Modeling of Single Mode Ridge Optical Waveguide Using Finite Different Method (FDM)

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Abstract- **This project is about numerical analysis of optical waveguide to analyze the propagation constant of the waveguides and obtain the TE field distributions. Numerical technique is essential in optical waveguide because they are easier to use and more transparent. To model the single mode optical waveguide structure, we have used Finite Different approach as one of the waveguide to simulate the electric field distribution. The waveguide allows guided wave propagation only if the thickness is greater than a critical cutoff thickness for each waveguide mode. From that, electric field distribution of 1550nm and 1310nm can be acquired and analysed. The waveguide structure of 1550nm shows better electric field distribution compared to 1310nm wavelength since the 1550nm waveguide can maximize the bandwidth system with low attenuating loss.**

***Keywords* - optical polymer, waveguide, Finite Different Method, electric field distribution**

I. Introduction

Optical waveguide devices already played important roles in telecommunications systems, and its importance will certainly grow in the future. An optical waveguide is a spatially inhomogeneous structure for guiding light. For example, for restricting the spatial region in which light can propagate. Usually, a waveguide contains a region of increased refractive index, compared with the surrounding medium, called cladding [2]. Optic communication systems widely operate in the wavelength windows at 850nm, 1300nm and 1550nm.

The optical waveguide is the fundamental element that interconnects the various devices of an optical integrated circuit, just as a metallic strip does in an electrical integrated circuit and clearly the optical waves travel in the waveguide in distinct optical modes. A mode, in this sense, is a spatial distribution of optical energy in one or more dimensions that remains constant in time [3]. An optical waveguide is very consequential in integrated optical circuit in establishing an optical device that enticing us to this project. The numerical analysis with various mathematical approaches is prominent to determine the waveguide structure is in finest condition and the propagation constant of the waveguides can be analyzed thus, the TE field distributions could obtained.

Optical waveguides, such as those illustrated in Fig.1 is considered to have dielectric boundaries extending to infinity. Optical waveguides modes are wave trapped in and around the core. They can be excited only by electric fields [3]. Optical waveguide consist of a core, which light is confined and a cladding or substrate surrounding the core [5]. For this project, we modeled ridge waveguide as demonstrated in Fig.1 and the characteristic of light was analyzed using a numerical method based on finite different method (FDM) approach. The finite difference method (FDM) is a numerical technique used in solving problem that uniquely defined by three things. They are partial differential equation such as Laplace's or Poisson' equation, a solution region boundary and initial conditions. The basic formulation that governs the propagation of light in the optical waveguide is Maxwell's equations and it is derived to obtain E-field [5].

In this project, we propose the waveguide structure is in trapezoidal cross structure illustrated in Fig.1 and the refractive index of polymer is 1.543 (baked using oven = 1.543) and refractive index of substrate, quartz is 1.5.

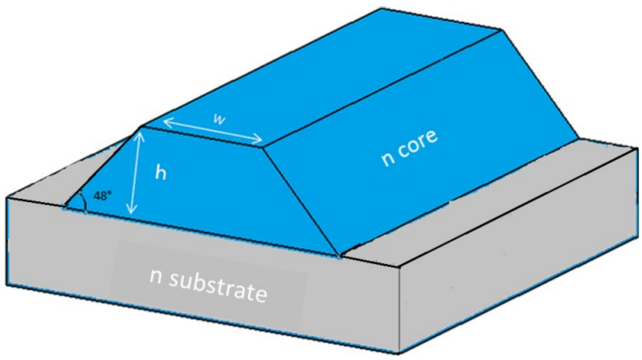


Figure 1: Ridge structure of waveguide.

II. Methodology

In this project, the process begins with understanding the behavior of light. In purpose of modeling the ridge optical waveguide, this enables us to investigate the characteristic of light in optical waveguide that influence the electric field distribution. Ridge waveguide structure consists of core index (is baked using oven which exhibit refractive index of 1.543) and substrate of polymer with refractive index of 1.5. Then, Maxwell equations must be solved in order to attain the electric field distribution. According to the Maxwell equations,

$$\nabla \times E = - \partial B / \partial t \quad (1)$$
$$\nabla \times H = \partial D / \partial t + J \quad (2)$$
$$\nabla . D = \rho \quad (3)$$
$$\nabla . B = 0 \quad (4)$$

where E and H are the electric and magnetic fields, B and D are the electric and magnetic flux densities, and J are electric charge and electric current densities, or their reduced form, the electromagnetic wave equation, with appropriate boundary conditions determined by the properties of the waveguide and cladding materials.

A finite difference solution to Poisson's or Laplace's equation, for example proceeds in three steps. First is dividing the solution region into a grid of nodes. Second, is approximating the differential equation and boundary conditions by a set of linear algebraic equations on grid points within the solution region, and finally, by solving this set of algebraic equations. The wave equation for the electric field can be presented as:

$$\nabla^2 E = \mu\varepsilon\, \partial^2 E/\partial t^2 \quad (5)$$

Now, considering a y-polarized TE mode which propagates in the z-direction and β as a propagation constant in longitude direction will then results:

$$\partial^2 Ey/\partial x^2 + \partial^2 Ey/\partial y^2 - \beta^2 Ey = - \omega^2 \mu\varepsilon Ey \quad (6)$$

Taking $k^2 = \omega^2\mu\varepsilon$ as the total propagation constant which combine the horizontal and vertical part, produces:

$$\frac{\partial^2 Ey}{\partial x^2} + \frac{\partial^2 Ey}{\partial y^2} + (k^2 - \beta^2)\, Ey = 0 \quad (7)$$

Knowing that k is a multiplication of free space propagation constant, $k_\circ$ and refractive index, n for respective layer, Equation (7) can be written in the form of :

$$\frac{\partial^2 Ey}{\partial x^2} + \frac{\partial^2 Ey}{\partial y^2} + (k_\circ^2 n^2 - \beta^2)\, Ey = 0 \quad (8)$$

Considering that Ey having component in x and y direction E(x,y), numerical approach of Taylor's expansion is applied to Equation (8) where the differential components are obtained as follows:

$$\frac{\partial^2 E}{\partial x^2} = \frac{E(i+1,j)+E\,(i-1,j) - 2E(i,j)}{\Delta x^2} \quad (9)$$

$$\frac{\partial^2 E}{\partial y^2} = \frac{E\,(i,j+1)+E\,(i,j-1)-2E(i,j)}{\Delta y^2} \quad (10)$$

Electric field can be obtained by combining Equation (8), (9) and (10):

$$E(i,j)= \frac{E\,(i+1,j)+E\,(i-1,j)+(\frac{\Delta x^2}{\Delta y^2})^2(\,E(i,j+1)+E(i,j-1)\,)}{2\left(1+\left(\frac{\Delta x^2}{\Delta y^2}\right)^2\right)- \Delta x^2(k_\circ^2 n^2(i,j)- \beta^2)} \quad (11)$$

Where i and j represent the mesh point corresponding to x and y directions respectively. If equation (8) is multiplied with and operating double integration towards x and y, it will result:

$$\beta^2 = \frac{\iint Ey\left(\left(\frac{\partial^2 Ey}{\partial x^2}+\frac{\partial^2 Ey}{\partial y^2}\right)+ k_\circ^2 n^2 Ey\right)\partial x\,\partial y}{\iint Ey^2 \partial x \partial y} \quad (12)$$

Due to difficulties in interpreting small differences of effectives index values, a more sensitive comparison is made by introducing a normalized propagation constant,

$$b = \frac{{n_{eff}}^2 - {n_{substrate}}^2}{{n_{guides}}^2 - {n_{substrate}}^2} \quad (13)$$

n_{eff} denotes the effective refractive index of the propagation mode which is related to propagation constant of the mode β and the wavenumber $k_\circ$ where the $k_\circ = 2\Pi/\lambda$ by

$$n_{eff} = \sqrt{\frac{\beta^2}{k_\circ}} \quad (14)$$

So, from these formulae we can attain the electric field distribution and determine all the parameter such as normalized propagation constant, propagation constant and the effective refractive index. In the modeling analysis of ridge channel waveguide, Eigen function of the wave equation need to be solved to know the propagation constant to ensure that the guidance of light in the waveguide and field distribution in the waveguide to inspect the field confinement in the waveguide.

In the application of Finite Different Method to solve equation (8), the electric field distribution and the refractive index, n is considered to be a discrete value at respective x- and y coordinate and bounded in a box, which represent the waveguide cross section. The box is divided into smaller rectangular area with a dimension of Δx and Δy at the x and y directions respectively. Brief description is given in Fig.2 where the waveguide cross section area is divided into M x N grid lines, which corresponds to the mesh size of Δx and Δy.

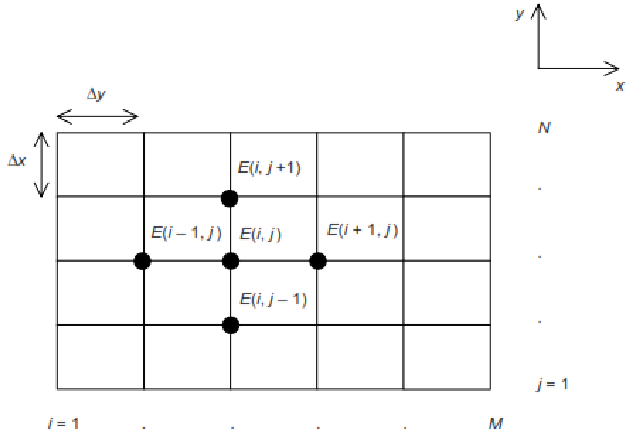


Figure 2: Presentation of the axis, meshes and grid lines for finite difference calculation (waveguide cross-section).

According to the Fig.3, a MATLAB program begins by defining constant parameters. Refractive index of polymer, cladding and substrate are constants parameters that designated as nf, nc and ns and the waveguide thickness is h.

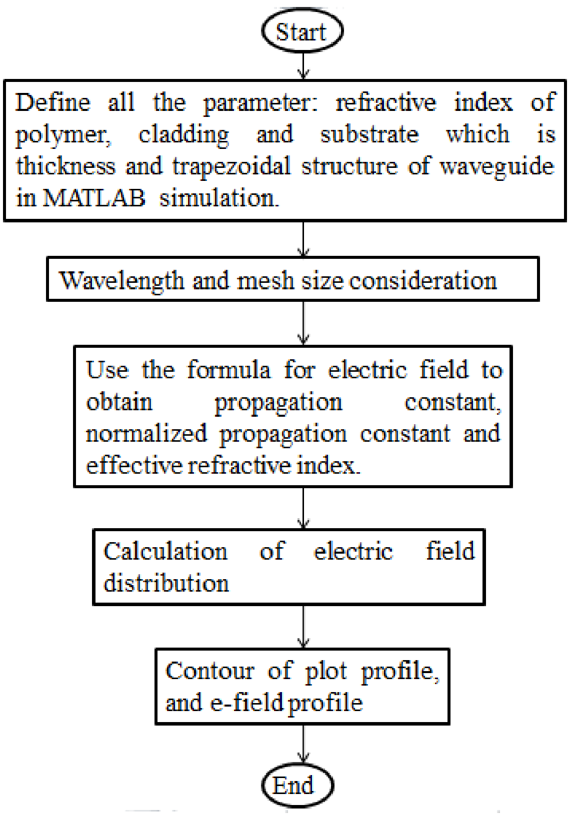
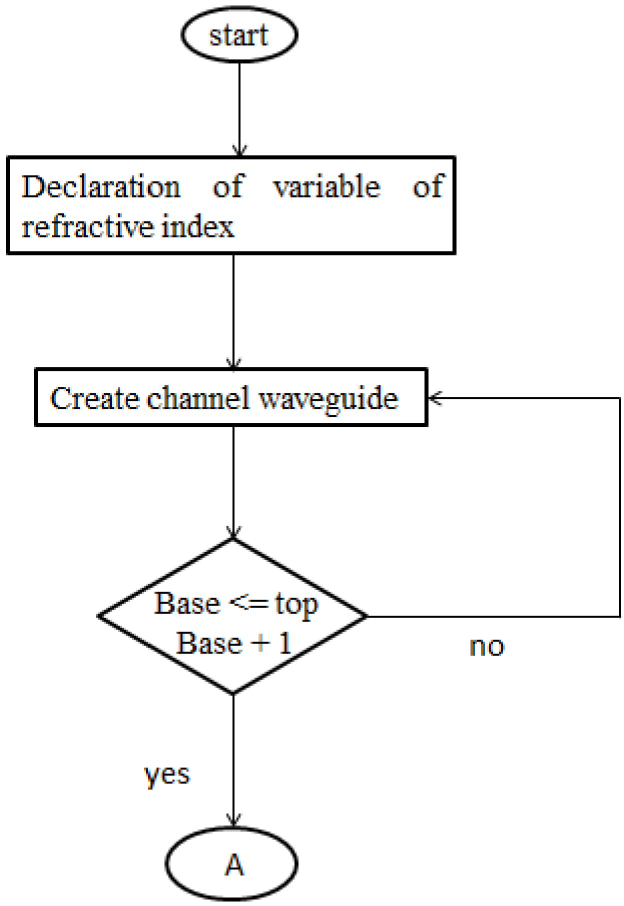


Figure 3: Process of modeling optical waveguide.

The distribution of evanescent is influenced by waveguide depth and thickness. Since this waveguide is trapezoidal structure, it is require defining the configuration designs such as height, width and their angle. Wavelength and mesh size are necessary where consideration of these two factors in relation the intensity gradient provides results which will be useful for the employment of the waveguide structure. For the purpose of completeness, we need the derivation of all the main equations which obtain the electric field distribution, normalized propagation constant, propagation constant and the effective refractive index. In this case, the direction of wave propagation is taken as the +z direction and only the TE mode is considered.

The simulation of program is using MATLAB software through Finite Different Method (FDM). In this development, knowledge about optical waveguide is very important especially the types of channel waveguide as mention in Fig.1 and the differences between single mode and multimode is the beam of light in mesh. Besides, the refractive index can change the propagation of light. In order to obtain an optimum structure of optical waveguide polymer based like Fig.1, analytical and numerical analysis are needed. In numerical analysis, understand the Maxwell's equation is very important to developed programming simulation which to be carried out using MATLAB software. The algorithm for the program is shown below:



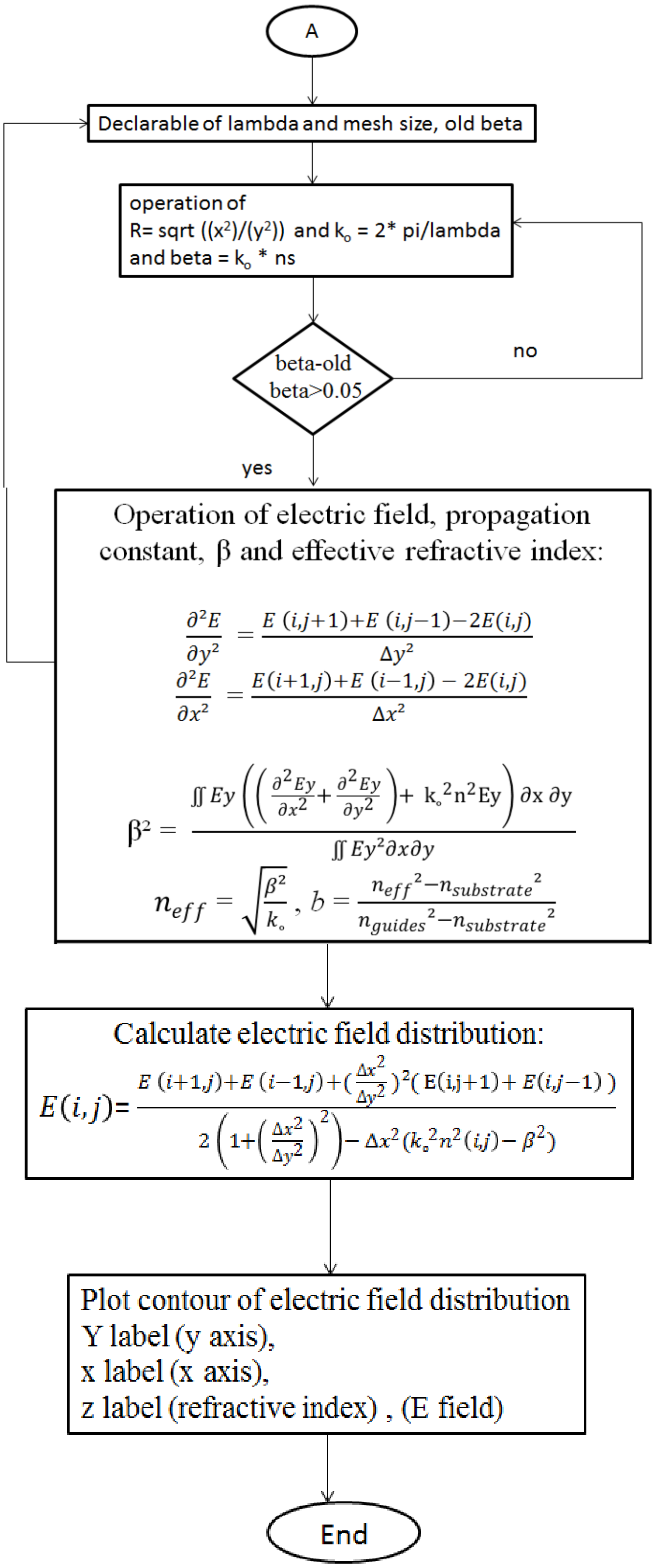


Figure 4: Algorithm of the program.

In this simulation, the thickness was varied while the width is a fix value. Through the simulation, different contour was exist depends on wavelength values which are 1310nm and 1550nm. Besides that, mesh size was tested in this project to explore detail about the waveguide based on the value of normalized propagation constant, propagation constant and effective refractive index.

III. RESULT

The result of Fig.5 consist of E-field Contour Plot and E-field Profile for Oven, nf = 1.543 and Quartz, ns = 1.500. The simulation was performed by wavelength of 1310nm and 1550nm with the mesh size of 0.125µm at the x-axis and 0.175µm at the y-axis. Fig.5, 6, 7 with 1550nm illustrated in three conditions which light guided in optical waveguide.

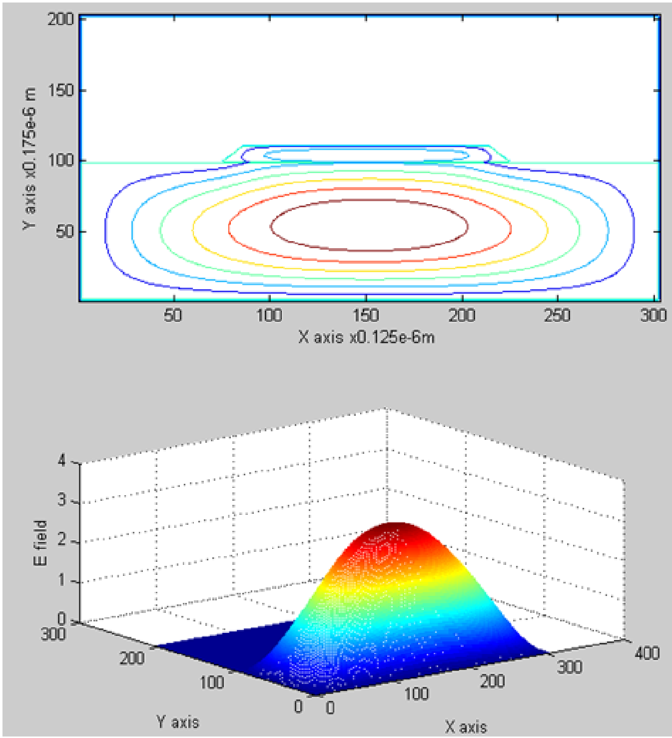


Figure 5: The figure of E-field Contour Plot and E-field Profile with mesh size x = 0.125µm, y = 0.175µm and h = 110 x 0.175µm for wavelength 1550nm.

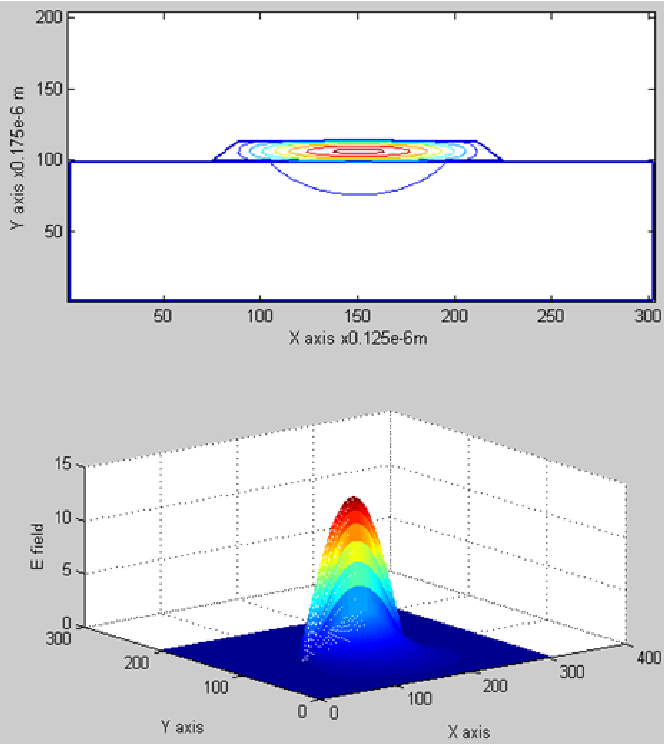


Figure 6: The figure of E-field Contour Plot and E-field Profile with mesh size x = 0.125µm, y = 0.175µm and h = 113 x 0.175µm for wavelength 1550nm.

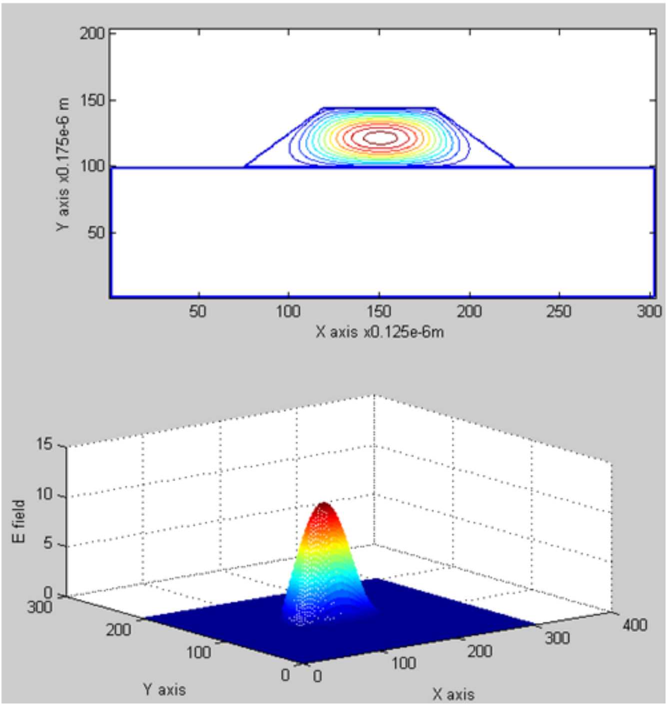


Figure 7: The figure of E-field Contour Plot and E-field Profile with mesh size x = 0.125µm, y = 0.175µm and h = 143 x 0.175µm for wavelength 1550nm.

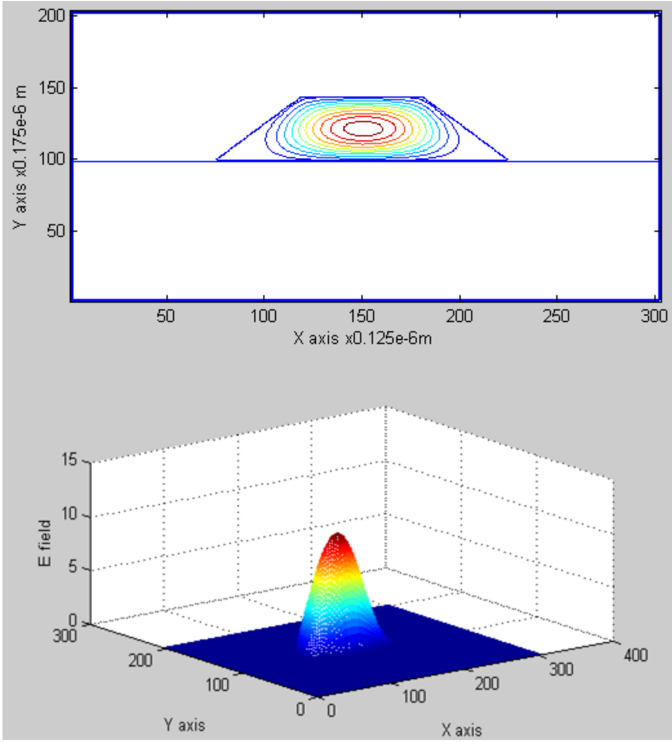


Figure 8: The figure of E-field Contour Plot and E-field Profile with mesh size x = 0.125µm, y = 0.175µm and h = 143 x 0.175µm for wavelength 1310nm.

Among these conditions, Fig.7 shows that the light is fully trapped while Fig.5 and Fig.6 display that the light is guided in leaking conditions. The leaking is happened because the thickness applied is not suitable for the light to confine better. Referred to the Fig.8, when the wavelength of 1310nm was used, the mode profile of the light exists in an appropriate trapped. Compare to the Fig.7 and Fig 8, the intensity are different each other because the wavelength changes are affecting the electric field distribution. Waveguides required operating at relatively high optical intensities and short wavelength [7]. The simulations show that different wavelength will give diverse contour.

For the mesh size, simulation in MATLAB shows that the smaller mesh size will increase the accuracy of the Effective Index and Normalized Propagation Constant. The mesh size also determine the sharpness of the plotting either more accurate or not. But if the mesh size is too small, the simulation process will took longer time [3]. Table 1 and Table 2 below show the different mesh size for dissimilar wavelength and the time taken for them during simulation. The mesh size was change to the 0.15µm in y-axis and 0.1µm in x-axis. By eliminating the complexity of boundary condition derivation, the accuracy of the result can be improved by decreasing the mesh size which can make the scanned points fall at the boundary as much as possible. The computational time been taken for smaller mesh size are much longer than the time taken for bigger mesh size. This is because the conventional finite different formula assumes the continuity of the field at a dielectric interface. Since the mesh size produce different time for every single simulation for each wavelength, so we can observe that more time are needed in smaller mesh size same goes to the thicknesses of waveguides.

Table 1: Time taken in simulation with mesh size for λ = 1550nm.

Thickness, h (x 0.175μm)	Δx = 0.125μm Δy = 0.175μm	Δx = 0.1μm Δy = 0.15μm
113	18.8265	46.8237
123	16.9374	16.5690
133	13.2201	14.1757
143	11.5816	12.5720

Table 2: Time taken in simulation with mesh size for λ = 1310nm.

Thickness, h (x 0.175μm)	Δx = 0.125μm Δy = 0.175μm	Δx = 0.1μm Δy = 0.15μm
113	15.6875	20.4736
123	13.2286	15.7118
133	12.2936	14.7201
143	10.8629	11.9358

The next simulation was developed to verify the thickness of waveguide changes with the different wavelength. The results for E-field Contour Plot and E-field Profile are shown below. The mesh size has been chosen as 0.1μm for x-axis and 0.15μm for y-axis. Based on Fig.9, the light is confined in the substrate in the waveguide with 0.17μm thickness and it is not a good waveguide due to the thickness is near to the critical cutoff thickness, so the electric field distribution is not fully trapped in the guiding region. From the simulation results, we notice that Fig.10, Fig.11 and Fig.12 are good waveguide since the lights of electric field distribution are confined well.

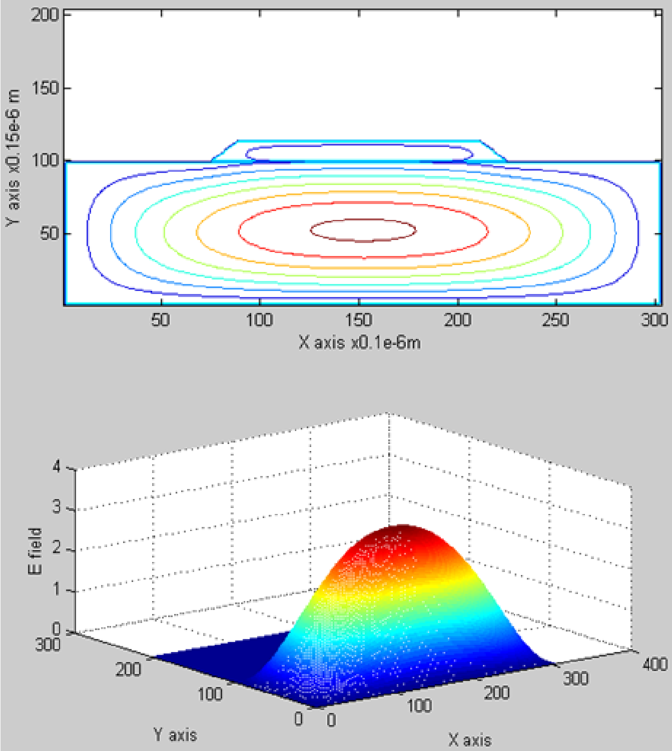


Figure 9: The figure of E-field Contour Plot and E-field Profile with mesh size x = 0.1μm, y = 0.15μm and h = 113 x 0.15μm for wavelength 1550nm.

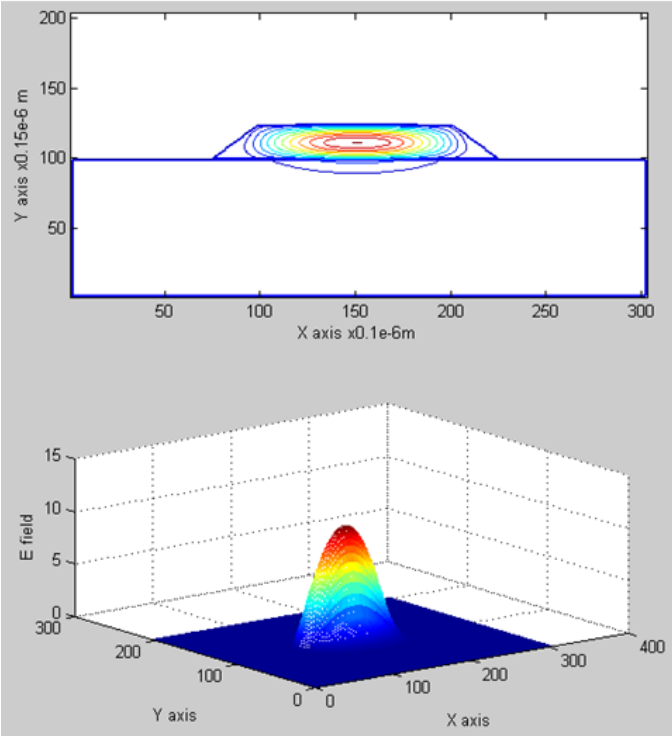


Figure 10: The figure of E-field Contour Plot and E-field Profile with mesh size x = 0.1μm, y = 0.15μm and h = 123 x 0.15μm for wavelength 1550nm.

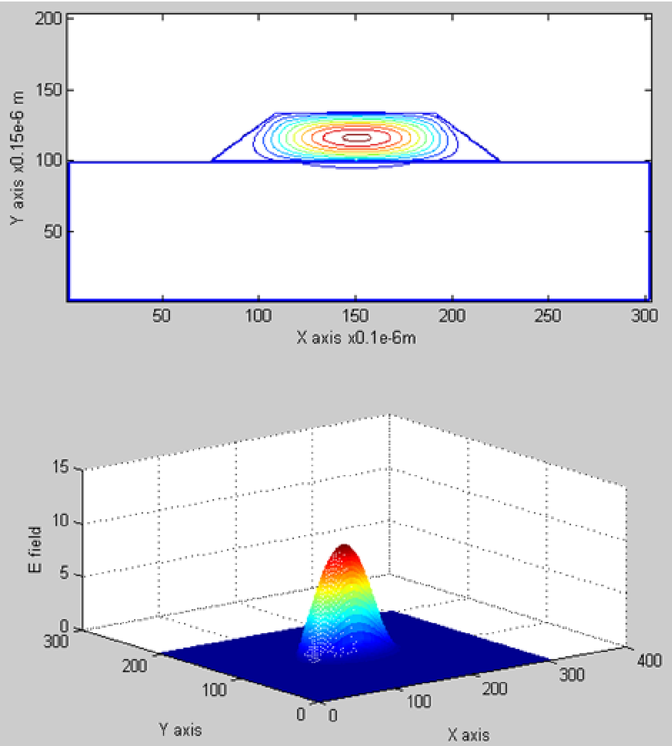


Figure 11: The figure of E-field Contour Plot and E-field Profile with mesh size x = 0.1µm, y = 0.15µm and h = 133 x 0.15µm for wavelength 1550nm.

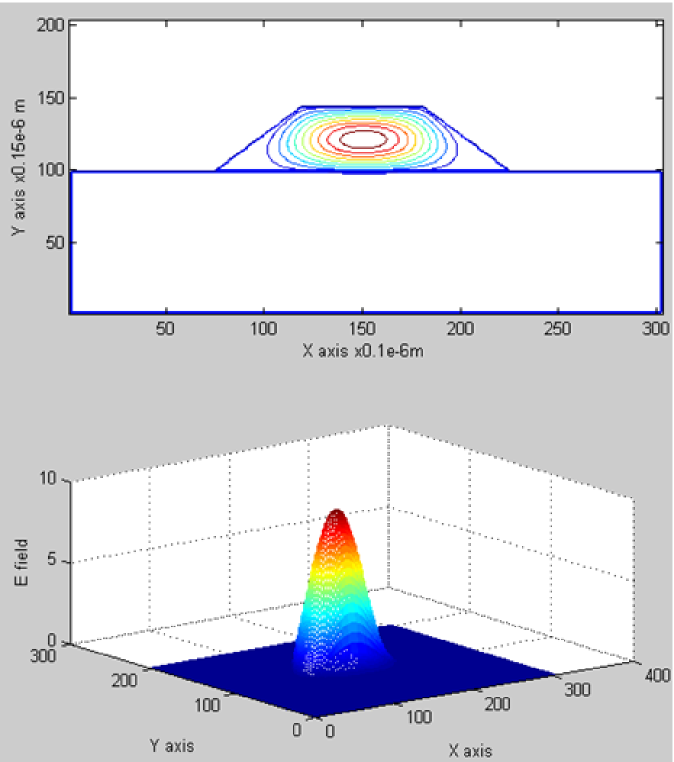


Figure 12: The figure of E-field Contour Plot and E-field Profile with mesh size x = 0.1µm, y = 0.15µm and h = 143 x 0.15µm for wavelength 1550nm.

For the thickness, h a sample of data for multiple thicknesses was taken and Table 1 and Table 2 show the values for normalized propagation constant, propagation constant and effective refractive index.

Table 3: Data Samples of Multiple Thicknesses for 1550nm

Thickness, h (x 0.175µm)	Normalized propagation constant, b	Propagation constant, β	Refractive effective index, n_{eff}
113	0.6769	6.1990	1.5292
123	0.7388	6.2098	1.5319
133	0.7798	6.2168	1.5336
143	0.8072	6.2210	1.5348

Table 4: Data Samples of Multiple Thicknesses for 1310nm

Thickness, h (x 0.175µm)	Normalized propagation constant, b	Propagation constant, β (µm)	Refractive effective index, n_{eff}
113	0.7580	7.3514	1.5327
123	0.8058	7.3611	1.5347
133	0.8372	7.3675	1.5361
143	0.8580	7.3718	1.5370

According to the Table 4 and Table 5, the values of normalized propagation constant, b increase as the thickness of the waveguide are increase. Same goes to the effective refractive index, n_{eff} which is increase slightly. Based on the theory, since the effective refractive index values are in range of in between the core refractive index and substrate refractive index, it is proving that it is in guided mode [5]. So, because of the value effective refractive index n_{eff} is around 1.543, the waveguide can be labeled as a guided mode waveguide.

Referred to the same table result, the value of propagation constant, β will be increase as the increases of the thickness. The increases show make the propagation light within the structure fully trapped into core region, resulting propagation loss to decrease. The propagation constant, β also use to calculate the value of effective refractive index, n_{eff} and normalized propagation constant, b [3].

For the cut-off thickness, it is use to make sure that the waveguide structure in this project is in single mode. The waveguide allows guided wave propagation only if the thickness is greater than a critical cutoff thickness for each waveguide mode. Since the thickness followed the cut off thickness, so the waveguide structure in this project is in single mode.

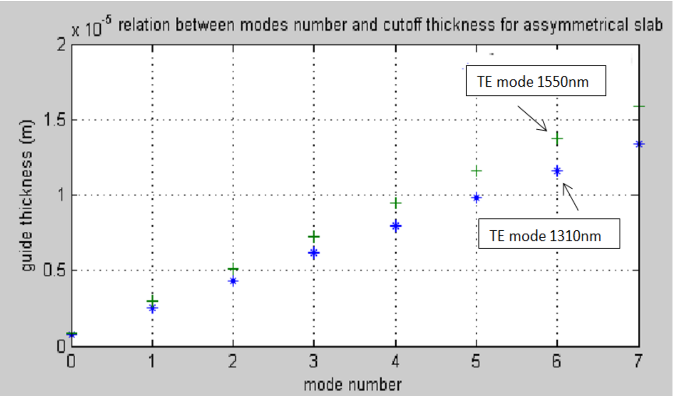


Figure 13: Cut-off thickness graph for wavelength of 1550nm and 1310nm.

According to the Fig.13, the graphs show the relation between modes number and the cut-off thickness for asymmetrical slab. The blue color represent for the wavelength of 1310nm while the green color denote for the wavelength of 1550nm. Based on that graphs, the cut-off thickness for the wavelength of 1550nm is between 1µm and 3µm while the cut-off thickness value for wavelength of 1310nm is about 0.9µm and 2.6µm.

IV. CONCLUSION

The finite different method is used to investigate about electric field distribution. So, the constant values of refractive index for core and cladding waveguide was used in determine the propagation constant by using this method. The important varied parameters are core thickness and ridge width waveguide. The value of effective refractive index n_{eff} was affected through all these parameter, same goes to the value of normalized propagation constant, b. The small difference refractive index between the core and cladding can give better performance of electromagnetic field pattern [5]. Hence, a single mode can be propagating within the waveguide and core region also will be radiating the electric field energy into it. Moreover, the normalized propagation constant b is more sensitivity than the value of ß and that the reason normalized propagation constant b is always adopted to investigate this type of waveguide [5].

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