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GEOMETRIC PROPERTIES OF NEW SUBCLASSES OF UNIVALENT AND BI-UNIVALENT FUNCTIONS ASSOCIATED WITH SUBORDINATION AND GENERALISED DIFFERENTIAL OPERATOR

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ABSTRACT

Let A denote the class of normalised analytic functions f on the unit disc $D = \{z \in \mathbb{C} : |z| < 1\}$, in the form $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$. Let S be the class of functions $f \in A$ that are univalent in D. A function f in A is univalent if it is a one-to-one mapping. Based on Duren (1983), the theorem of Koebe's one-quarter showed the images of D for every univalent function, $f \in S$ enclosing 1/4 radius disc. Hence, every function $f \in S$ has an inverse f^{-1}, which is defined by $f^{-1}(f(z)) = z, z \in D$, and $f(f^{-1}(\omega)) = \omega$ $(|\omega| < r_0(f);\ r_0(f) \geq \frac{1}{4}$, with the power series expansion $g(\omega) = f^{-1}(\omega) = \omega - a_2\omega^2 + (2a_2^2 - a_3)\omega^3 - (5a_2^3 - 5a_2a_3 + a_4)\omega^4 + \cdots$. Meanwhile, a function f is said to be bi-univalent in D if both f and its inverse f^{-1} are univalent in D. Let Σ denote the class of bi-univalent functions in D. This thesis is concerned with the geometric properties of the class of analytic univalent and bi-univalent functions in the complex plane. Thus, there are three major objectives to be achieved and the results are presented in three different chapters. The first objective is to apply the subordination method on the class of analytic univalent and bi-univalent functions involving radius and coefficient problems. The radii of starlikeness for concave univalent functions related to certain rational functions, lune, cardioid, and the exponential equation are obtained. Then, the Third Hankel Determinants $|a_3(a_2a_4 - a_3^2) + a_4(a_2a_3 - a_4) + a_5(a_3 - a_2^2)|$ and Generalised Zalcman Conjecture $|a_n a_m - a_{n+m-1}|$ of bi-univalent functions are acquired. Next, continuation from the subordination method in the previous objective, the second objective investigated the coefficient problems for the class of bi-univalent functions by employing subordination and a generalised differential operator. The initial coefficients $|a_2|$ and $|a_3|$ of Taylor-Maclaurin series associated with Fibonacci numbers, Chebyshev polynomials and Chebyshev polynomials with respect to symmetric and symmetric conjugate points defined by the Al-Oboudi operator are attained. Hence, the results obtained were an improvement and more generalised from the recent studies. Finally, the last objective is to introduce a new differential operator. Hence, motivated by the generalised differential operator in the second objective, this study proposed a new generalised differential operator by utilising the binomial series. This new operator is applied in obtaining the inclusion properties for the class of analytic univalent functions. Then, the results on covering theorem, distortion theorem, rotation theorem, growth theorem, and the radius of convexity for functions of the class of bi-univalent functions were derived by utilising the new differential operator. The final chapter of this thesis discussed on the suggestion problems for future research. In conclusion, this thesis introduces new subclasses of univalent and bi-univalent functions and extends several classical results in geometric function theory. The findings contribute significantly to the understanding of these classes, offering new techniques and insights for future research.

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CHAPTER ONE
INTRODUCTION

1.1 Introduction

Complex analysis and geometric function theory (GFT) play a crucial role in both mathematics and physics, serving as foundational concepts in these fields. Complex analysis, also known as the theory of functions with complex variables, was intensively studied in the nineteenth century. Complex analysis is a branch of mathematics focused on complex functions, derivatives, and the intricacies of the complex plane. A complex function is a function that receives a complex number as input and another complex number as output. These functions can be represented as a mapping from points in the complex plane to other points in the complex plane. The derivatives of these functions are also very interesting and are used to study the behaviour of complex functions near points in the complex plane. The complex plane is a two-dimensional coordinate system in which the x-axis represents the real part, and the y-axis represents the imaginary part of a complex number.

Complex analysis also studies the properties of complex functions such as analytic functions, complex integrals, and power series representations. If a complex function is differentiable at every point in its domain, it is called analytic. The derivatives of analytic functions have significant effects on the behaviour of the function. For example, the Cauchy-Riemann equations are a set of differential equations that complex analytic functions must satisfy. These equations have several important implications for the behaviour and characteristics of analytic functions, specifically demonstrating that these functions have the properties of local conformance and the ability to preserve angles between curves.

Complex integration is another important concept in complex analysis. Complex functions can be integrated along curves in the complex plane, just as real functions can be integrated along curves in the real plane. The contour integral is the most common form of the complex integral, where a function is integrated along a closed path in the complex plane. A key result in this area is Cauchy's theorem, which states that the value of a contour integral depends only on the homotopy class of the curve and not on the actual curve itself.