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INFLUENCE OF CORIOLIS FORCE ON CONTROLLED DOUBLE-DIFFUSIVE MARANGONI CONVECTION IN NANOFLUID LAYERS

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ABSTRACT

Controlled double-diffusive Marangoni convection in a rotating nanofluid layer, heated from below, is studied. Various types of lower-upper boundary conditions, including free-free and rigid-free, are considered. The nanofluids model incorporates the mechanisms of Brownian motion and thermophoresis in nanofluids. The stability of the nanofluids model is analysed using a linear stability analysis based on the normal mode technique. The eigenvalue problem is numerically solved using the Galerkin technique and computational simulations are carried out using Maple software. The influences of several parameters are examined and presented graphically, including modified diffusivity ratio, nanoparticles concentration, solutal Rayleigh number and Soret effects. These effects are found to contribute to the advancement of Marangoni convection, a phenomenon that occurs due to variations in surface tension along the interface of a nanofluid layer. Conversely, the presence of the Coriolis force (due to the rotation), controller gain, and Dufour effects are observed to slow down the process of Marangoni convection.

Keywords: *Controller Gain, Coriolis Force, Double-Diffusive Coefficients, Nanofluids Layer and Marangoni Convection.*

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1. Introduction

Nanofluids, which have captured significant attention from researchers, are a cutting-edge class of engineered fluids with substantial significance in the realms of science and technology. They are colloidal mixtures composed of nanosized particles (ranging from 1 to 100 nanometers) suspended in base fluids (Choi & Eastman, 1995). Buongiorno (2006) revealed that among these mechanisms, Brownian motion and thermophoresis play vital roles in the transport of nanofluids. Tzou (2008a, 2008b) utilized Buongiorno's model to examine the issue of thermal instability in a layer of nanofluids. Nield and Kuznetsov (2010) studied the initiation of convection in a nanofluid layer with a finite depth. Yadav et al. (2011) conducted a linear stability analysis of Rayleigh-Benard convection in a layer with a finite depth. Menni et al. (2020) reviewed the use of nanofluids to improve the thermal performance of heat exchangers. Their application in heat pipes—including conventional, pulsating, and closed two-phase thermosyphon systems—has been shown to increase thermal efficiency and reduce thermal resistance. In plate-fin heat exchangers, nanofluids with nanoparticles such as Al_2O_3, CuO, ZnO, TiO_2, and Fe_3O_4 improve the heat transfer coefficient, enabling more compact and efficient designs. The effectiveness of these nanofluids depends on factors such as nanoparticle size, shape, concentration, and the flow regime, while their advantages include higher thermal conductivity as well as improved spreading, wetting, and stability. Farhana et al. (2020) focused on the thermophysical properties of nanofluids, specifically nanocellulose-aqueous ethylene glycol nanofluids. Ahuja and Sharma (2020) reviewed nanofluid convection instability, highlighting the studies on the effects of rotation, magnetic fields, Hall currents, and local thermal non-equilibrium (LTNE) effects under various boundary conditions. Natural convection in a nanofluid layer with various effects has been investigated theoretically by many researchers to understand the influence of physical parameters on flow stability, heat transfer characteristics, and transport mechanisms (Gholamalizadeh et al., 2020; Khalid et al., 2020, 2021, 2022, 2023).

The instability of Marangoni convection induced by a surface tension gradient arises whenever the temperature gradient across the layer exceeds a certain critical value. Pearson (1958) conducted the first theoretical study and reported that there exist polygonal cellular patterns in paint layers even if the paint is on the underside of a horizontal plane surface. Linear stability analysis of Marangoni convection with free-slip boundary conditions at the bottom was first investigated by Boeck and Thess (1997). The onset of oscillatory Marangoni convection in a variable viscosity fluid layer with the effect of controller gain was studied theoretically using linear stability analysis by Kechil and Hashim (2009). Abidin et al. (2019) studied the effect of the controller on the steady Marangoni convection in a fluid layer bounded above by a deformable free surface and below by a rigid surface with variable viscosity using linear stability analysis. Linear stability analysis has been implemented upon normal mode techniques to study the effect of magnetoconvection in a nanofluid layer with an internal heat source (Chand & Rana, 2015; Khalid et al., 2017, 2019). In supporting such complex analytical and numerical investigations, Mansor et al. (2025) highlighted that mathematical software enhances conceptual understanding, analytical efficiency, and critical thinking, while facilitating technology integration in mathematical modelling. These findings align with the present study, where mathematical software is employed as an essential tool to support stability analysis and interpretation of convection phenomena, and can be applied by many researchers to strengthen analytical accuracy and research productivity.

Convective instability is significantly influenced by the Coriolis force resulting from the rotational motion of fluid layers. Chandrasekhar (1961) studied the Rayleigh-Benard convection in a fluid exhibiting a linear temperature profile, exploring the influence of rotation both with and without its effect. Yadav et al. (2011, 2013) have investigated the issue of thermal instability in rotating layers of nanofluids, considering the effects of Brownian motion, thermophoresis, and other relevant aspects of the problem. Furthermore, the convective instability in rotating fluid layers induced by buoyancy and thermocapillary has been extensively studied by many authors, thus attracting significant interest from the engineering industries and applications (McConaghy and Finlayson, 1969; Takashima and Namikawa, 1971; Friedrich and Rudraiah, 1984; Douiebe et al., 2001). Yadav et al. (2014, 2016) investigated the influence of rotation on the nanofluid layer and revealed that an increase in the nondimensional Taylor number parameter, derived from the dimensional equation, led to an elevation in thermal critical Rayleigh number and enhanced the stability of the system. Rafeek et al. (2020) demonstrated that internal heat sources destabilized double-diffusive convection by enhancing molecular diffusion, whereas rotation stabilizes the system by delaying convection onset. The effects of throughflow, Darcy, Taylor, and Peclet numbers play significant roles in controlling system stability. Kopp and Yanovsky (2022) theoretically examined convection onset in electrically conducting nanofluids within a Darcy–Brinkman porous layer under a helical magnetic field, showing that rotation and axial magnetic fields stabilize convection, while azimuthal magnetic fields can destabilize it, depending on Rossby number.

The ability to control intricate heat transfer flows is crucial in both technology and fundamental sciences. The application of feedback control in thermal instability was examined by Tang and Bau (1993a, 1993b), revealing that its utilization can effectively enhance the thermal critical Rayleigh number and consequently stabilize the system. Tang and Bau (1995, 1998a, 1998b) and Howle (1997a, 1997b, 1997c) conducted experimental demonstrations, showcasing that feedback control effectively stabilized the system. Bau (1999) employed a linear controller to intentionally postpone the occurrence of Marangoni-Benard instability. Interestingly, the study revealed that employing similar control strategies was also effective in delaying the onset of Rayleigh-Benard convection. Furthermore, numerous researchers have investigated the integration of feedback control (Siri et al., 2009; Yadav et al., 2020) with other factors to address instabilities caused by buoyancy and surface tension.

Despite extensive studies on Marangoni convection and nanofluid heat transfer, the combined influence of rotational effects and double-diffusive mechanisms in nanofluid layers remains insufficiently explored. In particular, the role of the Coriolis force in modifying controlled double-diffusive Marangoni convection has received limited attention in the existing literature. Therefore, the novelty of this study lies in examining the coupled effects of Coriolis force, thermosolutal gradients, and nanoparticle-enhanced fluids on the stability characteristics of Marangoni convection. The presence of nanoparticles further alters the transport properties of the fluid, which may significantly influence the onset of convection under rotational conditions. The main objective of this study is to investigate the influence of the Coriolis force on controlled double-diffusive Marangoni convection in nanofluid layers. Specifically, the study analyzes how rotation affects the onset and stability of convection driven by both temperature and concentration gradients at the fluid surface. This study builds upon the work conducted by Nield and Kuznetsov (2010) by incorporating the Marangoni convection into the nanofluids layer system for the lower-upper boundary conditions of free-free and rigid-free. By utilizing a linear stability analysis based on the normal mode technique and employing the Galerkin technique to obtain the eigenvalue solution, we investigate the onset of instabilities. The relevant parameters are numerically computed using Maple software and presented through graphical presentations.

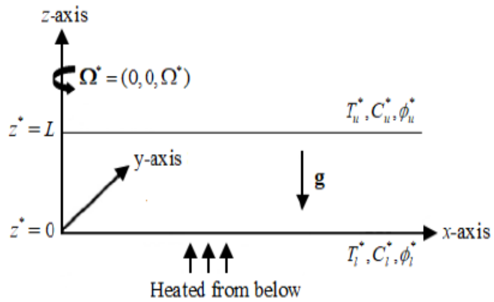

Figure 1. Physical Configuration and Coordinate System

2. Methodology

Consider a horizontal layer of a rotating incompressible nanofluid of thickness L confined between the planes $z^* \in [0, L]$ is heated from below and subjected to controller gain, K and double-diffusive coefficients as shown in Figure 1. A Cartesian coordinate system is used, with the z-axis pointing vertically upward. It is assumed that the upper free surface is non-deformable, σ^* is the surface tension, which acts as a function of temperature and solute concentration in the form

$$\sigma^* = \sigma_0^*\left[1-\beta_T^*\left(T^*-T_u^*\right)-\beta_C^*\left(C^*-C_u^*\right)\right]. \tag{1}$$

The nanofluid layer rotates about the vertical axis at a constant angular velocity $\boldsymbol{\Omega}^* = (0,0,\Omega)$. The stability of a horizontal rotating nanofluid layer in the presence of feedback control, K and double-diffusive coefficients is examined. The fixed temperature, solute concentration, and nanoparticles volumetric fraction of nanoparticles at the lower and upper walls are denoted, respectively, by T_l^*, C_l^* and ϕ_l^* at $z=0$ and T_u^*, C_u^* and ϕ_u^* at $z=L$. The governing equations under the Boussinesq approximation of this model, with the effects of rotation, feedback control and double-diffusive coefficients, are written as below

$$\nabla^* \cdot \boldsymbol{u}^* = 0, \tag{2}$$

$$\begin{aligned}\rho_f\left[\frac{\partial \boldsymbol{u}^*}{\partial t^*}+\left(\boldsymbol{u}^*\cdot\nabla^*\right)\boldsymbol{u}^*\right] &= \boldsymbol{g}\left[\phi^*\rho_p+\left(1-\phi^*\right)\rho_f\left[1-\alpha_T\left(T^*-T_u^*\right)-\alpha_C\left(C^*-C_u^*\right)\right]\right]\\ &\quad -2\rho_f\left(\boldsymbol{\Omega}^*\times\boldsymbol{u}^*\right)-\nabla^*p^*+\mu\nabla^{*2}\boldsymbol{u}^*,\end{aligned} \tag{3}$$

$$\begin{aligned}(\rho c)_f\left[\frac{\partial T^*}{\partial t^*}+\left(\boldsymbol{u}^*\cdot\nabla^*\right)T^*\right] &= \kappa\nabla^{*2}T^*+(\rho c)_p\left[D_B\nabla^*\phi^*\cdot\nabla^*T^*+\left(\frac{D_T}{T_u^*}\right)\nabla^*T^*\cdot\nabla^*T^*\right]\\ &\quad +(\rho c)_f\left(D_{TC}\nabla^{*2}C^*\right),\end{aligned} \tag{4}$$

$$\left[\frac{\partial C^*}{\partial t^*}+\left(\boldsymbol{u}^*\cdot\nabla^*\right)C^*\right] = D_S\nabla^{*2}C^*+D_{CT}\nabla^{*2}T^*, \tag{5}$$

$$\left[\frac{\partial \phi^*}{\partial t^*}+\left(\boldsymbol{u}^*\cdot\nabla^*\right)\phi^*\right] = D_B\nabla^{*2}\phi^*+\frac{D_T}{T_u^*}\nabla^{*2}T^*, \tag{6}$$

where $\boldsymbol{u}^* = \left(u^*, v^*, w^*\right)$ is the velocity, T^* is the temperature, C^* is the solute concentration, ϕ^* is the volumetric fraction of nanoparticles, ρ is the density of the base fluid, ρ_f is the nanofluid's density at the reference temperature T_l^*, ρ_p is the nanoparticles mass density, t^* is the time, p^* is the pressure, μ is the viscosity, $\boldsymbol{g}$ is the gravitational force, α_T is the thermal coefficients, α_C is the solutal volumetric coefficient, c is the specific heat, c_p is the specific heat of the nanoparticle, κ is the nanofluids thermal conductivity, D_{TC} is the Dufour diffusivity, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient, D_S is the solutal diffusivity, and D_{CT} is the Soret diffusivity respectively.

Equations (2)-(6) are nondimensionalized using the following transformation

$$\begin{gathered}\left(x^*,y^*,z^*\right)=L\left(x,y,z\right),p^*=\frac{p\mu\alpha_f}{L^2},t^*=\frac{tL^2}{\alpha_f},\phi=\frac{\phi^*-\phi_l^*}{\phi_u^*-\phi_l^*},C=\frac{C^*-C_u^*}{\Delta C^*},\\ \psi_z^*=\frac{\psi_z\alpha_f}{L},\left(u^*,v^*,w^*\right)=\frac{\alpha_f}{L}\left(u,v,w\right),T=\frac{T^*-T_u^*}{\Delta T^*},\end{gathered}\tag{7}$$

where $\alpha_f=\dfrac{\kappa}{\left(\rho c\right)_f}$ is the thermal diffusivity. The nondimensionalized (2)-(6) takes the form

$$\nabla\cdot\boldsymbol{u}=0,\tag{8}$$

$$\frac{1}{\Pr}\left[\frac{\partial\boldsymbol{u}}{\partial t}+\boldsymbol{u}\cdot\nabla\boldsymbol{u}\right]=-\nabla p-\nabla^2\boldsymbol{u}-Rm\hat{\boldsymbol{e}}_z+\frac{Rs}{Le}C\hat{\boldsymbol{e}}_z+RaT\hat{\boldsymbol{e}}_z-Rn\phi\hat{\boldsymbol{e}}_z-\sqrt{Ta}\left(\boldsymbol{u}\times\hat{\boldsymbol{e}}\right),\tag{9}$$

$$\left[\frac{\partial T}{\partial t}+\boldsymbol{u}\cdot\nabla T\right]=\nabla^2T+Df\nabla^2C+\frac{N_B}{Ln}\nabla\phi\cdot\nabla T+\frac{N_AN_B}{Ln}\nabla T\cdot\nabla T,\tag{10}$$

$$\left[\frac{\partial C}{\partial t}+\boldsymbol{u}\cdot\nabla C\right]=\frac{1}{Le}\nabla^2C+Sr\nabla^2T,\tag{11}$$

$$\left[\frac{\partial\phi}{\partial t}+\boldsymbol{u}\cdot\nabla\phi\right]=\frac{1}{Ln}\nabla^2\phi+\frac{N_A}{Ln}\nabla^2T,\tag{12}$$

where $\hat{\boldsymbol{e}}_z=\left(0,0,1\right)$ is the unit vector in the z-direction, $\Pr=\dfrac{\mu}{\alpha_f\rho_f}$ is the Prandtl number, $Rm=\dfrac{\left[\rho_p\phi_l^*+\rho_f\left(1-\phi_l^*\right)\right]gL^3}{\mu\alpha_f}$ is the basic density Rayleigh number, $Rs=\dfrac{\rho_fg\alpha_CL^3\Delta C^*}{\mu D_S}$ is the solutal Rayleigh number, $Le=\dfrac{\alpha_f}{D_S}$ is the Lewis number, $Ra=\dfrac{\rho_fg\alpha_TL^3\Delta T^*}{\mu\alpha_f}$ is the thermal Rayleigh number, $Rn=\dfrac{\left(\rho_p-\rho_f\right)\left(\phi_u^*-\phi_l^*\right)gL^3}{\mu\alpha_f}$ is the nanoparticles concentration Rayleigh number, $Ta=\left[\dfrac{2\Omega^*\rho_fL^2}{\mu}\right]^2$ is the Taylor number, $N_B=\dfrac{\left(\rho c\right)_p}{\left(\rho c\right)_f}\left(\phi_u^*-\phi_l^*\right)$ is the modified particle density increment, $N_A=\dfrac{D_T\Delta T^*}{D_BT_u^*\left(\phi_u^*-\phi_l^*\right)}$ is the modified diffusivity ratio, $Ln=\dfrac{\alpha_f}{D_B}$ is the nanofluids Lewis number, $Df=\dfrac{D_{TC}\Delta C^*}{\alpha_f\Delta T^*}$ is the Dufour parameter, and $Sr=\dfrac{D_{CT}\Delta T^*}{\alpha_f\Delta C^*}$ is the Soret parameter.

The normal mode expansion of the dependent variables is assumed in the form

$$\left(w',T',C',\phi',\psi_z'\right)=\left[W\left(z\right),\Theta\left(z\right),\eta\left(z\right),\Phi\left(z\right),\Psi\left(z\right)\right]e^{\left[i\left(a_xx+a_yy\right)+st\right]},\tag{13}$$

then, substituting Equation (13) into Equations (8)-(12). At the onset of instability, the system reaches a state of neutral stability where disturbances neither grow nor decay with time. Therefore, by letting $s=0$, the critical conditions for the onset of convection can be determined.

In the present study, the analysis is restricted to stationary convection, implying that the instability develops without oscillatory behaviour. Therefore, we obtain

$$\left(D^2-a^2\right)^2 W-a^2 Ra\,\Theta-a^2\frac{Rs}{Le}\eta+a^2 Rn\,\Phi-\sqrt{Ta}D\Psi=0, \tag{14}$$

$$W+\left[D^2-a^2-2\frac{N_A N_B}{Ln}D+\frac{N_B}{Ln}D\right]\Theta-\frac{N_B}{Ln}D\Phi+Df\left(D^2-a^2\right)\eta=0, \tag{15}$$

$$W+Sr\left(D^2-a^2\right)\Theta+\frac{1}{Le}\left(D^2-a^2\right)\eta=0, \tag{16}$$

$$W-\frac{N_A}{Ln}\left(D^2-a^2\right)\Theta-\frac{1}{Ln}\left(D^2-a^2\right)\Phi=0, \tag{17}$$

$$\left(D^2-a^2\right)\Psi+\sqrt{Ta}DW=0, \tag{18}$$

where $a=\sqrt{a_x^2+a_y^2}$ is the wavenumber and $D=\frac{d}{dz}$. Following the proportional feedback control by Bau (1999), the continuously distributed actuators and sensors are arranged in such a way that for every sensor, there is an actuator positioned beneath it. The determination of a control can be accomplished using the proportional-integral-differential controller of the form

$$q(t)=r+K\left[e(t)\right],e(t)=\hat{m}(t)+m(t), \tag{19}$$

where r is the calibration of the control, $e(t)=\hat{m}(t)+m(t)$ is an error or deviation from the measurement $\hat{m}(t)$, from the desired reference value $m(t)$, while K is the scalar controller gain where $K=K_P+K_D\frac{d}{dt}+K_L\int_0^t dt$, and K_P is the proportional gain, K_D is the differential gain, and K_L is the integral gain. Based on Equation (19), for one sensor plane and proportional feedback control, the actuator modifies the heated surface temperature using a proportional relation between the upper $z=1$, and the lower $z=0$, thermal boundaries for the perturbation field

$$T'(x,y,0,t)=-KT'(x,y,1,t), \tag{20}$$

where T' denotes the deviation of the temperature of the fluid from its conductive state. Equations (14)-(18) are solved subject to appropriate boundary conditions. For Marangoni convection, considering the proportional controller K positioned at the lower boundary of the nanofluid layer, we will have as below.

For the lower free and upper free boundaries

$$W=D^2W=\Theta(0)+K\Theta(1)=\eta=\Phi=\Psi=D\Psi=0 \text{ at } z=0, \tag{21}$$

$$W=D^2W+a^2Ma\,\Theta=D\Theta=\eta=\Phi=D\Psi=0 \text{ at } z=1. \tag{22}$$

For lower rigid and upper free boundaries

$$W=DW=\Theta(0)+K\Theta(1)=\eta=\Phi=\Psi=D\Psi=0 \text{ at } z=0, \tag{23}$$

$$W=D^2W+a^2Ma\,\Theta=D\Theta=\eta=\Phi=D\Psi=0 \text{ at } z=1. \tag{24}$$

The Galerkin-type weighted residuals method is applied to find the approximate solution to the system. The variables are written in a series of basis functions

$$W=\sum_{i=1}^{N}A_iW_i,\Theta=\sum_{i=1}^{N}B_i\Theta_i,\eta=\sum_{i=1}^{N}C_i\eta_i,\Phi=\sum_{i=1}^{N}D_i\Phi_i,\Psi=\sum_{i=1}^{N}E_i\Psi_i, \tag{25}$$

where A_i,B_i,C_i,D_i and E_i are constants and the basis functions $W_i,\Theta_i,\eta_i,\Phi_i$ and Ψ_i where $i=1,2,3,...,$ will be chosen corresponding to the free-free and rigid-free lower-upper boundary conditions. Substituting Equation (25) into Equations (14) - (18) and making the expressions on the left-hand sides of those equations (the residuals) orthogonal to the trial functions, a system of *5N* linear algebraic equations with *5N* unknowns are obtained. The determinant of the coefficients produces the eigenvalue equations for the system. One can regard *Ra* and *Ma* as the eigenvalues and can be found in terms of the other parameters. The eigenvalues problem obtained from the linear stability analysis is solved using Maple. The determinant of the coefficient matrix is then set to zero to derive the eigenvalue equation for the critical parameters. Maple is used to compute the critical values numerically for different parameter ranges. The solutions obtained are validated by comparing the limiting cases with previously reported results in the literature.

3. Results and Discussions

The onset of Marangoni convection in a rotating nanofluids layer heated from below with feedback control subjected to double-diffusive coefficients is examined for lower and upper boundaries of free-free and rigid-free respectively. The model used for nanofluids includes the combined effects of Brownian motion and thermophoresis mechanisms. Linear stability analysis is employed and the resulting eigenvalue, Marangoni number, *Ma* is extracted using the Galerkin technique. The parameters values are chosen according to the proposed range values by Chand & Rana (2015): $Rs = 100$, $Rn = 3$, $Sr = 0.2$, $Df = 0.2$, $N_A = 3$, $N_B = 0.01$, $Le = 0.4$ and $Ln = 90$. As for Taylor number and feedback control parameters, they are fixed at $Ta = 300$ and $K = 5$, respectively. The results obtained are presented graphically to illustrate the impact of various parameters on Marangoni number, *Ma* versus wave number, *a* in Figure 2-7. Additionally, the critical Marangoni number, Ma_c is plotted in Figure 8-11.

Table 1. Comparison of critical Rayleigh number, Ra_c for different values of *Ta* with Chandrasekhar (1961) and Yadav et al. (2016) in rigid-rigid boundary conditions for the limiting case of nanofluids layer

Ta	Ra_c		
	Chandrasekhar (1961)	Yadav et al. (2016)	Present Analysis
0	1707.80	1707.83	1707.76
10	1713.00	1712.74	1712.65
100	1756.60	1756.41	1756.33
500	1940.30	1940.26	1940.08
1000	2151.70	2151.39	2151.32
2000	2530.50	2530.18	2530.10
5000	3469.20	3468.58	3468.46
1000	4713.10	4712.13	4712.05

Test computations have been performed, and we compared our results with Chandrasekhar (1961) and Yadav et al. (2016) for regular fluid in the absence of feedback control, *K* and double-diffusive coefficients. The results of our analysis, as shown in Table 1, demonstrate good agreement with the results reported by Chandrasekhar (1961) and Yadav et al. (2016).

Figure 2 illustrates the plot of Marangoni number, *Ma* and the corresponding wavenumber, *a* for selected values of feedback control, $K = 1$, 3 and 5. The upward shift of the marginal stability curves as the *K* increases indicates that the controller stabilizes the motionless state for all wavenumbers. This phenomenon occurs because the sensors detect the deviations from the conductive state of the nanofluids layer prompting the actuators to suppress any disturbances (Bau, 1999). Figure 3 depicts the relationship between the Taylor number, *Ta* = 100, 200 and 300 in the plot of Marangoni number, *Ma* versus wavenumber a, respectively. As the *Ta* increases, the *Ma* also rises. This behaviour indicates that the Coriolis force resulting from rotation delays the onset of convection within the nanofluids layer. The rotational motion induces more vigorous movement of the nanofluids in the horizontal plane due to the vorticity introduced by the rotation mechanism. Consequently, the velocity of the nanofluids in the vertical plane decreases, resulting in a reduction in thermal convection.

Figures 4 and 5 illustrate the plot of two important effects of interdiffusion, namely Dufour and Soret effects that arise due to the combination of concentration gradient and temperature. The relationship between the Marangoni number, *Ma* and the wavenumber, *a* for different values of Dufour parameter, *Df* = 0.2, 0.4 and 0.6 is to stabilize the system. This is due to the energy flux that becomes more significant and increases the solute concentration by driving mass gradient within the system, thus delaying thermal convection. Conversely, increasing values of Soret parameter, *Sr* = 0.2, 0.4 and 0.6 on Marangoni number, *Ma* against wavenumber, *a* destabilized the system. The increase in *Sr* increases the temperature flux, which contributes to the acceleration of Marangoni convection.

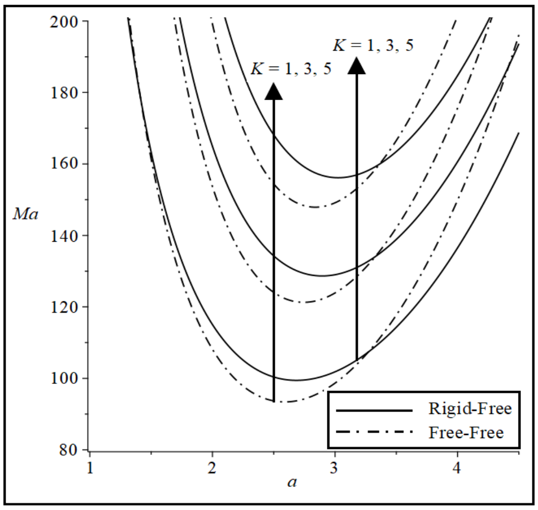

Figure 2. *Ma* versus *a* for various values of *K*

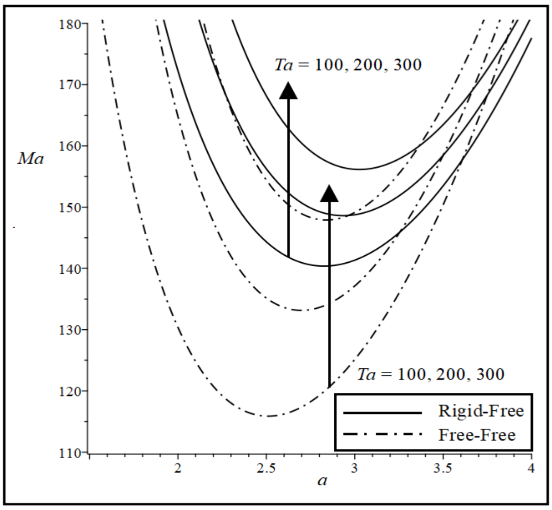

Figure 3. *Ma* versus *a* for various values of *Ta*

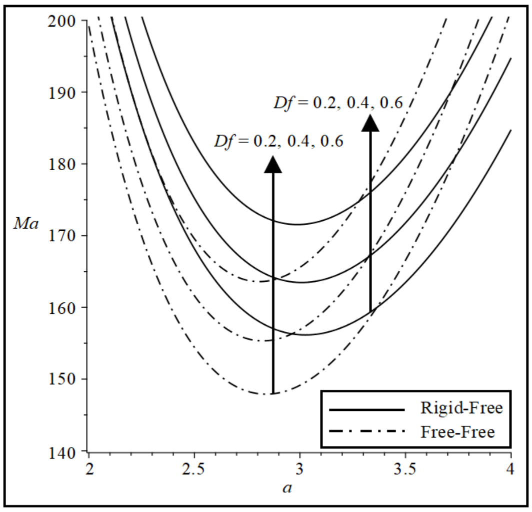

Figure 4. *Ma* versus *a* for various values of *Df*

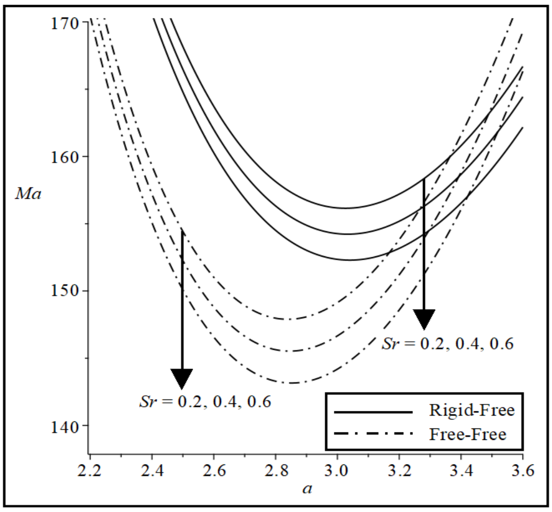

Figure 5. *Ma* versus *a* for various values of *Sr*

The influences of solutal Rayleigh number, *Rs* and nanoparticles concentration Rayleigh number, *Rn* on the onset of Marangoni convection nanofluids layer are illustrated in Figures 6 and 7. The increase values of *Rs* = 100, 150, 200 increase Marangoni number, *Ma* spontaneously and stabilized the system. Therefore, the onset of convection is postponed due to the greater amount of solute concentration than that of the solvent, leading to a decrease in the overall temperature within the system. Conversely, Marangoni number, *Ma* decreases rapidly as the values of *Rn* = 3, 4, 5 increases and destabilize the system. This is due to the increase in the volumetric fraction of nanoparticles that results in a combination of Brownian motion and thermophoresis diffusion within the system since they act as the primary mechanism to enhance thermal instability.

Figures 8 and 9 show the variation of critical Marangoni number, Ma_c, as a function of Taylor number, *Ta*, for different values of modified diffusivity ratio, N_A = 1, 9 and the variation of critical Marangoni number, Ma_c, as a function of feedback control, *K*, for different values of Soret parameter, *Sr* = 0.2, 0.8. It is observed that Ma_c increases with the increase in *Ta*, indicating that *Ta* delays the onset of convection even in the presence of the destabilizing effect of N_A as illustrated in Figure 8. At the same time, Figure 9 shows the increase in *K*, indicating that *K* effect delays the onset of convection even in the presence of the destabilizing effect of *Sr*.

The influence of Soret parameter, *Sr* = 0.2, 0.8, act as the reciprocal phenomenon to the Dufour parameter, *Df* as shown in Figure 10. The critical Marangoni number, Ma_c for the various values of the Dufour parameter increases slightly with the increase in values of Soret parameter. Therefore, the effect of increasing Soret parameter, *Sr* destabilized the system, but in the presence of the Dufour parameter, *Df* the destabilized system can be monitored and managed in the presence of the Dufour effect.

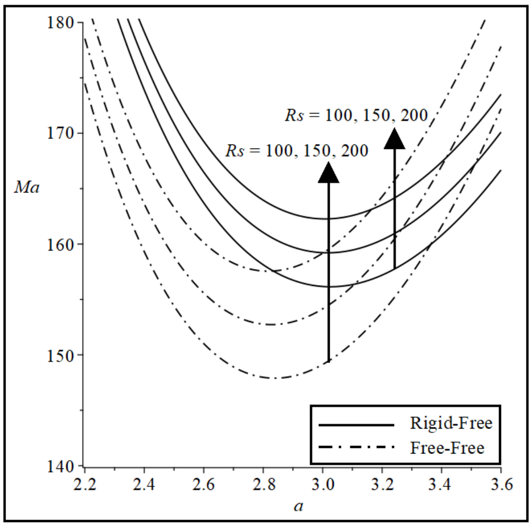

Figure 6. *Ma* versus *a* for various values of *Rs*

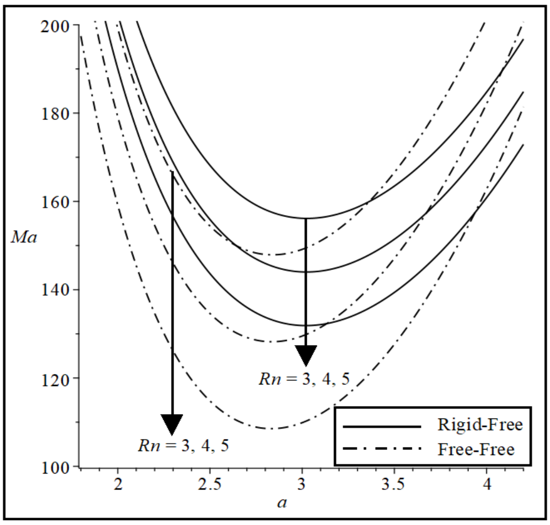

Figure 7. *Ma* versus *a* for various values of *Rn*

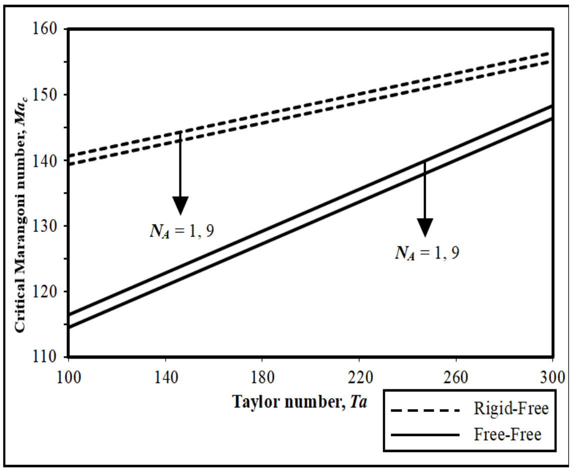

Figure 8. Ma_c in the function of *Ta* for various values of N_A

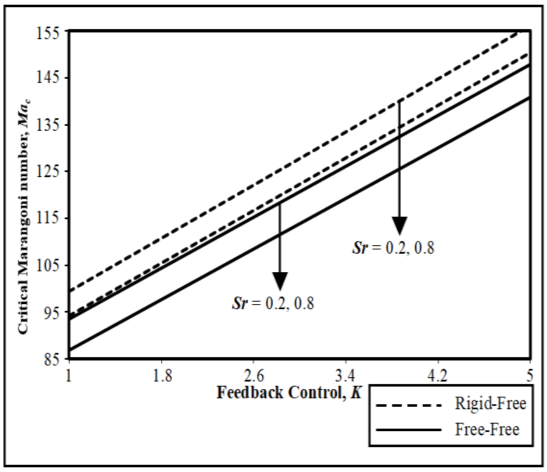

Figure 9. Ma_c in the function of *K* for various values of *Sr*

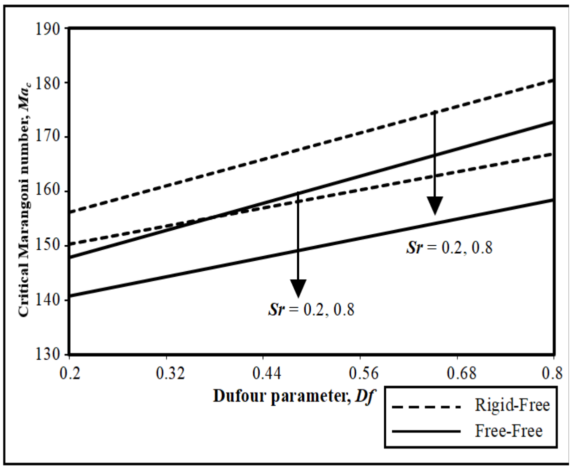

Figure 10. Ma_c in the function of *Df* for various values of *Sr*

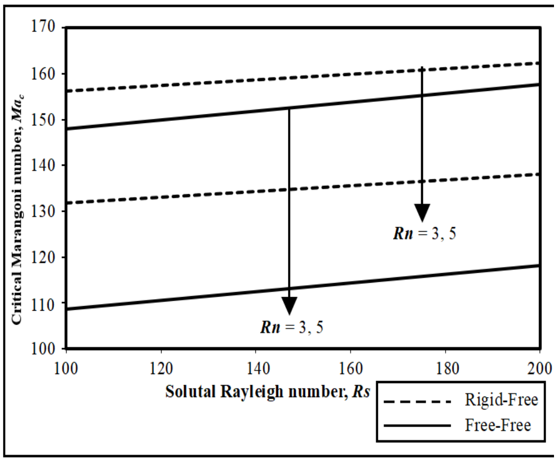

Figure 11. Ma_c in the function of *Rs* for various values of *Rn*

Finally, Figure 11 shows the variation of critical Marangoni number, Ma_c, as a function of solutal Rayleigh number, *Rs*, for different values of nanofluids' Rayleigh number, $Rn = 3, 5$. It is found that Ma_c increases with the increase in *Rs*, indicating that *Rs* delays the onset of convection even in the existence of the destabilizing effect of *Rn*.

4. Conclusion

The theoretical investigation focuses on the onset of Marangoni convection in a rotating nanofluids layer with feedback control under the influence of double-diffusive coefficients. The model incorporates the combined impact of Brownian motion and thermophoresis. The analysis utilizes linear stability analysis, obtaining the eigenvalue solution through normal mode analysis. The Galerkin method is applied to solve the eigenvalue solution, which is subsequently computed numerically using Maple software. The study considers the cases of lower-upper boundaries to be free-free and rigid-free.

It should be noted that the present study is limited to a linear stability analysis under the assumption of stationary convection and idealized boundary conditions. Nonlinear effects, oscillatory instability modes, and possible three-dimensional flow structures are not considered in this model. In addition, the analysis is theoretical and based on the nanofluid model of Nield and Kuznetsov, and therefore experimental validation is beyond the scope of the present work. Future studies may extend the present analysis by considering nonlinear effects, oscillatory convection, or experimental investigations to further validate the theoretical predictions.

The implementation of increasing values for the effects of feedback control, *K*, Taylor number, *Ta*, Dufour parameter, *Df*, and solutal Rayleigh number, *Rs* into the system leads to a lag on the onset of Marangoni convection, thereby stabilizing the system. Conversely, the implementation of increasing values for the effects of Soret parameter, *Sr*, nanoparticles concentration Rayleigh number, *Rn* and modified diffusivity ratio, N_A accelerate the onset of Marangoni convection, resulting in system destabilization. Meanwhile, the influence effect of modified particle density increment, N_B on the onset of Marangoni convection in the nanofluids layer system is negligible and can be disregarded (Yadav et al., 2013, 2014, 2016).

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Author Contribution

Izzati Khalidah Khalid conceptualized the study, supervised the overall research work, and oversaw the writing and revision of the manuscript. Mohd Rijal Ilias provided expert advice on the research methodology and contributed to the refinement of the analytical approach. Nurul Hafizah Zainal Abidin offered guidance on the programming aspects and assisted in the implementation of computational procedures. Ahmad Syukri Mohd Shukor conducted the simulations, generated the results, and assisted in data organization. Nur Asiah Mohd Makhatar reviewed the manuscript and provided constructive feedback to improve clarity and presentation. All authors read and approved the final manuscript.

Conflict of Interest

The authors have no conflicts of interest to declare.

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