Enhancing 3D Graph Visualisation in Multivariable Calculus Using Graphica3D

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Abstract: *One of the biggest challenges in learning multivariable calculus, particularly multiple integrals, is developing the skills to visualise, interpret and illustrate 2-dimensional (2D) and 3-dimensional (3D) graphs. These skills are crucial, as students must accurately identify integration regions and determine the limits of integration. However, many existing teaching materials for multivariable calculus often lack technological integration to support 3D graph visualisation, making it difficult for students to develop these essential skills. To address this gap, Graphica3D, an interactive educational module, has been created. The module integrates GeoGebra, an open-source mathematical software, to provide an interactive learning environment for exploring 2D and 3D graphs. In addition to explanatory notes on 3D graphs, Graphica3D features a GeoGebra applet that allows students to visualise various 3D graphs and their projections onto coordinate planes. The effectiveness of Graphica3D was evaluated through a survey of 37 engineering students enrolled in Further Calculus for Engineers at UiTM Cawangan Pulau Pinang. Before using the module, 40.5% of students admitted to struggling with 3D graph visualisation and sketching. After its implementation, all students agreed that the module enhanced their ability to visualise, interpret and sketch 3D graphs. The module's ease of use and interactive design make it a valuable supplement to traditional teaching methods, fostering a more visual and intuitive understanding of complex mathematical concepts.*

Keywords: *3-dimensional graphs, GeoGebra, interactive learning, multiple integrals, multivariable calculus*

1. INTRODUCTION

Multivariable calculus is a compulsory course in many degree programs in the sciences and engineering. One of the key topics in multivariable calculus is multiple integrals. Multiple integrals play a crucial role in mathematics and calculus, enabling the determination of plane areas, mass and volume calculations, moments of inertia, and surface areas of 3D objects (Milenković & Vučićević, 2024). One of the biggest challenges students encounter in learning multivariable calculus is developing the skills needed to visualise, interpret, and illustrate 3D geometrical shapes from algebraic equations. The application and practice of these skills in solving mathematical problems related to multiple integrals are crucial. Kashefi et al. (2011) reported that most students mainly struggled with interpreting and illustrating 3D graphs, which are important in determining integration limits and identifying regions of integration. Similarly, Gemechu et al. (2021) supported this finding by revealing that many students experienced difficulties associating algebraic equations with corresponding geometrical representations. This negatively impacted the students' ability to set up and solve multiple integration problems.

Generally, the traditional methods available for teaching multivariable calculus appear to have limitations and shortcomings in providing students with sufficient learning platforms to enhance their skills in interpreting, visualising, and illustrating geometric representations of algebraic equations. The traditional pencil-and-paper approach often fails to engage students in discovering and exploring geometrical concepts as effectively as technology-based methods (Sariyasa, 2017). Gemechu et al. (2021) further highlighted that traditional teaching methods tend to emphasise computational skills rather than conceptual understanding. As such, this suggests that innovative teaching strategies need to be designed to improve students' achievement by promoting visualisation and conceptual understanding, as well as encouraging the interactive learning of multivariable calculus.

Interactive tools like GeoGebra offer solutions to address the challenges faced by students and the gaps left by traditional teaching methods. GeoGebra is a powerful mathematical software that creates a flexible learning environment, allowing for the interactive exploration of mathematical equations. GeoGebra has features that permit students to manipulate mathematical variables and monitor the results immediately. This encourages students to actively experiment and explore mathematical concepts, thereby enhancing their learning outcomes (Jelatu et al., 2018). Ng and Rosli (2023) emphasised that GeoGebra not only aids students in understanding geometrical concepts but also assists educators in delivering mathematical content more clearly and effectively. Foo and Ng (2022) demonstrated that a GeoGebra-based module significantly improved students' comprehension of the topic of circles in mathematics. The study demonstrated that GeoGebra can be utilised to enhance students' understanding of abstract concepts and simplify complex ideas by providing interactive visualisation platforms.

As GeoGebra becomes a tool to support technical and interactive visual learning environments, flipbook meanwhile adds a creative touch by bringing lessons to life through visual effects and interactive storytelling. According to Hardiansyah (2016), the flipbook concept refers to a classic animation technique that involves applying a series of static images on paper to create the illusion of movement as the pages are flipped. The concept was initially used to generate animation stories without the help of advanced technologies. As technology has been growing rapidly, the concept has evolved too. This evolution results in the development of creative digital applications that incorporate animation effects into e-books. As such, flipbook is a great medium to improve the way students learn. Sari et al. (2024) suggested that flipbooks make lessons more engaging and enjoyable by displaying visual animation effects. It offers flexibility, allowing students to learn at their preferred time and location. It also supports links, images, videos, audio, and other functional supplementary materials that can be embedded into it. Saad et al. (2024) developed an educational flipbook containing step-by-step guidance, embedded videos, and other interactive materials, which were proven effective in enhancing students' interest in learning mathematics. Flipbooks also incorporate traditional and digital learning, assisting educators with a creative way to deliver lessons.

GeoGebra and flipbooks have technical, interactive and creative features. Therefore, integrating both as a learning device is one of the best combinations that can provide a promising way to inspire students to explore new knowledge and further assist them in understanding it effectively. Several studies have demonstrated the benefits of integrating both GeoGebra and flipbook in mathematics education. Azzahroh et al. (2024) demonstrated the integration of this approach to improve students' learning outcomes in quadratic functions. Their study showed that the students' mastery of the topic improved significantly after practising the integrated tools. Furthermore, Faizah et al. (2023) also developed an e-module that integrates GeoGebra and a flipbook for learning geometric transformations. They received positive feedback that their integrated module is effective and relevant as a practical solution to help students visualise abstract mathematical concepts and improve understanding. Jumroh et al. (2022) conducted research that also supported the integration of GeoGebra and flipbook as a learning device. They developed an e-module to enhance students' understanding of definite integral concepts. Their study proved that the integrated learning device benefits students in improving their comprehension of topics in definite integrals. Valid reviews from experts also supported their findings.

Based on these prior findings, this study aims to develop an educational tool, called Graphica3D, which leverages GeoGebra's powerful visualisation capabilities and the creative, multimedia-rich format of a flipbook to aid students in understanding abstract mathematical concepts taught in multivariable calculus. It is also hoped that the integrated learning tool encourages students' motivation to learn the knowledge. This approach aligns with previous studies that have shown significant improvements in mathematical learning outcomes when these tools were integrated into topics such as quadratic functions (Azzahroh et al., 2024), geometric transformations (Faizah et al., 2023), and definite integrals (Jumroh et al., 2022). Therefore, this study aims to further this foundation by focusing on the visualisation of 3D graphs, a fundamental component of multivariable calculus.

2. METHODS

The Graphica3D module was developed as an educational tool to address students' challenges in visualising and understanding 3D graphs. The key features and functionalities of an effective educational module were identified through a literature review, as well as feedback from lecturers and students. The development of the Graphica3D module consisted of three phases: design and conceptualisation, module development, and effectiveness testing.

2.1 DESIGN AND DEVELOPMENT OF THE GRAPHICA3D MODULE

Based on the gathered information, the essential features and functionalities required for the module were outlined. The module was designed to include interactive 3D graph visualisation, concise but straightforward explanatory notes on the 3D graphs, and accessibility across various devices.

The module was developed using GeoGebra, a free open-source mathematical software, to create an interactive learning environment. The GeoGebra applets were embedded in the module, enabling students to visualise standard 3D graphs in multivariable calculus, including planes, cones, cylinders, paraboloids, spheres, and hemispheres. Additionally, each 3D graph applet included a feature to project the graphs onto the three coordinate planes (xy-plane, yz-plane, and xz-plane), aiding students in understanding 2D projections of 3D surfaces. The Graphica3D module also includes notes explaining the mathematical functions of the 3D graphs. This can help students become familiar with the functions for each 3D graph.

The flow of user interaction within the Graphica3D module is structured to gradually build students' understanding of 3D graphs and their features. In the introduction, students are provided with an overview of the importance of visualising three-dimensional graphs when solving multiple integrals. Each section of the module focuses on a specific type of 3D graph, such as planes, cones, cylinders, paraboloids, spheres, and hemispheres. For each graph type, the module provides a brief description of its mathematical function, followed by an interactive GeoGebra applet. Students can manipulate the graphs to observe their shape from various perspectives by rotating, zooming, or changing the viewpoint.

Multiple mathematical functions are included for each type of graph to help students identify differences in algebraic forms and how these affect the 3D structure. Each applet also includes projection views on the xy-plane, yz-plane, and xz-plane, which help students better understand the relationships between 2D and 3D representations. Although the module is designed for sequential use, students are free to navigate through different graph types at their own pace. The flow used in the module – from explanation to hands-on manipulation – supports independent and visual learning, making abstract concepts more accessible and understandable.

To enhance accessibility, the Graphica3D module was designed as a digital flipbook, allowing students to access the material conveniently via laptops, tablets, or smartphones. The interactive applets and notes were arranged systematically to facilitate independent learning at one's own pace. To create the interactive flipbook, we utilised Heyzine Flipbooks, a free online platform that offers an intuitive and engaging interface. Figure 1 provides a preview of several pages from the Graphica3D module.

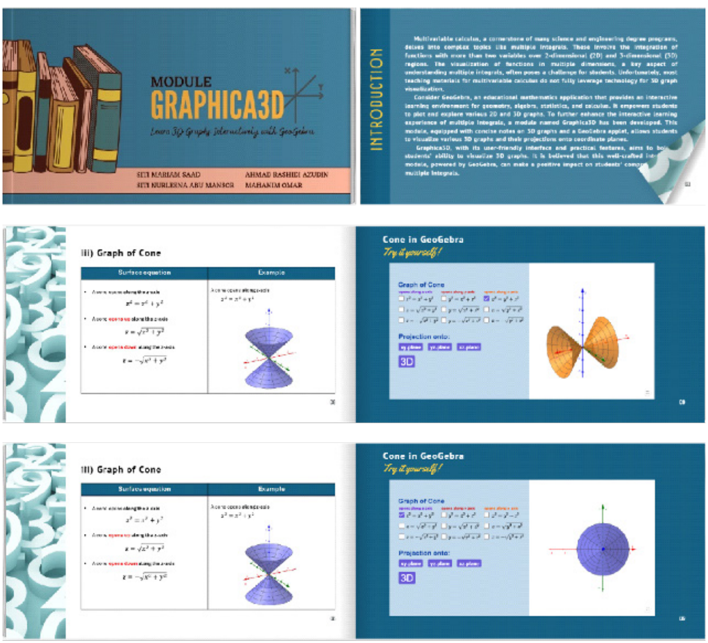

Figure 1: The Graphica3D Module

2.2 EVALUATION OF THE GRAPHICA3D MODULE

The effectiveness of the Graphica3D module in helping students visualise and sketch 3D graphs was evaluated through a survey. A total of 37 students enrolled in the Further Calculus for Engineers course at Universiti Teknologi MARA, Cawangan Pulau Pinang, participated in the study. A four-point Likert scale survey (1 = Strongly Disagree, 2 = Disagree, 3 = Agree, 4 = Strongly Agree) was distributed to students to gather their feedback on the usefulness of the module. The questionnaire was developed specifically for this study by the authors. The items were reviewed by two subject experts in multivariable calculus and mathematics education to ensure their clarity, relevance, and validity. Table 1 presents the questions asked in the survey.

No	Questions
1	The ability to sketch 3D graphs is important to solve multiple integrals.
1	I am skilled at sketching 3D graphs.
3	The Graphica3D module helps me learn how to sketch 3D graphs for various algebraic equations.
4	The Graphica3D module helps me in sketching 3D graphs from the given algebraic equations.
5	The Graphica3D module helps me in visualising 3D graph projections on coordinate planes.

Table 1: Assessment of Students' Experience with the Graphica3D Module

As illustrated in Figure 2, the respondents comprised 10 males (27%) and 27 females (73%). The majority were civil engineering students (17 students, 45.9%), followed by chemical engineering students (14 students, 37.8%), while the remainder were enrolled in electrical engineering.

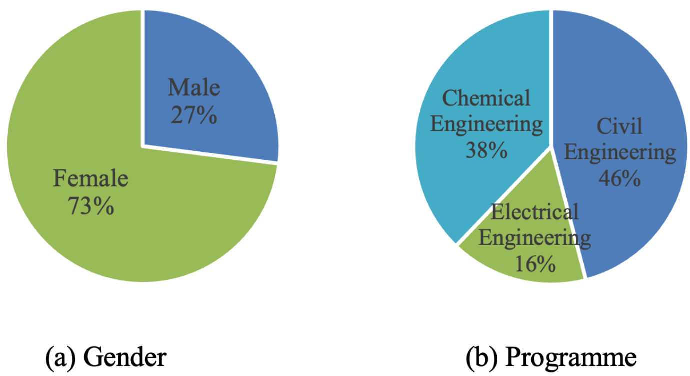

Figure 2: Summary of Respondents According to Gender and Programme

3. RESULTS AND DISCUSSIONS

3.1 STUDENTS' PERCEPTIONS OF 3D GRAPH SKETCHING SKILLS

The survey results reveal insights into students' perceptions and experiences with visualising and sketching 3D graphs using the Graphica3D module. As shown in Figure 3, the majority of students (70.3%) strongly agreed that having the ability to sketch 3D graphs is essential for solving problems related to multiple integrals. Regarding their skills in sketching 3D graphs, 54.1% agreed and 5.4% strongly agreed that they considered themselves skilled before being introduced to the Graphica3D module. However, 40.5% admitted to having difficulty in sketching 3D graphs, a relatively high percentage. According to Milenković et al. (2024), previous calculus courses tend to be more analytical than visual, which may explain why many students struggle to visualise and sketch graphs of two-variable functions.

The inability to visualise and sketch graphs of two-variable functions presents a significant barrier to students' understanding and problem-solving abilities in multiple integrals (Kashefi et al., 2010; Yeni et al., 2022). Solving multiple integrals extends beyond applying integration procedures correctly. Students must also identify integration domains (Alessio et al., 2022; Milenković &

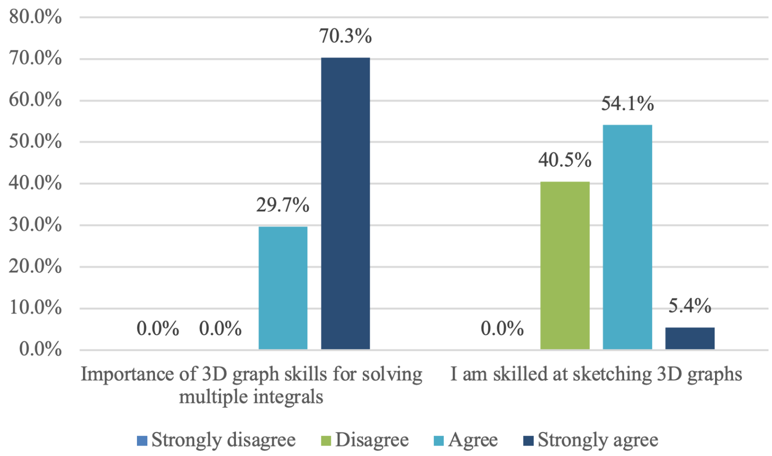

Figure 3: Students' Opinions on Sketching Skills for 3D Graphs

Vučićević, 2024). A weak understanding of graph concepts often leads to incorrect sketches of integration regions, which in turn cause difficulties in determining integration limits and solving multiple integration problems (Untarti & Kusuma, 2019; Gemechu et al., 2021).

To address this challenge, Martinez-Planell and Gaisman (2012, as cited in Gemechu et al., 2021) recommended exposing students to visual representations of functions of two variables to improve their comprehension of multiple integrals. Similarly, Milenković and Vučićević (2024) found that integrating GeoGebra into the learning environment significantly improved students' ability to solve multiple integral problems by enhancing their visualisation skills.

3.2 STUDENTS' FEEDBACK ON USING THE GRAPHICA3D MODULE 2.3

As displayed in Figure 4, all students either agreed (43.2%) or strongly agreed (56.8%) that the Graphica3D module helped them learn how to sketch 3D graphs. The combination of notes and GeoGebra applets enabled students to recognise algebraic equations and sketch the corresponding graphs—an aspect where all participants expressed agreement (45.9% agreed and 54.1% strongly agreed). This finding suggests that the interactive module effectively enhances students' understanding of 3D graph sketching.

Additionally, the module facilitated students' ability to illustrate projections of 3D graphs on different coordinate planes, with 59.5% agreeing and 40.5% strongly agreeing. The ability to visualise projections plays a crucial role in determining the limits of integration for double and triple integrals.

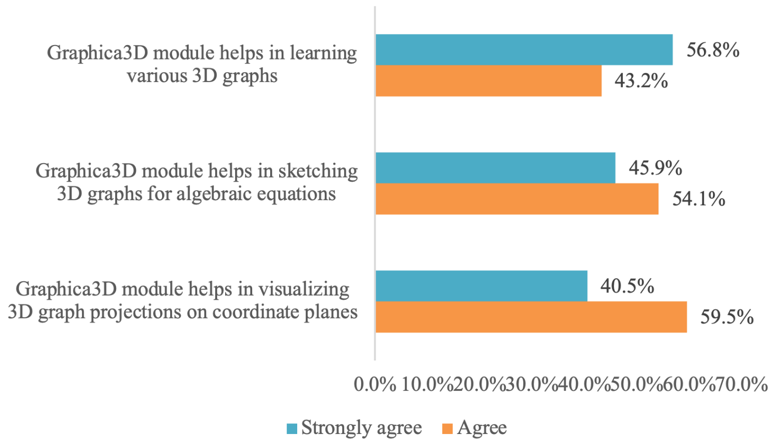

Figure 4: Students' Feedback on the Effectiveness of Graphica3D

Student feedback indicates that the Graphica3D module enhances their ability to visualise and sketch 3D graphs, ultimately strengthening their understanding of multiple integrals. This aligns with previous studies, which found that integrating GeoGebra with instructional materials aids students in learning the graphical representation of algebraic equations (Septian et al., 2021; Yeni et al., 2022).

Furthermore, Milenković et al. (2024) demonstrated that students who used GeoGebra-based applications achieved significantly better learning outcomes in multiple integral problems than those who relied on traditional teaching methods. This underscores the module's effectiveness as a supplementary learning resource, filling gaps in traditional classroom instruction by providing an interactive and visual learning experience.

With the Graphica3D module, students can independently explore various 3D graphs at their own pace. This aligns with Saputra and Fahrizal (2019), who argued that integrating GeoGebra into instructional materials empowers students with learning independence, even among those with no prior experience (de Carvalho et al., 2024).

4. CONCLUSION

This study examines the effectiveness of Graphica3D, an interactive educational module designed to enhance students' ability to visualise and understand abstract mathematical concepts in multivariable calculus. The implementation of Graphica3D among engineering students enrolled in multivariable calculus courses has yielded positive results. The findings indicate that the module improves students' ability to visualise and sketch 3D graphs, a crucial skill for understanding multiple integrals. Additionally, positive feedback suggests that the module serves as an effective supplementary learning tool. The promising outcomes of this study highlight the potential for further integration and development of Graphica3D to enhance learning experiences in multivariable calculus, particularly in the context of multiple integrals. Its interactive approach makes it a valuable supplement to traditional teaching methods, fostering a more visual and intuitive understanding of complex mathematical concepts.

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7. AUTHORS' CONTRIBUTION

Siti Mariam Saad: Writing – review and editing, Writing – original draft, Project administration, Methodology – designed and organised the research. Siti Nurleena Abu Mansor: Writing – review and editing, Methodology – developed the questionnaires, Data Analysis. Ahmad Rashidi Azudin: Writing – original draft. Mahanim Omar: Data Analysis, Writing – original draft. All authors offered valuable feedback and contributed to shaping the research, analysis, and manuscript.

8. CONFLICT OF INTEREST DECLARATION

We certify that the article is the Authors' and Co-Authors' original work. The article has not received prior publication and is not under consideration for publication elsewhere. This manuscript has not been submitted for publication, nor has it been published in whole or in part elsewhere. We attest that all authors have made significant contributions to the work, including the validity and legitimacy of the data and its interpretation, for submission to IJELHE.

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