

# JURNAL AKADEMIK

February 2005 Issue

**Attitudes Of Form Two Students  
Towards Learning Science In  
English: A Case Study Of Schools  
In Kota Samarahan**

**Caesar DeAlwis, Saira Joe And  
Sharifah Aishah Wan Kahar**

**The Sythesizing Process Of Work  
Motivation Between Western And  
Islamic Perspectives: An Analysis**

**Nursuria bt Mahrif,  
Prof. Ibrahim Mamat And  
Norizan b. Hj. Jili**

**Rethinking Industrial Relations  
In Malaysia**

**Kuldip Singh**

**The Influence Of Macroeconomic  
Factors On The Malaysian Equity  
Market**

**Jaafar Pyeman And  
Abdullah Sulaiman**

**Synthesis Of Ceramic Matrix  
Composite (Sic Based) Via  
Polymer Precursor Route**

**Hazman Seli And  
Zainal Arifin Ahmad**

**On Autoregressive Order Selection  
Criteria**

**Venus Khim-Sen Liew,  
Sie-Hoe Lau And  
Chin-Hong Puah**

**Penilaian Tren Pengundi-Pengundi  
Melayu / Melanau Muslim Dalam  
Pilihanraya Sarawak 2001**

**Prof Madya Shaharuddin  
Badaruddin,  
Abang Yusof Abang Spawi,  
Nadrawina Hj Isnin And  
Nursuria Mahrif**

**Sejauhmana Bilangan Lelaki Dan  
Wanita Di Tempat Kerja Boleh  
Mempengaruhi Persepsi Gangguan  
Seksual**

**Prof. Madya Dr. Sabitha  
Marican**



UNIVERSITI TEKNOLOGI MARA SARAWAK

*Usaha Taqwa Mulia*

## ON AUTOREGRESSIVE ORDER SELECTION CRITERIA

**Venus Khim-Sen Liew,**

Labuan School of International Business and Finance,  
Universiti Malaysia Sabah.

**Sie-Hoe Lau**

Faculty of Information Technology and Quantitative Science,  
Universiti Teknologi MARA, Sarawak Campus.

**Chin-Hong Puah**

Faculty of Economics and Business,  
Universiti Malaysia Sarawak.

### Abstract

This study investigates the performance of various commonly applied order selection criteria in selecting order of Autoregressive (AR) processes. The most important finding of this study is that Akaike's information criterion, Schwarz information criterion, Hannan-Quinn criterion, final prediction error and Bayesian information criterion perform considerably well in estimating the true autoregressive order, even in small samples. Besides, there is no significant gain in differentiating these criteria unless one has a considerably large sample size. This study contributes to the empirical literature by providing helpful guidelines regarding the use of order selection criteria in determining the autoregressive order.

### 1.0 INTRODUCTION

Many order selection criteria have been employed in economic studies to determine the Autoregressive (AR) order of time series variables. Briefly, an AR process of order  $p$  refers to a time series in which its current value is dependent on its first  $p$  lagged values and is normally denoted by AR ( $p$ ). Note that the AR order  $p$  is always unknown and therefore has to be estimated via various order selection criteria such as Aikake's information criterion (AIC) (Akaike, 1973), Schwarz's information criterion (SIC) (Schwarz, 1978) Hannan-Quinn's criterion (HQC) (Hannan and Quinn, 1979), the final prediction error (FPE) (Akaike, 1969), and the Bayesian information criterion (BIC) (Akaike, 1979). See Liew (2000) for an overview of these criteria.

These criteria have been popularly adopted in economic studies, see for example the work of Sarantis (1999, 2001) and Baum et al. (2001), who employed the AIC, Ahmed (2000) who used the AIC and BIC, Tan and Baharumshah (1999) who deployed the FPE, Yamada (2000) who used AIC and HQC and Xu (2003) who utilized the SIC in their empirical research. However, no special study has been carried out to contrast the performances of these order selection criteria, although few empirical studies (Taylor and Peel, 2000); Baum et al., 2001;

although few empirical studies (Taylor and Peel, 2000); Baum et al., 2001; Guerra, 2001) do notify the inconsistency of these criteria and their tendency to under estimate the autoregressive order . In this respect, this study has taken the initiative to conduct a simulation study on the empirical performances of these order selection criteria. In particular, this study aims at comparing the performances of various selected criteria, in their ability to identify the correct autoregressive order.

To preview our results, the major finding in the current study is that these criteria perform fairly well even in a relatively small sample with only 25 observations, in the sense that they managed to identify the correct order about 60% of the time and this performance increases as sample size grows. At other times, these criteria tend to underestimate the true order rather than to overestimate it. Consequently, in the spirit of parsimony, which is against the selection of too long an autoregressive order, these criteria can thus be safely applied. With the discovery of these empirical findings, this study contributes to the literature in providing useful insights on the use of order selection criteria in future empirical research.

The rest of this paper is organized as follows. Section 2 briefly describes the AR process, the order selection criteria and simulation procedure. Section 3 presents and discusses the results of this simulation study. Section 4 offers a summary of this study.

## 2.0 METHODOLOGY OF THIS STUDY

### 2.1 Autoregressive Process

Mathematically, an AR( $p$ ) process of a series  $y_t$  may be represented by

$$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \dots + a_p y_{t-p} + \varepsilon_t \quad (1)$$

where  $a_0$  is the intercept term and  $a_1, a_2, \dots, a_p$  are autoregressive parameters and  $\varepsilon_t$  are normally distributed random error terms with a zero mean and a finite variance  $\sigma^2$ .

The estimation of an AR ( $p$ ) process involves 2 stages: first, identify the AR order  $p$  based on certain rules such as order selection criteria, second, estimate the numerical values for intercept and parameters using regression analysis. This study is confined to the performances of various commonly used order selection criteria in identifying the true order  $p$ . In particular, this study generates AR processes with  $p$  arbitrary fixed at a value of 3 and uses these criteria to determine the order of each generated series as if the order is unknown.

The autoregressive parameters are independently generated from uniform distribution with values ranging from -1 to 1 exclusively. Measures are taken to ensure that the sum of these simulated autoregressive parameters is less than unity in magnitude ( $|a_1 + a_2 + a_3| < 1$ ) so as to avoid non-stationary AR processes.

<sup>2</sup> A related work by Liew (2000) studies the performance of an individual criteria, namely the Aikake's biased corrected information criterion. The current study is more comprehensive than Liew (2000) in the sense that more criteria are involved for the purpose of comparative study.

The error term is generated from standard normal distribution, whereas the intercept term is omitted without loss of generality. We simulate data sets for various usable sample sizes,  $S$ : 25, 50, 100, 200, 400, 800 and 1600. For each combination of processes and sample sizes, we simulated 1000 independent series for the purpose of order estimation. In every case, the initial value,  $y_0$  is arbitrarily set to zero. In our effort to minimize the initial effect, we simulate  $100 + S$  observations and discard the first 100 observations, leaving the last  $S$  observations for order estimation. The estimated order ( $\hat{p}$ ) is allowed to be determined from any integer ranging from 1 to 20 inclusively.

In this respect, we compute the values for all 20 orders for each specific criterion and ( $\hat{p}$ ) is taken from the one that minimizes that criterion. Note that each criterion independently selects one ( $\hat{p}$ ) for the same simulated series.

## 2.2 Order selection criteria

The order selection criteria to be evaluated include:

- (a) Akaike's information criterion,

$$AIC_p = -2T [\ln(\hat{\sigma}_p^2)] + 2p; \quad (2)$$

- (b) Schwarz's information criterion,

$$SIC_p = \ln(\hat{\sigma}_p^2) + [p \ln(T)]/T; \quad (3)$$

- (c) The Hannan-Quinn criterion,

$$HQC_p = \ln(\hat{\sigma}_p^2) + 2 T^{-1} p \ln[\ln(T)]; \quad (4)$$

- (d) The final prediction error,

$$FPE_p = \hat{\sigma}_p^2 (T - p)^{-1} (T + p) \text{ and} \quad (5)$$

- (e) The Bayesian information criterion,

$$BIC_p = (T-p) \ln[(T-p)^{-1} T \hat{\sigma}_p^2] + T[1 + \ln(\sqrt{2\pi})] + p \ln[p^{-1} (\sum_{i=1}^T y_i^2 - T \hat{\sigma}_p^2)], \quad (6)$$

where  $\hat{\sigma}_p^2 = (T - p - 1)^{-1} \sum_{i=p}^T \hat{\varepsilon}_i^2$ ,  $\varepsilon_i$  is the model's residuals and  $T$  is the sample

size and  $p$  is the number of parameters fitted in the model. Note that the cap sign (^) indicates an estimated value. Liew (2000) provides an overview of these criteria, whereas details are given in, for instance, Brockwell and Davis (1996) and the references therein.

The main task of this study is to compute the probability of each of these criteria in correctly estimating the true autoregressive order. Note that this probability takes a value between zero and one inclusively, with a probability of zero, meaning that the criterion fails to pick up any true order and thereby is a poor criterion. On the other hand, a probability of one implies that the criterion manages to correctly select the true order in all cases and hence is an excellent criterion.

Besides, we also inspect the selected orders of the estimated order for 1000 simulated series of known order (that is,  $p = 3$ ), so as to gain deeper understanding of the performance of various criteria. We will refer to the situation whereby a criterion selected at lower orders than the true ones as an underestimate, whereas overestimate would mean the selection of higher orders than the true ones.

### 2.3 Simulation procedure

Briefly, the simulation procedure involves three sub-routines. The first sub-routine generates a series of from the AR process, whereas the second sub-routine selects the autoregressive order of the simulated series and the third sub-routine evaluates the performance of the order selection criteria. The algorithm for the simulation procedure for each combination of sample size  $S$  and AR order  $p$  is outlined as follows:

1. Independently generate  $a_1$ ,  $a_2$  and  $a_3$  from a uniform distribution in the range  $(-1, 1)$ , conditioned on  $|a_1 + a_2 + a_3| < 1$ .
2. Generate a series of size  $100 + S$  from the AR process as represented in Equation (1) of order  $p = 3$  with  $a_0 = 0.0$  and  $a_1$ ,  $a_2$  and  $a_3$ , obtained from Step 1. Initialize the starting value,  $y_0 = 0.0$ . Discard the first 100 observations to minimize the effect of the initial value.
3. Use each selection criterion to determine the autoregressive order ( $\hat{p}$ ) for the last  $S$  observations of the series simulated in Step 2. Five selection criteria are involved.
4. Repeat Step 1 to Step 3 for  $B$  times, where  $B$  is fixed at 1000 in this study<sup>3</sup>.
5. Compute the probabilities of (i) correct estimate, which is computed as  $\#(\hat{p} = P)IB$ ; (ii) underestimate, which is computed as  $\#(\hat{p} < P)IB$ ; and (iii) overestimate, which is computed as  $\#(\hat{p} > P)IB$ , where  $\#(\bullet)$  denotes numbers of time event ( $\bullet$ ) happens.

---

<sup>3</sup>Since the values of autoregressive parameters as well as error terms in each series are randomly generated in the simulation procedures, we effectively have  $B$  different series of AR (3) series.

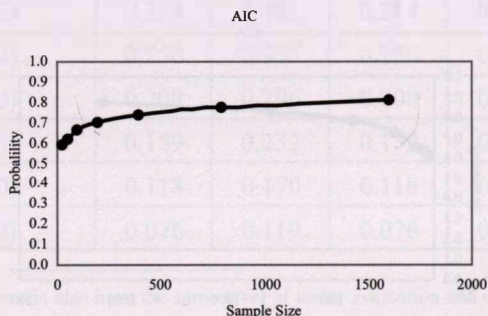
### 3.0 RESULTS AND DISCUSSION

The probability of various criteria in correctly estimating the true order of the AR process is tabulated in Table 1. Generally, Table 1 shows that AIC, SIC, FPE, HQC and BIC perform considerably well in estimating the true autoregressive order, in all cases. For example, for the case where the sample size equals 25, the probability for each of the above criteria, in that order, is 0.591, 0.584, 0.591, 0.603 and 0.596. This means that out of 1000 simulated series of known order, AIC, SIC, FPE, HQC and BIC respectively have correctly identified the true order 591, 584, 591, 603 and 596 times. Thus, 60% of the times, these criteria have successfully selected the true order. Thus, we may conclude that these criteria perform fairly well in picking up the true order.

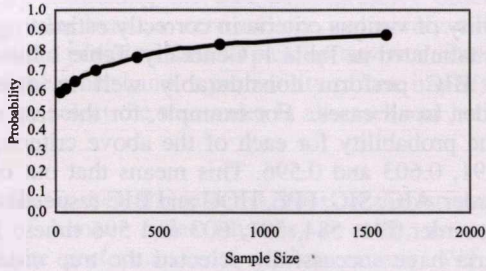
**Table 1: Probability of correctly estimated the true order of AR process.**

Sample Size (Logarithmic Scale)	Order Selection Criteria				
	AIC	SIC	FPE	HQC	BIC
25 (1.40)	0.591	0.584	0.591	0.603	0.596
50 (1.70)	0.619	0.605	0.619	0.632	0.608
100 (2.00)	0.659	0.640	0.659	0.650	0.651
200 (2.30)	0.695	0.700	0.695	0.728	0.709
400 (2.60)	0.734	0.759	0.734	0.782	0.781
800 (2.90)	0.768	0.823	0.768	0.825	0.832
1600 (3.20)	0.803	0.873	0.803	0.873	0.879

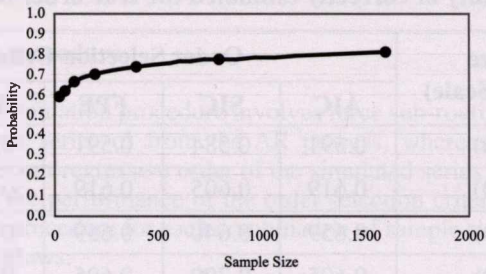
Table 1 also shows that these criteria perform better as the sample size grows. With a sample size of 1600, the probability concerned for each of the same five criteria has reached a value of 0.803, 0.873, 0.803, 0.873 and 0.879 respectively. The observed improvement in performance for each of these five criteria as the sample size grows is clearly depicted in Figure 1.



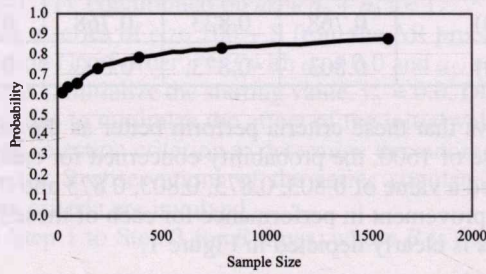
SIC



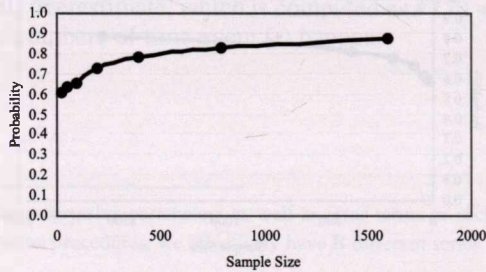
FPE



HQC



HQC



The third finding revealed by Table 1 is that AIC and FPE (both constructed by Akaike) seem to have identical performance in terms of their ability to correctly locate the true order. In fact, a closer inspection of the selected order for each simulated series (results not shown) discovered that they consistently choose the same order at all times<sup>4</sup>. One would expect AIC to improve over FPE as it was proposed by Akaike to overcome the inconsistency of the latter (Akaike, 1973). However, such improvement is not observed in this study.

An interesting question in mind is whether we can identify the best criterion in selecting the AR order. However, it is difficult to just rely on Table 1 regarding this matter, as no criterion is found to consistently perform better than the rest in all cases. Nonetheless, both HQC and BIC do perform substantially better than others, in cases where the sample size is equal to or larger than 200. This suggests that differentiating between order selection criteria may pay off only when one has large enough sample size (200 and above).

Further analysis of the distribution of the selected orders is conducted and the results are summarized in Tables 2 and 3. Table 2 reveals that for a relatively small sample size, AIC, SIC, FPE, HQC and BIC have under-estimated the true order with a probability of around one-third. Nonetheless, the probability of under estimation reduces as the sample size grows, to the extent of a negligible effect for a sample size as large as 1600. This finding is may be clearly seen from Figure 1.

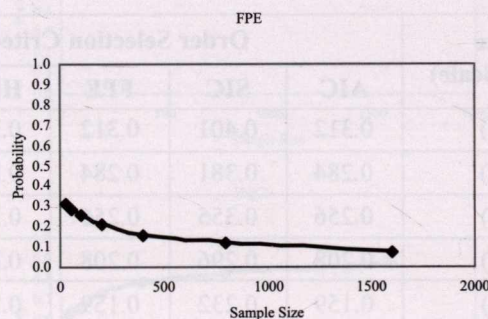
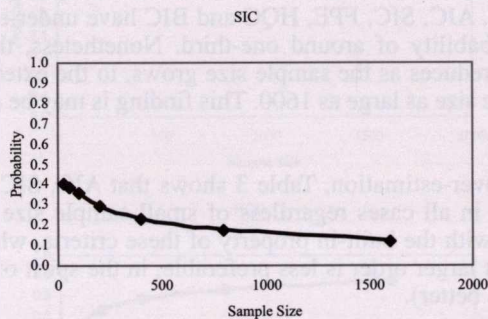
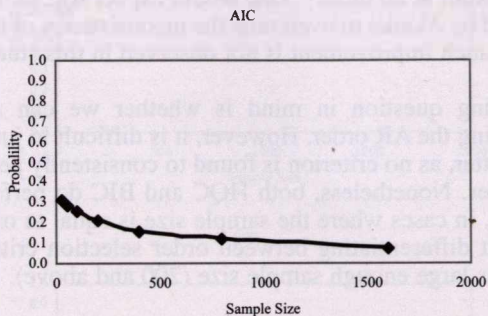
Regarding over-estimation, Table 3 shows that AIC, SIC, FPE, HQC and BIC is negligible in all cases regardless of small sample size. This empirical finding is in line with the built-in property of these criteria, which are designed in such a way that larger order is less preferable, in the spirit of parsimony (that is, the simpler the better).

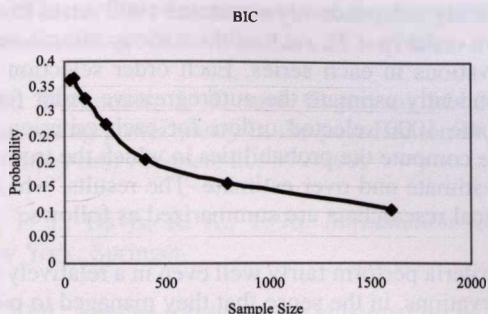
**Table 2: Probability of under estimating the true order of the AR process.**

Sample Size (Logarithmic Scale)	Order Selection Criteria				
	AIC	SIC	FPE	HQC	BIC
25 (1.40)	0.312	0.401	0.312	0.355	0.364
50 (1.70)	0.284	0.381	0.284	0.333	0.368
100 (2.00)	0.256	0.355	0.256	0.316	0.328
200 (2.30)	0.208	0.296	0.208	0.244	0.275
400 (2.60)	0.159	0.232	0.159	0.189	0.204
800 (2.90)	0.118	0.170	0.118	0.146	0.158
1600 (3.20)	0.076	0.119	0.076	0.093	0.107

<sup>4</sup> Hence, these two criteria also have the same level of under estimation and over estimation as will be shown in Tables 2 and 3 later.

**Figure 2: Performances of various criteria in under estimating the true order.**





**Table 3: Probability of overestimating the true order of the AR process.**

Sample Size (Logarithmic Scale)	Order Selection Criteria				
	AIC	SIC	FPE	HQC	BIC
25 (1.40)	0.097	0.015	0.097	0.042	0.040
50 (1.70)	0.097	0.014	0.097	0.035	0.024
100 (2.00)	0.085	0.005	0.085	0.034	0.021
200 (2.30)	0.097	0.004	0.097	0.028	0.016
400 (2.60)	0.107	0.009	0.107	0.029	0.015
800 (2.90)	0.114	0.007	0.114	0.029	0.010
1600 (3.20)	0.121	0.008	0.121	0.034	0.014

#### 4.0 SUMMARY

The determination of autoregressive order for a time series is especially important in economics studies. Various order selection criteria such as the Aikaike's information criterion (AIC), Schwarz's information criterion (SIC), Hannan-Quinn's criterion (HQC), the final prediction error (FPE) and the Bayesian information criterion (BIC) have been employed for a while by researchers in this respect. As the outcomes of these criteria may influence the ultimate findings of a study, a thorough understanding of the empirical performance of these criteria is warranted. This simulation study is specially conducted to shed light on this matter.

The current study independently simulated 1000 series from autoregressive processes of known order ( $p = 3$ ), each of the various sample sizes ranging from 25 to 1600 observations in each series. Each order selection criterion is then allowed to independently estimate the autoregressive order for each simulated series, yielding some 1000 selected orders for each criterion. Based on these selected orders, we compute the probabilities in which the true order is correctly identified, under estimate and over estimate. The results, which provide useful insights for empirical researchers are summarized as follows.

First, these criteria perform fairly well even in a relatively small sample ( $S$ ) with only 25 observations, in the sense that they managed to pick up the correct order about 60% of the time. Second, this performance increases as the sample size grows. Third, with relatively small samples, there is no gain in differentiating between these criteria but HQC and BIC are found to outdo the rest for sample sizes reaching 200 and beyond. Fourth, the probability of underestimation can be as high as one-third in the case of  $S=25$ , but this probability decreases as sample size grows. The problem of overestimation, however, is negligible in all cases.

To conclude, this study finds that depending on the sample size, at least 60% of the time, the order selection under study is able to correctly identify the true order of a given autoregressive process. Overestimation is negligible in all cases, whereas underestimation does occur but this problem tends to reduce as sample size grows. The findings in this simulation study may be taken as useful guidelines for future research in the use of order selection criteria in determining the autoregressive order.

## 6.0 ACKNOWLEDGEMENTS

Constructive suggestions from anonymous reviewers that improve substantially the presentation of this paper are gratefully acknowledged. However, the authors would like to blame each other for any remaining errors.

## 7.0 REFERENCES

Ahmed, M. 2000. 'Money –income and money-price-causality in selected SAARC countries: some econometric exercises'. *Indian Economic Journal*, 48, 55 – 62.

Akaike, H. 1969. 'Fitting autoregressive models for prediction'. *Annals of the Institute of Statistical Mathematics*, 21, 243 – 247.

Akaike, H. 1973. 'Information theory and an extension of the maximum likelihood principle', in B. N. Petrov and F. Csaki (eds.), *Proceedings of 2nd International Symposium on Information Theory Akademiai Kiado, Budapest*, 267 – 281.

Akaike, H. 1979. 'A Bayesian extension of the minimum AIC procedure of Autoregressive model fitting'. *Biometrika*, 66, 237 – 242.

Baum, C.F., Barkoulas, J. T. and Caglayan, M. 2001. 'Nonlinear adjustment to purchasing power parity in the post-Bretton Woods era'. *Journal of International Money and Finance*, 20, 379-399.

Brockwell, P. J., Davis, R. A., 1996. *Introduction to Time Series and Forecasting*. New York: Springer.

Guerra, R. 2001. *Nonlinear adjustment towards Purchasing Power Parity: the Swiss France - German Mark case*. Working Paper, Department of Economics, University of Geneva.

Hannan, E. J. and Quinn, B. G. 1979. 'The determination of the order of an autoregression'. *Journal of Royal Statistical Society*, 41, 190 – 195.

Liew, K. S. 2000. *The performance of AICC as order determination criterion in the selection of ARMA time series models*. Unpublished thesis, Department of Mathematics, Universiti Putra Malaysia.

Sarantis, N. 1999. 'Modelling non-linearities in real effective exchange rates', *Journal of International Money and Finance*, 18, 27 – 45.

Sarantis, N. 2001. 'Nonlinearities, cyclical behaviour and predictability in stock markets: international evidence'. *International Journal of Forecasting*, 17, 439 – 482.

Schwarz, G. 1978. 'Estimating the dimension of a model'. *Annals of Statistics*, 6, 461 – 464.

Tan, H. B. and Baharumshah, A. Z. 1999. 'Dynamic causal chain of money, output, interest rate and prices in Malaysia: evidence based on vector error-correction modeling analysis', *International Economic Journal*, 13, 103 – 120.

Taylor, M. P. and Peel, D. 2000. 'Nonlinear adjustment, long-run equilibrium and exchange rate fundamentals.' *Journal of International Money and Finance*, 19, 33-53.

Xu, Z. 2003. 'Purchasing power parity, price indices, and exchange rate forecasts'. *Journal of International Money and Finance*, 22, 105 – 130.

Yamada, H. 2000. 'M2 demand relation and effective exchange rate in Japan: a cointegration analysis'. *Applied Economics Letters*, 7, 229 – 232.