Dynamic Economic Dispatch Solution Using Particle Swarm Optimization

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Abstract - **This paper presents particle swarm optimization (PSO) technique to solve the dynamic economic dispatch (DED) problem for the determination of the optimal schedule of output powers from differents units (generator) at the minimum cost possible over a certain period. The total of the output power will satisfy the load demand for a period that has been divided into a number of small intervals of 24 hours. The DED is a dynamic problem of economic dispacth that takes into the consideration of the ramp rate limits of the generating units. The results showed that PSO technique was capable of obtaining high quality solution with a great efficiency for DED problems.**

Keywords: Dynamic Economic Dispatch (DED), Particle Swarm Optimization (PSO), Ramp Rate Limits.

I. INTRODUCTION

Dynamic economic dispatch (DED) is a dynamic problem of power system due to the large variation of load demand. The main objective of DED is to determine the optimal output power generation from different units (generator) that meet the load demands at minimum cost for a certain period of time interval while satisfying all the constraints. DED is not only the most accurate formulation but also the most difficult dynamic optimization process[1]. DED is an extension of the static economic dispatch (SED). SED can only handle a single load level and may fail to deal with large variations of the load demand due to the dynamic constraints of the generator which called ramp rate limit that is used to maintain the life of the generators[2].To overcome this problem, DED is implemented by dividing the economic dispatch period into a number of small time intervals under the ramp rate constraints. [3].

Ramp rate limit refers to dynamic constraints of a generator that makes the DED problem become more complex.This dynamic problem gives the differences between SED and DED because SED only consider the operational limit which include minimum and maximum power generation of each unit while DED consider both operational limit and ramp rate limit which include ramp up and ramp down of a generator for a period of time. Due to the ramp-rate constraints of a generator, the operational decision at hour t may affect the operational decision at a later hour[4].

A survey of literature on the DED solution methods showed that various traditional mathematical optimization and artificial intelligent (AI) techniques have been applied to solve the DED problem. The traditional mathematical methods are such as linear programming (LP), nonlinear programming (NLP) and quadratic programming (QP). It is observed that the traditional mathematical methods have some limitations to solve DED problems. The traditional methods suffer with large execution time and the methods are unable to find the optimal solution within the reasonable execution time due to their programming charateristics[5].

The methods based on AI techniques such as algorithm (GA), particle swarm optimization (PSO), simulated annealing (SA), and Differential Evolution (DE) also have been developed. Results showed that they are more effective than the traditional mathematical methods in solving DED with ramp rate constraints [6].

In this paper PSO has been applied to fullfill the objectives of the DED problem. The constraints that are considered for the proposed method are the operating limit, real power balance constraint with transmission losses and ramp rate limits. The optimization algorithm has been succesfully implemented in MATLAB programming using Intel(R) Core(TM)2 Duo CPU, 2.20GHz with 3GB RAM personal computer. The proposed technique was tested on a 26-bus system containing 6 generator units, and 46 transmission lines.

II. PROBLEM FORMULATION

A. Objective function

The main objective of DED is to determine the optimal schedule of generator unit with minimum cost possible over the entire period while satisfying all the constraints.The objective function of DED is formulated as below:

$$\min C_T = \sum_{t=1}^{T}\sum_{i=1}^{n} C_{it}(P_{it}) \qquad (1)$$

Where $t = 1,2, \dots \dots T$

Where C_T is the total operating cost over the whole dispatch period T ; t is the number of hours in period T; n is the number of generator units; C_{it} (P_{it}) is the fuel cost of i^{th} unit at time t in \$/h and P_{it} is the real power output of generating unit i at time period t in MW.
The fuel cost C_i of generating unit i at any time interval t is normally expressed as a quadratic function as

$$C_i(P_{it}) = a_i + b_i P_{it} + c_i P_{it}^2 \quad (2)$$

Where a_i, b_i and c_i are the cost coefficients of generating i^{th} unit.

B. Constraints

The objective function above is minimized subject to variety of below constraints.

1) Equality Constraint

This constraint is based on the principle of power balance equation that total generation of real power (MW) at any time t should satisfy the load demand at time t and also the transmission power loss at time t. This constraint is mathematically expressed as

$$\sum_{i=1}^{n} P_{it} = P_{Dt} + P_{Lt} \quad (3)$$

Where P_{Dt} is the total power demand at time t and P_{Lt} is the total transmission loss at time t.

The general form of transmission loss formula based on Kron's loss formula is given by

$$P_{Lt} = \sum_{i=1}^{n} \sum_{j=1}^{n} P_{it} B_{ij} P_{jt} + \sum_{i=1}^{n} B_{oi} P_{it} + B_{oo} \quad (4)$$

Where Bij , Boi, Boo are the transmission loss coefficients.

2) Inequality Constraint

Also called real power operating limit, the constraint are as follows

$$P_{it}\min \leq P_{it} \leq P_{it}max \quad (5)$$

Where $P_{it\,min}$ and $P_{it\,max}$ is the minimum and maximum real power output of generating unit i at time period t in MW.

3) Dynamic Constraint

The dynamic constraint due to ramp rate limit of generating units are given by[7]

a) When generation increases

$$P_{it} - P_{i(t-1)} \leq U_{Ri}$$

b) When generation decreases

$$P_{i(t-1)} - P_{it} \leq D_{Ri} \quad (6)$$

The constraint from (5) after including ramp rate limit can be described by

$$(P_{it\min}, P_{i(t-1)} - D_{Ri}) \leq P_{it} \leq (P_{it\max}, P_{i(t-1)} + U_{Ri}) \quad (7)$$

Where $P_{i(t-1)}$ is the previous real power output of generating unit i; U_{Ri} is the ramp up limits of *ith* unit in MW/h and D_{Ri} is the ramp down limits of *ith* unit in MW.
To modify the real power output P_{it}, the formula below is used [8],

$$P_{it} = P_{i(t-1)} - D_{Ri}\text{, if } P_{it} < P_{i(t-1)} - D_{Ri} \quad (8)$$

and

$$P_{it} = P_{i(t-1)} + U_{Ri}\text{, if } P_{it} > P_{i(t-1)} + U_{Ri} \quad (9)$$

4) Fitness Function

The fitness function formula[9] that consists of total fuel cost function, C_T and power balance constraint P_b as in Equations (2) and (3) is shown below :

$$Fitness = \frac{1}{C_T + P_{bt}} \quad (10)$$

Where $P_{bt} = \sum_{i=1}^{n} P_{it} - P_{Dt} - P_{Lt}$

III. METHODOLOGY

A. Particle Swarm Optimization (PSO)

Kennedy and Eberhart developed a PSO algorithm based on the behavior of individual (particles) of a swarm inspired from fish schooling and bird flocking[10]. It uses a number of particles that form a swarm that randomly flying in the search space looking for the global optimal position to get food. In a PSO system, particles fly around in a multidimensional search space. It has been observed that members within the group particles share the information experiences among them and this will lead to more faster searching time and accuracy. During the flight, each particles adjusts its position according to its own experience and the experience of neighboring member particles and making use of the best position encountered by itself and its neighbors to search the food[11]. The particle which at present is globally the best particle producing are set to be the best performance at that time and it represent the minimum of the cost function achieved so far. Assume that x and v denote a particle position and its corresponding flight speed (velocity) in a search space respectively. The *ith* particle is represented as $X_i = (X_{i1}, X_{i2}, \dots X_{id})$ in the dimensional search space.The best previous position of a particle is recorded and represented as

$$pBest_i = (pBest_{i1}, pBest_{i2}, \ldots .pBest_{id})$$

The index of the best particle among all the particles in the group is represented as $gBest_d$. The rate of velocity for the particle *i* is represented as

$$v_i = (v_{i1}, v_{i2}, \ldots .v_{id}).$$

Mathematically,velocities of the particles are modified according to the following equation :

$$V_{ij}^{k+1} = V_{ij}^{k} + C_1(pBest - X_{ij}^{k})R_1 + C_2(gBest - X_{ij}^{k})R_2 \qquad (11)$$

Where $i = 1, 2\ldots.n$ and $j = 1, 2\ldots.d$

To ensure the convergence of PSO in obtaining optimal solution, Eberhart and Shi indicate that application of a constriction factor may be necessary[12]. The constriction factor is formulated as below:

$$V_{ij}^{k+1} = K[V_{ij}^{k} + C_1(pBest - X_{ij}^{k})R_1 + C_2(gBest - X_{ij}^{k})R_2] \qquad (12)$$

Where

$n = number\ of\ particles\ in\ d$
$d = dimensional\ search\ space$
$K = constriction\ factor$
$V_{ij}^{k+1} = the\ updated\ particle\ velocity$

$V_{ij}^{k} = velocity\ of\ ith\ particle\ at\ iteration\ k$
$X_{ij}^{k} = current\ position\ of\ ith\ particle\ at\ iteration\ k$
$k = pointer\ of\ iterations(generation) so\ far$
$pBest = best\ position\ of\ the\ ith\ particles\ recorded$
$gBest = best\ position\ of\ the\ group\ until\ iteration\ k$
$C_1, C_2 = acceleration\ coefficients$
$R_1, R_2 = random\ number\ in\ the\ range\ (0,10)$

and

$$K = \frac{2}{|2 - C - \sqrt{C^2 - 4C}|} \qquad (13)$$

Where $C = C_1 + C_2$ and $C > 4$

Each individual particle update its current position to the next new position according to the following equation:

$$X_{ij}^{k+1} = X_{ij}^{k} + V_{ij}^{k+1} \qquad (14)$$

Where X_{ij}^{k+1} is the updated particle's position.

B. *DED based on PSO Technique*

The following steps describe how PSO was implemented to the DED problem.

Step 1: *Initialization of parameters*
Specify all the PSO parameters including number of particles *n*, acceleration constants *C1* and *C2*, particle's velocity and position limit, *Xmin, Xmax, Vmin* and *Vmax*, the operating limit, ramp rate limit, load demand, B-coefficients and the fuel cost coefficients of each unit.

Step 2: Initialize the particle's velocities and position in random manner. In this problem, the real power outputs of each units are taken as the particles in PSO. A particle will represented the real power output for 6 units of the generator. The particles are randomly generated between the maximum and minimum operating limits for all the generators according to (5). Losses are ignored at this step.

Step 3: *Fitness evaluation*

a) Check the real power output for each unit from the particle using operating limit equation from (5).
b) Calculate P_L using (4) with the B-loss coefficients.
c) Calculate the total output power from each unit after considering P_L. Check the total ouput power with the load demand at that time.
d) Calculate the fuel cost using (2).
e) Evaluate the fitness of each particles using (10).

Step 4: Set the pBest from the fitness evaluation which is refer to the minimum total fuel cost of that time.

Step 5: Set the gBest from the best value among the pBest.

Step 6: *Optimization of the particles.*

a) Modify the velocity of each particles according to equation (12). Evaluate the velocity according to the V_{min} and V_{max} from the Table 5. If the $V_{ij} < V_{min}$, then $V_{ij} = V_{min}$ and if the $V_{ij} > V_{max}$, then $V_{ij} = V_{max}$.
b) Modify the position of each particles using equation (14). Evaluate the new position according to the X_{min} and X_{max} from the Table 5. If the $X_{ij} < X_{min}$, then $X_{ij} = X_{min}$ and if the $X_{ij} > X_{max}$, then $X_{ij} = X_{max}$.

Step 7: Repeat Step 3 while considering the ramp rate constraint.The new position of the particles which refer to the real power ouput for each unit must satisfy the ramp rate constraints described by Equations (6) and (7). If the real power output generating break the limit, then it must be modified towards the near margin of the acceptable solution by Equations (8) and (9).

Step 8: Evaluate the fitness of the new positon of each particles. Check if the current value pBest is greater than the previous pBest, set the new value as the pBest. Then next, if the new pBest is greater than previous gBest, set it as the new gBest.

Step 9: Check the number of iterations,*itermax* which was set in Table 5. If it reaches the maximum,go to Step 9. Otherwise, the optimization process will be repeated start from Step 3.

Step 10: The individual of particles that generates the latest gBest is the optimal generation of real power output from each unit with the minimum of total generation cost at time *t*.

Flow chart for the DED based on PSO step is described in Figure1 below:

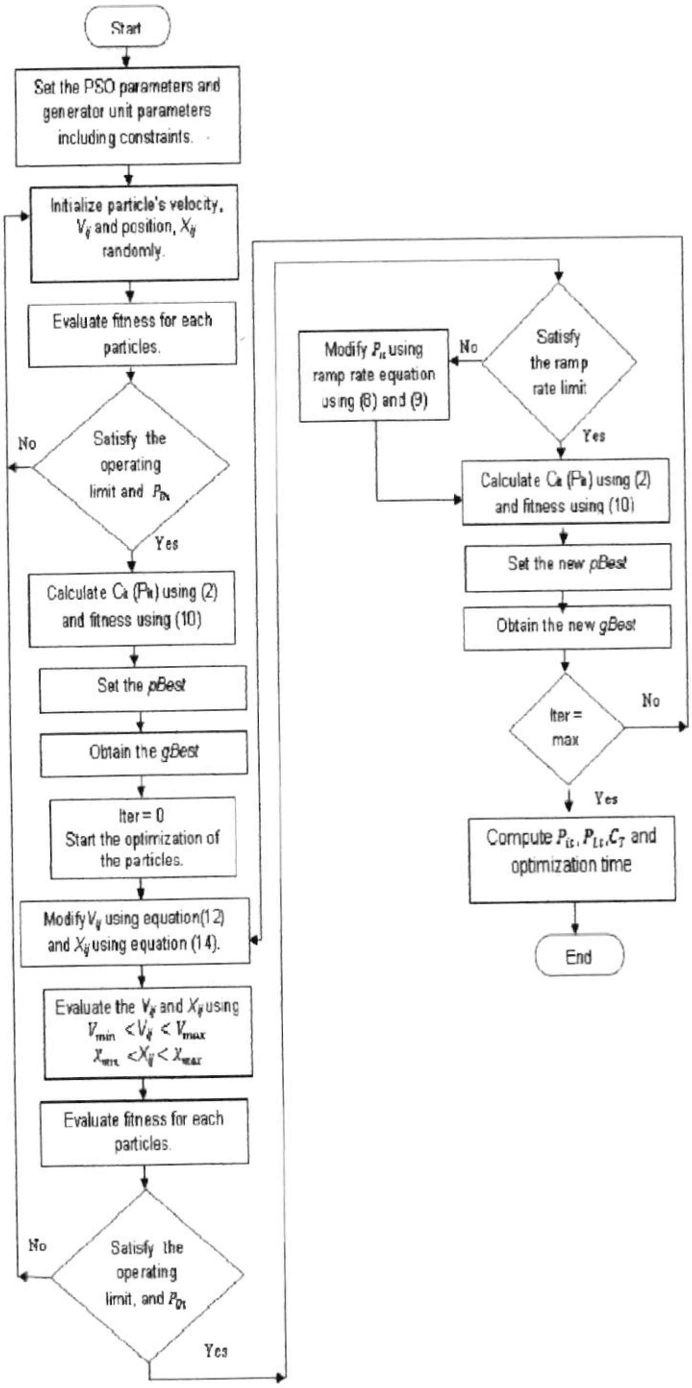

Figure 1. Flow chart for DED based on PSO

IV. RESULT AND DISCUSSION

A. Description of Test System

In order to justify the proposed method, the dynamic constrained economic dispatch was solved for IEEE 26-bus system and results are presented in this section. There are 6-generator units, 26 buses and 46 transmission lines.The cost coefficients and power generation limits for the test case are given in Table 1, ramp limits are given in Table 2 and the load demand for the time intervals of 24 hours is given in Table 3. Program in MATLAB was developed to perform DED using PSO and executed using Intel(R) Core(TM)2 Duo CPU, 2.20GHz with 3GB RAM personal computer. Losses are included in this case. The B-loss coefficients in p.u. on a 100 MVA base is given in Table 4 and are obtained from Newton-Raphson power flow solution. .

Table 1. Generating Unit Capacity and Coefficients

Unit	P_i^{min} (MW)	P_i^{max} (MW)	a_i ($/MWh^2)	b_i ($/MWh)	c_i
1	100	500	0.0070	7.0	240
2	50	200	0.0095	10.0	200
3	80	300	0.0090	8.5	220
4	50	150	0.0090	11.0	200
5	50	200	0.0080	10.5	220
6	50	120	0.0075	12.0	190

Table 2. Ramp Rate limits

Unit	P_i^0 (MW)	UR_i (MW/h)	DR_i (MW/h)
1	440	80	120
2	170	50	90
3	200	65	100
4	150	50	90
5	190	50	90
6	110	50	90

Table 3. Load Demand for 24 Hours

Hour	1	2	3	4	5	6	7	8
Load	955	942	935	930	935	963	989	1023
Hour	9	10	11	12	13	14	15	16
Load	1126	1150	1201	1235	1190	1251	1263	1250
Hour	17	18	19	20	21	22	23	24
Load	1221	1202	1159	1092	1023	984	975	960

Table 4. B-loss coefficients

$$B_{ij} = \begin{bmatrix} 0.0017 & 0.0012 & 0.0007 & -0.0001 & 0.0005 & -0.0002 \\ 0.0012 & 0.0014 & 0.0009 & 0.0001 & -0.0006 & -0.0001 \\ 0.0007 & 0.0009 & 0.0031 & 0.0000 & 0.0010 & -0.0006 \\ -0.0001 & 0.0001 & 0.0000 & 0.0024 & -0.0006 & -0.0008 \\ -0.0005 & -0.0006 & -0.0010 & -0.0006 & 0.0129 & -0.0002 \\ -0.0002 & -0.0001 & -0.0006 & -0.0008 & -0.0002 & 0.0150 \end{bmatrix}$$

$$B_{oi} = 1.0e^{-3}[-0.3908 \quad -0.1297 \quad 0.7047 \quad 0.0591 \quad 0.2161 \quad -0.6635]$$

$$B_{oo} = 0.056$$

The PSO parameters are selected to achieve best solution and are set as follows:

Table 5. PSO parameters

Number of Particles, *n*	30
d	6
itermax	1000
x_{min}	1
x_{max}	0
Vmax	1
Vmin	-1
C_1	2.05
C_2	2.05

B. Simulation Results

The optimal dispatch of real power output generation (MW) associated with power loss (MW) and optimal fuel cost with optimization time for the given load demand of 24 hours using the proposed method was summarize in Table 6. The sum of total generating power in each interval satisfies all the constraints given by (3), (5) and (7).

Table 6. Optimal MW Generation for each unit, Transmission loss and Fuel Cost for 24 hours

Hours	Load Demand (MW)	P1 (MW)	P2 (MW)	P3 (MW)	P4 (MW)	P5 (MW)	P6 (MW)	PL (MW)	Fcost $/h
1	955	378.7865	120.9963	214.3768	81.23653	113.2434	54.36775	8.007231	11429.95
2	942	378.3374	121.7369	210.5498	78.83131	105.9282	54.42426	7.807122	11267.54
3	935	375.2674	121.7826	208.0915	78.54844	109.0326	50.00279	7.724556	11178.16
4	930	370.3298	118.356	210.3912	79.05634	109.3992	50.10954	7.642126	11116.11
5	935	375.2674	121.7826	208.0915	78.54844	109.0326	50.00279	7.724556	11178.16
6	963	377.654	123.316	214.5862	91.20177	114.244	50.0198	8.021848	11529.03
7	989	370.3366	122.3909	219.8512	103.015	130.5098	51.26202	8.356091	11862.5
8	1023	395.6922	138.08	225.627	98.63225	123.9003	50.04816	8.979677	12327.16
9	1126	421.4357	150.8363	246.0532	110.028	143.2666	65.06689	10.68678	13624.46
10	1150	423.1397	153.2557	244.3766	120.0567	147.1314	72.98482	10.94493	13939.85
11	1201	429.1572	163.5467	256.7596	124.4787	160.1319	78.8727	11.94573	14617.06
12	1235	443.5756	163.1349	259.8921	144.1797	155.5566	80.91339	12.25231	15073.55
13	1190	435.8808	158.0042	247.2453	134.4319	154.6414	71.34332	11.5465	14470.82
14	1251	434.8715	173.7079	256.1604	147.6677	163.7646	87.36051	12.53261	15289.8
15	1263	500	163.0229	252.8439	124.0891	158.5029	77.79943	13.25741	15475.07
16	1250	499.7515	162.9188	250.2812	125.7388	153.654	70.62552	12.96956	15301.6
17	1221	444.5517	166.6264	257.8312	123.1841	153.9856	87.11701	12.29395	14885.59
18	1202	425.6404	160.9045	254.6463	134.1853	158.4967	79.90849	11.78087	14630.84
19	1159	425.2159	157.3386	245.1608	118.555	152.1124	71.79838	11.18122	14058.67
20	1092	395.9927	140.87	227.4026	86.55751	131.9793	119.6921	10.49385	13223.62
21	1023	395.6922	138.08	225.627	98.63225	123.9003	50.04816	8.979677	12289.41
22	984	385.9431	128.0097	214.8595	93.81221	119.726	50	8.350427	11793.51
23	975	378.2419	129.372	213.2962	93.05378	118.8416	50.3888	8.194237	11680.41
24	960	381.037	120.0404	215.7035	87.26036	113.9982	50	8.039605	11491.2

The solution of DED using PSO was compared to the solution using Newton Raphson method [13] in Table 7. The Newton Raphson method test system data is similar to the one used in this paper that is 26 buses system containing 6 generator units and 46 transmission lines. The comparison was analyzed at the hour 1 of 955MW load demand. From the result, while satisfying the real power balance constraint, the total fuel cost for PSO method is 11429.95 $/h and total fuel cost for Newton Raphson method is 11448.43$/h. From the comparison it can be seen that DED using PSO solution has a very good accuracy with high quality result because the total fuel cost for PSO method is lower than total fuel cost for Newton Raphson method. Moreover, the total transmission loss, P_L using PSO method is also lower than the Newton Raphson method. On other word, PSO method is very reliable in determining the solution for DED problem that is to determine the optimum real power generation output of each unit and minimize the total fuel cost.

Table 7. Comparison of PSO and Newton Raphson Result

Method	**PSO**	**Newton Raphson**
$P_{demand}(MW)$	955	955
$P_1(MW)$	378.7865	382.2625
$P_2(MW)$	120.9963	125.2776
$P_3(MW)$	214.3768	215.054
$P_4(MW)$	81.23653	88.2079
$P_5(MW)$	113.2434	103.6762
$P_6(MW$	54.36775	50
Total Generation, P_T (MW)	963.0072	964.4782
$P_L(MW)$	8.007231	9.47816
Fuel Cost ($/h)	11429.95	11448.43

For further analysis, the number of particles has been varied for other 3 values which are 40, 50 and 60 in order to evaluate the effect of number of particles towards the real power generation output and the total fuel cost, C_T . As shown in the Table 8, the solution showed that increasing the number of particles will result in a decreasing of the total fuel cost and its confirms that, PSO method is capable to give a high quality solution to DED problem.

Table 8. Result for the Variation Number of Particles.

Number of particles, n	Total Fuel Cost, C_T($/h)
40	313734.1
50	310456.2
60	307456.6

V. CONCLUSION

In this paper, Particle Swarm Optimization (PSO) technique was implemented to solve the dynamic economic dispatch (DED) problem. The dynamic characteristic of the generator unit that is ramp rate limit is considered for the unit generation operation in the proposed method. The DED planning must gives the optimal generation dispatch while satisfying the system load demand and the operational constraints for generator unit including the ramp rate limit.The results show that the proposed method was capable of obtaining high quality solution with a great efficiency in DED problems in determining the optimal schedule of output powers from differents generator units at the minimum cost possible over the 24 hours period interval of load demand.

VI. RECOMMENDATION

There are several addition and development that can be done on DED problems in order to have high quality and accurate solutions. The improvement that can be done is such as taking into account other generator constraints such as spinning reserve requirement and emission constraint. All the constraints will give more accurate result to the solution of DED problems. In addition, accurate modeling of DED problem will be improved when the valve point loadings effects in the generating units are taken into account. Valve point effect are are usually modelled in two form which is i)consider the prohibited zones as the inequality constraint and ii) implement the effect as the non-smooth cost function for the fuel cost function[14].

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