# UNIVERSITI TEKNOLOGI MARA

# GENERALISED L-R INTUITIONISTIC FUZZY NUMBER FOR RIVER WATER POLLUTION CLASSIFICATION

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# ABSTRACT

Real-world problems are full of uncertainties, and in most cases, decisions are made in a situation of uncertainty. Uncertainty occurs due to ambiguous, vague, inconsistent, and imprecise information. Therefore, in order to model the uncertainty information comprehensively, a mathematical set theory is needed to cater all the information. The current intuitionistic fuzzy number still lacks in the comprehensiveness of decision-making evaluation due to the limitation of its left and right functions that only use linear functions as the left and right functions. Therefore, the L-R intuitionistic fuzzy number (LRIFN) introduces a more comprehensive approach by incorporating non-linear functions for left and right membership and non-membership functions, showing its capacity to represent the human thinking (decision-maker) which does not always linear. However, the existing L-R intuitionistic fuzzy number does not involve the decision-makers' perspective which has different levels of knowledge, experience, and background. Therefore, this research aims to introduce a generalised L-R intuitionistic fuzzy number (GLRIFN), which consider the different heights of the core for membership and non-membership degrees which can be determined by the confidence, reliability, or sureness level of the decision-maker. The GLRIFN, a generalisation of LRIFN, is more comprehensive and reliable for a real problem in resolving uncertainty information in the decision-making process. Other than that, the C++ programming language is developed to ease the interpretations and analysis of union and intersection operations of the GLRIFN. Furthermore, developing relation properties of GLRIFN is crucial to enhance the capability of handling complex uncertainty information in GLRIFN. Moreover, addressing uncertainty information effectively requires a comprehensive understanding of the underlying principles of GLRIFN. Therefore, more significant finding of the research must be determined with uncertain information such as arithmetic operations, aggregation operator, distance measure, and similarity measure. In addition, this research proposed the GLRIFN based TOPSIS (GLRIFN-TOPSIS) for river water pollution classification for several rivers in Johor, Malaysia, namely Kim Kim River, Sayong River, Telor River, Pelepah River, and Bantang River from 2019 to 2021. Overall result consistently shows that the Kim Kim River is the most polluted river classified in this research. The GLRIFN-TOPSIS is appropriate, flexible, and realistic for classifying river water pollution. Finally, GLRIFN allow for better uncertainty quantification by considering the confidence level. The adaptability and precision of GLRIFN make them applicable not only to environmental assessments but also to a broader range of fields where uncertainty quantification is crucial for informed decision-making.

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# TABLE OF CONTENTS

# CHAPTER 1
# INTRODUCTION

## 1.1 Research Background

Real-world challenges are frequently uncertain, and decisions usually have to be made under these uncertain circumstances. The challenges may come from uncertainty, complexity of decisions, inherent unpredictability, et cetera. The complexity arises from the decisions that are frequently formulated within an uncertain environment. This is due to the inherent unpredictability of various factors. Uncertainty introduces an element of risk, making decision-making a challenging task for individuals and organisations alike. Uncertainty means the events whose results cannot be foreseen precisely ahead of time (Liu, 2007). For example, the prediction of "tossing a coin," "rolling a die," "stock price," et cetera, which will not be precise in the future. Decision-making is the process of selecting a possible course of action from all available alternatives (Khani & Rainer, 2016; Lai et al., 1994). It involves assessing information, evaluating potential outcomes, and ultimately choosing the most suitable option based on a combination of rational analysis, personal values, and situational considerations (Sola, 2018).

Uncertainty occurs due to ambiguous (being open to multiple interpretations), vague (lacking clarity and specifics), inconsistent (showing contradictions or lack of agreement), and imprecise (lacking accuracy and precision) information. In such a manner, uncertainty and information are very closely related. Therefore, in order to model the uncertainty information, a mathematical set theory is needed to cater all the information. However, until now, there is still a debatable issue among scholars regarding the best mathematical model to cater for the problem of uncertainty. In line with the issue, Zadeh (1965) introduced the fuzzy set with a single membership function, $\mu_A(x)$, that expresses the membership function or truth information. A fuzzy set is defined as those sets whose boundaries are not clear. In fuzzy set, the range of values of the membership function is the closed unit interval [0,1].

After that, the double membership functions exist, which is an intuitionistic fuzzy set (IFS) that considers the membership and non-membership functions. The intuitionistic fuzzy set, which is the extension of fuzzy set that has been introduced by