A Modified Integer Programming Model for Cleaning Service Scheduling

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ABSTRACT

This paper analyzes the problem faced by a cleaning service company, which records and pays out a large amount of overtime each month. With only one shift per day, the labor force is required to work overtime in order to satisfy the daily work demand. The objective of this paper is to determine the optimum schedule to satisfy the labor requirements at a minimum cost. Tiberwala et al.'s (1972) single-shift integer programming model for scheduling workers with two consecutive days off per week has been used as the starting point for this research. This paper modifies the model to incorporate two and three shift schedules with one day off per week. Three alternative workforce schedules are then compared and evaluated. The first alternative maintains the traditional one-shift schedule, currently being practiced by the company. The second and third alternatives switch to a two-shift schedule and three-shift schedule, respectively, with overlapping working hours linking the shifts. In comparison to the existing one-shift schedule, the three-shift schedule is expected to produce savings of 21.4 per cent equivalent to RM 23,592 annually.

***Keywords**: Scheduling, Integer Programming, Cleaning Services, Workforce*

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Introduction

In today's competitive business environment, cleaning service companies strive for quality and cost-effectiveness. Efforts are directed towards optimizing the service systems in order to better utilize resources and become more cost-effective. Service systems are heavily affected by their worker scheduling systems, which are designed to maintain the company's operation at a minimum cost. Reducing the cost of operation requires optimal use of the required and available resources.

Scheduling is a key business component, which influences the utilization of cleaning resources in that it is used to identify the resources required for the cleaning jobs. A schedule is then developed based on available resources and job demands. Essentially scheduling controls the utilization of the cleaning resources.

Labor scheduling organizes work so it can be executed in the most efficient manner. It requires getting the right people in the right place to perform the required task. The benefits of proper scheduling include cost savings due to efficient use of labor hours, increased service satisfaction from faster execution of jobs and reduced stress from better work performance. In short, labor scheduling ensures the allocation of needed resources and the sequence in which they are needed so that job demands can be fulfilled in the shortest time with the least cost.

The cleaning services company considered in this paper deals primarily with daily cleaning services that are necessary to keep buildings orderly and clean. Specifically, the company is obliged to clean a market building daily and currently operates seven days a week in one single shift, which starts at 0700 and concludes at 1500. Currently, workers are on duty six days a week, which may or may not include weekends.

In order to remain competitive in the industry, there is a need for the management to handle ever-increasing cleaning job demands with limited human resources. After working hours workers are allowed to complete their remaining cleaning tasks on an unrestricted overtime basis until 2100 every day. The workers receive their monthly basic salary of RM 600 on top of which they also receive overtime pay on an hourly basis of RM 2.90 and RM 5.80 for a normal day and for a public holiday, respectively.

The current working system is implemented based solely on the management's observations and its discretion; no systematic methods have been implemented in scheduling the workers. Hence, the company has experienced excessive operating costs due to unlimited overtime payments to the workers. A consistent increase in labor costs requires the management to determine the best workforce schedule to respond to present challenges and optimize effectiveness and efficiency.

This paper presents a mathematical model for cleaning workforce scheduling at the aforementioned cleaning service company. The main objective of this project is to determine the optimum working schedule that satisfies the labor requirements at a minimum operating cost. The other objective is to determine the operating cost difference between the present system and the proposed shift system.

Based on the study of job demands, overtime costs, current workforce size and other relevant data, it is recommended that the company switches to a three-shift schedule system. In comparison to the existing one-shift schedule, a three-shift schedule is expected to produce savings of 21.4 per cent equivalent to RM 23,592 annually.

Literature Review

Perpetually accelerating industrial growth and increasing labor costs have led to continued interest in personnel scheduling. Various research works have used mathematical optimization methods to model staffing problems.

The work of Baker and Magazine [1] provides early examples of using mathematical programming in solving rostering problems. The work considers the maximum work stretch constraint, but gives limited recognition of varying demand with the day of the week. Numerous works have since been performed on scheduling, which apply Integer Programming (IP) to find an optimal solution, such as Judice *et al.* [2], who proposed an integer programming formulation to minimize the costs of human resources needed in a process, which was linked to the lot sizing production plan, by matching staff to the work requirements. The result provided an important decision making support system for the company in question.

Hung [3] has discussed the condition needed for a feasibility of a labor mix problem and he presented a simple one-pass method that provides the least cost labor mix. Billionnet [4] has extended Hung's work by solving the problem using integer programming approach. He considered a hierarchical workforce in which a higher qualified worker can substitute for a lower qualified one, but not vice versa. The finding of the study has shown that the mathematical programming language (MPL) and the integer programming solver XA are able to provide an immediate implementation with low computation times. The work was further extended by Seckiner [5] who studied the hierarchical workforce problem under compressed workweeks. If in Billionnet's model, all workers are assigned to one shift type, Seckiner has modified the integer programming model by allowing alternative shifts in a day during the week, with the aim to save worker costs. Besides cost reduction, the multiple-shift model also provides benefits such as commuting reduction and creates more balance work-life proportion for workers.

Burke *et al.* [6] presents a hybrid multi-objective model that combines IP and variable neighborhood search (VNS) to deal with highly-constrained nurse rostering problems in modern hospital environments. The problem was formulated using a multi-objective IP model to solve easy handled sub problem by only including the constraints. An alternative heuristic model was then formulated and a basic VNS approach was proposed to improve the IP solution. The results demonstrate that the hybrid approach is able to outperform other approaches in solving today's complex hospital environments.

Alfares [7] described a real life aircraft maintenance labor scheduling study. The objective was to determine the optimum maintenance workforce size and schedule to satisfy labor requirements with minimum cost. The approach utilized a unique IP formulation to obtain an optimum seven-day workweek schedule with no increase in workforce size. Alfares [8] applied a similar approach, using an algorithm, to obtain an optimum seven-day work schedule for Afam Power Station.

The obvious advantage of using IP techniques is its ability to determine whether the problem is feasible or not. During analysis IP incurs less interruption in obtaining an optimal solution and any adjustment needed in the future can be readily incorporated into the model in the form of constraints. In addition, the IP formulations naturally allow the incorporation of numeric constraints and objectives into planning domains [9].

Modeling

The data for this project was collected on daily basis from April to July 2008 for which two working hour durations were considered. The first is normal working hours, which starts at 0700 until 1500, and the second is overtime, which starts at 1500 until 2100.

In order to design a working shift that satisfies the labor requirements, one needs to determine the daily labor demands for every hour for each day of the week. This was accomplished by considering the daily morning market that normally ends in the afternoon and the large volume of shoppers at weekends. In addition, the cleaning supervisor was asked to estimate the number of laborers required to satisfy the labor requirements by considering the daily activities on the site and the actual work recorded in the logbook. As a result, it was possible to determine the labor demands for certain time periods in a day, Table 1.

Table 1: Daily Maintenance Labor Demands for Certain Time Periods

Day, *i*	Mon, *1*	Tue, *2*	Wed, 3	Thu, *4*	Fri, *5*	Sat, *6*	Sun, *7*
0700-0900	4	5	4	5	4	6	6
0900-1300	6	7	6	7	6	8	8
1300-1500	8	9	8	9	8	10	10
1500-1700	4	4	4	4	4	4	4
1700-2100	2	2	2	2	2	2	2

Homogeneous workforce scheduling techniques can be applied to this problem due to the uniformity of the work performed by the laborers throughout the day at the site. The IP model to be used in this problem applies a single-shift integer programming model formulated by Tiberwala *et al.* [10] for scheduling workers with two consecutive days off per week.

$$\text{Minimize } Z = \sum_{i=1}^{7} x_i \tag{1}$$

Subject to:

$$\left(\sum_{j=1}^{7} x_j\right) - x_{i-1} - x_i \geq r_i \qquad i = 1,2,\ldots,7 \tag{2}$$

$$x_i, r_i \geq 0 \text{ and integer,} \qquad i=1,2,\ldots,7 \tag{3}$$

where

x_i = number of workers assigned to days-off pattern *i*, off on day *i* and *i*+1

r_i = minimum number of workers required on day *i*.

Since the company policy specifies that the minimum days off per week for any worker must be at least one day, Tiberwala's formula is modified accordingly to schedule workers one day off per week.

Three alternative shift schedules, which satisfy the labor demands stated in Table 1, are examined and evaluated with respect to their operating costs. The first schedule maintains the traditional one-shift schedule currently being practiced by the company, the second and third schedules implement a two-shift and three-shift schedule, respectively, with overlapping working hours linking the shifts.

Results and Discussion

Alternative 1: Current One-shift Schedule

At present, a laborer will be on duty six days a week, which may or may not include weekends. The current working schedule is presented in Table 2. The working shift starts from 0700 to 1500 hours and has a fixed monthly salary of RM 600. The worker may then extend the working period from 1500 up to 2100 hour whilst being paid on an overtime basis. The overtime pay rate on a normal day is RM 2.90 per hour and RM 5.80 per hour on public holidays. From historical data, the combined laborers' average monthly salary is RM 9166.08.

Table 2: The Current One-shift Working Schedule

Time (hour)	0700	1500	2100
Normal working hours			
Overtime hours			

Alternative 2: Two-shift Schedule

The volume of shoppers at the market is huge between 1000 and 1300, but after 1300 the number of people starts to decrease, leaving behind large volumes of rubbish. Therefore, on any day, 1300 to 1500 is considered the peak period and requires the more cleaners.

In order to satisfy the greater need for workers during this time, a two-shift schedule with overlapping working hours during the peak period is proposed, Table 3.

Hiring more workers will increase operating costs and the company wishes to minimize the number of workers, whilst providing seven-day service coverage. Since the cleaning workers perform the job at the same rate, homogeneous workforce scheduling can be applied to the company working system.

Table 3: A Two-shift Working Schedule

Time (hour)	0700	1300	1500	2100
Morning Shift				
Evening Shift				

The one-shift Tiberwala's IP formula is modified to reflect the new two-shift schedule. The objective function Z in (1), which has a single set of workers assigned to days-off pattern i, is modified such that x_i is replaced with two new variables u_i and v_i, which correspond to the morning and evening shifts, respectively, (4), is minimized with respect to the total number of laborers.

The company policies state that each worker must have one day of rest per week, therefore modification requires exclusion of a second-day off per week, which is denoted x_{i-1} in (2). Equations (5) and (6) show the modified constraints for one day off per week. Since there are two constraints in the model, p_i and q_i denote the minimum number of workers required on day i for the morning and evening shifts, respectively.

$$\text{Minimize } \sum_{i=1}^{7} u_i + v_i \tag{4}$$

Subject to:

$$\left(\sum_{j=1}^{7} u_j\right) - u_i \geq p_i, \qquad i = 1,2,\ldots,7 \tag{5}$$

$$\left(\sum_{j=1}^{7} v_j\right) - v_i \geq q_i, \qquad i = 1,2,\ldots,7 \tag{6}$$

$$u_i, v_i, p_i, q_i \geq 0 \text{ and integer}, \qquad i = 1,2,\ldots,7 \tag{7}$$

where

u_i = number of morning shift workers, off on day i.
v_i = number of evening shift workers, off on day i.
p_i = minimum number of morning shift workers required on day i.
q_i = minimum number of evening shift workers required on day i.

In order to determine the minimum number of shift workers required, the figures in Table 1 will be used as the basis to determine laborer demand by shift, Table 4.

The two variable sets, u_i and v_i, can be solved independently since the constraints in (5) and (6) are independent of each other. The optimum modified model solution can be obtained using any available mathematical

Table 4: Daily Maintenance Laborer Demands for Each Shift of Each Day

Shift Name	Time	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Morning	0700-1500	6	7	6	7	6	8	8
Evening	1300-2100	4	4	4	4	4	4	4

software. For the purposes of this project *QM for Windows* was used to solve the modified IP model and yields a value of 13 for the objective function *Z* and the labor assignment values as shown in Table 5.

Since all 13 laborers are working in regular 8-hour shifts, there will is no need for overtime, thereby minimizing operating costs. The workers receive a standard monthly salary of RM 600 and therefore the combined laborers' monthly salary is RM 7,800. This equates to a saving of RM 1,366 every month over the traditional one-shift schedule, although it is noted that number of laborers has increased from 10 to 13.

Table 5: Morning Shift and Evening Shift Labor Assignments

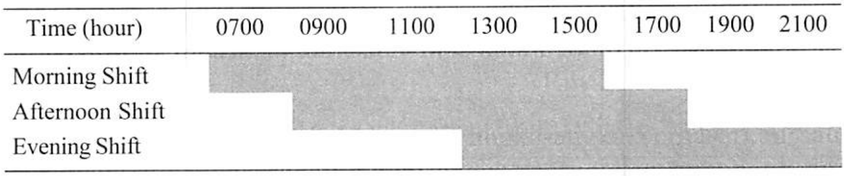
	Morning Shift					Evening Shift				
Day	u_1	u_2	u_3	u_4	u_5	v_3	v_4	v_5	v_6	v_7
Mon	off	1	2	1	2	1	1	1	1	1
Tue	2	off	2	1	2	1	1	1	1	1
Wed	2	1	off	1	2	off	1	1	1	1
Thu	2	1	2	off	2	1	off	1	1	1
Fri	2	1	2	1	off	1	1	off	1	1
Sat	2	1	2	1	2	1	1	1	off	1
Sun	2	1	2	1	2	1	1	1	1	off

Alternative 3: Three-shift schedule

The two-shift schedule requires an additional three laborers, it is conceivable that the company may wish to reduce this number and further reduce monthly expenses. In order to provide seven-day coverage, which has and sufficient workers during the 1300 to 1500 period, but is more economical, a three-shift schedule was attempted.

The shifts are skewed towards the earlier part of the day, since this period demands more laborers. The three-shift schedule is presented in Table 6, in which the shift periods are the Morning Shift (0700-1500), Afternoon Shift (0900-1700) and Evening Shift (1300-2100).

Table 6: A Three-shift Working Schedule



The IP model formulated is similar to the modified two-shift model developed except that there are three variable sets denoting the three different shifts. Under this working schedule, w_i, x_i and y_i denote the number of workers who will be off on day *i*, for the Morning, Afternoon and Evening shifts, respectively. There are three constraints which represent the minimum number of workers required on day *i* for each of the three shifts. The second-day off per week, which is denoted by x_{i-1} in (2) is again excluded to reflect a one day-off per week schedule. The right-hand side for constraint (2) is replaced by a_i, b_i and c_i for the Morning, Afternoon and Evening Shifts, respectively.

$$\text{Minimize } Z = \sum_{i=1}^{7} w_i + x_i + y_i \tag{8}$$

Subject to:

$$(\sum_{j=1}^{7} w_j) - w_i \geq a_i, \qquad i = 1,2,\ldots,7 \tag{9}$$

$$(\sum_{j=1}^{7} x_j) - x_i \geq b_i, \qquad i = 1,2,\ldots,7 \tag{10}$$

$$(\sum_{j=1}^{7} y_j) - y_i \geq c_i, \qquad i = 1,2,\ldots,7 \tag{11}$$

$$w_i, x_i, y_i, a_i, b_i, c_i \geq 0 \text{ and integer}, \quad i = 1,2,\ldots,7 \tag{12}$$

where

w_i = number of Morning Shift workers, off on day *i*.
x_i = number of Afternoon Shift workers, off on day *i*.
y_i = number of Evening Shift workers, off on day *i*.
a_i = minimum number of Morning Shift workers required on day *i*.
b_i = minimum number of Afternoon Shift workers required on day *i*.
c_i = minimum number of Evening Shift workers required on day *i*.

The figures in Table 1 were again used to provide the minimum number of shift workers required daily. The daily laborer demands determined for each shift are presented in Table 7. The optimum solution obtained from *QM for Windows* yields a value of 12 for the objective function Z; the corresponding labor assignment values are presented in Table 8.

Table 7: Daily Labor Demands for Each Shift of Each Day

Shift	Time	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Morning	700-1500	4	5	4	5	4	6	6
Afternoon	900-1700	2	2	2	2	2	2	2
Evening	1300-2100	2	2	2	2	2	2	2

Table 8: Morning Shift, Afternoon Shift and Evening Shift Labor Assignments

	Morning Shift				Afternoon Shift			Evening Shift		
Day	w_2	w_3	w_4	w_5	x_5	x_6	x_7	y_5	y_6	y_7
Mon	1	2	1	2	1	1	1	1	1	1
Tue	off	2	1	2	1	1	1	1	1	1
Wed	1	off	1	2	1	1	1	1	1	1
Thu	1	2	off	2	1	1	1	1	1	1
Fri	1	2	1	off	off	1	1	off	1	1
Sat	1	2	1	2	1	off	1	1	off	1
Sun	1	2	1	2	1	1	off	1	1	off

With a minimum total number of laborers of 12, the combined laborers' monthly salary is reduced to RM 7,200. This is a further saving of RM 600 per month compared to the two-shift schedule and RM 1,966 compared to the traditional one-shift schedule. Moreover, in implementing this schedule the company would only need to hire an additional two workers, instead of three as in the proposed two-shift schedule.

Figure 1 presents the comparison of the number of labors between the three proposed schedules. The two-shift schedule would require management to hire an additional three workers, whereas the three-shift schedule would only require a further two. In order to assist the decision making process, Figure 2 presents the conceptual percentage savings that could be made by the company if the proposed two- and three-shift schedules are adopted. The three-shift schedule would theoretically offer 21.4 % worth of savings, which equates to RM 23,592 of savings per year. By comparison the two-shift schedule would yield 14.9 % worth of

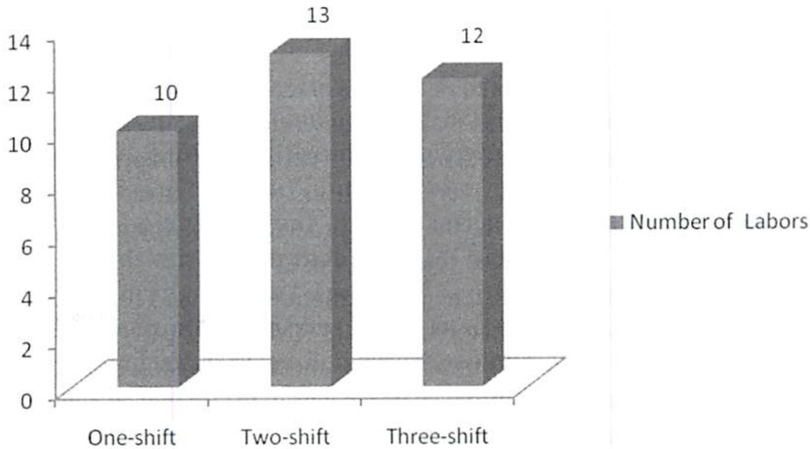


Figure 1: The Number of Labors for the Proposed Schedules

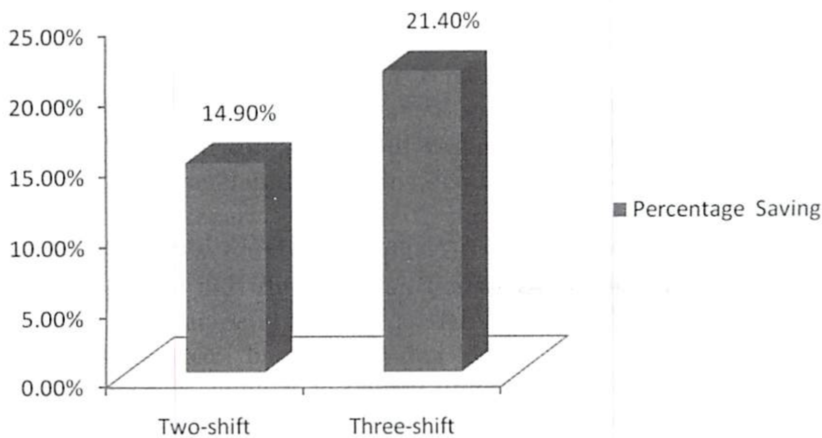


Figure 2: Percentage Savings for the Proposed Schedules

savings. It is therefore recommended that the three-shift schedule is adopted by the company's management, because it eliminates overtime payments and significantly reduces operating costs.

Conclusion

The objective of this paper is to determine an optimum schedule, which satisfies the labor requirements for a cleaning service company at minimum cost. The traditional one-shift schedule currently being practiced incurs large quantities of overtime every month in order to satisfy the work demands. A three-shift schedule has been proposed, which satisfies both the labor demands and reduces operating costs.

The proposed three-shift schedule eliminates the need for overtime and is expected to yield estimated savings of RM 23,592 per year or 21.4 % of the combined laborers' annual salary under the existing traditional shift schedule. From an employees' perspective, the proposed schedule creates a better balance between work and life, and from an employer's perspective sufficient resources are available during peak hours, no overtime is required nad they save money.

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