (Ampang) and rural (Hulu Langat) river systems exhibit chaotic rainfall of comparable magnitude. The K values of the 0-1 Test (Figure 2) and LE values (Figure 3) of all the stations are highly correlated, indicating that the chaotic behavior of precipitation is not affected by the urban or rural character of the environment. The results suggest that the rainfall process is inherently chaotic, independent of infrastructure, anthropogenic modification, or land use.

Earlier conjectures implied that rural locations might exhibit stronger chaotic behavior due to the lack of human impact and inherent hydrological variability. The findings, however, indicate that urban locations also preserve high chaos levels. This is due to the intricate interaction between natural precipitation processes and engineered drainage infrastructure, which can contribute to heightened uncertainty instead of forming stable hydrological regimes (Sapini et al., 2017). Urban infrastructure, including storm drainage systems and flood controls, modifies the hydrodynamics of flow without removing the intrinsic chaotic nature inherent in precipitation. Rural river systems, despite being relatively subject to fewer anthropogenic modifications, still retain the non-linear dynamics owing to the intrinsic complexities in watershed hydrology and effects of climatic factors (Sivakumar, 2000).

The same level of chaos engaged in both settings underscores the intricacy in forecasting floods and managing water resources in varying landscapes. The research refers to the use of chaos theory-based flood forecast models in all settings, as opposed to presuming that urban and rural settings require fundamentally different prediction mechanisms. Given the fact that chaos prevails in both cases, the application of real-time monitoring systems coupled with non-linear prediction methods may enhance the quality of flood risk analysis (Fauzi et al., 2022). All these findings point to the necessity of adaptive responses to flooding according to the intrinsic uncertainty of rainfall features, so that both urban and rural settlements are adequately prepared for such extreme events.

Conclusion and Recommendations

Using the 0-1 Test and Lyapunov Exponent (LE) methods, this study investigates chaos of rainfall patterns in the Ampang and Hulu Langat river basins in urban and rural settings, respectively. Results confirm that rain dynamics in both contexts are chaotic with respect to initial conditions and non-linear. The study revealed that the level of chaos in river systems remains nearly the same in both urban and rural areas. This suggests that while urbanization alters water flow and flooding patterns, it does not substantially change the chaotic nature of rainfall itself. While we were able to get a more accurate data quality based on the application of the Single Exponential Smoothing Technique, the stochastic nature of rainfall continued to be a challenging factor. These findings underscore the limitations of linear assumption hydrological models, adding force to demands for non-linear flood prediction approaches considering chaotic rainfall variability. By incorporating chaos theory into hydrological studies, this article comes with opportune insights for improving flood prediction models and disaster preparedness interventions, particularly in Malaysia's flood-prone zones.

Since rainfall has an inherent uncertainty, flood forecasting models must incorporate non-linear approaches to enhance predictability. That both urban and rural areas are characterized by the same chaotic behavior indicates that land use change and development do not eliminate the intrinsic unpredictability of rain. This justifies the requirement for adaptive, site-specific flood mitigation strategies that combine real-time monitoring and sophisticated forecasting models. Future studies should expand the analysis framework to encompass more river basins in contrasting climatic regimes to substantiate these results even more. Additionally, the conjunction of machine learning techniques, recurrent neural networks (RNN) and long short-term memory (LSTM) models, with chaos-induced approaches can improve predictability by modeling intricate temporal interdependencies in precipitation data effectively. Besides, the inclusion of other chaos detection techniques, like the Brock-Dechert-Scheinkman (BDS) test, can enhance statistical verification and reinforce the soundness of chaos identification in hydrological research.

Practically, policymakers and disaster management institutions need to incorporate chaos-based hydrological models in national flood forecasting systems. The use of real-time monitoring systems that integrate chaos-informed predictive models would have the potential for greatly enhancing early warning mechanisms and disaster response measures. Urban environments need to prioritize sustainable drainage practices to curb the risk of flooding, while rural environments need to ensure that natural water retention systems remain intact to effectively manage riverine flow. Interdisciplinary cooperation between hydrologists, climate scientists, and urban planners will be essential to translate these research outcomes into successful policy interventions. By enhancing our knowledge of unstable rainfall patterns, this study opens the door for more adaptive flood management practices, thereby enabling communities to become more resilient to extreme weather events in a time characterized by rising climate variability.

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