

**NUMERICAL METHODS FOR SOLVING HIGHER-ORDER
ORDINARY DIFFERENTIAL EQUATIONS**

FATIN NOR ATHIRAH BINTI MUHAMAD

**Thesis submitted in fulfilment
of the requirement for the degree of
Bachelor of Science
Mathematical Modelling and Analytics (Hons.)
College of Computing, Informatics and Mathematics
Universiti Teknologi MARA**

February 2025

ABSTRACT

The higher-order ordinary differential equation (ODE) is essential for accurately describing complex systems that cannot be captured by first or second-order models. This research employs various numerical methods to solve higher-order ODEs for fourth- and fifth-order, including both homogeneous, non-homogeneous and stiff equations. The methods include Euler's Method, Heun's Method, the Fourth-Order Runge-Kutta method, Fifth-Order Runge-Kutta method and Runge-Kutta-Fehlberg method. To enhance the accuracy of approximate solutions for higher-order ODE, the numerical methods are combined with Richardson's and Aitken's extrapolation, respectively. The approximate solutions for each method are presented in tables and visualised through graphs. The error for each method is recorded in tables and illustrated through graphs to identify the most effective method, while the CPU time is measured to evaluate the computational cost. The results showed that the combination of Runge-Kutta fifth-order method and Runge-Kutta-Fehlberg method with Aitken's extrapolation gives the best result in solving the fourth-, fifth-order for both homogeneous and non-homogeneous. For stiff equation, Runge-Kutta-Fehlberg method with Aitken's extrapolation give the lowest error compared to Runge-Kutta fifth-order method with Aitken's extrapolation. For, the combination of Richardson's extrapolation can effectively reduce the computational cost but its accuracy is not good as combining with Aitken's extrapolation.

ACKNOWLEDGEMENT

In the name of Allah, the Most Merciful and Compassionate, I am deeply grateful for His blessings and guidance, which have given me the strength and determination to complete this thesis. I would like to extend my heartfelt thanks to my supervisor, Dr. Nurul Ainina Redwan, for her invaluable guidance, encouragement, and patience throughout this journey. Her support has been instrumental in the completion of my work. I am also thankful to the lecturers of Universiti Teknologi MARA for their assistance and dedication in providing me with the knowledge and tools needed for my studies. Lastly, I am profoundly thankful to my family for their unconditional love, prayers, and encouragement. Their unwavering support has been my greatest strength and inspiration throughout this journey.

TABLE OF CONTENT

	Page
DECLARATION BY THE SUPERVISOR	i
DECLARATION BY CANDIDATE	ii
ABSTRACT	iii
ACKNOWLEDGEMENT	iv
TABLE OF CONTENT	v
LIST OF TABLES	viii
LIST OF FIGURES	viii
LIST OF ABBREVIATIONS AND SYMBOL	xii
CHAPTER ONE : INTRODUCTION	1
1.1 Introduction	1
1.2 Background of the Study	1
1.3 Problem Statement	3
1.4 Objectives	4
1.5 Significance of the Project	4
1.6 Project Benefit	5
1.7 Scope of the Project	6
1.8 Definition of Terms and Concepts	6
1.9 Organisation of the Project	8
CHAPTER TWO: LITERATURE REVIEW	10
2.1 Introduction	10

2.2	Initial Values Problem	10
2.2	Stiff Equation	11
2.3	Numerical Method	12
2.3.1	Euler's Method	13
2.3.2	Modified Euler's Method (Heun's Method)	14
2.3.3	Fourth-order Runge-Kutta method (RK4)	15
2.3.4	Butcher's Fifth-order Runge-Kutta method (RK4)	15
2.3.5	Runge-Kutta-Fehlberg method (RK45)	16
2.3.6	Richardson's extrapolation method (RE)	17
2.3.7	Aitken's extrapolation method (AE)	18
	CHAPTER THREE: METHODOLOGY	19
3.1	Introduction	19
3.2	Mathematical Formulation	19
3.2.1	Basic Concept for Higher-Order Linear Differential Equation	19
3.2.2	Homogeneous and Non-homogeneous Equation	20
3.2.3	Solving Higher-Order Ordinary Differential Equations	21
3.2.4	Euler's Method (EM)	22
3.2.5	Heun's Method (HM)	24
3.2.6	Fourth-Order Runge-Kutta method (RK4)	26
3.2.7	Fifth-Order Runge-Kutta method (RK5)	29
3.2.8	Runge-Kutta-Fehlberg method (RK45)	33