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MERGING LANES: WHERE E-LEARNING DIVERSITY MEETS FUTURE TRENDS

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A COMPARATIVE STUDY OF CLASSICAL AND AESTHETIC BÉZIER CURVES IN ENGINEERING SURFACE DESIGN

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ABSTRACT

Computer Aided Geometric Design (CAGD) is important in engineering and design for creating smooth and complex surfaces used in areas like automotive, aerospace, and computer graphics. Classical Bézier curves and surfaces are commonly used because they are easy to control using control points and have good geometric properties. However, one main limitation is that any change to the control points affects the entire shape, making it difficult to adjust specific parts of a surface. To solve this issue, aesthetic Bézier methods introduce shape parameters that allow better control of the curve and surface, improving smoothness and visual appearance. While many studies focus on curves, there is still limited research on how these methods perform for full surface construction. Therefore, this study compares classical Bézier and aesthetic Bézier approaches in engineering surface design by analysing their mathematical properties, flexibility, and effects on surface smoothness and visual quality using computational modelling. The results aim to help engineers and designers choose the most suitable method for producing high-quality and visually pleasing surfaces.

Keywords: *Computer Aided Geometric Design (CAGD), Classical Bézier curves, Aesthetic Bézier Methods, Surface Modelling, Shape Control and Smoothness*

Introduction

Parametric curves form a fundamental concept in Computer Aided Geometric Design (CAGD), where curves are expressed as mathematical functions of one or more parameters. By defining coordinates independently as functions of a parameter, typically (t), parametric representations provide the flexibility to model complex geometric shapes such as circles, splines, and free-form curves. This capability makes parametric curves highly suitable for a wide range of applications in engineering, computer graphics, and industrial design. Common parametric curve representations include Bézier curves, Non-Uniform Rational B-Splines (NURBS), and rational Bézier curves, all of which play a crucial role in modern CAD/CAM systems. (Pérez et al., 2018)

From parametric curves, more complex geometric entities such as parametric surfaces can be constructed by introducing an additional parameter. These surfaces can be generated through transformations of curves, including translation and rotation, allowing for the modelling of continuous

and smooth shapes. Among these representations, the Bézier curve stands out as a widely used parametric polynomial curve defined by a set of control points. Its ability to provide intuitive geometric control without necessarily passing through all control points makes it particularly effective for designing smooth and visually appealing shapes in engineering and design applications. (Yang et al., 2015)

Over time, various extensions and alternatives to the classical Bézier curve have been developed to address its limitations and enhance its flexibility. Early developments such as the Timmer parametric cubic and Hermite curves introduced different forms of control, including tangent-based shaping and piecewise smoothness. Rational Bézier curves further extended the classical formulation by incorporating weights, enabling the exact representation of conic sections and improving shape control. These developments highlight the continuous effort to refine curve modelling techniques to meet the increasing demands of precision and adaptability in engineering design. (Gobithasan & Ali, 2004)

More advanced formulations, including cubic trigonometric and hyperbolic Bézier curves, introduce shape parameters that allow local adjustment of curves without altering control points. These non-polynomial approaches provide enhanced flexibility and are capable of representing more complex geometric behaviours, such as oscillatory or exponential shapes. Additionally, hybrid and fractional Bézier curves combine polynomial, trigonometric, and fractional basis functions to achieve improved curvature control while preserving essential geometric properties. Although these models offer greater design freedom, they also introduce higher mathematical and computational complexity. (Yan & Liang, 2011)

In engineering construction, Bézier curves and surfaces are extensively used for modelling smooth free-form geometries, such as automotive bodies, aircraft components, and mechanical parts. Their intuitive control through control points enables designers to achieve both functional performance and aesthetic quality. Furthermore, Bézier-based representations are essential in manufacturing processes, including CNC machining and tool path generation, where smoothness and precision are critical. Given their importance, this study focuses on examining and comparing classical and aesthetic Bézier approaches, particularly in terms of their effectiveness in constructing high-quality engineering surfaces.

Methodology

This study focuses on the construction and analysis of cubic Bézier curves in the context of engineering surface design. The methodology involves both mathematical formulation and computational visualization to evaluate fundamental properties and behavior of cubic Bézier curves.

The cubic Bézier curve is defined parametrically as:

$$C(t) = \sum_{i=0}^n B_i^n(t)P_i, t \in [0,1] \tag{1}$$

$$B_{0,3}(t) = (1 - t)^3, \tag{2}$$

$$B_{1,3}(t) = 3t(1 - t)^2, \tag{3}$$

$$B_{2,3}(t) = 3t^2(1 - t), \tag{4}$$

$$B_{3,3}(t) = t^3. \tag{5}$$

where P_i are the control points and $B_i^n(t)$ are the Bernstein basis functions. The study specifically considers cubic Bézier curves (degree 3) with four control points to illustrate key properties. The fundamental properties examined include:

- **Boundary Interpolation:** The curve interpolates its first and last control points, verified analytically by evaluating $C(0)$ and $C(1)$.
- **Convex Hull Property:** The curve lies entirely within the convex hull formed by its control points, based on the non-negativity and partition of unity properties of Bernstein polynomials.
- **Boundary Tangent Property:** Tangent vectors at the endpoints depend only on the first and last control points, ensuring smooth connection to adjacent curves or surfaces. Analytical derivations of the tangent vectors are performed using the derivative of the cubic Bézier formula.

Figure 1 shows the cubic Bézier basis functions plotted over the parameter $t \in [0,1]$. The Bernstein basis functions exhibit the key properties of non-negativity, partition of unity, and symmetry.

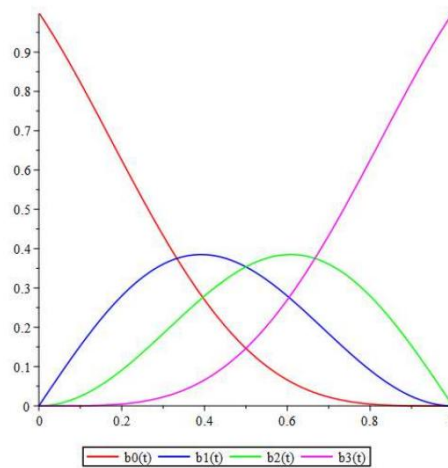


Figure 1: Cubic Bernstein basis function

By using the cubic Bernstein basis functions, a cubic Bézier curve can be expressed as follows:

$$C(t) = \sum_{i=0}^3 P_i B_{i,3}(t), \tag{5}$$

where P_i represents the control points in R^2 , and $B_{i,3}(t)$ denotes the cubic Bernstein basis functions. The cubic Bézier curve generally satisfies key properties, including endpoint interpolation, endpoint tangents, and the convex hull property. Figure 2 illustrates an example of a cubic Bézier curve. It should be noted that modifying the shape of the curve requires adjusting the positions of its control points.

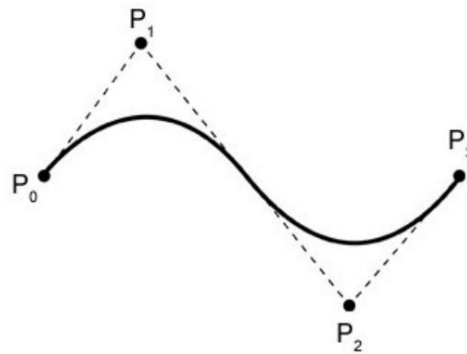


Figure 2: Cubic Bézier curve

Results and Discussion.

Engineering surfaces are designed to represent the complex geometries commonly encountered in practical design applications. In this study, all engineering surfaces are generated using Bézier curve representations to provide smoothness, geometric consistency, and intuitive control through a finite set of control points. The parameters u and v are defined within the interval $[0,1]$, unless otherwise specified. Since engineering surfaces are formulated as parametric extensions of Bézier curves, they inherently retain all the fundamental properties of classical Bézier curves, ensuring smooth, stable, and geometrically consistent surface representations.

A ruled surface is generated by linearly interpolating between two Bézier boundary curves, $C_1(u)$ and $C_2(u)$, which are defined as

$$C_1(u) = \sum_{i=0}^n P_i B_i^n(u), C_2(u) = \sum_{i=0}^n Q_i B_i^n(u). \tag{6}$$

The ruled surface is then expressed as

$$R(u, v) = (1 - v)C_1(u) + vC_2(u), u, v \in [0,1]. \tag{7}$$

This formulation creates a surface by connecting corresponding points on the two boundary curves through straight lines, providing a simple and effective method for constructing parametric surfaces.

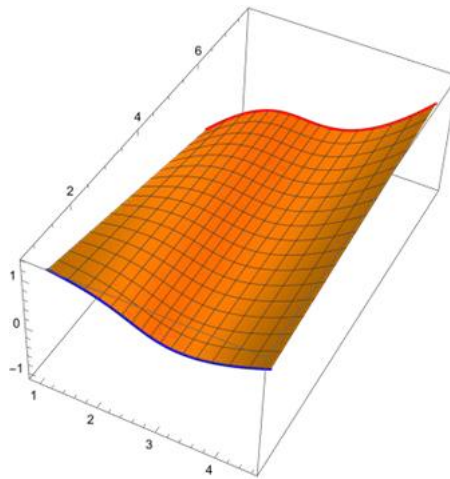


Figure 3: Ruled Surface of Two Cubic Bézier Curve

Figure 3 illustrates a ruled surface created by linearly interpolating between two cubic Bézier curves. The red curve represents $C_1(u)$, while the blue curve represents $C_2(u)$. For each parameter value $u \in [0,1]$, a straight line connects the corresponding points on $C_1(u)$ and $C_2(u)$, forming a smooth surface that spans between the two boundary curves.

A swung surface is generated by scaling and rotating a Bézier profile curve along a trajectory curve. Mathematically, the swung surface can be expressed as

$$S(u, v) = (x(u)r(v), y(u)r(v), z(v)), \quad u, v \in [0,1]. \quad (7)$$

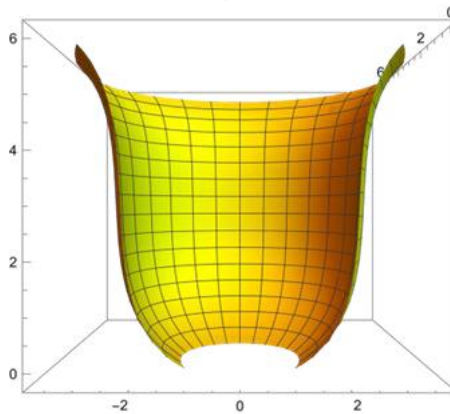


Figure 3: Swung Surface of Bézier Curve

Figure 4 illustrates a swung surface created by combining a profile curve with a trajectory curve. The profile curve is positioned and transformed according to the trajectory curve, resulting in a parametric surface whose coordinates are determined by both the profile and trajectory curves.

Bezier curves based on Bernstein basis functions can be extended to construct various engineering surfaces while preserving key properties such as boundary interpolation, convex hull, and

endpoint tangency. This framework provides a structured approach for analyzing and comparing classical and aesthetic Bézier surface constructions.

Conclusion

This study highlights the versatility of Bézier curves, defined using Bernstein basis functions, in the construction of engineering surfaces. By extending these curves into parametric surfaces, key geometric properties such as boundary interpolation, the convex hull property, and endpoint tangency are inherently preserved, ensuring smoothness, stability, and geometric consistency in surface design. The methodological framework presented allows for systematic construction, visualization, and analysis of both classical and aesthetic Bézier surfaces. While classical Bézier surfaces offer intuitive control through fixed control points, aesthetic Bézier surfaces introduce shape parameters that enhance local flexibility, curvature control, and visual fairness without altering global geometry. The comparative analysis demonstrates that incorporating aesthetic parameters improves surface smoothness and adaptability, making it a promising approach for engineering applications where both functional performance and aesthetic quality are critical. Overall, this work provides a structured foundation for selecting and evaluating Bézier-based surface modeling techniques, supporting designers and engineers in creating high-quality, visually appealing, and geometrically robust free-form surfaces.

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