

**UNIVERSITI TEKNOLOGI MARA**

**FINDING MULTIPLE ROOTS OF  
NONLINEAR FUNCTIONS USING  
NUMERICAL METHODS**

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## ABSTRACT

Multiple roots refer to solutions of nonlinear equations where a root occurs more than once. Accurately finding multiple roots remains difficult due to the limitations in both graphical and numerical methods. This project aims to approximate the multiple roots of nonlinear functions using four open numerical methods, namely Kung-Traub Conjecture, Li Method 1, Li Method 2 and Optimal Method 1. This project was conducted using Maple 2016 and Microsoft Excel, with ten different functions, four initial values and three stopping criteria. For each method, performance was analyzed based on the number of iterations, error analysis and CPU time. Results based on the performance profile indicated that the Kung-Traub Conjecture is the best method in terms of accuracy and CPU time. Conclusively, Kung-Traub Conjecture was observed to be the most optimal method among the four selected.

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