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Enhancing Heat Transmission with Ternary Nanofluid in Mixed Convection Flow Under the Impact of Convective Heat Flux

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ABSTRACT

Heat transfer remains a major issue in the automotive, renewable energies, electronic cooling, refrigeration, and thermal storage industries. To enhance heat transfer, ternary nanofluid, a novel type of nanofluid, is explored. The current study examines a two-dimensional and steady mixed convection flow of a ternary nanofluid over an elongated sheet. In light of the variety of applications it has, the investigation of titanium dioxide (TiO_2), aluminum oxide (Al_2O_3), and silica oxide (SiO_2) based in water with convective boundary conditions is becoming increasingly prevalent in innovation and research. The partial differential equations are reduced to ordinary differential equations through similarity transformations, and numerical solutions are obtained using the bvp4c solver. The paper offers discussions and graphical depictions of temperature and velocity profiles across important variables, including mixed convection parameters and Biot numbers. When compared to hybrid nanofluids, the ternary nanofluid has a bigger influence on the efficiency of heat transmission. The significance of SiO_2 nanoparticles despite having the least thermal conductivity is because of its stability and chemical inertness, meanwhile TiO_2 is a smaller size particle which helps to reduce agglomeration rate thus increased thermal conductivity augmentation. Al_2O_3 is extensively utilized in commonplace items including cooling systems, solar collectors, and lubricants. Its vast range of uses has lately caught the attention of material science specialists.

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INTRODUCTION

Oil, water and ethylene glycol are some of the widely used conventional liquids for cooling in various industries such as solar collectors (Elsheikh et al., 2018), car radiators (Subhedar et al., 2018), refrigerators (Bellos & Tzivanidis, 2018), and cooling of electronic equipment (Bahiraei & Heshmatian, 2018). However, given the poor thermophysical properties of these traditional liquids, researchers were led to explore an innovative way of cooling fluids called nanofluid. Nanofluid is developed with a suspension of 100 nm diameter sized nanoparticles in water by Choi & Eastman (1995). The enhanced thermal conductivity and rheological properties made possible by nanofluid with various geometries were found in Moghaddaszadeh et al. (2019) and Krishnakumar et al. (2018).

As demand for higher thermal conductivity of nanofluid increases, scientists and researchers have expanded their interest into nanofluid containing multiple types of nanoparticles. Therefore, a hybrid nanofluid was introduced. An extra nanoparticle with improved thermal conductivity characteristics is dispersed throughout the nanofluid to create hybrid nanofluids. One major advantage of adopting a hybrid nanofluid is that the overall enhancement of heat transport can be significantly improved by integrating the right nanoparticles. The improved thermal conductivity of hybrid nanofluid has drawn numerous scientists to study its potential in solving real-world heat transfer problems. For example, Bhatti et al. (2022) investigated the hybrid nanofluid comprises of silica oxide (SiO_2) and diamond (C) nanoparticles suspended in water thermal conductivity for application in solar collectors. It is found that the hybrid nanofluid enhances the thermal and optical efficiency of solar collector energy conversion systems. Meanwhile Ali & Xianjun (2020) used aluminum oxide (Al_2O_3) and titanium oxide (TiO_2) hybrid nanofluids to improve the thermal stability and heat transfer efficiency of engine oils in vehicles. On the other hand, Mahamude et al. (2022) experimentally studied the use of hybrid nanofluids graphene/waste cotton to improve heat exchange efficiencies in flat plate solar collectors.

The major drawback concerning hybrid nanofluid is their chemical instability. According to Yasmin et al. (2023), the problems that restrict commercialization efforts include the hybrid nanofluids' agglomeration and sedimentation as well as the nanoparticle suspension times. Therefore, a ternary nanofluid (TNF) was introduced as a potential solution for enhancing the rheological behavior, as well as improving its thermophysical and heat transfer. By adding a third nanoparticle in the nanofluid, the limitations that are observed in hybrid nanofluid can be rectified. The improvement of thermal conductivity of TNF has been successfully proven theoretically as well as experimentally by researchers. Ramadhan et al. (2020) has shown that adding an appropriate third nanoparticle to a hybrid nanofluid is shown to improve its rheological behavior, thus improving its stability. Considerable research is being conducted by utilizing TNF to improve the heat exchange process. Fangfang et al. (2023) showed that TNF is superior to hybrid nanofluid in terms of thermal conduction in blood flow through the arteries, which then helps to remove the toxic plague. In addition, Hanapiah et al. (2024) found that the TNF comprising graphene oxide, aluminum oxide (Al_2O_3), and silica oxide (SiO_2) had a greater heat transfer rate than the base fluid.

Scientists are currently investigating the behaviour and thermal properties of TNF to identify its potential advantages, limitations, and suitability for industrial applications. Humphrey (2022) reported that metallic nanoparticles such as zinc (Zn), iron (Fe) and silver (Ag) have high thermal conductivity but low stability. Meanwhile, Muneeshwaran et al. (2021) reported that metallic oxide such as zinc oxide (ZnO), aluminum oxide (Al_2O_3), and titanium oxide (TiO_2) nanoparticles have higher stability with relatively lower thermal conductivity. Mehta et al. (2022) discussed that the addition of TiO_2 in nanofluid Al_2O_3-water helps in improving the overall thermal conductivity of the nanofluid. Besides, the incorporation of the TiO_2 leads to a lower agglomeration rate due to smaller particle size, ultimately resulting in higher thermal conductivity augmentation. The presence of SiO_2 further helps with the stability as Dong et al. (2022) discover that the synthesized SiO_2 nanoparticles in water are extremely stable after 36 months. Manjunatha et al. (2022) studied the combination of $TiO_2 - SiO_2 - Al_2O_3$ in base fluid water for environmental purification, while Bilal et al. (2023) studied the same combination of $TiO_2 - SiO_2 - Al_2O_3$ in Carreau Yasuda fluid to improve

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energy profile of an engine oil. Other experimental works that utilizes $TiO_2 - SiO_2 - Al_2O_3$ with water due to their high thermal conductivity includes Gangadevi & Vinayagam (2018), Pryazhnikov et al. (2017), Chiam et al. (2017) and Sundar et al. (2014). Furthermore, Ramadhan et al. (2019) explored the combination of $TiO_2 - SiO_2 - Al_2O_3$ TNF as a cooling fluid in car radiator and find maximum 39.7% improvement of heat transfer rate for the coolant side. Therefore, TNF consisting of $TiO_2 - SiO_2 - Al_2O_3$ nanoparticles suspended in water is chosen in this present study.

Applications of TNF in a surface with a stretching condition is useful in an extrusion process such as paper manufacturing, fibre production, wire or plastic drawing and hot rolling. Fatima et al. (2018) compared the effect of thermal radiation and Cattaneo-Christov (CC) heat flux on the mixed convective heat transport of hybrid and TNF radiation across a stretched sheet. Bilal et al. (2023) studied the TNF of TiO_2-SiO_2-Al_2O_3 flowing over a vertical stretched sheet with the influence of magnetic dipole and heat transmission. They discovered that TNF had a higher tendency to enhance thermal energy transfer than hybrid and nanofluids. Meanwhile, TNF flow on a curved surface that is stretching and shrinking was numerically examined by Mahmood et al. (2023). As a third nanoparticle was introduced to the hybrid nanofluid, the authors saw improvements in temperature distribution and velocity profile. They discovered that in TNF instances, the transmission of heat and momentum is more significant than the hybrid nanofluid.

The impact of mixed convection parameters in a hybrid nanofluid has been examined by a number of scientists in recent years. Rostami (2018) investigated the impact of mixed convection close to a vertical plate stagnation point while Khan & Pop (2020) examined the impact of mixed convection and an entropy formation on across a stretchy spinning disk. Najm (2022) looked at how mixed convection affected the hybrid nanofluid flow's ability to transport heat in a cylindrical body. Farooq (2022) later investigated how the number of Biot and mixed convection factors affected the thermal profile of hybrid nanofluid flowing over a stretched sheet. However, it appears that no previous research has investigated the velocity and thermal behavior of TNF flow in mixed convective subject to convective boundary conditions. The novelty of the present study is the impact of the convective heat flux and mixed convection parameter on the TNF $TiO_2 - SiO_2 - Al_2O_3$ nanofluid flowing past an elongated sheet with convective heat flux at wall. Applying a relevant similarity transformation into the governing equations turn it into a non-dimensional equations. The boundary value problem is subsequently computed using bpv4c package in MATLAB software. Graphical representations are used to illustrate and evaluate the results.

MATHEMATICAL FORMULATION

Considering a heat transfer phenomenon in a steady, incompressible, two-dimensional TNF induced by a mixed convection flow over stretched sheet subject to convective heat flux at boundary. The water-based TNF is suspended with Al_2O_3, TiO_2, and SiO_2. Parallel to the *x*-axis is the sheet that is stretched with the velocity, $U_w = ax$. Meanwhile perpendicular to the axis of the stretched sheet is *y* with a T_w denoted as a fixed temperature of the wall. T_∞ indicates the surrounding temperature of the fluid while T denotes the temperature of the TNF. The mathematical model of TNF flow passing a stretched sheet subject to mixed convection and convective heat flux at boundary are derived, following the works of Manjunatha et al. (2022).

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho_{tnf}\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = \mu_{tnf}\frac{\partial^2 u}{\partial y^2} + g\left(T - T_\infty\right)\left(\rho\beta\right)_{tnf} \tag{2}$$

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$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=k_{tnf}\left(\frac{\partial^2 T}{\partial y^2}\right) \tag{3}$$

Here, the dynamic viscosity and density of TNF were represented by μ_{tnf} and ρ_{tnf}. The mixed convection effect is the product of gravitational force, g and the temperature changes between the fluid and ambient $\left(T-T_\infty\right)$ and the thermal expansion coefficient, $\left(\rho\beta\right)_{tnf}$ of TNF. The dynamics of TNF as shown in Fig 1 occur within the boundary condition.

$$y=0: \quad u=U_w=ax \qquad v=0 \qquad -k_{tnf}\frac{\partial T}{\partial y}=h_f\left(T_w-T\right) \tag{4}$$

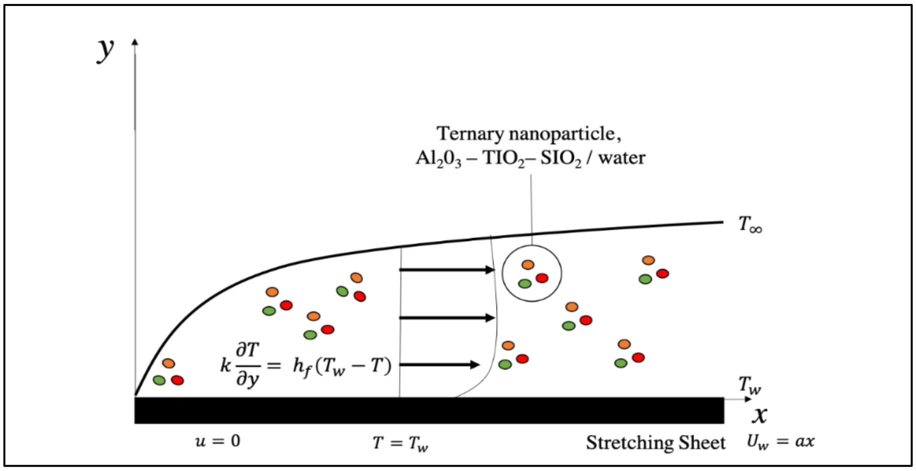

Fig. 1. Physical configuration of ternary nanofluid on a stretching sheet.

Here, a is the stretching parameter that is a constant value, k is the thermal conductivity of TNF and h_f represents the heat transmission coefficient of the fluid while and $\left(T_w-T\right)$ is the heat difference between the wall and the ambient temperature. The heat gains or losses from convection with an adjacent wall can be model by taking into account convective heat flux at boundary (Jha & Samaila, 2022). Meanwhile as boundary begin further from the stretched sheet, where $y\to\infty$, conditions are formulated as (Manjunatha et al., 2022).

$$y\to\infty: \quad u=0 \qquad T=T_\infty \tag{5}$$

The mechanical quantities of the fluid are represented by the local Nusselt number Nu_x and local skin friction coefficient C_f are defined as (Mahdy, 2012):

$$Nu_x=\frac{xq_w}{k_f\left(T_w-T_\infty\right)} \qquad C_f=\frac{\tau_w}{\rho_f U_w^2} \tag{6}$$

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where heat flux at wall, q_w and the sheer stress of surface along the $x-$ axis, τ_w is defined as follows.

$$q_w = -k_{tnf}\left(\frac{\partial T}{\partial x}\right) \qquad \tau_w = \mu_{tnf}\left(\frac{\partial u}{\partial x}\right) \tag{7}$$

Following the study of Bilal et al. (2023), the TNF physical characteristics, such as density $\left(\rho_{nf}\right)$ and viscosity $\left(\mu_{nf}\right)$ follows a mixture rule while the heat capacity $\left(\rho C_\rho\right)_{nf}$ and the thermal conductivity $\left(k_{nf}\right)$ were defined as shows in Table 1.

Table 1. Physical characteristics of ternary nanofluid

Properties	Nanofluid	Nanofluid
Density	NF	$\rho_{nf} = \rho_f\left(1-\varphi_1\right)+\varphi_1\rho_1$
	HNF	$\rho_{hnf} = \left(1-\varphi_2\right)\left(\rho_f\left(1-\varphi_1\right)+\varphi_1\rho_1\right)+\varphi_2\rho_2$
	TNF	$\rho_{tnf} = \left(1-\varphi_3\right)\left\{\left(1-\varphi_2\right)\left(\rho_f\left(1-\varphi_1\right)+\varphi_1\rho_1\right)+\varphi_2\rho_2\right\}+\varphi_3\rho_3$
Viscosity	NF	$\mu_{nf} = \dfrac{\mu_f}{\left(1-\varphi_1\right)^{2.5}}$
	HNF	$\mu_{hnf} = \dfrac{\mu_{nf}}{\left(1-\varphi_1\right)^{2.5}+\left(1-\varphi_2\right)^{2.5}}$
	TNF	$\mu_{tnf} = \dfrac{\mu_{tnf}}{\left(1-\varphi_1\right)^{2.5}+\left(1-\varphi_2\right)^{2.5}+\left(1-\varphi_3\right)^{2.5}}$
Thermal Conductivity	NF	$\dfrac{k_{nf}}{k_f} = \dfrac{k_1+2k_f-2\phi_1\left(k_f-k_1\right)}{k_1+2k_f+\phi_1\left(k_f-k_1\right)}$
	HNF	$\dfrac{k_{hnf}}{k_{nf}} = \dfrac{k_2+2k_{nf}-2\phi_2\left(k_{nf}-k_2\right)}{k_2+2k_{nf}+\phi_2\left(k_{nf}-k_2\right)}$
	TNF	$\dfrac{k_{tnf}}{k_{hnf}} = \dfrac{k_3+2k_{hnf}-2\phi_3\left(k_{hnf}-k_3\right)}{k_3+2k_{hnf}+\phi_3\left(k_{hnf}-k_3\right)}$
Heat capacity	NF	$\left(\rho C_\rho\right)_{nf} = \left(\rho C_\rho\right)_f\left(1-\varphi_1\right)+\varphi_1\left(\rho C_\rho\right)_1$
	HNF	$\left(\rho C_\rho\right)_{hnf} = \left(1-\varphi_2\right)\left[\left(\rho C_\rho\right)_f\left(1-\varphi_1\right)+\varphi_1\left(\rho C_\rho\right)_1\right]+\varphi_2\left(\rho C_\rho\right)_2$
	TNF	$\left(\rho C_\rho\right)_{tnf} = \left(1-\varphi_3\right)\left\{\left(1-\varphi_2\right)\begin{bmatrix}\left(\rho C_\rho\right)_f\left(1-\varphi_1\right)+ \\ \varphi_1\left(\rho C_\rho\right)_1\end{bmatrix}+\varphi_2\left(\rho C_\rho\right)_2\right\}+\varphi_3\left(\rho C_\rho\right)_3$

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Here, φ refers to the volume fraction. Water (fluid), Al_2O_3-water (nanofluid), Al_2O_3-TiO_2-water (hybrid nanofluid) and Al_2O_3-TiO_2-SiO_2-water (ternary nanofluid) were indicated by a subscript f, nf, hnf and tnf while Al_2O_3, TiO_2 and SiO_2, nanoparticle is referred by a subscript $(*)_1$, $(*)_2$ and $(*)_3$, respectively. Table 2 listed for each nanoparticle, the constants of its specific heat capacity (C_p), density (ρ), and thermal conductivity (k) (Ramadhan et al., 2019):

Table 2. Thermophysical constants of the base fluid and nanoparticles

Nanoparticle	$C_p\,(J/kg-K)$	$\rho\,(kg/m^3)$	$k\,(W/m-K)$
H_2O	4179	997	0.613
Al_2O_3	880	3890	30
TiO_2	683	397	4.8
SiO_2	680	217	1.3

The following similarity variables were adopted into Equation 1 to 3 into a set of non-dimensional ODEs (Manjunatha et al., 2022).

$$\eta = \left(\frac{a}{\upsilon}\right)^{\frac{1}{2}} y \qquad \psi = (a\upsilon)^{\frac{1}{2}} x f(\eta) \qquad \theta = \frac{T - T_\infty}{T_f - T_\infty} \tag{8}$$

where $\upsilon_{tnf} = \dfrac{\mu_{tnf}}{\rho_{tnf}}$ is the kinematic viscosity, $\alpha_{tnf} = \left(\dfrac{k_{tnf}}{(\rho C_p)_{tnf}}\right)$ is the TNF's thermal diffusivity. θ is a dimensionless temperature with similarity variable η while $\psi(x, y)$ the stream function. Two equations to ensure the satisfaction of the continuity Equation 1 is introduced, which are $u = \dfrac{\partial \psi}{\partial y} = axf'(\eta)$ and $v = -\dfrac{\partial \psi}{\partial x} = -(a\upsilon)^{\frac{1}{2}} f(\eta)$. Imposing Equation 8 into Equation 2 to 5, we obtain;

$$\frac{\mu_{tnf}}{\mu_f} f'''(\eta) + \frac{\rho_{tnf}}{\rho_f}\left[f(\eta) f''(\eta) - f'(\eta)^2 \right] + \lambda\theta = 0 \tag{9}$$

$$\theta'' + \Pr f(\eta)\theta'(\eta) = 0 \tag{10}$$

with dimensionless boundary conditions

$$f'(0) = 1 \qquad f(0) = 0 \qquad \theta'(\eta) = \mathrm{Bi}\left(\theta(0) - 1\right) \tag{11}$$

$$f'(\infty) = 0 \qquad \theta(\infty) = 0 \tag{12}$$

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where the constant λ is the mixed convection parameter defined as $\lambda = \dfrac{Gr_x}{\mathrm{Re}_x^2}$. Meanwhile the Grashof number is defined as $Gr_x = \dfrac{g\left(\rho\beta\right)_{tnf}\left(T_w - T_\infty\right)}{\rho_{tnf}a^2 x}$. Prandtl number is defined as $\mathrm{Pr} = \left(\dfrac{\upsilon}{\alpha}\right)_f$ is the convective parameter is defined as $Bi = \dfrac{h_f}{k_f}\sqrt{\dfrac{\upsilon_f}{a}}$. Applying the Equation 8 into Equation 6, yielding;

$$\mathrm{Re}_x^{-\frac{1}{2}} C_f = \left(\frac{\mu_{tnf}}{\mu_f}\right) f''(0) \qquad -\mathrm{Re}_x^{\frac{1}{2}} Nu_x = \left(\frac{k_{tnf}}{k_f}\right)\theta'(0) \tag{13}$$

where the local Reynold number is $\mathrm{Re}_x = \dfrac{U_w x}{\upsilon_f}$.

RESULTS AND DISCUSSIONS

In this study, the behavior of TNF in a mixed convection subject to convective heat flux at boundary past a stretching sheet is analyzed. The parameters investigated, including mixed convective parameter λ and Biot number, Bi are discussed with the support of graphs and tabulated data. The current method is verified by contrasting the newly computed numerical results with the prior study. Table 3 portrayed the comparison of the numerical outputs by setting the parameters volume fraction, $\varphi_1 = \varphi_2 = \varphi_3 = 0$, mixed convection parameter, $\lambda = 0$ and $\mathrm{Bi} = 1000$. The value of $-\theta(0)$ shows an excellent agreement with less than 1.5% deviation compared to previous study for various values of Pr.

Table 3. Comparison of $-\theta(0)$ for limiting case of $\varphi_1 = \varphi_2 = \varphi_3 = 0$ and Bi = 1000 for various Pr values

Pr	Gorla & Sidawi (1994)	Khan & Pop (2010)	Makinde & Aziz (2011)	Present study
0.70	0.5349	0.4539	0.4539	0.4542
2.00	0.9113	0.9113	0.9114	0.9105
7.00	1.8954	1.8954	1.8954	1.8918
20.00	3.3539	3.3539	3.3539	3.3427

According to experimental study by Sharifpur et al. (2018), the ideal concentration of nanoparticles for improving heat transfer of nanofluid is below 5%. Therefore, the volume fraction studied for TiO_2 and Al_2O_3 in the present study is set as 1% each while the third nanoparticles, SiO2 is set at 2% as suggested by Dong et al. (2022). Fig 2 illustrates the impact of λ on $\theta(\eta)$. As λ increase, it is note that the $\theta(\eta)$ increase as well. This can be explained by the physical definition of λ. This can be physically explained by the definition of λ which is the ratio of Gr_x to the Re_x. As Gr_x is inversely related to viscous force, increase in λ results in a weakening of viscous force and dominance of buoyancy force. As buoyancy force increases, so does the temperature difference of the fluid. An increase $\theta(\eta)$ in convective transport within the fluid can be explained by the high temperature difference as λ increases. Additionally, it was discovered that $\theta(\eta)$ for TNF was higher than HNF. As shown in Fig 3, it is shown that the $f(\eta)$ of

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TNF reduces as the value of λ increase. This observation aligns with the results reported by Waini et al. (2020) and Asghar et al. (2022) where they studied HNF flow in mixed convection environment.

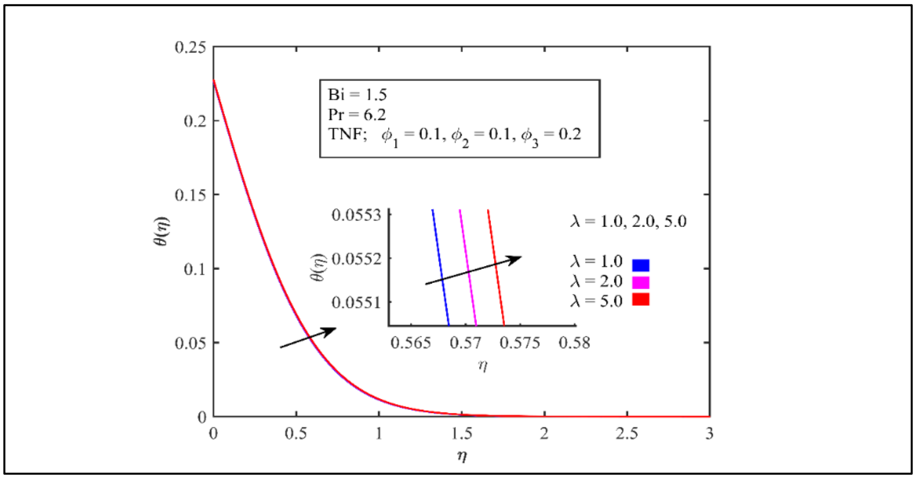

Fig. 2. $\theta(\eta)$ when $\lambda = 1.0, 2.0, 5.0$.

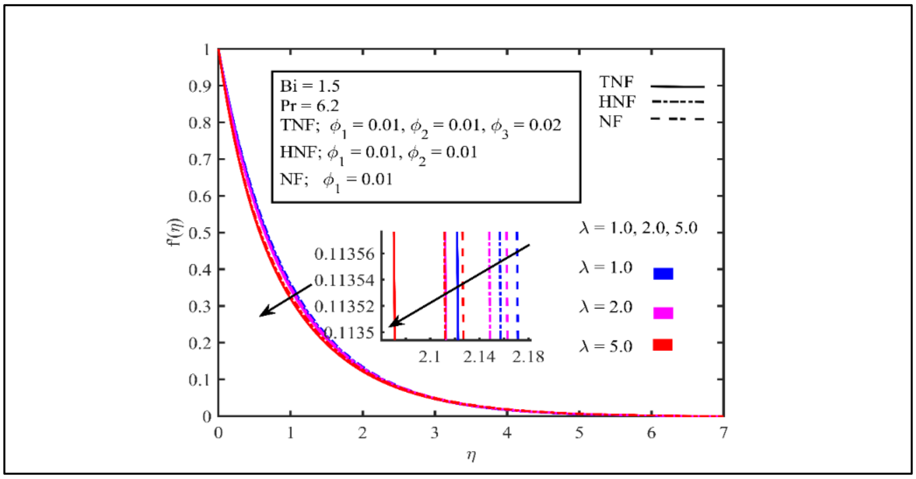

Fig. 3. $f(\eta)$ when $\lambda = 1.0, 2.0, 5.0$.

The effect of increasing Bi on $\theta(\eta)$ and $f(\eta)$ are shown in Figs 4 and 5, respectively. As Bi increases, a rise in the $\theta(\eta)$ is observed in Fig 4. This finding can be explained by the rise of heat transfer coefficient as Bi increase. This leads to a surge in the amount of heat transferred from the sheet to the fluid. This outcome is aligned with Aly (2019) analysis of the impact of Bi on the flow of hybrid nanofluids.

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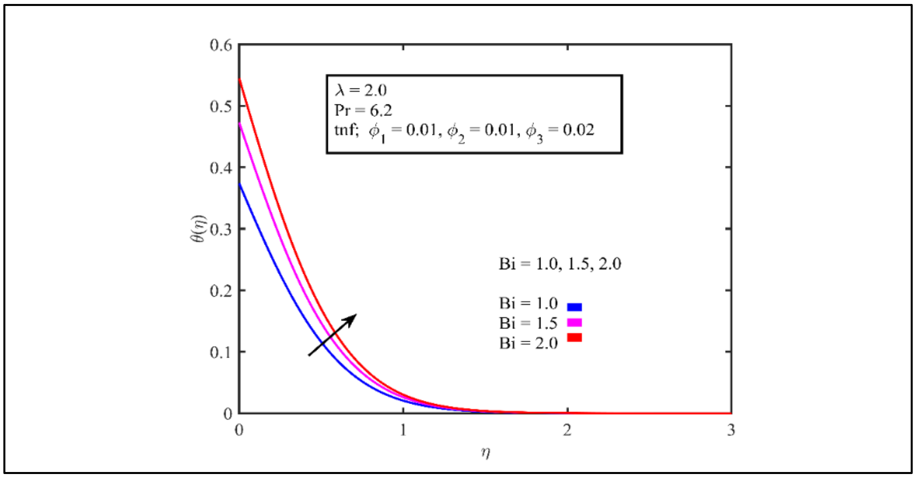

Fig.4. $\theta\left(\eta\right)$ when $\mathrm{Bi}=1.0, 2.0, 5.0$.

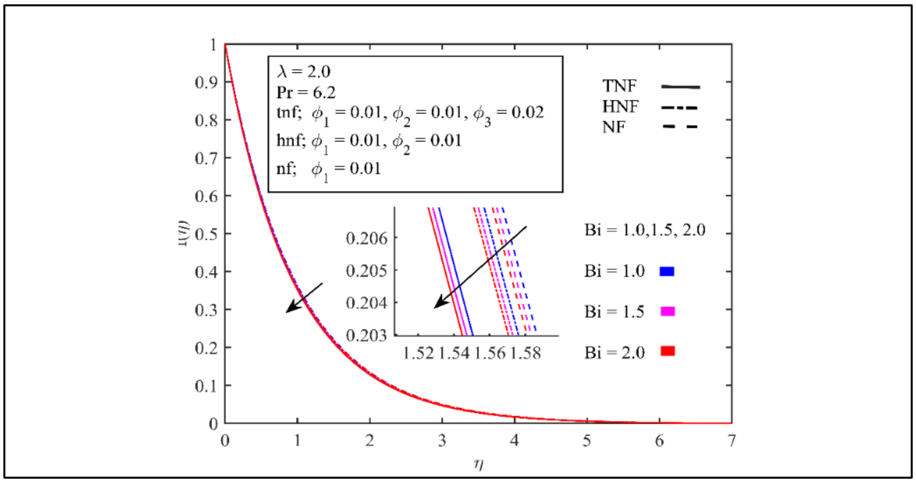

Fig. 5. $f\left(\eta\right)$ when $\mathrm{Bi}=1.0, 2.0, 5.0$.

A rise in Bi results in a fall in the $f\left(\eta\right)$, as seen in Fig 5. Physically, the kinematic viscosity of the fluid increase in tandem with a rise of Bi. This led to more viscous force that resists the fluid particles motion in TNF produced as kinematic viscosity rise. Consequently, $f\left(\eta\right)$ stunted with growing Bi. These patterns are in line with the study of convective heat flux in hybrid nanofluid by Khan & Pop (2020). Furthermore, as Bi rises, TNF shows a slower velocity than HNF.

Table 4 illustrates, greater Bi results in increased skin friction and Nusselt number. The involving parameters, namely λ and Bi over $-\mathrm{Re}_x^{\frac{1}{2}} Nu_x$ and $\mathrm{Re}_x^{-\frac{1}{2}} C_f$ are presented in Table 4. The behavior of Bi for local skin friction depicted that the values of local skin friction drastically drop off for higher Bi. Physically, it indicates that when the TNF condensed with nanoparticles across the stretched sheet, it escalates the shear stress of the nanofluid, thereby reducing the values of local skin friction. Table 4 also

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shows the changes in Nusselt number for the various λ for HNF and TNF. TNF is observed to have a small upsurge in the Nusselt number in comparison to HNF.

Table 4. Comparative analysis of HNF and TNF at various values of parameters λ and Bi over $-\mathrm{Re}_x^{\frac{1}{2}} Nu_x$ and $\mathrm{Re}_x^{-\frac{1}{2}} C_f$

Investigated parameters		HNF		TNF	
λ	Bi	$-\mathrm{Re}_x^{\frac{1}{2}} Nu_x$	$\mathrm{Re}_x^{-\frac{1}{2}} C_f$	$-\mathrm{Re}_x^{\frac{1}{2}} Nu_x$	$\mathrm{Re}_x^{-\frac{1}{2}} C_f$
1.0		-0.8012	-1.1193	-0.7883	-1.1405
2.0		-0.7968	-1.2463	-0.7838	-1.2663
5.0		-0.7795	-1.6722	-0.7654	-1.6911
	1.0	-0.6308	-1.1922	-0.6226	-1.2134
	1.5	-0.7968	-1.2463	-0.7838	-1.2663
	2.0	-0.9170	-1.2863	-0.8996	-1.3052

CONCLUSION

This study analyses the numerical solutions for the TNF flow in a mixed convection subject to convective boundary conditions over a stretched sheet. The bvp4c package in MATLAB software is utilized to obtain the numerical solution, which is derived by converting the governing PDEs into nonlinear ODEs using the related similarity transformation. Notable conclusions drawn from this analysis include:

(i) As the mixed convection parameters, λ and Bi, increase, the temperature field also increases.
(ii) As the mixed convection parameters, λ and Bi, increase, it is found that the velocity field decreases.
(iii) The Nusselt number and local skin friction decreased as Bi increased.
(iv) When compared to hybrid and nanofluid, the TNF's thermal conductivity is enhanced by the inclusion of the third nanoparticles, which raises the Nusselt number.

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CONFLICT OF INTEREST STATEMENT

The authors declare that there are no competing interests with the funders and concur that this study was carried out free from financial, commercial, or self-benefitting conflicts.

AUTHORS' CONTRIBUTIONS

The authors affirm that each contributor participated equally in every aspect of this work. The final manuscript was reviewed and approved by all authors. The contributions of the authors to the paper are outlined as follows: Naqiah Yunus**: writing - original draft preparation**; Nur Ilyana Kamis: **data**

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validation; Sharidan Shafie: **conceptualization, methodology;** Lim Yeou Jiann, Noraihan Afiqah Rawi: **writing – review and editing, supervision**. Each author approved the final text of the manuscript after evaluating the findings.

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