

**UNIVERSITI TEKNOLOGI MARA**

**HYBRID CONJUGATE GRADIENT  
METHOD UNDER EXACT LINE  
SEARCH FOR AN ULTRASOUND  
INVERSE PROBLEM**

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## ABSTRACT

The Conjugate Gradient (CG) method is one of the popular methods that many researchers often use in solving unconstrained optimization problems. It has been proven that the CG method is preferred for solving optimization problems due to its simplicity, low memory requirements, and global convergence properties. While the CG method is widely used for solving large-scale problems, this method often faces limitations in real-world applications due to slow convergence and a lack of robustness in the presence of numerical tests. These problems reduce its efficiency and robustness when applied in complex systems. Therefore, this research focuses on the performance of the hybrid CG method under exact line search and its implementation in the ultrasound inverse problem. There are four hybrid CG coefficients used in this study, which are Polak-Ribiere-Polyak (PRP) as the main CG coefficient to hybrid with Hestenes-Stiefel (HS), Dai-Yuan (DY), Norrlaili-Rivaie-Mustafa-Ismail (NRMI) and Linda-Aini-Mustafa-Rivaie (LAMR). Fifteen test functions with different initial points and variable ranges from 2, 4, 10, 100, 1000 and 10,000 are selected to conduct the numerical tests. The numerical results are measured based on the number of iterations (NOI) and CPU time. These four hybrid CG coefficients successfully solve all fifteen test functions, thereby determining their efficiency and robustness. Therefore, the PRPLAMR coefficient is determined as the best hybrid CG coefficient in terms of lesser number of iterations (NOI) and central processing unit (CPU) time. Lastly, the PRPLAMR coefficient is implemented in ultrasound inverse problems, achieving excellent performance with the PSNR value of 27.03 dB and the SSIM value of 0.9079 under a low noise level.

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